UNIVERSIDAD COMPLUTENSE
FACULTAD DE CIENCIAS MATEMATICAS

CONSEJO SUPERIOR
DE INVESTIGACIONES CIENTIFICAS

INSTITUTO DE ASTRONOMIA Y GEODESIA
(Centro Mixto C.S.I.C. - U.C.M.). MADRID

Publicación núm. 183

A NEW METHOD FOR EARTH TIDE ANALYSIS

por

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PRESENTADO EN LA REUNIÓN DEL GRUPO DE TRABAJO «HIGH PRECISION TIDAL DATA PROCESSING», BONN, 14-16 OCTUBRE, 1992

MADRID
1992

A NEW METHOD FOR EARTH TIDE DATA ANALYSIS

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Abstract:

The method suggests to use a sophisticated scheme for the approximation of the drift of the tidal records. The drift is represented separately in subintervals of the record through polynomials of a variable power. An optimum power is estimated for every subinterval by testing statistical hypotheses through the criteria of Fisher and Akaike (AIC) and multiple analyses of all data for every value of the polynomial's power. The experimental results show that the drift curve has a rather complicated behavior even in data obtained through a superconducting gravimeter. For this reason relatively short subintervals of the order of 48 hours are to be prefered for the approximation of the drift.

1. Introduction.

Let Y is an Earth tide observation, i.e. an ordinate of a record like those on Fig. 1. The general model for Y is

(1) $$Y = G + D + \varepsilon$$

where G is the pure tidal component of the record, D is the drift and ε is the error of the observation. If the popular slang of the communication technique is used, G is the useful signal of the record, ε is the noise and D - somewhat between signal and noise.

The task of the tidal analysis is to estimate the parameters upon which depends G, or, which is the same, to get the estimate $\tilde{G}$ of G. Form this point of view D is a function of the time which describes the zero-line of the record.

The same data Y can be used for non-tidal problems like recent crustal movements, superficial deformations, volcano and earthquake precursors etc. These problems need the determination of the second component of (1), i.e. the estimate $\tilde{D}$ of D. For example as precursors can be considered some accelerations and jumps of $\tilde{D}$ (by the way, we can look for precursors in G as well).

In order to get $\tilde{G}$ we need to separate it from D. This can be done only if we obtain, in one or another way, the estimate $\tilde{D}$. Thus we see that the tidal analysis of Y has to satisfy both tidal and non-tidal problems.

Evidently, we shall have good estimates $\tilde{G}$ and $\tilde{D}$ if we can reduce the effect of the noise ε on both of them. It is out of question to estimate ε directly. We can only define the residuals,

$$r = Y - \tilde{G} - \tilde{D}, \tag{2}$$

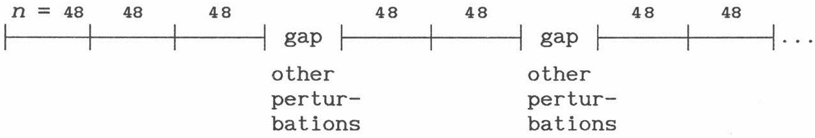
and set up a reasonable condition about them. When ε is a white noise most reasonable is the condition of the Method of the Least Squares (here and further MLS),

$$\sum r^2 = \text{Min}. \tag{3}$$

For the use of (3) we need analytical models of G and D. There are no problems with the model of G but we have not a good theory about D. That is why a crucial moment of the analysis is how it deals with the drift.

In a classical effort to apply the MLS Horn (1959) suggested to approximate the drift by a polynomial over the whole interval of the observations. The method of Horn could not accept gaps and data like those on Fig. 1.b.

In another application of the MLS (Venedikov, 1966, Melchior and Venedikov, 1968, see also Melchior, 1978), which will be called here M67, a more flexible model of the drift was accepted. The record to be processed is subdivided into intervals, we shall call them Δ, of length n = 48 hours as it is shown on the following scheme



Intervals Δ in the method of analysis M67.

The drift is approximated independently in every Δ by a polynomial of low fixed order (Figure 2). This kind of approximation admits any changes in the drift as well as any gaps between the Δ.

In M67 the hourly ordinates Y are replaced as observations by a pair of filtered numbers for every Δ and every one of the main tidal bands D (diurnal), SD (semidiurnal) and TD (terdiurnal). This is made in order to avoid the correlation of ε and get a severe estimate of the precision, different for the D, SD and TD tides.

Afterwards several methods were proposed, e.g. (Chojnicki, 1973, Schüller, 1977, Jentzsch, 1977) as well as a more flexible variant of M67 (Venedikov, 1984). However, none of these methods could show significant advantages with respect to M67.

A most interesting modern approach to the problem was made by the method used in Mizusawa (Ishiguro, Akaike, Ooe & Nakai, 1983, Tamura, Sato, Ooe & Ishiguro, 1991). In this method the gaps define natural intervals of the record. In each interval we get a separately estimated drift. The drift model is related with a condition for every sequence of three ordinates.

Here we shall propose a method (here and further M92) which is partly a generalization of M67, partly its alternative. The motives for its creation are the following.

First, there are modern observations of high quality with very stable drift. For them the model used by M67, which is good for an irregular drift, may be not necessary. We can hope that the drift can be approximated in intervals Δ much longer than 48 hours.

Second, for such observations the correlation of the noise may be not so important. We can try to avoid the use of the filtered numbers and return to the hourly ordinates.

Finally, now we have powerful computers which allow the development of very sophisticated methods. We can make very complicated experiments and we are, in a way, obliged to exploit these possibilities.

However, a more sophisticated method does not yet means a better method and we do not suggest necessarily to replace M67 by M92. What seems quite sure, it could be useful to apply in parallel both methods.

2. The tidal model.

If G_t is the value of G at time t, we have the theoretical model

$$(4) \qquad G_t = \sum_k \delta H_k \cos(\Phi_k + \omega_k t + \kappa)$$

where each term represents a tidal wave with an index k.

The quantities ω_k (angular velocity), H_k (theoretical amplitude) and Φ_k (initial theoretical phase) are well known. The parameters δ (amplitude factor) and κ (phase shift) are the unknown parameters of G whose estimates are the object of the analysis. They can be considered as functions of the angular velocity ω,

$$(5) \qquad \delta = \delta(\omega_k) \quad \text{and} \quad \kappa = \kappa(\omega_k).$$

A problem is the great number of the tides in (4) - in the modern tidal potential developments of Tamura (1987) and Xi Quinwen (1985) there are more than 1000 tidal waves.

This is easy to overcome this problem by using that (5) are slowly varying functions. We can shape groups of tides, say g groups, with close ω_k and accept that δ and κ are constants within a group. If $k = k_j$ are the indices of the tides in the j-th group, we can set up

$$(6) \qquad \delta(\omega_k) = \delta_j = \text{const. and } \kappa(\omega_k) = \kappa_j = \text{const.}$$
$$\text{for } k = k_j,\ j = 1,2,\ldots g$$

and introduce the unknowns (Venedikov, 1961, 1966)

$$(7) \qquad \xi_j = \delta_j \cos \kappa_j \quad \text{and} \quad \eta_j = -\delta_j \sin \kappa_j$$

With them the expression (4) becomes,

(8) $$G_t = \sum_{j=1}^{g} (\xi_j \, c_j(t) + \eta_j \, s_j(t)) \quad \text{where}$$

$$c_j(t) = \sum_{k=k_j} H_k \cos(\Phi_k + \omega_k t) \text{ and } s_j(t) = \sum_{k=k_j} H_k \sin(\Phi_k + \omega_k t)$$

The grouping or the separation of the tides should be made in dependence on the total length of the record. It should guarantee a linear independence (close to orthogonality) of the functions $c_j(t)$ as well as $s_j(t)$.

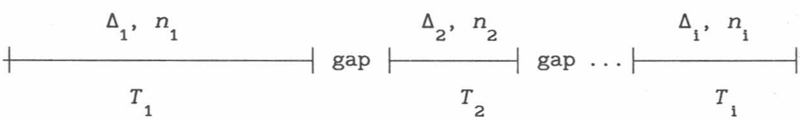
For a more convenient manipulation with (8) we can replace it by

(9) $$G_t = \sum_{j=1}^{m} a_{tj} x_j,$$

where x_j and a_{tj}, $j = 1,2,\ldots m = 2g$, replace the unknowns (7) and the functions (8) respectively.

3. Model of the drift.

The model of D in M92 is related with the scheme



Intervals Δ_i in the method of analysis M92.

The record is subdivided into N intervals denoted by Δ_i, with a central epoch T_i and, generally, a variable length n_i hours. Within any interval Δ_i are no gaps so that n_i is also the number of the hourly ordinates in Δ_i.

In the following the letter i ($i = 1,2,\ldots N$) will used exclusively as an index of an interval Δ_i.

In every Δ_i we have separate approximation of the drift through

(10) $$D_t = \sum_{k=0}^{v_i} z_{ki} p_{tk} \quad \text{for a given } \Delta_i.$$

At the moment we shall accept that

(11) $$p_{tk} = (t-T_i)^k \quad \text{for } \Delta_i$$

so that (10) is a polynomial of power v_i. The coefficients z_{ki} of the polynomial are unknowns specific for Δ_i. The power v_i is also specific for every Δ_i. The selection of v_i will be discussed a little bit later.

