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**PRICE VOLATILITY UNDER ALTERNATIVE MONETARY INSTRUMENTS\***

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ABSTRACT

When private agents have difficulty in interpreting price fluctuations, they are led into suboptimal allocations of resources. Consequently, price uncertainty is an undesirable feature of a business cycle. However, the way how monetary policy is implemented may influence the size of the unpredictable component of price fluctuations and hence, the welfare of the private agents in the economy. This paper addresses the long standing issue of the optimal choice of a monetary instrument under uncertainty. In a money-in-the-utility function model, it is shown that this is far from being a purely monetary issue, and also that the optimal choice of instrument depends on the fiscal policy in effect. If the Government collects enough taxes, relative to its expenditures, a nominal interest rate policy produces a more stable price level, the opposite being true when taxes are low, relative to Government expenditures.

RESUMEN

Cuando los agentes económicos privados tienen dificultades para interpretar las fluctuaciones de precios, se producen asignaciones subóptimas de los recursos, por lo que la incertidumbre respecto a los precios constituye una característica no deseable de los ciclos económicos. Sin embargo, la forma de llevar a cabo la política monetaria puede ejercer una cierta influencia en la magnitud del componente no predecible de las fluctuaciones de precios y, por tanto, en el bienestar de los agentes privados de la economía. En este trabajo se aborda el tema de la elección óptima de los instrumentos monetarios en presencia de incertidumbre. Se muestra, en un modelo que incluye el dinero en la función de utilidad, cómo este tema está lejos de ser un tema puramente monetario, así como que dicha elección óptima depende de la política fiscal desarrollada. Si el gobierno recauda impuestos suficientes en relación a sus gastos, una política de tipo de interés nominal da lugar a un nivel de precios más estable; siendo cierto lo contrario cuando los ingresos por impuestos son bajos en comparación con los gastos.

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## 1. INTRODUCTION

After decades of discussion on the ability of monetary policy to influence real variables in the economy, there does not exist a general agreement on whether those effects exist, how important they are and for how long are they noticed. There seems however to be a wide consensus on the fact that a less than careful monetary policy may introduce serious distortions on economic decisions. In particular, excessive price fluctuations could be an important source of uncertainty for private agents, who might be led into a suboptimal allocation of resources. One way to conceptualize the potential problem resides in Lucas (1973) where it is shown how the inability of the private agents to perfectly forecast whether perceived price shocks are economy-wide or market-specific, prevents them from taking the optimal decision, which they would have chosen under perfect information. From that point of view, price uncertainty is undesirable, so one of the goals of monetary policy has to be not to become an additional source of price fluctuations.

However, almost everybody would agree that the way how monetary policy is implemented may have important short run effects on the evolution of the relevant variables in the economy and, in particular, on the price level. This preoccupation about unnecessary policy induced distortions has produced a long standing discussion on the optimal way to conduct monetary policy. A fundamental level of analysis has tried to characterize conditions under which a monetary policy designed to control a money aggregate is to be preferred to one of controlling interest rates, and viceversa.

In his seminal paper, Poole (1970) considered an IS-LM framework, reaching the conclusion that when the source of uncertainty arises from the real sector, a policy of controlling the money supply is to be preferred, being an interest rate policy best when uncertainty comes from the monetary sector. As it has been the case with other studies, Poole (1970) assumes that the monetary authority has minimizing output fluctuations as his objective. Sargent and Wallace (1975) reconsidered a natural rate version of the IS-LM model under rational expectations, to show that a deterministic monetary rule was as good as any other. They also reproduced the standard result of macroeconomic models in the classical tradition about the indeterminacy of the price level under an interest rate pegging policy. Because of that, they seemed to favor money pegging policies, and among them, a deterministic one, since it is easy to understand and has the same implications as the more complicated ones.

Surprisingly enough, not much attention has recently been paid to the discussion of this important monetary theory issue. We address again the question, although we focus on the implied characteristics of the price series, rather than on output. Hence, the purpose of this paper is to analyze the relative importance of the price fluctuations that emerge under a money supply pegging policy versus an interest rate pegging policy. At a difference of previous treatments, this question is discussed in the context of a general equilibrium model, so that quantities, as well as prices, are endogenously determined. That strategy forces us to specifying functional forms for preferences, technology and the like, so an important issue is always the robustness of the obtained results under sensible changes in the economic environment. The advantage of this approach is that we do not leave any important piece of the model to be imposed from the outside, which could completely condition its implications.

A clear precedent to the issues dealt with in this paper is Sims (1983) where, among other points, the question was raised as to the degree of price stability that could be achieved by alternative monetary policies. Sims argued that in the continuous time version of a model like the one in this paper, with time separable preferences, consumption would not have differentiable paths. But the marginal utilities provided by real balances and consumption are closely related in general equilibrium, so that a policy of pegging the money supply to a given target might have a negative impact on the size of price fluctuations in an stationary equilibrium. In comparison, an interest rate pegging policy would force the price level to anticipate future real returns to capital. In Sims words: "...it is not clear a priori which is likely to produce a more erratic time path for the price level".

We use recent work by Leeper (1991) and Sims (1991), to point out that the optimal choice of monetary instrument is by no means a purely monetary issue. The objective of the paper is then to provide a general equilibrium model under uncertainty, in which private agents have a nontrivial demand for money, and the Government follows a previously specified financing strategy. Prices and interest rates are endogenously determined from the equilibrium conditions. These are a set of stochastic, nonlinear difference equations that cannot be written in closed form, and require the use of a solution method to extract the model's qualitative and quantitative properties.

The economy is described in section 2, where the equilibrium is characterized. In section 3 we discuss the effects of implementing a policy of controlling the money supply, whereas in section 4 we analyze the effects of implementing a policy of controlling nominal interest rates. Stationary steady states are characterized in section 5, where some

theoretical results are described. The stability properties of the model are examined in section 6. Since it needs to be solved numerically, the choice of parameter values is discussed in section 7, together with a description of the solution method and the empirical results. The paper closes with some conclusions and possible extensions for further work.

## 2. THE ECONOMY

### 2.1 The model

We consider a modified version of the Sidrauski (1967) growth model with production and a continuum of identical, infinitely lived consumers. Each one of them derives utility from the single consumption commodity in the economy, as well as from holding real balances. The two arguments enter separably in the utility function. Such a specification implies that money is neutral in important ways that will clearly appear in our discussion. Although our analysis can and should be extended to nonneutral environments, it is important to first have an answer to our proposed monetary policy design issue in a simpler framework<sup>1</sup>.

Each consumer has access to a technology to transfer resources over time, which we assume to be stochastic in nature. The government in this economy finances a given sequence of expenditures  $G_t$  by issuing currency  $M_t$ , as well as by lump-sum taxes  $T_t$  and risk-free bonds  $B_t$ . When these are bought in an amount of  $B_t$  dollars at time  $t-1$ , they pay  $(1+i_t)B_t$  dollars at time  $t$ . The interest rate  $i_t$  is announced at time  $t-1$  to be the nominal rate of return to be delivered at time  $t$  with certainty. Government expenditures have no effect on the utility of private agents or on the available technology. The price level, measured in dollars per time  $t$  good, is denoted by  $P_t$ . Holding money from  $t$  to  $t+1$  yields a gross rate of return of  $P_t/P_{t+1}$ . Since all agents are alike, distributional issues are not relevant in this model.

