

International Journal of Modern Physics A
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Precise dispersive data analysis of the $f_0(600)$ pole

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We review how the use of recent precise data on kaon decays together with forward dispersion relations (FDR) and Roy's equations allow us to determine the sigma resonance pole position very precisely, by using only experimental input. In addition, we present preliminary results for a modified set of Roy-like equations with only one subtraction, that show a remarkable improvement in the precision around the σ region. We also improve the matching between the parametrizations at low and intermediate energy of the S_0 wave, and show that the effect of this on the sigma pole position is negligible.

Keywords: Roy's equations, dispersion relations, sigma, scalar mesons, meson-meson scattering

1. Introduction

The values quoted in the Particle Data Table for the sigma or $f_0(600)$ resonance mass and width, based on both pole position and Breit-Wigner parameter determinations are very widely spread, with an estimated mass and half width of ¹:

$$\sqrt{s_\sigma} \equiv M_\sigma - i\Gamma_\sigma/2 \simeq (400 - 1200) - i(250 - 500) \text{ (MeV)}. \quad (1)$$

This large uncertainty is mainly due to the fact that old data sets for pion-pion scattering are poor and often contradictory. Moreover, the choice of data sets varies among different works. To make things worse, there is quite a variety of different ways to extrapolate the data on the real axis to the complex plane, and the pole position of the sigma is greatly affected by model dependences.

This said, model independent techniques for extrapolating amplitudes from the real axis onto the complex plane exist in the form of dispersion relations, which allow us to analytically continue an amplitude away from the real axis provided we know its imaginary part for physical values of the energy. These dispersive techniques have already been successfully used for predicting the position of the sigma pole, with a remarkable agreement among the different works:

$$440 - i 245 \text{ MeV} \quad \text{Dobado, Pelaez (1997)}^2 \quad (2)$$

$$470 \pm 50 - i 260 \pm 25 \text{ MeV} \quad \text{Zhou et al. (2005)}^3 \quad (3)$$

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In particular, there exists a dispersive representation that incorporates crossing exactly, written by Roy ⁴, which involves only the partial wave amplitudes. Roy's equations have already been used to predict the position of the sigma pole from the theoretical predictions of ChPT ⁵, obtaining:

$$\sqrt{s_\sigma} = 441_{-8}^{+16} - i 272_{-19.5}^{+9} \text{ MeV} \quad (4)$$

In addition, the data coming from the E865 collaboration at Brookhaven ⁶, and especially the recently published data from NA48/2 ⁷ provide us with very precise data on pion-pion scattering at very low energies. These allow us to obtain very reliable parametrizations of the S0 wave at low energy ⁹, from which the scattering lengths can be directly extracted ¹⁰ with a remarkable precision and in good agreement with the theoretical predictions of ChPT ⁵.

Our aim is thus to perform a dispersive analysis, including all available experimental data, in order to give a precise and model independent determination of the sigma pole position, by using exclusively data, analyticity and crossing symmetry. We use both Forward Dispersion Relations (FDR) and Roy's equations, without assuming ChPT, so that we can actually test its predictions.

2. Approach and results

The details on the parametrizations used for the data have been explained fully in Ref. ¹¹, that we will denote by KPY08. It is enough to say here that two different sets of parameters are considered:

- *Unconstrained Fits to Data* (UFD), in which each partial wave is fitted independently. This set satisfies both FDR and Roy's equations within the experimental errors in all waves except the Roy equation for the S2 wave, for which the deviation is about 1.3σ , and the antisymmetric FDR above 930 MeV by a couple of standard deviations.
- *Constrained Fits to Data* (CFD), obtained by constraining the fits to satisfy simultaneously FDR and Roy's equations, so that all waves are correlated. The CFD set provides a remarkably precise and reliable description of the experimental data, and at the same time satisfy the analytic properties remarkably well.

These two sets provide a reliable parametrization for the imaginary part of the partial waves that we need as input for Roy's equations.

An elastic resonance has an associated pole on the second Riemann sheet of the complex plane S-matrix, which, as it is well known, corresponds by unitarity to a zero on the first sheet. As usual then, we just need to look numerically for zeroes of the S-matrix on the physical sheet, $S_0^0(s) = 1 + 2i\sigma(s)t_0^0(s)$, where the analytic extension of the partial wave amplitudes away from the real axis is given by Roy's equations, whose domain of validity has been shown to cover the region of the complex plane where the sigma lies ⁵.