In the practical application of M92, Δ_i are defined by:

(i) in any case by the gaps, like in the method of Mizusawa, and, optionally,

(ii) by the epochs of the calibrations,

(iii) by epochs given as input data, related with places where we suspect changes in the drift's behaviour,

(iv) in dependence on two integers n_{min} and n_{max} which $n_{min} \le n_i \le n_{max}$ or

(v) by choosing a length n and $n_i = n =$ const. In this case we have the scheme of M67 with $n = 48$ replaced by the chosen value of n.

Let us return to the powers v_i in (10).

We must be able to work with v_i depending on i, i.e. with polynomials (10) of different order for every Δ_i. When n_i is variable, we can expect higher v_i for longer Δ_i and lower v_i for shorter Δ_i. We may need different v_i corresponding to the changes in the drift properties and behaviour, independently of whether the length n_i is variable or $n_i = n =$ const.

Evidently, we are not able to choose, before the processing, a set of fixed values v_i, $i = 1,2,...N$. That is why M92 presumes a variation of v_i separately for every Δ_i and selection of optimum values. This will be discussed in paragraphs 5 and 6. Now, in the next paragraph, we shall consider the use of the models (9) and (10) for getting the estimates of the unknowns only for a given fixed set of powers $v_1, v_2, ...v_N$.

4. Observational equations and their solution after the MLS.

If we joint (9) and (10) into (1) we get the observational equations

(12) $$Y_t = \sum_{j=1}^{m} a_{tj} x_j + \sum_{k=0}^{v_i} p_{tk} z_{ki} \quad \text{for } \Delta_i,\ i = 1,2,\dots N$$

where Y_t is an ordinate at time t. Here the error ε is ignored but we keep it in mind.

According to (11) p_{tk} is related with T_i and Δ_i and it would be more correct to write p_{itk}. In order to avoid too many indices we shall drop away the index i.

In a given Δ_i we have n_i hourly ordinates Y_t. For them the argument time t takes the values

(13) $$t = T_i - \nu_i,\ T_i - \nu_i + 1, \dots\ T_i + \nu_i, \quad \text{where} \quad \nu_i = (n_i - 1)/2.$$

We shall deal first with the equations (12) for a given Δ_i and we shall introduce the following matrices related with the values (13) of t:

(14) $(n_i \times 1)$ dimensional column vector

$$\mathbf{Y}_i = \|Y_t\|, \quad t = T_i - \nu_i,\ T_i - \nu_i + 1, \dots T_i + \nu_i,$$

(15) $(n_i \times m)$ dimensional matrix

$$\mathbf{A}_i = \|a_{tj}\|, \quad j = 1,2,\dots m,$$
$$t = T_i - \nu_i,\ T_i - \nu_i + 1, \dots T_i + \nu_i \text{ and}$$

(16) $(n_i \times v_i + 1)$ dimensional matrix

$$\mathbf{P}_i = \|p_{tk}\|, \quad k = 0,1,\dots v_i$$
$$t = T_i - \nu_i,\ T_i - \nu_i + 1, \dots T_i + \nu_i.$$

Further we shall arrange the unknowns in the column vectors:

(17) $$\mathbf{z}_i = \|z_{ki}\|, \quad k = 0,1,\dots v_i, \quad \text{dimension } (v_i + 1 \times 1),$$

$$\mathbf{x} = \|x_j\|, \quad j = 1,2,\dots m, \quad \text{dimension } (m \times 1),$$

where $\mathbf{x}$ represents the tidal unknowns (ξ and η) which are common for all Δ_i while $\mathbf{z}_i$ represents the drift unknowns, which are related with Δ_i, i.e. which depend on i. The number of the elements of $\mathbf{z}_i$, v_i+1, is also depending on i.

By using these matrices we can rewrite (12) in the matrix form

(18) $$\mathbf{Y}_i = \mathbf{A}_i\mathbf{x} + \mathbf{P}_i\mathbf{z}_i \quad \text{for } \Delta_i,\ i = 1,\ldots N.$$

In principle, (18) can be directly solved by the MLS. However, this is difficult even for a powerful computer because the number of the drift unknowns in all $\mathbf{z}_i$ can be very high. For example, if we use fixed length intervals with $n_i = 48$ and polynomials of fixed power $v_i = 2$, a record of length one year will provide 546 drift unknowns.

This inconvenience can be avoided if we separate the unknowns $\mathbf{z}_i$ from $\mathbf{x}$. Namely, we can orthogonalize $\mathbf{P}_i$, then $\mathbf{A}_i$ towards $\mathbf{P}_i$, in the following classical way.

We replace $\mathbf{P}_i$ by a matrix

(19) $$\mathbf{Q}_i = \mathbf{P}_i\mathbf{S}_i, \quad \text{with } \mathbf{S}_i\mathbf{S}_i^* = (\mathbf{P}_i^*\mathbf{P}_i)^{-1}$$

where $\mathbf{S}_i$ is an upper triangular matrix of dimensions $(v_i+1 \times v_i+1)$, $\mathbf{Q}_i$ has the same dimensions as $\mathbf{P}_i$ and * means the transpose of a matrix.

We replace also $\mathbf{A}_i$ by the matrix

(20) $$\mathbf{B}_i = \mathbf{A}_i - \mathbf{Q}_i(\mathbf{Q}_i^*\mathbf{A}_i)$$

which has the dimensions of $\mathbf{A}_i$.

It is easy to see that $\mathbf{Q}_i$ is orthonormal and $\mathbf{B}_i$ is orthogonal to both $\mathbf{Q}_i$ and $\mathbf{P}_i$, i.e.

(21) $$\mathbf{Q}_i^*\mathbf{Q}_i = \mathbf{I} \quad \text{and} \quad \mathbf{B}_i^*\mathbf{Q}_i = \mathbf{B}_i^*\mathbf{P}_i = \mathbf{0}$$

where $\mathbf{I}$ is a unit matrix and $\mathbf{0}$ is a zero matrix. The properties (21) are observed separately for every Δ_i.

Now with $\mathbf{Q}_i$ and $\mathbf{B}_i$ we transform the equations (18) into

(22) $$\mathbf{Y}_i = \mathbf{A}_i\mathbf{x} + \mathbf{P}_i\mathbf{z}_i = \mathbf{B}_i\mathbf{X} + \mathbf{Q}_i\mathbf{Z}_i,\ i = 1,2,\ldots,N,$$

where $\mathbf{Z}_i$ and $\mathbf{X}$ are new unknowns which replace $\mathbf{z}_i$ and $\mathbf{x}$. From here and the expressions (19) and (20) about $\mathbf{Q}_i$ and $\mathbf{B}_i$ we get

(23) $$\mathbf{X} = \mathbf{x} \quad \text{and} \quad \mathbf{Z}_i = \mathbf{S}_i^{-1}(\mathbf{z}_i + (\mathbf{Q}_i{*}\mathbf{A}_i)\mathbf{X})$$

i.e. the tidal unknowns $\mathbf{x}$ are not changed, but the drift unknowns $\mathbf{z}_i$ are transformed into $\mathbf{Z}_i$.

Now we can apply the MLS by taking into account (21). Then we get the following systems of normal equations for the estimates $\tilde{\mathbf{Z}}_i$ of $\mathbf{Z}_i$ and $\tilde{\mathbf{X}}$ of $\mathbf{X}$:

First N matrix equations for the drift unknowns

(24) $$(\mathbf{Q}_i{*}\mathbf{Q}_i)\tilde{\mathbf{Z}}_i = \mathbf{Q}_i{*}\mathbf{Y}_i, \quad i = 1,2,\ldots N,$$

then a matrix equation for the tidal unknowns

(25) $$(\mathbf{B}{*}\mathbf{B})\tilde{\mathbf{X}} = \mathbf{B}{*}\mathbf{Y}, \qquad \mathbf{B} = \|\mathbf{B}_i\|, \quad \mathbf{Y} = \|\mathbf{Y}_i\|, \quad i = 1,\ldots N$$

Here $\mathbf{B}$ is obtained through the arrangement of $\mathbf{B}_i$ $(i = 1,\ldots N)$ into a single matrix and $\mathbf{Y}$ - through the arrangement of $\mathbf{Y}_i$ into a single column vector. If $\tilde{n}$ is the sum of n_i, i.e. the total number of the data, $\mathbf{B}$ has the dimensions $(\tilde{n} \times m)$ and $\mathbf{Y}$ - $(\tilde{n} \times 1)$.

From the solution of (24) and (25) we can get the estimates

(26) $$\tilde{\mathbf{Z}}_i = (\mathbf{Q}_i{*}\mathbf{Q}_i)^{-1}(\mathbf{Q}_i{*}\mathbf{Y}) = \mathbf{Q}_i{*}\mathbf{Y}$$

(27) $$\tilde{\mathbf{X}} = (\mathbf{B}{*}\mathbf{B})^{-1}(\mathbf{B}{*}\mathbf{Y}),$$

Thus $\mathbf{Z}_i$ are obtained in a most simple way. However, if we want to get the original drift unknowns, i.e. the coefficients z_{ki} of the polynomials (10), we have to go back from $\mathbf{Z}_i$ to $\mathbf{z}_i$ through the computation of

(28) $$\tilde{\mathbf{z}}_i = \mathbf{S}_i(\tilde{\mathbf{Z}}_i - (\mathbf{Q}_i{*}\mathbf{A}_i)\tilde{\mathbf{X}})$$

For the computations it is important that we can get directly $\mathbf{B}{*}\mathbf{B}$ and $\mathbf{B}{*}\mathbf{Y}$ through $\mathbf{A}_i$ and $\mathbf{Q}_i$ after

(29) $$B^*B = \sum_{i=1}^{N} B_i^*B_i = \sum_{i=1}^{N} \left(A_i^*A_i - (Q_i^*A_i)^*(Q_i^*A_i)\right)$$

$$B^*Y = \sum_{i=1}^{N} B_i^*Y_i = \sum_{i=1}^{N} \left(A_i^*Y_i - (Q_i^*A_i)^*(Q_i^*Y_i)\right).$$

5. Variation of the drift model.

Practically this is done by choosing two parameters v_{min} and v_{max}. Then v_i, for every Δ_i, is let taking all values

(30) $$v_{min} \le v_i \le v_{max} .$$

A most natural value of v_{min} seems to be $v_{min} = 1$. This is a default value of M92 but it can be changed optionally.