The representative agent in the economy chooses sequences of consumption, capital, government bonds and money to solve the problem:

<sup>1</sup> An alternative would be to consider a cash-in-advance framework, with two consumption goods, one of which must necessarily be bought with money, as in Lucas and Stokey (1983), or a transaction costs technology, as in Sims (1991).

$$\text{Max}_{\{C_t, K_{t+1}, M_{t+1}, B_{t+1}\}} E_0 \sum_{t=0}^{\infty} \beta^t [ U(C_t) + \gamma V(m_t) ] \quad (1)$$

subject to :

$$C_t + K_{t+1} + \frac{M_{t+1} - M_t}{P_t} + \frac{B_{t+1}}{P_t} = F(K_t, \theta_t) + K_t + (1+i_t) \frac{B_t}{P_t} - T_t \quad (2)$$

where  $Y_t = F(K_t, \theta_t) = \theta_t K_t^\alpha$  is the output produced at the beginning of period  $t$ . Output depends on the stock of capital accumulated at the end of last period  $K_t$ , as well as on the realization of a random technology shock  $\theta_t$ . We consider this shock to have a fair amount of persistence, as represented by a first order autoregressive structure, with a coefficient close to one. To keep the model simple, we have abstracted from adjustment costs of capital or depreciation, although they could easily be included into the specification of a 'net' production function like  $F(K_t, \theta_t)$ . Time  $t$  consumption is denoted by  $C_t$ , whereas  $M_{t+1}$ ,  $B_{t+1}$  and  $K_{t+1}$  denote nominal money, bond holdings and the stock of capital at the end of period  $t$ . Agents observe the current period output, take the policy variables:  $G_t$ ,  $T_t$ ,  $M_t$ ,  $B_t$  and prices  $P_t$  as given, and solve the previous optimization problem on the basis of the information set:  $\Omega_t = \{Y_{t-1}, K_{t-1}, M_{t-1}, B_{t-1}, C_{t-1}, G_{t-1}, T_{t-1}, P_{t-1}, i_{t-1}, s \geq 0\}$ .

The government's budget constraint, in terms of the single representative consumer is:

$$\frac{B_{t+1} - (1+i_t)B_t}{P_t} + \frac{M_{t+1} - M_t}{P_t} = G_t - T_t \quad (3)$$

or:

$$b_{t+1} (1+\pi_t) - (1+i_t) b_t + (1+\pi_t) m_{t+1} - m_t = G_t - T_t \quad (4)$$

where  $m_t = M_t/P_t$  and  $b_t = B_t/P_t$  denote the real value of the stock of money and public debt, whereas the overall resource constraint, which comes from adding up the representative agent constraint (2) with that of the government (4), is:

$$F(K_t, \theta_t) = C_t + K_{t+1} - K_t + G_t \quad (5)$$

which is, of course, not independent of the other two. To obtain the equilibrium, we need to solve for the values of  $C_t$ ,  $K_{t+1}$ ,  $m_{t+1}$ ,  $\pi_t$ ,  $P_t$  and  $b_{t+1}$ . In our discussion, we assume that real Government expenditures are held constant, except by a serially uncorrelated shock which reflects a less than perfect ability on fixing the value of public consumption:

$$G_t = G + \epsilon_t^g \quad \text{for all } t \quad (6)$$

where  $\epsilon_t^g$  is i.i.d.,  $N(0, \sigma_g^2)$ .

There are therefore two sources of uncertainty in the model: the technology shock  $\theta_t$ , and the Government expenditures control error  $\epsilon_t^g$ .

## 2.2 The equilibrium

If we denote partial derivatives by subindices, the optimization problem (1) leads to the optimality conditions<sup>2</sup>:

$$U_{c_t} = \beta E_t [(1+F_{k_t}) U_{c_{t+1}}] \quad \text{for all } t \quad (7)$$

$$(1+i_{t+1})^{-1} = E_t \left[ \beta \frac{U_{c_{t+1}}}{U_{c_t}} \frac{P_t}{P_{t+1}} \right] \quad \text{for all } t \quad (8)$$

$$1 + r_{t+1} = \frac{U_{c_t}}{\beta E_t U_{c_{t+1}}} \quad \text{for all } t \quad (8')$$

$$\gamma \frac{\beta E_t (V_{m_t} / P_{t+1})}{U_{c_t} / P_t} = 1 - E_t \left[ \beta \frac{U_{c_{t+1}}}{U_{c_t}} \frac{P_t}{P_{t+1}} \right] \quad \text{for all } t \quad (9)$$

$$\lim_{t \rightarrow \infty} E_t [\beta^{t-1} U_{c_t} K_{t+1}] = 0 \quad (10)$$

<sup>2</sup>For these, see also as an example: DenHaan(1990), Sims(1991), McCallum(1990), Novales(1990).

$$\lim_{t \rightarrow \infty} E_t [\beta^{t-1} m_{t+1} U_{c_t} (1+\pi_{t+1})] = 0 \quad (11)$$

$$\lim_{t \rightarrow \infty} E_t \left[ \beta^{t-1} \frac{B_{t+1}}{P_{t+1}} U_{c_t} \right] = \lim_{t \rightarrow \infty} E_t [\beta^{t-1} b_{t+1} (1+\pi_{t+1}) U_{c_t}] = 0 \quad (12)$$

The two versions of (8) are obtained depending on whether we consider nominal bonds that pay a rate  $i_t$  as in our budget constraint (2), or real bonds that pay a rate of return equal to  $r_t$ <sup>3</sup>. The certainty equivalence version of (7) shows that the representative consumer saves in the form of capital up to the point where its marginal productivity is equal to the marginal rate of substitution between current and future consumption. Equations (8) and (8') show that the marginal rate of substitution over time must be equal to the real return on private bonds. Equation (9) gives the equilibrium behavior of the marginal rate of substitution between consumption and real balances, and it is an implicit money demand equation. It shows that the typical consumer cannot make himself better off by foregoing one unit of consumption today, holding its value as money, and spending that amount to purchase some commodity tomorrow. In its certainty equivalence version, produces negative correlations between real balances and real interest rates, nominal rates, and the rate of inflation.

The economy is in equilibrium if: a) agents behave according with the optimality conditions for problem (1) so that conditions (7) to (12) hold, b) their budget constraint (2) and that of the Government (4) (together with (6)) are satisfied, and c) they demand the same amount of money and debt which is being offered by the government. In addition, we have the definitions:  $m_t = M_t/P_t$ , and  $\pi_t = (P_{t+1}-P_t)/P_t$ .

By a steady state in the deterministic version of the model (i.e., under  $\theta_t = 1$  and  $\epsilon_{t+1} = 0$  for all  $t$ ) we understand an equilibrium trajectory along which  $C_t$ ,  $K_t$ ,  $Y_t$ ,  $m_t$ ,  $b_t$ ,  $G_t$ ,  $T_t$  and  $\pi_t$  are constant, since there is no technical progress or population growth. A stationary steady state in the stochastic economy is an equilibrium trajectory along which the mentioned variables experience bounded fluctuations around their deterministic steady

<sup>3</sup> In which case, the budget constraint would be:

$$C_t + K_{t+1} + \frac{M_{t+1} - M_t}{P_t} + B_{t+1} = F(K_t, \theta_t) + K_t + (1+r_t)B_t - T_t$$

state values. In steady state, the optimality conditions, the technology and the two budget constraints translate into:

$$\begin{aligned} 1 &= [1 + F_{K^*}] \beta \\ G^* + C^* &= F(K^*) \\ Y^* &= F(K^*) \\ 1 + i^* &= (1+r^*)(1+\pi^*) \\ 1 + r^* &= \beta^{-1} \\ \gamma \beta V_m^* / U_C^* &= 1 + \pi^* - \beta \\ b^*(\pi^* - i^*) + \pi^* m^* &= G^* - T^* \end{aligned}$$

where asterisks denote steady state values. As we will show later on, a policy of controlling the rate of growth of money supply will fix the steady state rate of inflation  $\pi^*$ ; together with values  $G^*$  and  $T^*$  given by a specific fiscal policy, we can solve the system for  $C^*$ ,  $m^*$ ,  $K^*$ ,  $Y^*$ ,  $r^*$ ,  $i^*$ , and  $b^*$ . Alternatively, a policy of fixing the nominal rate of interest  $i^*$ , together with  $G^*$ ,  $T^*$  values, will again allow for solving the above system for the seven remaining variables.

The important fact is that, given  $G^*$ , the first three equations determine the capital stock, consumption and output as a function of just  $\alpha$  and  $\beta$ , so we get the standard superneutrality result that these values are independent of the rate of inflation and the rate of money growth, which will generally be closely related (see McCallum (1990), for example). Money enters the steady state system just in the form of real balances, so that any change in the level of the money supply process will be accompanied with a proportional change in the price level. We can also see that any discount rate below one will produce positive steady state real interest rates, and nominal rates in excess of the rate of inflation. In summary, alternative settings for  $\pi^*$  will have no effect on the steady state values of the main real variables<sup>4</sup>.