Taking the UFD set as the input for Roy's equations, we find an S-matrix zero at $\sqrt{s} = (426 \pm 25) - i(241 \pm 17)$ MeV. However, Roy's equations are not completely satisfied by this data set, thus the pole position will be much more reliable if the input satisfies the equations, as it is the case for the CFD set. In this case we find:

$$\sqrt{s_\sigma} = (456 \pm 36) - i(256 \pm 17) \text{ MeV}, \quad (5)$$

which still has big uncertainties due to the strong dependence of Roy's equations on the scattering lengths, in particular of the a_0^2 , which is known with less precision. These values are, however, subject to further improvement and should be considered preliminary. It should also be noted that they are in perfect agreement with the theoretical prediction by Caprini *et al.* of $\sqrt{s_\sigma} = 441_{-8}^{+16} - i 272_{-19.5}^{+9}$.

3. Work in progress

The three authors of this work together with F. J. Ynduráin (see, i.e., ⁸ in this conference or Ref. ⁹) have derived a modified set of Roy-like equations which are based on once-subtracted dispersion relations – Roy's equations are twice subtracted. The motivation for these new equations is that their uncertainties are smaller than for standard Roy's equations in the region above ~ 400 MeV, which is of interest for our work. This allows us to obtain the position of the sigma pole from Constrained Fits to Data with higher accuracy than by constraining with the standard Roy's equations alone. In addition, they allow us to better describe the $f_0(980)$ region, as the errors there are now much smaller and some parametrizations could be now discarded.

We have already performed ⁹ a preliminary Constrained Fit to Data (CFD-II) in which these new equations are also imposed as new constraints within errors. Moreover, following a suggestion in ¹² we have improved the matching of the low and intermediate ($f_0(980)$) regions at $\sqrt{s} = 932$ MeV by imposing continuity not only on the phase shifts, but also on the derivative. This gives rise to a new set of parameters which better encode the experimental information together with unitarity, analyticity and crossing symmetry, therefore allowing us to obtain a more precise and reliable determination of the sigma pole, which for this preliminary CFD-II set is:

$$\sqrt{s_\sigma} = (459 \pm 47) - i(257 \pm 18) \text{ MeV, (preliminary from Roy Eqs.)} \quad (6)$$

$$\sqrt{s_\sigma} = (461 \pm 14) - i(255 \pm 14) \text{ MeV, (preliminary from GKPY)} \quad (7)$$

to be compared with the result from the GKPY equations of $\sqrt{s_\sigma} = (461 \pm 13) - i(254 \pm 14)$ MeV, obtained with the CFD data set in which the improved matching condition of a continuous first derivative was not imposed. We can see that, although the phase shift does change above 932 MeV (see Fig. 1), the pole position is almost unaffected. A full analysis including these new equations should be complete within the next few months.

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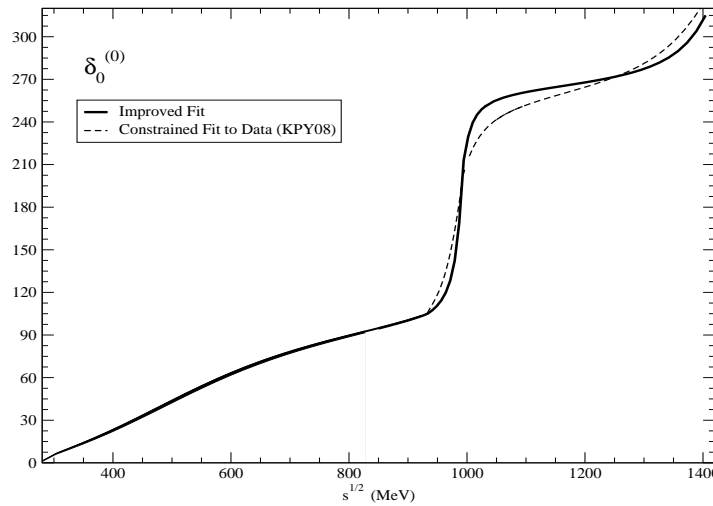


Fig. 1. S0 wave phase shifts. We show both the previous fit in KPY08 and a new preliminary fit that improves the matching between the conformal and K-matrix regions.

4. Acknowledgements

This work is dedicated to the memory of Prof. F. J. Ynduráin. We thank H. Leutwyler for his comment about the matching condition, and the organizers for creating the nice scientific atmosphere of the event and the Spanish research contracts PR27/05-13955-BSCH, FPA2004-02602, UCM-CAM 910309 and BFM2003-00856 for partial financial support.

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