The concrete variation of v_i, for $v_{min} = 1$ and $v_{max} = 4$, is demonstrated through the following scheme, where v'_i means an optimum value of v_i for Δ_i:

$v_{min} = 1$, $v_{max} = 4$, v'_i means an optimum value.

variation of v_1 in Δ_1

v_1	v_2	...	v_N
1	1	...	1
2	1	...	1
3	1	...	1
4	1	...	1
v'_1			

variation of v_2 in Δ_2

v_1	v_2	...	v_N
v'_1	1	...	1
v'_1	2	...	1
v'_1	3	...	1
v'_1	4	...	1
	v'_2		

...

variation of v_N in Δ_N

v_1	v_2	...	v_N
v'_1	v'_2	...	1
v'_1	v'_2	...	2
v'_1	v'_2	...	3
v'_1	v'_2	...	4
			v'_N

- We make the analysis of all data, i.e. all Δ_i, by choosing $v_1 = 1, 2, \ldots v_{max}$, while $v_2 = v_3 = \ldots = v_N = 1$
- Through statistical criteria we choose the optimum value v'_1 of v_1
- We make the analysis of all data, i.e. all Δ_i, by choosing $v_1 = v'_1$, $v_2 = 1, 2, \ldots v_{max}$, $v_3 = \ldots = v_N = 1$
- Through statistical criteria we choose

the optimum value v'_2 of v_2

and so on till we choose

the optimum value v'_N of v_N.

The whole procedure can be repeated by getting as initial values, instead of $v_i = v_{min} = 1$, the selected optimum values v'_i and look for new optimum values.

The analysis, i.e. the creation and the solution of the normal equations (24) and (25), is made for all indicated values of v_i. Thus, when the initial value is $v_{min} = 1$, we have to process a number of Nv_{max} analyses.

Such a quantity of computations is too big and needs as many as possible facilitations. A most important facilitation is the following. It consists in using the matrices created for a given variant of v_i for the next variant.

Let us suppose that we have made the computation for a given set of values of $v_1, v_2, \ldots v_N$, in particular for $v_i = v$ for a given interval Δ_i. Then the matrices $\mathbf{P}_i$ and $\mathbf{Q}_i$ related with Δ_i have $v + 1$ columns, i.e. they can be represented as

(31) $$\mathbf{P}_i = \|\mathbf{p}_o\ \mathbf{p}_1\ \cdots\ \mathbf{p}_v\|,\quad \mathbf{Q}_i = \|\mathbf{q}_o\ \mathbf{q}_1\ \cdots\ \mathbf{q}_v\|,$$

where $\mathbf{p}_k$ and $\mathbf{q}_k$ are columns of dimension $(n_i \times 1)$. Here, as far as $\mathbf{S}_i$ is triangular, $\mathbf{q}_k$ depends on $\mathbf{p}_o, \ldots \mathbf{p}_k$ only and it does not depend on $\mathbf{p}_{k+1}, \ldots \mathbf{p}_v$ $(k < v)$.

Let us now increase $v_i = v$ to $v_i = v + 1$. Then $\mathbf{P}_i$ and $\mathbf{Q}_i$ are to be replaced by

(32) $$\mathbf{P}'_i = \|\mathbf{p}_o\ \mathbf{p}_1\ \cdots\ \mathbf{p}_v\ \mathbf{p}_{v+1}\|,\quad \mathbf{Q}_i = \|\mathbf{q}_o\ \mathbf{q}_1\ \cdots\ \mathbf{q}_v\ \mathbf{q}_{v+1}\|.$$

Due to the above considerations, the first v+1 columns of $\mathbf{Q}_i$ remain the same and we need only to compute the additional $\mathbf{q}_{v+1}$.

For the new value $v_i = v + 1$ the estimates of the drift unknowns $\tilde{\mathbf{Z}}_i$ are to be replaced by

(33) $$\tilde{\mathbf{Z}}'_i = \mathbf{Q}'^{*}_i\mathbf{Y}_i$$

where $\tilde{\mathbf{Z}}'_i$ has $v + 2$ elements, while $\tilde{\mathbf{Z}}_i$ has $v + 1$ elements. The first

$v + 1$ elements of $\tilde{\mathbf{Z}}'_i$ remain the same as in $\tilde{\mathbf{Z}}_i$, while the new element of $\tilde{\mathbf{Z}}'_i$ is

$$(34) \qquad \tilde{Z}_{i\,v+1} = \mathbf{q}_{v+1}{}^*\mathbf{Y}_i .$$

We shall have a new matrix $\mathbf{B}'_i$ instead of $\mathbf{B}_i$, a new matrix $\mathbf{B}'$ instead of $\mathbf{B}$ and, instead of (25), new normal equations

$$(35) \qquad (\mathbf{B}'^*\mathbf{B}')\tilde{\mathbf{X}}' = \mathbf{B}'^*\mathbf{Y},$$

for new estimates $\tilde{\mathbf{X}}'$ of the same tidal unknowns.

For the practical application it is important that

$$(36) \qquad \mathbf{B}'^*\mathbf{B}' = \mathbf{B}^*\mathbf{B} - (\mathbf{q}_{v+1}{}^*\mathbf{A}_i)^*(\mathbf{q}_{v+1}{}^*\mathbf{A}_i)$$

$$\mathbf{B}'^*\mathbf{Y} = \mathbf{B}^*\mathbf{Y} - (\mathbf{q}_{v+1}{}^*\mathbf{A}_i)(\mathbf{q}_{v+1}{}^*\mathbf{Y}_i) .$$

For a given variant with normal equations (24) and (25) the sum of squares of the residuals can be obtained, according to the theory of the MLS, after

$$(37) \qquad S_o^2 = \mathbf{Y}^*\mathbf{Y} - \sum_{i=1}^{N} \tilde{\mathbf{Z}}_i{}^*\tilde{\mathbf{Z}}_i - \tilde{\mathbf{X}}^*(\mathbf{B}^*\mathbf{Y}), \quad \text{d.f. } f = \sum_{i=1}^{N} (n_i - v_i - 1) - m$$

where d.f. means degrees of freedom.

When $v_i = v$ is increased to $v_i = v + 1$, as above, the corresponding sum of squares becomes

$$(38) \qquad S_1^2 = \mathbf{Y}^*\mathbf{Y} - \sum_{i=1}^{N} \tilde{\mathbf{Z}}_i{}^*\tilde{\mathbf{Z}}_i - \tilde{Z}_{i\,v+1}^{\,2} - \tilde{\mathbf{X}}'^*(\mathbf{B}'^*\mathbf{Y}), \quad \text{d.f. } f_1 = f - 1$$

where $\tilde{\mathbf{X}}'$ are the new tidal estimates, related with $v_i = v + 1$.

The estimates of the variances of the observations are respectively

$$(39) \qquad \sigma_o^2 = S_o^2/f \quad \text{and} \quad \sigma_1^2 = S_1^2/f_1$$

where σ_o and σ_1 are the corresponding standard deviations or mean square errors. The estimation of the precision of $\tilde{\mathbf{X}}$ and $\tilde{\mathbf{X}}'$ can be obtained, according to the MLS, through σ_o and $(\mathbf{B}^*\mathbf{B})^{-1}$ and σ_1 and $(\mathbf{B}'^*\mathbf{B}')^{-1}$

respectively.

6. Selection of an optimum variant.

We shall denote by V_o the variant with $v_i = v$ and by V_1 the variant with $v_i = v+1$ for a fixed Δ_i. The difference in V_o and V_1 is in the models of the drift for Δ_i which are

(40) $$V_o: \quad D_t = z_o p_{to} + z_1 p_{t1} + \dots + z_v p_{tv}$$

$$V_1: \quad D_t = z_o p_{to} + z_1 p_{t1} + \dots + z_v p_{tv} + z_{v+1} p_{tv+1} \quad \text{for } \Delta_i$$

where the index *i* at z_{ki} is dropped away for convenience.

We see that V_o can be obtained from V_1 under the "zero hypothesis"

(41) $$H_o: \quad z_{v+1} = 0$$

Then the problem to select a better variant among V_o and V_1 is equivalent to the testing of H_o. If H_o is rejected then V_1 is to be chosen and, vice versa, if H_o cannot be rejected than V_o can be preferred.

This can be done by the method of analysis of variances. We have to compute the famous ratio *F* of Fisher, in this case

(42) $$F = (S_o^2 - S_1^2)/\sigma_o^2.$$

When H_o is true and V_o is to be chosen *F* should be a small number. When H_o is not true and V_1 is to be chosen, *F* should be a big number. We can decide what is a big or a small number according to the following.