<sup>4</sup> As pointed out in McCallum(1990), there is one real variable that is affected by  $\pi$ , namely  $m$ . In fact, it is not hard to show that in our setup, equation (9) implies  $dm/d\pi < 0$ , so that real balances will be lower for higher inflation rates (since the cost of holding money increases). Mc Callum argues that this effect from  $\pi$  to  $m$  should not be considered a violation of superneutrality, since the class of models in that such an effect does not appear is essentially empty.

## 2.3 Preferences

As shown in (1), we assume that preferences depend on money just through real balances, and are time separable as well as separable between the consumption good and real balances. As a particular specification, we assume in what follows that:  $V(m_t) = \log(m_t)$ , so that utility is everywhere increasing in real balances. We also assume that the utility derived from the consumption good is separable over time<sup>5</sup>.

## 3. CONTROLLING THE MONEY SUPPLY

Let us first examine the consequences on the equilibrium of this economy of maintaining a policy of a given constant gross growth rate  $g$  for the money supply. We add to the previous set of equations the money supply equation:

$$M_{t+1} = g M_t \quad t \geq 0 \quad (13)$$

which, dividing by  $P_t$  becomes:

$$m_{t+1} (1+\pi_t) = g m_t \quad t \geq 0 \quad (14)$$

and shows that, since in steady state real balances are constant, the gross rate of inflation is equal to  $g$ .

### 3.1 Price determination under constant money growth

As an illustration, we will first discuss the special case of a constant money supply ( $g=1$ ), to generalize later on to any value of  $g$ . From (14), a constant money supply implies zero steady state inflation. Each possible value of  $M$  then determines a different but unique price level, all of them with the same value for real balances. That is the essence of the neutrality property discussed in the previous section. Incorporating our assumptions on preferences, the implicit demand for money equation becomes:

<sup>5</sup> Of course, it will be interesting to check how robust our results turn out to be to this set of assumptions.

$$\gamma \beta \frac{E_t \left[ \frac{P_{t+1}}{M} \frac{1}{P_{t+1}} \right]}{U_c \frac{P_t}{P_t}} = 1 - E_t \left[ \beta \frac{U_{c,t}}{U_c} \frac{P_t}{P_{t+1}} \right] \quad (15)$$

and if we define:  $W_t = U_c/P_t$  which, under time separable preferences is an  $\Omega_t$ -measurable random variable for each  $t$ , we get:

$$E_t W_{t+1} = \frac{1}{\beta} W_t - \frac{\gamma}{M}$$

which, by successive substitutions, leads to:

$$\beta^k E_t W_{t+k} = W_t - \frac{\gamma}{M} \beta \frac{1-\beta^k}{1-\beta} \quad k \geq 1$$

or:

$$\beta^k \left[ E_t W_{t+k} - \frac{\gamma}{M} \frac{\beta}{1-\beta} \right] = W_t - \frac{\gamma}{M} \frac{\beta}{1-\beta} \quad k \geq 1 \quad (16)$$

This equation has a stationary solution:

$$W_t = \frac{\gamma}{M} \frac{\beta}{1-\beta} \quad \text{for all } t \quad (17)$$

along which:  $\lim_{k \rightarrow \infty} \beta^k E_t W_{t+k} = 0$ , and also a nonstationary<sup>6</sup> solution, with the

difference:  $W_{t+k} - [\beta\gamma/(M(1-\beta))]$  and hence,  $W_t$  itself, expected to diverge to infinity at a rate  $\beta$ . With a sensible utility function, that could happen just with prices going to zero. This latter solution to (16) can be written:

<sup>6</sup> By a nonstationary solution, we mean one in which prices may diverge to infinity or go to zero. The stationary solution to (16) generates zero inflation in the deterministic version of the model, the nonstationary solutions being associated with either positive or negative inflation rates.

$$P_{t,k} = \frac{U_{c,t}}{\frac{1}{\beta^k} \frac{U_c}{P_t} - \frac{1-\beta^k}{\beta^k} \frac{\gamma}{M} \frac{\beta}{1-\beta}} \quad k \geq 0 \quad (18)$$

With this price sequence,  $\beta^k E_t W_{t+k}$  would converge to a well defined constant when  $k$  goes to  $\infty$ . However, it is not hard to show that this trajectory violates the transversality condition for real balances, except in the special case in which  $U_c$  and  $P_t$  are related precisely as in (17), which is a special case of (18).

Therefore, a constant money supply leads to a unique stationary steady state given by (17), at which the price sequence is:

$$P_t = \frac{1-\beta}{\gamma \beta} M U_c \quad \text{for all } t \quad (19)$$

which produces zero steady-state inflation. With a constant money supply, prices act as a perfect signal for the total amount of commodity resources in the economy. The price level is high when consumption is low, and viceversa. But that implies that prices are required to make rapid and accurate adjustments to developments in consumption<sup>7</sup>. As Sims (1983) pointed out for the continuous time version of a model like this, 'with this kind of utility function, consumption will tend to have the property of being locally unpredictable, and prices will inherit that property'.

The price level given by (19) is optimal, satisfying the transversality condition for real balances<sup>8</sup>. Under this policy we have:

<sup>7</sup> Time separability of the utility provided by the consumption good is not crucial for these results. Under non-separability, we would define  $W_t = E_t U_{c,t}/P_t$  to get the same results, including a constant nominal rate equal to  $\beta^{-1}-1$ . The only minor difference is that the price sequence would now be:  $P_t = (1-\beta)M(E_t U_{c,t})/\gamma\beta$ , a function of the conditional expectation of the current marginal utility rather than its realized value, which would be unknown as of time  $t$ .

<sup>8</sup> Indeed, equation (11) becomes:

$$\lim_i E_t [\beta^{i-1} m_{t+i-1} U_{c,t+i-1} (1+\pi_{t+i})] = \lim_i E_t \left[ \frac{\beta^i}{1-\beta} \gamma \frac{U_{c,t} P_{t+i+1}}{U_{c,t+i} P_{t+i}} \right] = 0$$

since using (19), the square bracket can be easily seen to go to zero at a rate  $\beta$ .

$$(1+i_{t+1})^{-1} = E_t \left[ \beta \frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} \right] = E_t \left[ \beta \frac{U_{c,t+1}}{U_{c,t}} \frac{U_{c,t}}{U_{c,t+1}} \right] = \beta \quad (20)$$

which means that the direct response of prices to fluctuations in the marginal utility of consumption has the effect that the expected rate of inflation incorporates all the expected movements in consumption, producing a constant nominal rate<sup>9</sup>.

If a policy of a constant and deterministic, non-unit gross rate of money growth  $g$  is implemented so that  $M_t = gM_{t-1}$ , then we can iterate on the money demand equation (9) to obtain:

$$E_t W_{t+k} = \frac{1}{\beta^k} W_t - \frac{\gamma}{M_0 g^{t+1}} \left[ \frac{1}{\beta^{k-1}} + \frac{1}{\beta^{k-2}g} + \dots + \frac{1}{g^{k-1}} \right]$$

where  $W_t$  is the same process defined before. This equation can be written:

$$\beta^k \left[ E_t W_{t+k} - \frac{\beta}{M_0 g^{t+1}} \frac{\gamma}{1 - \frac{\beta}{g}} \frac{1}{g^k} \right] = W_t - \frac{\beta}{M_0 g^{t+1}} \frac{\gamma}{1 - \frac{\beta}{g}}$$

which has a stationary solution<sup>10</sup>:

$$W_{t+k} = \frac{U_{c,t+k}}{P_{t+k}} = \beta \frac{\gamma}{M_0 g^{t+1}} \frac{1}{1 - \frac{\beta}{g}} \frac{1}{g^k}$$

meaning that, in a stationary steady state, with stable consumption, prices grow or fall over time at a rate equal to  $g$ .