Considered as a random variable, *F* has the *F*-distribution of Snedecor with d.f. $1:f$ (practically $1:\infty$). For a given confidential probability *P* or level of significance $\alpha = 1-P$, like $P = 95\%$ or $\alpha = 5\%$, we can find a critical value F_{crit}. Then we can accept that the computed *F* is big or small if

(43) $$F \geq F_{crit} \quad or \quad F < F_{crit}$$

respectively.

A practical inconvenience is that through F we can easily compare only two variants. When we have to use many variations in the model function it becomes rather complicated the comparison of all variants, two by two.

Much more comfortable way to choose the optimum variant is the Akaike's Information Criterion (AIC) (Sakamoto, Ishiguro & Kitagawa, 1986) which has also a serious theoretical background. It can be computed after

$$\text{(44)} \qquad AIC = \tilde{n} \log 2\pi + \tilde{n} \log \sigma^2 + \tilde{n} + 2(\tilde{m} + 2)$$

where $\tilde{n}$ is the total number of the data, $\tilde{m}$ is the total number of all unknowns (including the drift ones) and σ^2 is the estimate of the variance. As an optimum variant can be accepted the case with the lowest computed (observed) AIC.

In the practical application of M92 the program search automatically the minimum of AIC for every Δ_i. If the minimum is found for the variant $v_i = v'$, then the quantity F is computed for the variants

$$\text{(45)} \qquad F_1 \text{ for } (v'-1, v') \quad \text{and} \quad F_2 \text{ for } (v', v'+1).$$

We have a confirmation of the solution taken on the basis of AIC if F_1 is a big number and F_2 - a small number, in the sense discussed above.

7. Separation of the main tidal groups.

In M67, as well as in the earlier methods of Lecolazet, Doodson-Lennon and Pertsev, there is a separation of the D and SD tides at the same stage as the elimination of the drift. More concretely, in M67 this means that in every interval of 48 hours the filters for the SD tides eliminate the D tides and the filters for the D tides eliminate the SD tides.

This is reasonable as far as there are variable meteorological waves, S_1 with period 24^h and S_2 with period 12^h.

In M92 a similar option is included in the following way for the separation of the SD tides from the S_1 D tide in every Δ_i.

Optionally, we can set up

(46) $p_{t1} = c_{S_1}(t)$ and $p_{t2} = s_{S_1}(t)$

where c and s are the functions (8) computed for the S_1 tidal group. Then p_{to} is kept as before, after (11), but the other polynomial terms are shifted as

(47) $p_{3k} = (t-T_i)^1,\ p_{4k} = (t-T_i)^2,\ \ldots$

Then, following just the same algorithm, together with the drift elimination, we get an elimination of the S_1 tide when SD tides are determined. Just in the same way, by replacing S_1 by S_2 in (46), we can get an elimination of S_2 when the D tides are determined.

Thus, in addition to the variants with different power for the drift, we get the variants with and without elimination of the S_1 or S_2 tide.

8. Practical application and some results.

M92 and the corresponding computer program has been applied on two series of data:

LAN - gravity observations by an LCR gravimeter installed in the volcanic geodynamic station Cueva de los Verdes in Lanzarote and

BRU - gravity observations with a superconducting gravimeter in Brussels (Observatoire Royal de Belgique).

It has to be immediately declared that the "first" hypothesis, made in paragraph 1 has not been justified. It appeared practically impossible, i.e. absolutely inefficient, the approximation of the drift in very large time intervals. Generally, when the drift was approximated in longer intervals we used to get higher variances (mean square errors).

This is a conclusion made through the analyses of the whole series. For a graphic demonstration we have used two pieces of monthly series without gaps:

LANM: 1000 hourly ordinates of LAN, Figures 3.a, 4.a and 5.a

BRUM: 770 hourly ordinates of BRU, Figures 3.b, 4.b and 5.b.

The units used on the plots are : abscissa - time in hours, ordinate - gravity in 0.1 μgal.

On Fig. 3 the drift D is approximated through a single polynomial of a fixed power $v = 4$, without the variation discussed in Paragraph 6. We see important residuals r which are evidently auto-correlated. There is, possibly, a deterministic signal in the r.

On Fig. 4 we see the approximation of D in LANM and BRUM again by a single polynomial but this time the power v is allowed to vary. The optimum power v' is selected automatically through the AIC criterion. The resulted v' is: for LANM $v' = 17$ and for BRUM $v' = 8$.

Here is a more complicated estimated $\tilde{D}$ which is charged by a part of the residuals from the case $v = 4$ on Fig.3. The residuals have a lower level but still important correlation, better seen for BRUM.

The comparison between Fig. 3 and 4 shows the importance of the variation of the power v as well as that the statistical criterion runs well, at least not too badly.

At the last Fig. 5, we have an approximation of D in intervals of 48 hours, in a way like in M67, with a variation of the power v for every interval. An interesting numerical result is that for the whole series LAN and BRU we have got the following mean values of the optimum powers

LAN: mean $v' = 1.71$, BRU: mean $v' = 2.44$.

Thus, somewhat unexpectedly, D in BRU has a more complicated behaviour than in LAN. This can be seen from the plotted curves of the estimated $\tilde{D}$.

Now we have a much lower level of the r with a much less evident correlation, for both LANM and BRUM.

Nevertheless, from the study of r for the whole series LAN and BRU, we have stated that the auto-correlation does not disappear totally.

On Fig. 6 we have the estimated auto-correlation of R for BRU when the intervals of the approximation are of length $n = 36 = \text{const}$. The picture is quite similar for $n = 48$ and much less favorable for higher values of n.

Here we see that we have an improvement of the auto correlation if the S_1 tide is removed (when SD are determined). This is an indication that the auto correlated noise has really a meteorological origin. Nevertheless, with or without the elimination of S_1, we have a non-negligible auto-correlation. This may affect considerably the estimates of the precision. Namely, we can expect lower estimated variances (mean square errors σ) than the actual ones.

Generally, one way to avoid a correlated noise of meteorological origin, is to incorporate meteorological data in the analysis. This problem needs deeper consideration and we shall not discuss it here.

Tables 1 and 2 represent the results for LAN and BRU respectively from the application of M67 and M92.

In all cases we have lower σ for M92 compared to M67. The ratio between σ for the two methods, grossly estimated, is

σ(M67)/σ(M92) ≅ 3:2 for LAN, SD tides
LAN, D tides
BRU, SD tides
≅ 4:1 for BRU, D tides

even higher than 4:1 in the last case.

The ratio 3:2 is not too big and, with some optimism, we can believe that we have a real raise of the precision due to a new powerful technique of the analysis. However, 4:1 is a too big ratio, too beautiful, to be true.

When these results are interpreted it is necessary to take into account that in M92 we have not a separate estimation of the precision as in M67. For the computation of σ and its interpretation a very important condition is the independence of the errors ε, i.e. the assumption that ε is a white noise. If actually this condition is not observed, it is possible M92 to provide lower σ, in particular for the D tides.

One should think carefully what is better - to have lower or higher estimates of the precision. When the computed σ are lower than the actual errors we can be mislead to make interpretations and conclusions going too far. When the computed σ are higher we are prevented from this but some details in the results can escape from our attention.

Anyway, our final conclusion is that M92 can be a good numerical and mathematical tool for the analysis of the Earth tide data. However, it should be used carefully, in parallel with other methods, e.g. M67. What seems quite certain, M92 can procure a rather good and detailed description of the drift function. The latter can be useful for studying problems of the non-tidal Earth deformations.

Acknowledgments

This work has been done in the Institute of Astronomy and Geodesy in Madrid. One of the authors (A.P.Venedikov, Institute of Geophysics, Sofia) has been supported by the Sabbatical Program of the University Complutense of Madrid.

The data from the suppercondictive gravimeter have been kindly offered by Prof. P.Melchior and Dr. B.Ducarme from the Belgian Royal Observatory.

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TABLE 1 *Comparison between the methods of analysis M67 and M92.*

Lanzarote, volcanic station Cueva de los Verdes, gravimeter LCR
Data: 14.05.1987/12.06.1991, intervals of length n = 48

D tides	*amplit.factor δ*		*phase shift κ*	
	M92	*M67*	*M92*	*M67*
σQ_1	1.2059	1.3036	6°.283	0°.552
	± 413	± 772	±1.954	±3.389
$2Q_1$	1.2528	1.2637	-1.410	-1.660
	132	237	605	1.074
σ_1	1.2097	1.2433	1.090	0.602
	110	195	519	898
Q_1	1.1823	1.1831	-2.136	-2.202
	18	30	84	146
ρ_1	1.1527	1.1600	-2.709	-1.021
	93	158	460	781
O_1	1.1500	1.1500	-1.684	-1.661
	3	6	16	38
τ_1	1.2057	1.0330	-6.299	-1.009
	345	574	1.643	3.184
NO_1	1.1535	1.1488	-0.699	-0.355
	40	66	198	327
χ_1	1.1585	1.0993	1.648	0.577
	220	360	1.088	1.873
π_1	1.1519	1.0995	2.809	0.834
	143	231	709	1.205
P_1	1.1321	1.1320	0.215	0.191
	8	14	42	69
S_1	0.7495	0.9180	24.29	11.00
	509	843	4.087	5.259
K_1	1.1172	1.1166	0.205	0.201
	3	4	12	21
ψ_1	0.9763	1.0261	-18.11	-9.869
	340	552	1.998	3.081
φ_1	1.1162	1.0877	5.329	5.413
	200	325	1.023	1.711
ϑ_1	1.2279	1.1430	-3.738	0.190
	222	363	1.038	1.818
J_1	1.1486	1.1494	1.251	0.467
	42	69	212	346
SO_1	1.2429	1.1122	0.800	-0.626
	245	405	1.131	2.084
OO_1	1.1681	1.1507	0.367	0.786
	53	88	261	440
ν_1	1.1686	1.1166	2.550	0.655
	265	451	1.303	2.317

SD tides	*amplit.factor δ*		*phase shift κ*	
	M92	*M67*	*M92*	*M67*
ε_2	1.0529	1.0792	-8°.862	-6°.847
	± 210	± 390	±1.142	±2.069
$2N_2$	0.9630	0.9888	-2.000	-2.022
	67	118	397	684
μ_2	0.9958	0.9836	-4.360	-3.782
	57	100	327	582
N_2	0.9960	0.9970	0.780	0.855
	9	15	51	88
ν_2	0.9963	1.0008	1.472	1.251
	48	81	276	462
M_2	1.0179	1.0173	2.167	2.160
	18	3	9	16
λ_2	1.0571	1.0765	3.832	1.997
	239	381	1.295	2.029
L_2	1.0666	1.0677	3.368	4.114
	61	96	325	518
T_2	1.0368	1.0424	5.838	4.863
	63	98	346	541
S_2	1.0689	1.0677	4.001	3.991
	4	6	19	31
K_2	1.0594	1.0583	3.795	3.862
	11	17	58	91
η_2	1.0947	1.0712	5.606	4.800
	167	261	871	1.393
$2K_2$	1.1023	1.0810	0.499	2.447
	413	651	2.147	3.451
M_3	1.1018	1.0958	1.697	1.757
	103	76	537	396

TABLE 2 *Comparison between the methods of analysis M67 and M92.*

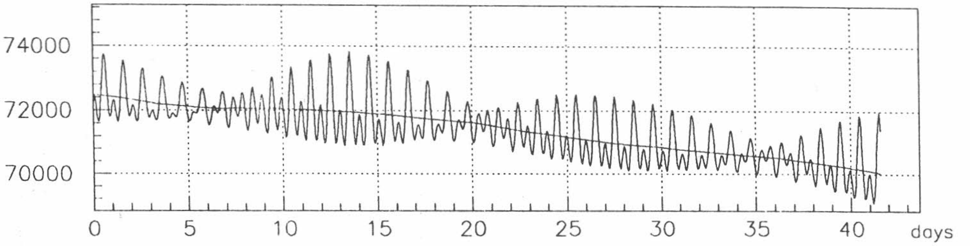
Brussels, superconducting gravimeter, P. Melchior, B. Ducarme.
Data: 21.04.1982/25.11.1987, intervals of length n = 48^h

D tides	amplit. factor δ		phase shift κ	
	M92	M67	M92	M67
σQ_1	1.2745	1.2575	-0°.262	-0°.332
	± 105	± 487	± 470	±2.219
$2Q_1$	1.1611	1.1642	-1.749	-1.885
	33	150	162	737
σ_1	1.1540	1.1524	-1.181	-1.152
	27	122	134	608
Q_1	1.1605	1.1603	-0.469	-0.473
	4	19	20	093
ρ_1	1.1644	1.1769	-0.647	-0.780
	22	98	108	477
O_1	1.1630	1.1634	-0.088	-0.086
	08	3	3	17
τ_1	1.1415	1.1026	-1.948	-5.174
	73	319	366	1.660
NO_1	1.1668	1.1682	0.844	0.717
	10	42	47	208
χ_1	1.1779	1.1710	0.481	-0.292
	51	225	250	1.102
π_1	1.1830	1.1856	0.503	0.169
	32	138	151	668
P_1	1.1643	1.1648	0.150	0.152
	19	8	9	40
S_1	1.3160	1.3218	-4.203	-5.406
	112	494	498	2.141
K_1	1.1505	1.1505	0.147	0.142
	06	3	2	12
ψ_1	1.2509	1.2740	0.596	-0.125
	76	332	345	1.491
φ_1	1.1928	1.1848	0.675	0.494
	44	193	210	933
ϑ_1	1.1470	1.1540	-1.133	-0.988
	51	226	254	1.124
J_1	1.1671	1.1651	0.557	0.504
	10	44	47	214
SO_1	1.1220	1.1309	0.974	0.917
	57	257	290	1.304
OO_1	1.1748	1.1805	-0.033	0.133
	13	60	65	294
ν_1	1.1666	1.1755	0.314	0.253
	66	311	326	1.515

SD tides	amplit. factor δ		phase shift κ	
	M92	M67	M92	M67
ε_2	1.1307	1.1463	3°.420	3°.341
	± 98	± 180	± 494	± 900
$2N_2$	1.1473	1.1494	2.324	2.444
	32	55	157	276
μ_2	1.1946	1.1981	4.365	4.150
	26	46	126	219
N_2	1.1767	1.1770	2.807	2.821
	4	7	20	34
ν_2	1.1776	1.1760	2.672	2.807
	22	37	107	180
M_2	1.1950	1.1949	2.482	2.481
	08	1	3	6
λ_2	1.1492	1.1432	3.062	3.571
	109	174	544	870
L_2	1.1952	1.1946	1.565	1.592
	22	34	104	165
T_2	1.2081	1.2091	1.224	1.134
	29	45	138	215
S_2	1.2141	1.2143	1.123	1.133
	17	3	8	12
K_2	1.2118	1.2112	1.216	1.208
	5	8	25	39
η_2	1.1868	1.1806	0.424	0.967
	89	139	430	672
$2K_2$	1.2348	1.2420	-2.993	-2.165
	224	350	1036	1.614
M_3	1.0751	1.0781	0.620	0.824
	64	59	342	312

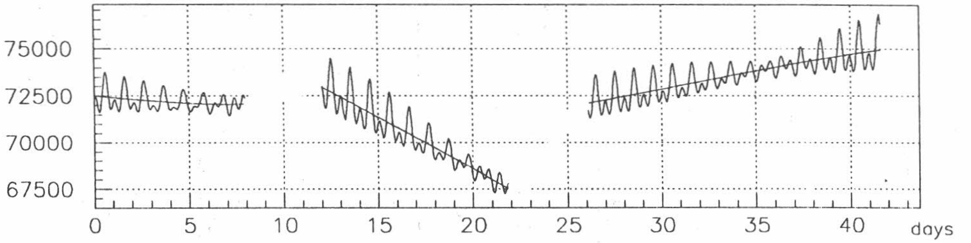
06/10/92 15.12

Fig. 1.a



Observed tidal variations of the gravity with a drift

Fig. 1.b



Observed tidal variations of the gravity with gaps and irregular drift
unit = 0.1 microgal

Figure 1

Fig. 2.

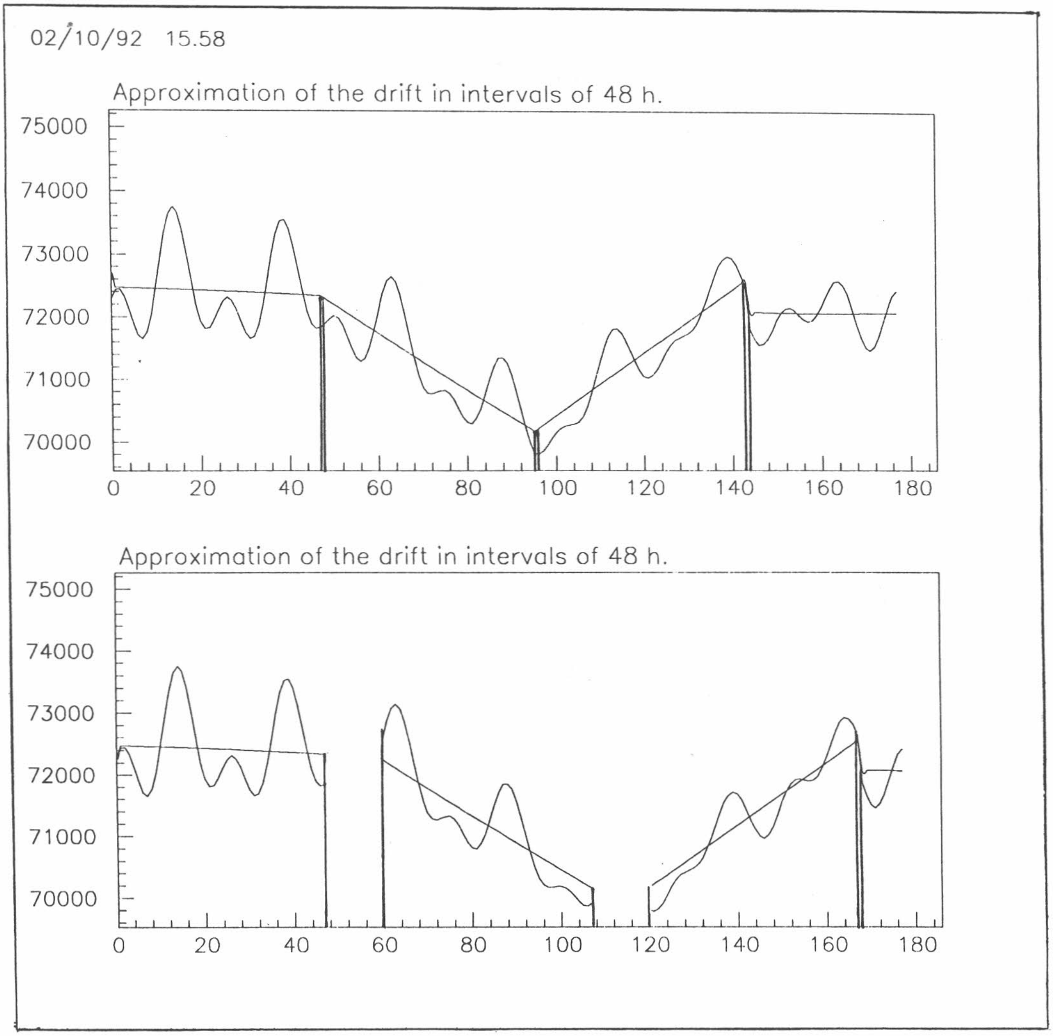

02/10/92 15.58
Approximation of the drift in intervals of 48 h.
75000
74000
73000
72000
71000
70000
0 20 40 60 80 100 120 140 160 180
Approximation of the drift in intervals of 48 h.
75000
74000
73000
72000
71000
70000
0 20 40 60 80 100 120 140 160 180