This solution can also be written:

<sup>9</sup>This is a special case of the more general result suggested in Sims (1983) that the nominal rate would generally move with consumption, having itself in a continuous time representation, nondifferentiable paths.

<sup>10</sup> Which is, of course, the same solution one would obtain by inverting the lag operator on  $U_c/P_t$  in (9).

$$M_0 g^t \frac{U_{c,t}}{P_t} = \frac{M_t}{P_t} U_{c,t} = \frac{\beta \gamma}{g - \beta} \quad \text{for all } t$$

that is:

$$P_t = \frac{g - \beta}{\gamma \beta} M_t U_{c,t} \quad \text{for all } t \quad (21)$$

which shows how, in fact, the money stock and the price level evolve over time at the same rate<sup>11</sup>. However, it is not hard to show (see Appendix 1), that a monetary equilibrium exists just for values of  $g$  above  $\beta$ . Prices will be increasing or decreasing over time, depending on whether  $g$  is above or below 1. When  $g$  is equal to 1, equation (21) collapses to (19), the solution we found in the constant money supply case. Nominal interest rates are again constant, and equal to  $g/\beta$ ; the mentioned constraint guaranteeing the existence of monetary equilibrium, implies that nominal rates will be positive.

### 3.2 The coordination of fiscal and monetary policy

We have seen in section 2.2 how a combination of Government expenditures and tax processes can be used to determine a steady state in the deterministic version of the model. As usual though, there is no guarantee that the economy will converge to it. We discuss in this section the class of fiscal policy processes ( $G_t$ ,  $T_t$ ) which will produce a stable equilibrium. This feasibility issue has recently been discussed in Sims(1991) and Leeper(1991), among others. The question needs to be addressed examining the government budget constraint (3), rewritten as:

$$\begin{aligned} \frac{B_{t+1}}{P_t} &= (1+i) \frac{B_t}{P_t} - \frac{M_{t+1} - M_t}{P_t} + G_t - T_t = \\ &= (1+i) \frac{B_t}{P_t} - (g-1) \frac{M_t}{P_t} + G_t - T_t \end{aligned} \quad (22)$$

<sup>11</sup> There is also a non stationary solution to the difference equation which generalizes (18), and is not an equilibrium, except in its special version (21).

where we have already incorporated a constant nominal interest rate of  $i$ . If the Government maintains constant lump-sum taxes/transfers  $T$ , then a constant money supply ( $g=1$ ) makes (22) a non-stable stochastic difference equation on real debt, which would make it grow at a rate  $1+i = \beta^{-1}$ . Hence, the transversality condition (12) that the stock of real debt cannot grow as fast as  $\beta^{-1}$  would fail to hold.

To achieve an equilibrium under a constant money supply, the authority needs to implement an active fiscal policy that levies taxes each period as a function of the stock of real debt:  $T_t = h(b_t)$ , and that can be done in a variety of ways (Sims(1991)). Any linear policy:  $T_t = T_M + a_M b_t$ , with  $i < a_M < 1+i$ , in which the variable component of the tax collection is greater than the interest service of the debt, but less than its face value plus interest, all in real terms, would make (22) to generate a stable, positive path for  $B_t$ <sup>12</sup>:

$$B_{t+1} = [(1+i) - a_M] B_t + (G_t - T_M) P_t \quad (23)$$

If the money supply is let to grow at a rate greater than one, then fiscal policy gets a hand from the inflation tax on real balances. Incorporating the constant value of the nominal rate of interest, the Government budget constraint rewritten as:

$$P_t (T_t - G_t) + (g-1) M_t = B_{t+1} - (g/\beta) B_t \quad (24)$$

indicates how the single period surplus, together with the inflation tax must roughly exceed the interest charge of Government bonds to achieve stability<sup>13</sup>. If the money supply contracts over time, fiscal policy will have to be more restrictive.

#### 4. EQUILIBRIUM UNDER AN INTEREST RATE PEGGING POLICY.

We now consider policies in which the authority conducts monetary policy so that the nominal rate of interest stays constant over time. Since real rates are constant in

<sup>12</sup> These policies are more demanding than necessary, since they force fiscal policy to be pegged to the business cycle. For the existence of a stationary steady state, it is enough that lump-sum taxes depend linearly on  $b^*$  rather than on  $b_t$ .

<sup>13</sup> We know that a monetary equilibrium exists only if  $g > \beta$ , which makes the lagged coefficient in (24) to be above one.

an stationary steady state, policies designed to maintain constant nominal rates can be naturally compared to those designed to maintain constant rates of inflation. Analytically, it is easier to deal with the inverse of the nominal rate, and consider rules of the form:

$$E_t \left[ \beta \frac{U_{c,t}}{U_c} \frac{P_t}{P_{t+1}} \right] = \frac{1}{1+\delta_0} \quad (25)$$

where  $\delta_0$  is a nonnegative constant.

The left hand side of (25) is, according to (8), the inverse of the equilibrium value of the nominal rate that is being offered on the one period Government bonds. Being a forecasting formula, it can also be interpreted as reflecting a commitment on the part of the monetary authority to watch the current period's situation of economy-wide resources and issue money so that the implied price behavior and, more specifically, its correlation with consumption will be such that the private sector can optimally build forecasts as mentioned.

#### 4.1 Price behavior under constant nominal interest rates

Equation (25) can be considered as a difference equation in prices with  $U_c$  as the driving process, having as a possible solution<sup>14</sup>:

$$P_{t+1} = \beta (1+\delta_0) \frac{E_t U_{c,t}}{U_c} P_t \quad (26)$$

starting from any initial price  $P_0$ . The price sequence given by (26) is a solution to (25) in which consumers actually know, at time  $t$ , what the price level will be at time  $t+1$ . It is

<sup>14</sup> If we substitute the proposed solution at the left hand side of (25), we get:

$$E_t \left[ \beta \frac{U_{c,t}}{U_c} \frac{P_t}{P_{t+1}} \right] = E_t \left[ \beta \frac{U_{c,t} P_t}{U_c \beta (1+\delta_0) \frac{E_t U_{c,t}}{U_c} P_t} \right] = \frac{1}{1+\delta_0} E_t \left[ \frac{U_{c,t}}{E_t U_{c,t}} \right] = \frac{1}{1+\delta_0}$$

so that (26) is indeed, a solution to the difference equation.

the minimum conditional variance solution to (25), that is, the one solution that involves the minimum uncertainty as of time  $t$ <sup>15</sup>. But the price sequence given by:

$$P_{t+1} = \beta (1+\delta_0) \frac{U_{c,t}}{U_c} P_t \quad (27)$$

from a starting price  $P_0$ , is another solution to (25). In this case, however, the time  $t+1$  price level is unknown to private agents as of time  $t$ . It is to them a random variable which is correlated with consumption in such a way that makes (25) to be an optimal forecasting formula. It can be called the maximum conditional variance solution.

In fact, there is a continuum of solutions to the stochastic difference equation (25). Any price sequence of the form:

$$P_{t+1} = \beta (1+\delta_0) \frac{U_{c,t}}{U_c} \frac{1}{(1+\xi_{t+1})} P_t \quad (28)$$

where  $\xi_{t+1}$  is a random variable with  $E_t \xi_{t+1} = 0$ , is a solution to (25), as can be checked by direct substitution. The minimum variance solution comes from (28) when  $1+\xi_{t+1} = U_{c,t+1}/E_t U_{c,t+1}$ , whereas the maximum conditional variance solution comes from this general expression when  $1+\xi_{t+1} = 1$ <sup>16</sup>.

If the monetary authority chooses  $\delta_0$  so that:  $\beta(1+\delta_0) > 1$  ( $< 1$ ) we would have in steady state an increasing (decreasing) price sequence, with a gross rate of inflation (deflation) precisely equal to the value of the product  $\beta(1+\delta_0)$ . If, for instance:  $\delta_0=0$ , then prices will decrease at a rate  $\beta$ <sup>17</sup>. If we choose  $P_0$  to be the same as in a policy of

<sup>15</sup> In our case, that minimum conditional variance is zero, since the time  $t+1$  price level becomes known at time  $t$ . It is also the only analytical solution to (25) which makes  $P_{t+1}$  to be known at time  $t$ .

<sup>16</sup> In the minimum conditional variance solution,  $\text{Var}_t(1+\xi_{t+1}) = \text{Var}_t \eta_{t+1} / (E_t U_{c,t+1})^2$ , where  $\eta_{t+1}$  denotes the innovation in the marginal utility of consumption, an expression that can be thought of as the square of the conditional coefficient of variation of the marginal utility of consumption. In the maximum conditional variance solution,  $\text{Var}_t(1+\xi_{t+1}) = 0$ .

<sup>17</sup> Again, these statements would be exact for the deterministic version of the model. In the stochastic version, prices would fluctuate around a negative trend line with slope  $\beta$ .

controlling the money supply to grow at a rate  $g = \beta(1+\delta_0)$ , the maximum conditional variance solution generates not only the same inflation, but also the same price series as that money growth control policy.

Any steady state rate of inflation equal to  $g$  can be achieved by choosing  $\delta_0$  such that:  $\beta(1+\delta_0)=g$ . Taking any of the price sequences in (28) to the money demand equation (9), we get:

$$M_{t+1} = \gamma \beta \frac{1+\delta_0}{\delta_0} \frac{P_t}{U_c} = \frac{\gamma \beta}{g-\beta} g \frac{P_t}{U_c} \quad (29)$$

In the absence of random shocks, this is the same relation between money supply, prices and the marginal utility of consumption as in the money pegging case, although here the money stock fluctuates around a time trend of  $g$ .

Additionally, taking any price sequence from (28) into the definition of the nominal rate, we get:

$$1+i_t = E_t \left[ \beta \frac{U_{c,t}}{U_c} \frac{P_t}{P_{t+1}} \right] = E_t \left[ \beta \frac{U_{c,t}}{U_c} \frac{(1+\xi_{t+1}) P_t}{\beta(1+\delta_0) \frac{U_{c,t}}{U_c} P_t} \right] = \frac{1}{1+\delta_0}$$

so that expectations are, of course, fulfilled.

However, even though each price sequence in (28) is a solution to (25), they may not be equilibrium price sequences. For that, they must be compatible with stationary sequences for real balances and bonds, a crucial issue that we analyze in the next subsection. There, we will show that there is a single equilibrium price sequence in the family (28).

#### 4.2 Price determination under nominal interest rate policies

A last observation concerns the possible indeterminacy of the price level under interest rate pegging policies. We have found that each member of (28) produces a solution to (25) only as a function of an initial, arbitrary price  $P_0$ . In the zero inflation case, the whole price sequence will fluctuate around  $P_0$ , while in general, they will do so just after a deterministic exponential trend of size  $\beta(1+\delta_0)$  is taken out of the series. That

dependence on the initial price level might suggest the price indeterminacy which arises in some models under nominal rate policies (see Sargent(1979), for example). However, an argument parallel to that in Sims(1991), shows that a nominal interest rate policy in this model actually determines the initial price level. That makes unnecessary suggestions like those in McCallum(1981) that, in order to determinate the price level, the value of the interest rate used as an instrument of monetary policy should be chosen as a function (presumably direct) of the money stock.

The argument showing the price level determination starts by dividing through the Government budget constraint (22) by  $M_{t+1}/P_t$ , to get:

$$\frac{B_{t+1}}{M_{t+1}} - (1+\delta_0) \frac{B_t}{M_t} \frac{M_t}{M_{t+1}} + \left[ 1 - \frac{M_t}{M_{t+1}} \right] = (G_t - T_t) \frac{P_t}{M_{t+1}} \quad (30)$$

On the other hand, the demand for money under constant nominal rates and log-utility becomes:

$$\gamma \beta \frac{P_t C_t}{M_{t+1}} = \frac{\delta_0}{1+\delta_0} \quad (31)$$

which guarantees that if we measure money 'velocity'  $v_t$  as:  $v_t = P_t C_t / M_{t+1}$ , then  $v_t$  is constant:  $v_t = v_0 = \delta_0 / (\gamma \beta (1 + \delta_0))$ , and we have:

$$\frac{1}{1+i_t} = E_t \left[ \beta \frac{C_t}{C_{t+1}} \frac{P_t}{P_{t+1}} \right] = \frac{1}{1+\delta_0} \quad (32)$$

and using the definition of velocity, we get:

$$E_t \left[ \frac{M_{t+1}}{M_{t+2}} \right] = \frac{1}{\beta(1+\delta_0)} = \frac{1}{1+\pi^*} \quad (33)$$

If we now take conditional expectations  $E_{t-1}$  in (30), and use the latter expression, we arrive at:

$$E_{t-1} \left[ \frac{B_{t+1}}{M_{t+1}} \right] - \frac{1}{\beta} \frac{B_t}{M_t} + 1 - \frac{1}{\beta(1+\delta_0)} = v_0 E_{t-1} \left[ \frac{G_t - T_t}{C_t} \right] \quad (34)$$

which has a unique, stable solution:

$$\frac{B_{t+1}}{M_{t+1}} = \frac{\pi^*}{1+\pi^*} \frac{\beta}{1-\beta} - v_0 E_t \left[ \sum_{i=1}^{\infty} \beta^i \frac{G_{t+i} - T_{t+i}}{C_{t+i}} \right] \quad (35)$$

which implies that  $B_{t+1}/M_{t+1}$  is stationary, so long as the single period deficit  $G_{t+i} - T_{t+i}$  is not expected to explode, or  $C_{t+i}$  go to zero at a rate faster than  $\beta$ . In the absence of random shocks, the nominal stock of bonds, as well as the money supply, will grow at the rate of inflation. Then, going back to (30), we get:

$$\frac{M_t}{M_{t+1}} \left[ 1 + (1+\delta_0) \frac{B_t}{M_t} \right] = 1 + \frac{B_{t+1}}{M_{t+1}} - v_0 \frac{G_t - T_t}{C_t} \quad (36)$$

which implies a stationary path for  $M_t/M_{t+1}$ , although not necessarily for the money supply itself. That means that the nominal stock of both, money and bonds, will generally be nonstationary under a policy of fixing a constant nominal rate of interest. As pointed out in Sims(1991), that does not mean that they are indeterminate. On the contrary, the expected present value of the ratio of Government deficit to private consumption will determine the ratio of bonds to money every period from (35); that, taken to (36), will determine the money supply growth and hence, both  $M_{t+1}$  and  $B_{t+1}$  as a function of the beginning of period values  $B_t$  and  $M_t$ . Finally, the constant velocity condition determines the price level  $P_t$ .<sup>18</sup>

#### 4.3 Fiscal policy under a nominal rate pegging policy

Contrary to a money growth control policy, here constant lump sum taxes:  $T = T_t$  for all  $t$ , are compatible with a stationary equilibrium in which the stock of real bonds

<sup>18</sup> This argument does not depend on the form of the utility of the consumption good, so long as it is independent of the one for real balances. It does depend on the latter, since all we can conclude in the general case is that:  $E_t(V_{mt+2}/V_{mt+1}) = (1+\pi^*)^{-1}$ .



behaves with bounded oscillations around its deterministic steady state value. From (35), a necessary condition to guarantee positive bond holdings in steady state is:

$$G^* - T_1 < \frac{\pi^*}{1+\pi^*} \frac{C^*}{v_0} = \frac{\pi^*}{1+\pi^*} m^*$$

so that lump-sum tax collections are at least as large as the part of Government expenditures that is not covered by the inflation tax. In particular, to have positive bond holdings under deflation, the constant lump-sum taxes must exceed the average value of Government expenditures. Of course, if one accepts the possibility that the Government makes loans to the public rather than the other way around, negative bond holdings are not a problem. In the next sections we will present one such an equilibrium as an illustration.