Fig. 3.a

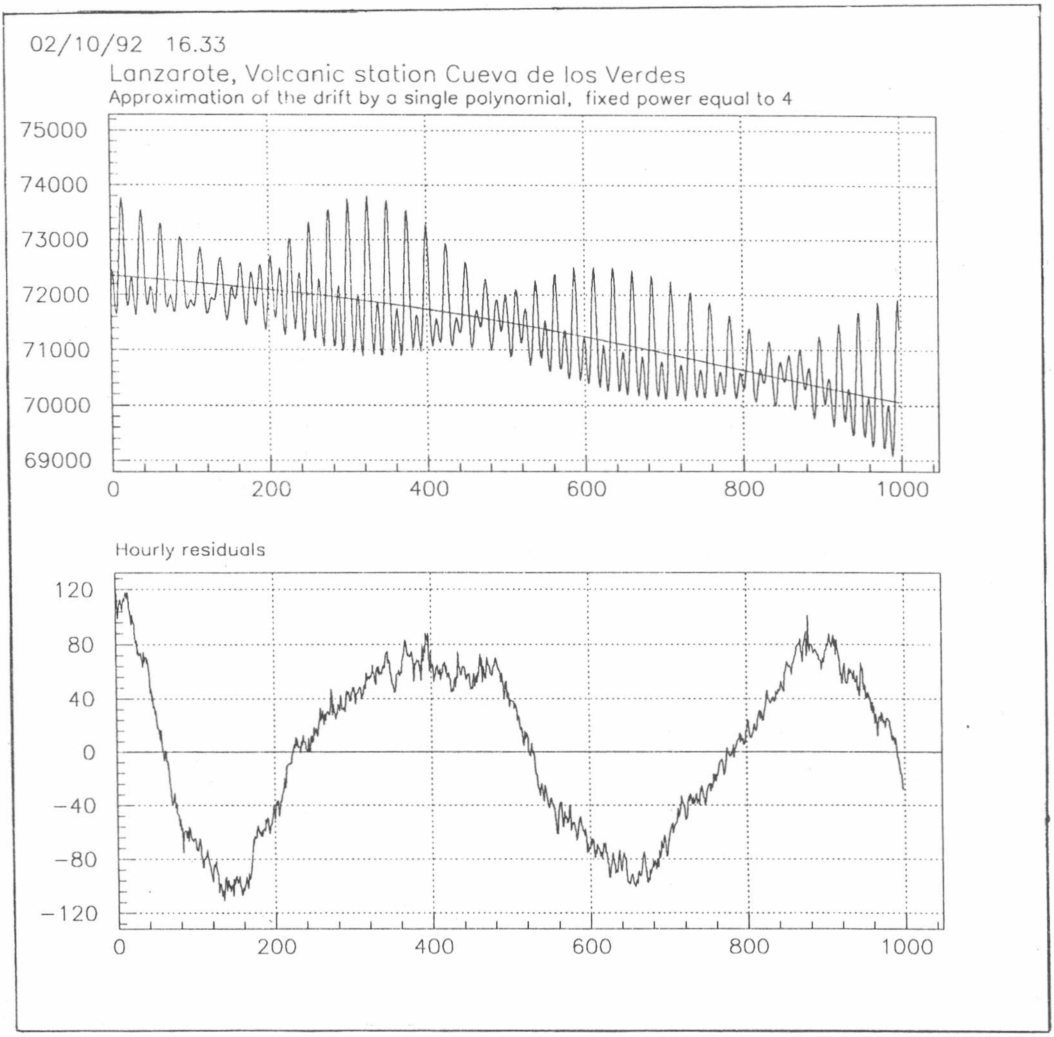
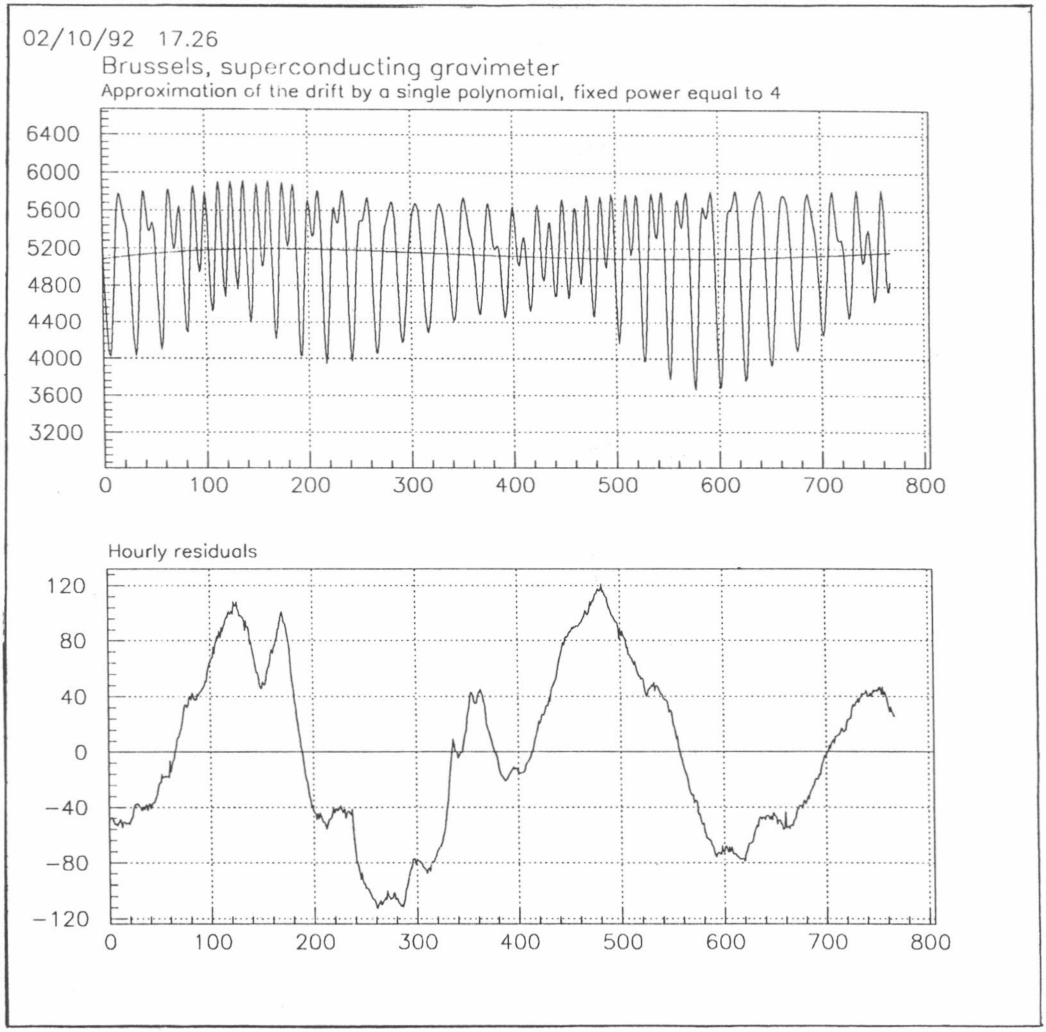

02/10/92 16.33
Lanzarote, Volcanic station Cueva de los Verdes
Approximation of the drift by a single polynomial, fixed power equal to 4
75000
74000
73000
72000
71000
70000
69000
0
200
400
600
800
1000
Hourly residuals
120
80
40
0
−40
−80
−120
0
200
400
600
800
1000

Fig. 3.b



02/10/92 17.26
Brussels, superconducting gravimeter
Approximation of the drift by a single polynomial, fixed power equal to 4
6400
6000
5600
5200
4800
4400
4000
3600
3200
0
100
200
300
400
500
600
700
800
Hourly residuals
120
80
40
0
−40
−80
−120
0
100
200
300
400
500
600
700
800