It is also possible to achieve a stationary equilibrium with tax policies that link the amount collected each period to the real value of Government debt. If we assume a linear specification:  $T_t = T_1 + a b_t$ , it is not hard to show that the only constraint for an stationary equilibrium to exist is for the constant  $a$  to stay below:  $(1-\beta)(1+\delta_0)$ .

## 5. STATIONARY STEADY STATES.

We compare in this section some of the statistical properties of the equilibrium price series that emerge under the two alternative ways of implementing monetary policy. Our maintained assumption is that the monetary authority cares about the degree of price volatility that will be derived under each possible strategy. We will generate equilibrium price series with the same mean, and compare their second order moments, repeating the exercise for different policy parameters. The money neutrality embedded in the model guarantees that the consumption, savings and production decisions will be the same under the two policy mechanisms. That common reference makes unnecessary more sophisticated comparison criteria than some price volatility statistics<sup>19</sup>.

Because of this neutrality property, the question we analyze in this paper could be reinterpreted as to the degree of price volatility that would be introduced by either monetary policy in order to generate a given process for per capita consumption, or for total resources. Given that the steady state values of the real and nominal interest rates, the inflation rate and real balances are the same under the two policies, private agents will tend to be indifferent between the two mechanisms since, for a common consumption process, the possible welfare differences will just arise from differences on the level of real balances<sup>20</sup>. We will first make these results more specific by presenting some welfare comparisons between the steady states that would be achieved under different policy mechanisms. Being steady state results, they fully apply to the deterministic version of the model. Since they just have to do with first order moments, they apply to the stochastic equilibria just in an average sense.

<sup>19</sup> At least, a more thorough discussion on the possible loss functions of the monetary authority would be needed. For instance, if the output sequences generated under each policy arrangement were different, it would seem necessary to include some of its statistical properties among the comparison criteria.

<sup>20</sup> That is a consequence of the utility function depending on just the first order moment of the distribution of real balances. However, it does not mean that higher order moments could not be included in the utility function, giving raise to a more complex policy analysis problem.

### 5.1 The time aggregate utility function in steady state.

We saw in section 3 that if the money supply grows over time at a rate  $g > \beta$ , we have in steady state:

$$\left(\frac{P}{M}\right)^* = \frac{g - \beta}{\gamma \beta} U_c^*$$

so that real balances are a decreasing function of the rate of money growth  $g$ . Since the inflation rate is equal to the rate of growth of the money supply, we have:

Lemma 1: The deterministic steady states that arise under all constant money supply policies are equivalent from the point of view of the welfare of the private agents, with independence of the chosen value for the money stock. More generally, given a constant money growth rate  $g$ , consumers are indifferent in steady state as to the level of the money stock.

Lemma 2: Given two policies (labelled P and P') of constant money growth at rates  $g$  and  $g'$ , with  $\beta < g < g'$ , consumers prefer the deterministic steady state that arises under P to the one obtained under P'.

Under a policy that keeps a constant nominal interest rate:  $\delta_0 = \beta^{-1} - 1$ , equation (28) shows that there is zero steady state inflation, and dividing through (29) by  $P_{t+1}$ , real balances can be seen to be equal to:

$$\left(\frac{M}{P}\right)^* = \gamma \beta \frac{1}{U_c^*} \frac{1 + \delta_0}{\delta_0} = \frac{\gamma}{\delta_0} \frac{1}{U_c^*} = \frac{\gamma \beta}{1 - \beta} \frac{1}{U_c^*}$$

which is clearly the same result we would get from (19), so that:

Lemma 3: The deterministic steady state that arises under a constant nominal rate policy:  $\delta_0 = \beta^{-1} - 1$  is welfare equivalent to the ones that are achieved under any of the constant money supply policies. All of them produce a zero steady state inflation rate.

For all other positive values  $\delta_0$  we have inflation or deflation. The steady state

value of real balances would now be:

$$\left(\frac{M}{P}\right)^* = \gamma \beta \frac{1}{U_c^*} \frac{1 + \delta_0}{\delta_0}$$

which is decreasing in  $\delta_0$ , so that:

Lemma 4: In the deterministic economy, with a monotonic utility function of real balances, private agents are better off with low nominal rates of interest.

These constant, positive nominal interest rate policies generate a steady state inflation (or deflation) rate equal to  $\beta(1 + \delta_0)$ , which is above  $\beta$ . That is the same inflation rate that is produced by a money supply that grows at a gross rate  $g = \beta(1 + \delta_0)$  so that the previous result is just a restatement of lemma 2. More generally:

Lemma 5: Private agents are indifferent between the steady states that emerge from a policy of keeping a constant, positive nominal rate of interest, and a constant money growth policy that generates the same rate of inflation or deflation<sup>21</sup>.

Proof: It is enough to divide through (29) by  $P_{t+1}$  and use the steady state relation:  $\pi^* = \beta(1 + \delta_0)$ , to see that the steady state values of real balances that come from (21) and (29) are the same.

The previous results state that, in the deterministic economy, consumers prefer a lower nominal rate of interest. The one-to-one correspondence between the constant value of the nominal rate and the steady state rate of inflation makes this proposition equivalent to saying that consumers prefer deflation to inflation, and more to less deflation. In the limit, the authority might want to choose as an objective a zero nominal rate, corresponding to a deflation rate of  $\beta$ . This goes of course along the lines of the Friedman's rule of a zero nominal interest rate<sup>22</sup>. However, as we mentioned in

<sup>21</sup> Technically, we would need to make sure that the rate of money growth  $g$  exceeds the discount rate  $\beta$ , but the equal rate of inflation condition:  $g = \beta(1 + \delta_0)$ , together with the non-negativity of the nominal rate  $\delta_0$ , guarantees that.

<sup>22</sup> In an stationary steady state, the gross real interest rate is equal to  $\beta^{-1}$ . Together with a gross deflation rate of  $\beta$ , that would amount to a zero nominal interest rate.

section 3.2, in the limiting case, as the nominal rate of interest goes to zero, there is no equilibrium with unbacked currency in this economy, so that that a first-best equilibrium is not well defined. That is due to the monotonicity of the utility of real balances.

Two observations need to be made: First, in the stochastic equilibrium, the price series fluctuate around their deterministic steady states, and the previous results clearly do not consider the importance of such deviations. So long as the fluctuations that experience both price series are purely random around the same mean, then for long periods, the deterministic indifference results from the time aggregate utility functions should carry over to the stochastic economy. But if the monetary authority cares about the second order moments of the price series, it will not be as indifferent between the two policy mechanisms as consumers are. However, it is essentially impossible to obtain closed form expressions for higher order moments of the two price series, so that we will make the comparisons in the general stochastic case by numerically solving the model, in the next section.

Secondly, the fact that the steady state real balances under the two policies are the same<sup>22</sup> does not mean that the two series will be the same in the stochastic economy. The opposite is true: the two series for real balances experience fluctuations around a common steady state value, but differ in value at each point in time. In fact:

$$\frac{m_t^M}{m_t^I} = \frac{1 + \pi_{t-1}}{g} \frac{U_{c,t}}{U_c}$$

which is equal to 1 in steady state.

## 5.2 Variance comparisons.

In the previous section, we have discussed how the representative agent ranks all the feasible steady states. We now proceed in this section to evaluate the alternative monetary policies from the point of view of the policy maker. To do so, we need to assume

<sup>22</sup> So long as they are obtained for the same steady state inflation (lemma 5)

that the monetary authority has a given loss function<sup>24</sup>. As a starting point, we let us assume that when volatility is considered a negative characteristic of a price sequence, the policymaker is worried about the variance of the price series that arises under a given monetary policy.

To establish some comparison between the two policies, let us first notice that in the case of a perfect instrument control, we have under a policy of controlling the money supply:

$$\frac{P_{t+1}}{P_t} = \frac{M_{t+1}}{M_t} \frac{U_{c,t}}{U_c} = g \frac{U_{c,t}}{U_c} \quad (37)$$

whereas under an interest rate pegging policy, we have:

$$\frac{P_{t+1}}{P_t} = \frac{1 + \delta_0}{1 + r_t} = \beta (1 + \delta_0) \frac{U_{c,t}}{U_c} \frac{1}{1 + \xi_{t+1}} \quad (38)$$

which includes an extra factor relative to (37).

However, it is not clear that the policy maker should really worry about the sample variance, that is, the historical variance of prices, rather than about their predictability, that is, the accuracy of the conditional variance as a predictor of future prices. In that sense, it is important to examine the relative values of the forecast errors in the two price series that emerge under the alternative policy implementation mechanisms.

A nominal interest rate policy would then be preferred if and only if:

$$\text{Var}_t \left[ \frac{U_{c,t}}{U_c} \right] \geq \text{Var}_t \left[ \frac{U_{c,t}}{U_c} \frac{1}{1 + \xi_{t+1}} \right]$$

which will depend on the conditional correlations between the marginal rate of substitution and the  $1 + \xi_{t+1}$  variable. As we mentioned in section 4.3, in the minimum variance solution:  $1 + \xi_{t+1} = U_{c,t+1}/E_t U_{c,t+1}$ , which should be negatively correlated with the marginal

<sup>24</sup> Of course, one possibility might be to believe that the policymaker just cares about consumers' welfare. In that case, the previous results can be seen as a first approximation to characterizing an optimal policy, as well as establishing some comparisons between any two alternative monetary policy implementation schemes.

rate of substitution, so long as current consumption is a good proxy for expectations of future consumption. A nominal interest rate policy would then tend to be best; under the maximum conditional variance,  $1 + \xi_{t+1} = 1$ , hence uncorrelated with the marginal rate of substitution, and a money control policy might be better.

Even though these arguments are made for the rate of inflation rather than the price level, they suggest that there might be some advantage for interest rate policies, specially in terms of conditional variances. In the next section we actually compute numerical unconditional and conditional second order moments for prices to compare the two classes of policies.

## 6. STABILITY ANALYSIS

Not all the solutions to the equations that define the policy mechanism need to be equilibrium prices; for that, they must be compatible with stable trajectories for the rest of the variables in the economy. The situation is not different from standard versions of the neoclassical growth model, in which it is well known how a stable manifold can be characterized, so that the economy will converge towards the steady state just if it is on that path, diverging to either nonfeasible or suboptimal allocations in any other case. If the economy is not on the stable manifold at a given point in time, a large enough value in one of the structural shocks (technology, government expenditures, monetary policy) could make it deviate ever increasingly from the steady state equilibrium.

We are interested on the properties of the economy as it experiences stochastic, but stable, fluctuations around its deterministic steady state, so we want to avoid precisely the situation we just described. To try to characterize the stable manifold, we need to linearly approximate the model. It can then be represented as a dynamic linear system, whose stability properties can be analyzed by well known methods.

The fundamental dynamic structure of the model is made up of: a) the equality between marginal rates of substitution and transformation (6), b) the money demand relation (8), c) the feasibility constraint (5), d) the Government budget constraint (4), e) the process assumed for Government expenditures (6) and f) the equations defining the monetary policy in effect (13) or (25), in each case.

The Government budget constraint is the only equation involving bond holdings, which is the variable to be solved for, once we know the values of the rest of the vector. Being a nontrivial dynamic equation in per capita bond holdings, it is by no means

true that stable paths for all variables except  $b_t$  guarantee a stable path for  $b_t$  as well. This is the problem we discussed in sections 3 and 4, mentioning the general need for a link between lump-sum taxes and the real value of the outstanding debt. We now discuss whether some more additional restrictions are needed to guarantee stability under each alternative monetary policy.

### 6.1 Stability under a money supply control policy

Under such a policy, we add to the mentioned set of equations the dynamics of real balances, given by (14). The linear approximation<sup>25</sup> around steady state values is a system of first order difference equations:

$$\Gamma_0 x_t = \Gamma_1 x_{t-1} + \psi \quad t \geq 1$$

in the vector  $x_t = (\tilde{c}_t, \tilde{K}_{t+1}, \tilde{m}_{t+1}, 1 + \tilde{\pi}_t, \tilde{G}_t, \tilde{b}_{t+1})$ , where tildas denote deviations with respect to the deterministic steady state. In this system,  $\psi$  is a vector of constants.

We now need to obtain the left eigenvalues and eigenvectors of this system<sup>26</sup>. The stable manifold is the subspace of  $R^7$  orthogonal to the  $q$  unstable eigenvectors of the system above, i.e., those corresponding to the eigenvalues whose modulus exceeds one. The stable manifold is therefore characterized by the equations:  $z_i' x_t = 0$ , where  $z_i$ ,  $i = 1, \dots, q$  denote the unstable left eigenvectors of the product matrix:  $(\Gamma_0)^{-1} \Gamma_1$ . These stability conditions imply that the shadow prices are time invariant linear functions of the state variables.

In our case, all the six eigenvalues turned out to be real and positive, two of them being greater than one. The corresponding eigenvectors, applied to  $x_t$  produced the constraints:

<sup>25</sup> In their deterministic versions

<sup>26</sup> This is Vaughan's eigenvector method to solve the algebraic matrix Riccati equation, discussed in Hansen and Sargent(1991). It is also the argument used to characterize the stable trajectories in Sims(1991).

$$\begin{aligned} m_t &= \tau_1 (C_t - C^*) + m^* \\ K_{t+1} &= \tau_2 (C_t - C^*) + K^* \end{aligned} \quad (39)$$

which characterize the hyperplane orthogonal to the two unstable directions. The stationary equilibrium must be in this hyperplane at each point in time.

Characterizing the stable manifold can also shed some light on the conceptual properties of the model. First, although the coefficients  $\tau_1$  and  $\tau_2$  in these stability conditions will generally change with the chosen values of the structural parameters, it is interesting to note that in our specific problem, the value of  $\tau_1$  was always equal to:  $(g-\beta)/(\gamma\beta)$ , the ratio of steady state real balances to consumption. With our logarithmic utility function, that stability condition is just the constant velocity condition that we discussed in section 3. It is also, of course, the same relation between real balances and the marginal utility of consumption that we reached after an analytical stability argument in (19) and (21).

The second stability condition imposes a constant value for the relative size of the consumption and capital stock fluctuations around their deterministic steady states. In our case, that  $\tau_2$  ratio turns out to depend just on the steady state values of both variables, staying constant for different policy parameters.

We should bear in mind that there is a third stability condition, linking tax collections to the value of the stock of real debt, as discussed above. That relation is already incorporated in the system of equilibrium conditions, which is why taxes  $T_t$  do not explicitly appear in the vector  $x_t$ .

### 6.2 Stability under a nominal interest rate control policy

Under a monetary policy of fixing the nominal return on Government assets, we substitute (25) for (14) in the system above, obtaining again two stability conditions: one is the same relationship between consumption and capital stock fluctuations as before, the other being of the form:

$$\bar{b}_t + \tau_3 (\bar{m}_t + \bar{G}_t) = \tau_4 \bar{C}_t \quad (40)$$

which is a pricing rule, linking the stocks of physical and financial assets to consumption

and prices<sup>27</sup>. The coefficients on real balances and Government expenditures fluctuations turned out to always be equal to each other, which shows up in (40).

Once again, tax collection must be related to the real value of the outstanding debt in a given manner, in order to guarantee stability.

## 7. MODEL CALIBRATION.

In this last section we present some quantitative evidence on the degree of price volatility that is induced on the price level by alternative ways of conducting monetary policy. The two policy mechanisms we have considered differ on whether the monetary authority decides to focus on controlling the money supply and let interest rates adjust so that the money market stays in equilibrium or, alternatively, control the nominal rate of interest and let the money stock adjust.