Fig. 4.a

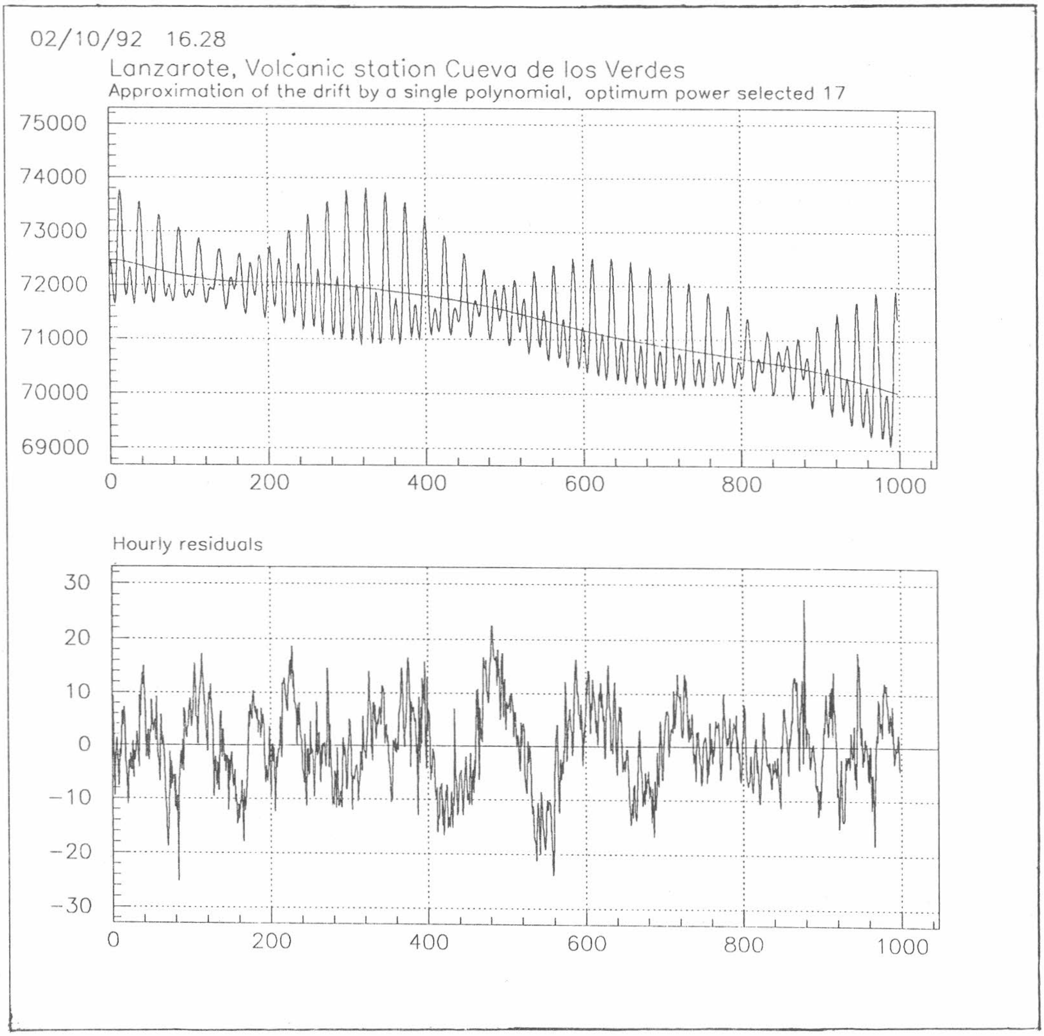
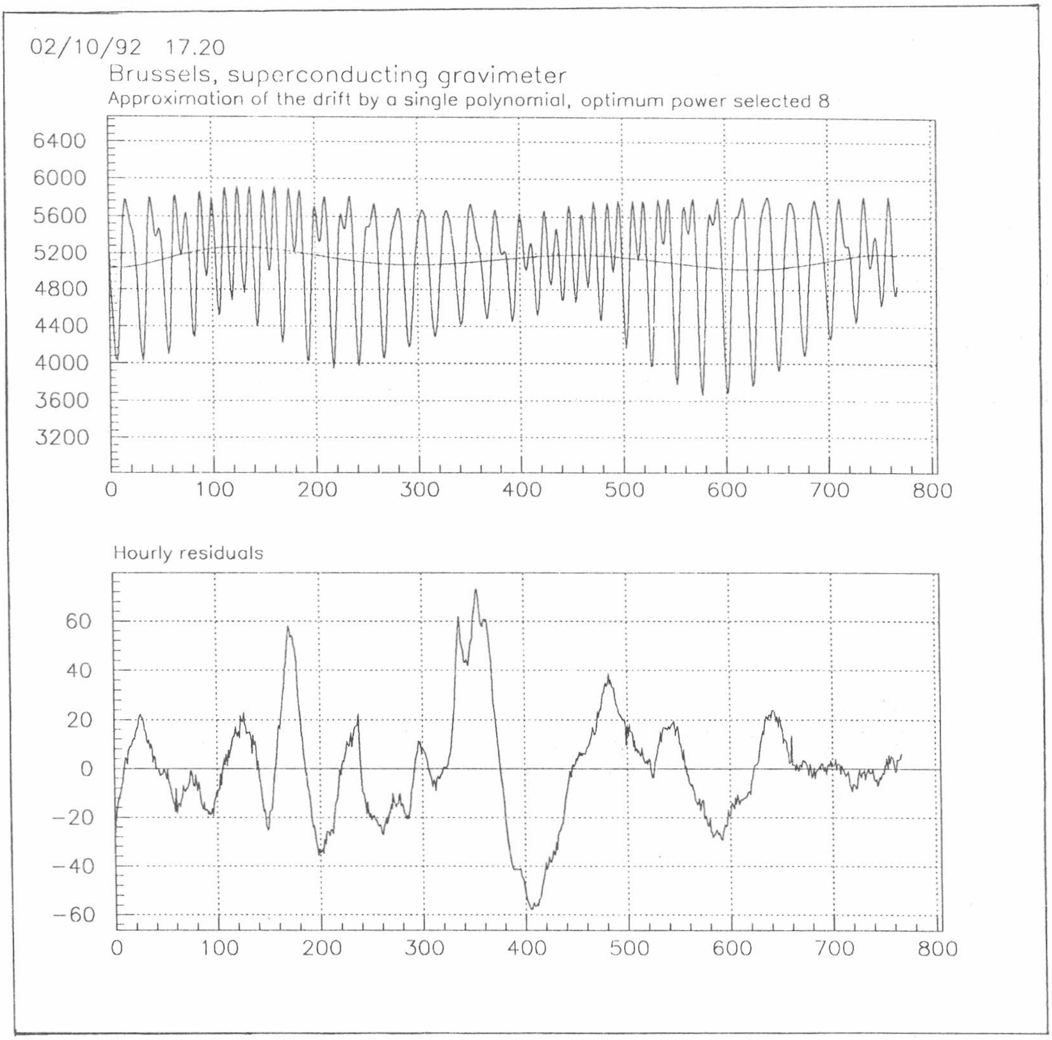
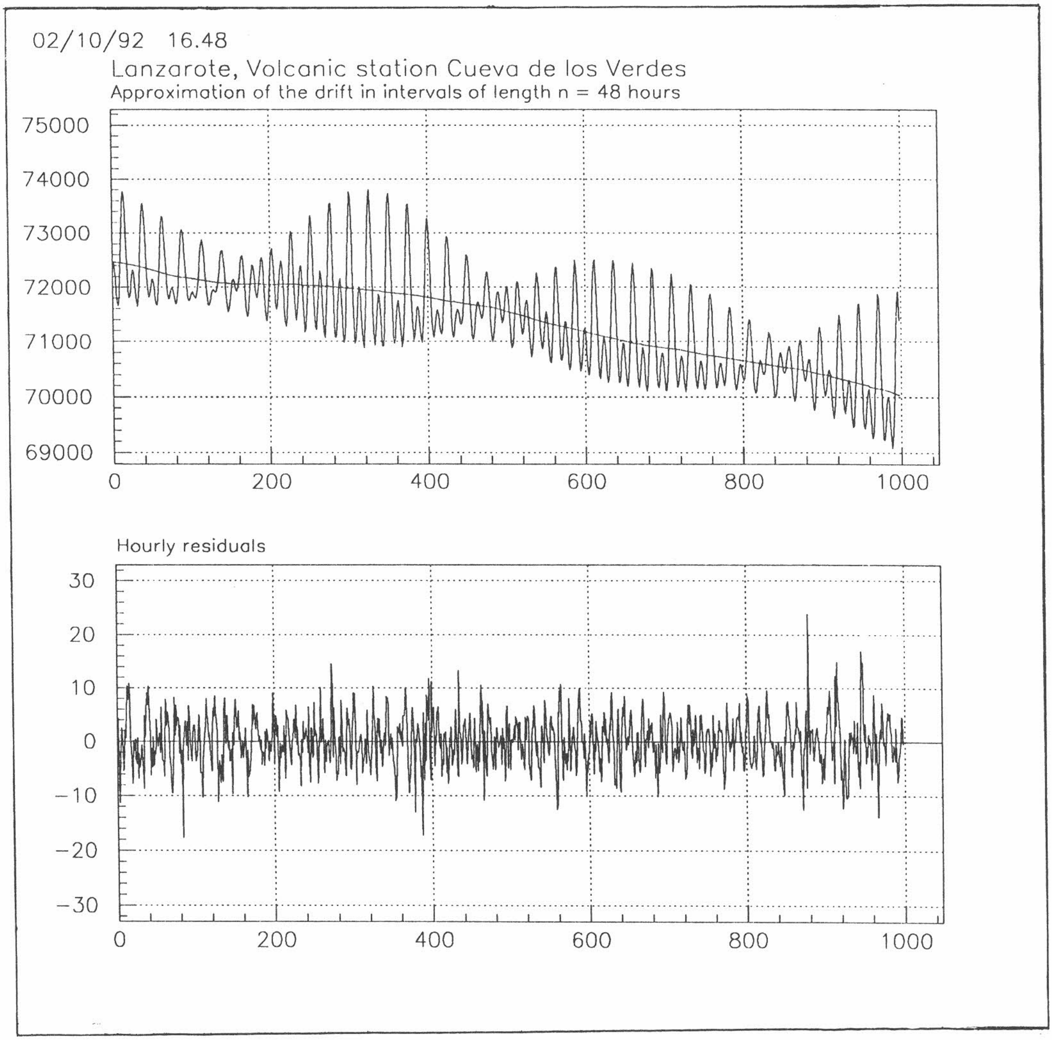

02/10/92 16.28
Lanzarote, Volcanic station Cueva de los Verdes
Approximation of the drift by a single polynomial, optimum power selected 17
75000
74000
73000
72000
71000
70000
69000
0
200
400
600
800
1000
Hourly residuals
30
20
10
0
−10
−20
−30
0
200
400
600
800
1000

Fig. 4.b



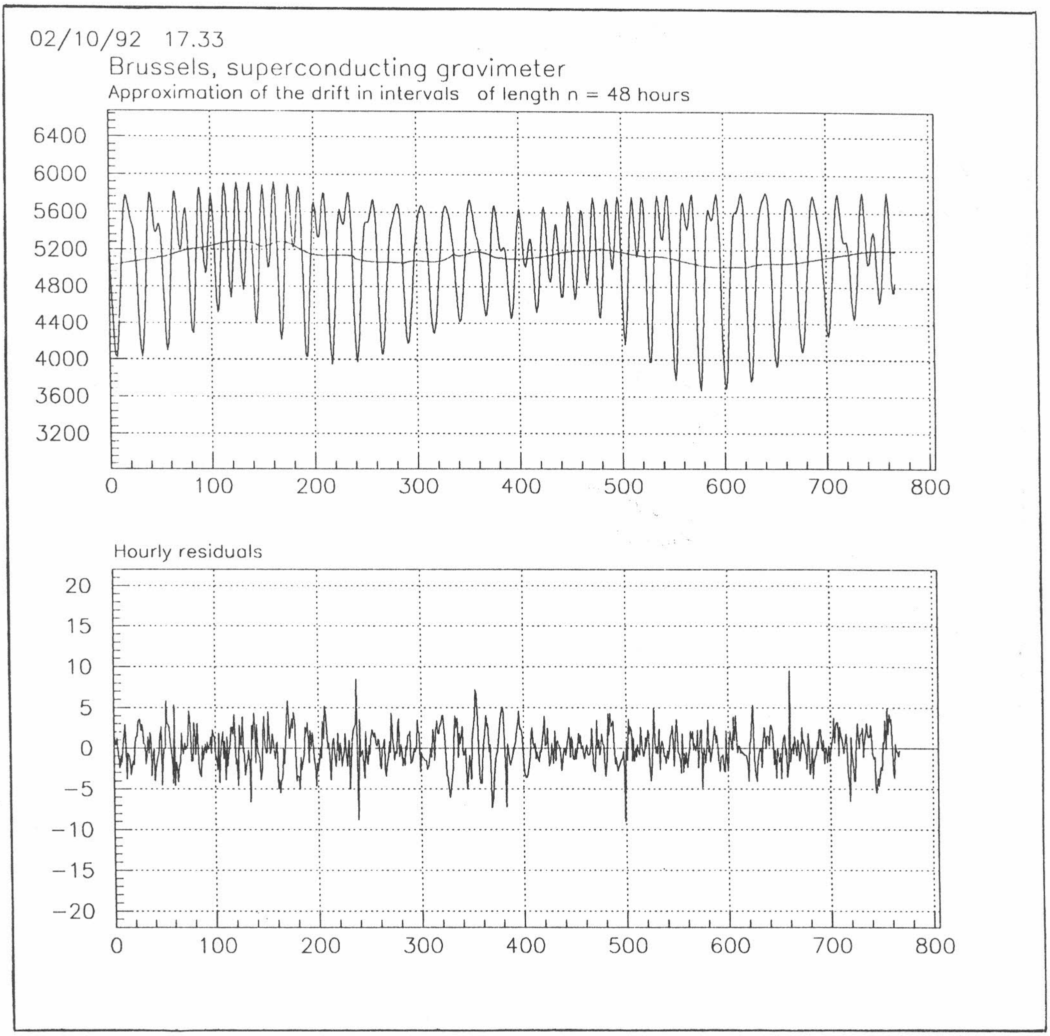
02/10/92 17.20
Brussels, superconducting gravimeter
Approximation of the drift by a single polynomial, optimum power selected 8
6400
6000
5600
5200
4800
4400
4000
3600
3200
0
100
200
300
400
500
600
700
800
Hourly residuals
60
40
20
0
−20
−40
−60
0
100
200
300
400
500
600
700
800

Fig. 5.a



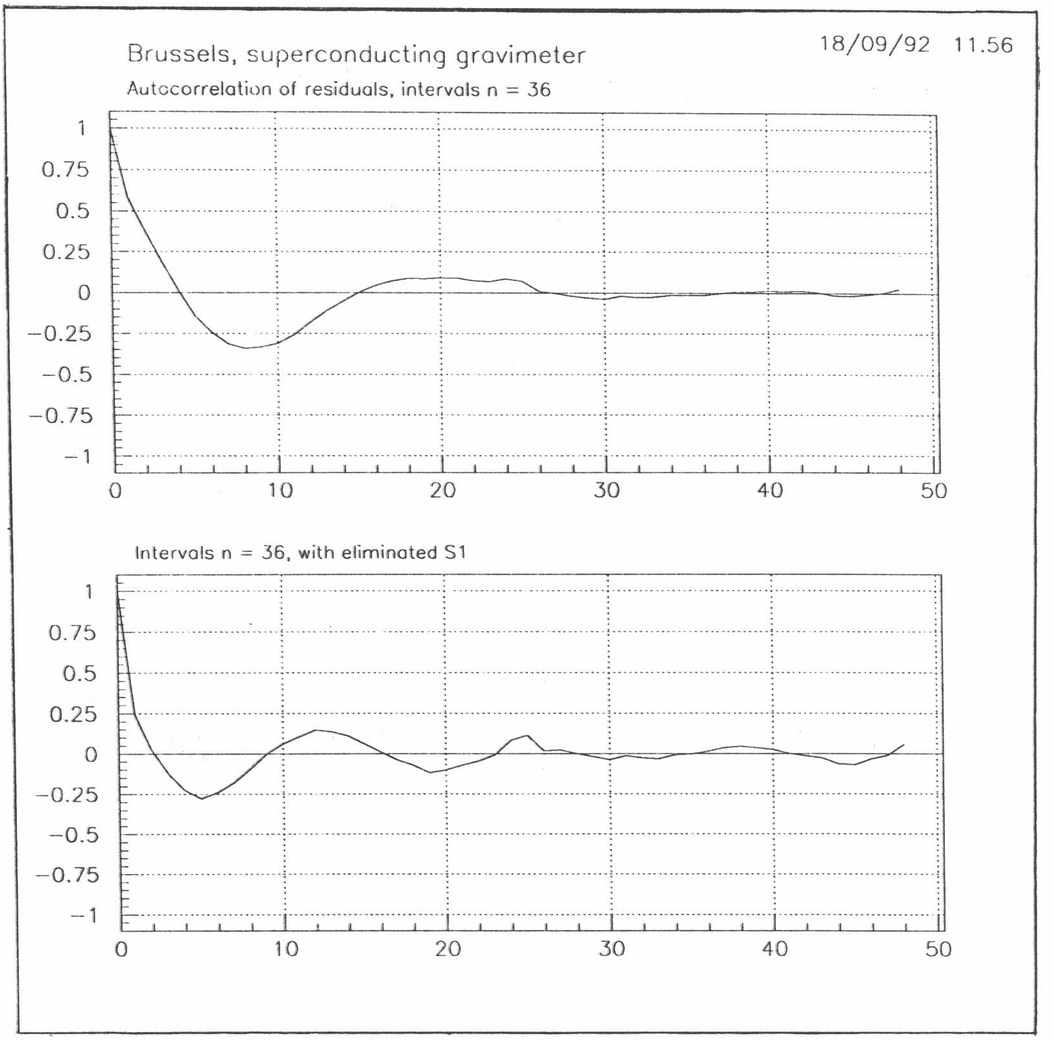
02/10/92 16.48
Lanzarote, Volcanic station Cueva de los Verdes
Approximation of the drift in intervals of length n = 48 hours
75000
74000
73000
72000
71000
70000
69000
0
200
400
600
800
1000
Hourly residuals
30
20
10
0
−10
−20
−30
0
200
400
600
800
1000

Fig. 5.b



02/10/92 17.33
Brussels, superconducting gravimeter
Approximation of the drift in intervals of length n = 48 hours
6400
6000
5600
5200
4800
4400
4000
3600
3200
0
100
200
300
400
500
600
700
800
Hourly residuals
20
15
10
5
0
−5
−10
−15
−20
0
100
200
300
400
500
600
700
800

Fig. 6.



Brussels, superconducting gravimeter
18/09/92 11.56
Autocorrelation of residuals, intervals n = 36
1
0.75
0.5
0.25
0
−0.25
−0.5
−0.75
−1
0
10
20
30
40
50
Intervals n = 36, with eliminated S1
1
0.75
0.5
0.25
0
−0.25
−0.5
−0.75
−1
0
10
20
30
40
50

PUBLICACIONES DEL INSTITUTO DE ASTRONOMIA Y GEODESIA DE LA UNIVERSIDAD COMPLUTENSE — MADRID

(Antes Seminario de Astronomía y Geodesia)

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Depósito Legal: M. Sep. 894 - 1958
ISSN: 0213 - 6198
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