### 7.1 Solution method.

The main difficulty in deriving the equilibrium properties in our model is that it is characterized by a set of nonlinear, stochastic difference equations which do not have a closed form solution. Hence, one needs to use a method to obtain numerical solutions from such a system of equilibrium conditions. To illustrate the way to get equilibrium realizations from the model, let us substitute the two conditional expectations in (7) and (8) by their realized values and the corresponding expectation errors:

$$U_{c_t} = \beta [1 + \alpha \theta_{t+1} K_{t+1}^{\alpha-1}] U_{c_{t+1}} + u_{1,t+1} \quad (41)$$

and:

$$\frac{1}{1+i_t} = \beta \frac{U_{c_{t+1}}}{U_{c_t}} \frac{1}{1+\pi_t} + u_{2,t+1} \quad (42)$$

<sup>27</sup> Notice that, with a logarithmic utility of consumption, this can be seen as a relation between state variables like  $b_t$ ,  $m_t$ ,  $G_t$ , and the shadow price of the utility maximization problem, which is equal to the marginal utility of consumption.

which allow us to write the demand for money equation (9) as:

$$\gamma \beta \frac{1}{U_c} \frac{1}{g} \frac{1}{m_t} = 1 - \frac{U_{c,t}}{U_c} \frac{1}{1+\pi_t} + u_{2,t+1} \quad (43)$$

in the case of a money growth control policy, or:

$$\gamma \beta \frac{1}{m_{t+1}} \frac{1}{1+\pi_t} U_c = \frac{\delta_0}{1+\delta_0} + u_{2,t+1} \quad (44)$$

under a policy of fixing the nominal interest rate. However, we have already seen how, so long as we keep the assumption of a perfect control of either the rate of money growth or the nominal rate of interest, the right hand side of the money demand equation is perfectly predictable under both policies, so that the expectation error  $u_{2,t+1}$  is identically zero, for all  $t$ .

To solve the model forwards, we can start by drawing sample realizations for the two exogenous stochastic processes: the technology shock  $\theta_t$ , and the shock on Government expenditures  $\epsilon_t^G$ , and use the equilibrium conditions, together with the stability restrictions we exhibited in the previous section, to obtain realizations for the rest of the variables in this economy. In our simulations, we assumed the technological shock  $\theta_t$  to either be purely transitory, i.e., white noise, or quite persistent, following an AR(1) structure with coefficient .90. In both cases it was supposed to have mean 1, and variance denoted by  $\sigma_\theta^2$ . A realization of the pair  $(\theta_t, G_t)$ , together with the stability condition that gives  $K_{t+1} = K(C_t)$ , allows us to obtain the consumption series from the budget constraint (5). After that, we easily obtain the series for the capital stock and output. It is clear from this description how the realized values of these variables are independent of the chosen monetary policy. That is a consequence of the strong neutrality embedded into the model, and would go away in more general specifications.

Under a money growth control policy, the money supply mechanism gives us  $M_t$ , and we get prices from the constant velocity condition. Plugging the chosen values for the fiscal parameters  $T_M$  and  $a_M$  in the Government budget constraint, we obtain nominal bonds  $B_{t+1}$  as a function of  $G_t$ ,  $M_{t+1}$ ,  $P_t$ , and the constant value of the nominal rate of interest. All we need are initial nominal stocks of both, money and bonds.

Under a policy of controlling the nominal interest rate, we have the same determination of the real sector variables (Government expenditures, consumption and

capital) as under the money growth control policy. They will be functions of the two shocks:  $\theta_t$  and  $\epsilon_t^G$ . After estimating an AR representation for the ratio of Government deficit to consumption, we used the procedure described in section 4 to obtain, in this order:  $B_{t+1}/M_{t+1}$ ,  $M_{t+1}$ ,  $B_{t+1}$  and  $P_t$ . At each point in time we have as initial conditions:  $P_{t-1}$ ,  $B_t$ ,  $M_t$  as well as  $C_{t-1}$  and  $K_t$ ; the system of equilibrium conditions will then give us values of  $m_t$  and  $\pi_{t-1}$ . After that, we can generate a realization for the expectation error  $u_{1t}$  in (41), as well as for the  $\xi_t$  variable in (28). It is of interest to check that neither of them are serially correlated.

By contrast, the backwards solving method described in Sims (1990) and used in Novales (1990) and Sims(1989), among others<sup>28</sup>, consists on taking as a starting point a serially uncorrelated process of variance  $\sigma_1^2$  for the expectations error  $u_{1,t+1}$ , together with one of the structural shocks,  $\theta_t$  say, to obtain the value of  $\epsilon_t^G$ , as well as the decision and state variables for the period<sup>29</sup>. This is the procedure we used to obtain the results in the next section. However, there is clearly a one-to-one mapping between the two solution strategies: one should choose one or the other depending on how strongly he/she feels about the assumed stochastic structure for the starting processes in each case.

## 7.2 Choice of parameter values

We will just present numerical results for the case  $\tau = 1$ , i.e., a logarithmic specification for the utility of the consumption good. The parameter  $\alpha$  in the production function was chosen equal to 1/3, and the value of  $\gamma$  in the utility function was 1. The starting money supply was always  $10^6$  units. Steady state Government expenditures were taken to be 20% of output, so that consumption takes the remaining 80%, in line with actual data. Although we have already seen that consumers prefer deflation to inflation or

<sup>28</sup> This procedure exploits the fact that for most Pareto problems like the one in this paper, it is easier to postulate a given stochastic representation for some of the decision variables, and solve for the implied stochastic structures for the state variables, rather than the other way around.

<sup>29</sup> Another alternative implementation strategy would start from a given process for one of the decision rules, an AR(2) for consumption estimated from actual data, say, together with either  $\theta_t$  or  $\epsilon_t^G$ , to derive the other shock and the expectation error  $u_{1t}$ . It is quite clear that there would then be a one-to-one mapping between consumption and either the technology shock or the Government expenditures innovations, so that the two procedures are equivalent, even though it is not obvious what the mapping would be between the space of ARIMA representations for the two processes.

constant prices, we will present results corresponding to the three situations. We consider annual inflation rates of 6%, 4%, 2% and 0%, as well as a deflation of -2%. The value of  $\beta$  was chosen 0.99, since we consider that we are generating quarterly data, and an annual real interest rate of about 4% seems reasonable. These choices imply steady state values of 5.74 for per capita output, 4.60 for consumption, 189.57 for the stock of capital, and 1.15 for Government expenditures, irrespective of the values chosen for the monetary and fiscal parameters.

The monetary policy parameters  $g$  and the nominal rate  $\delta_0$  were chosen so that the two policies would generate the same inflation or deflation rate. The fiscal policy parameters  $T_1$ ,  $T_M$ ,  $a_1$ ,  $a_M$  were chosen so that total tax collections under both monetary policies be the same:  $T_M + a_M b_M^* = T_1 + a_1 b_1^*$ . That condition also guarantees that the steady state holdings of real bonds will be the same under both policies. In all the results we present, we used constant lump-sum taxes under a nominal rate policy ( $a_1=0$ ), and different values for  $T_1$  as the ratio of total taxes to Government expenditures:  $T_1/G^* = 1.05, 1.20, 2.0, 5.0$ . Equations (30) and (34) show that total tax collections must exceed Government expenditures for the ratio  $B_{t+1}/M_{t+1}$  to be stationary. Hence, the situations we considered oscillate between a minimum in which total taxes collected exceed Government expenditures by just 5%, to a case in which taxes were 5 times as large as public expenditures.

Table 1 shows the steady state values that arise for prices and the stocks of real money and bonds under the alternative policy rules considered. It also shows total tax collections in column 7, as well as the values of the coefficients in the stability constraints:  $\tau_1$ ,  $\tau_3$  and  $\tau_4$  in columns 8 to 10; the value of  $\tau_2$ , being constant, it is not shown in the table. As one would expect from solving the steady state system of equations in section 2, the table shows that real balances fall and real bond holdings go up when inflation increases: higher inflation raises the opportunity cost of holding real balances, and consumers substitute bonds for money. Steady state real balances are invariant to the size of the deficit, whereas real bond holdings increase when the Government deficit falls. The steady state relations in section 2 show that, contrary to real balances, real bonds are a source of Government expenses (since the nominal rate is always in excess of the inflation rate) so, for a given inflation tax, higher direct tax revenues can come together with higher real debt issuing, as well as the opposite.

Table 2 shows the steady state nominal interest rate values that correspond to each value of the rate of inflation, as well as the price level and the value of the lifetime utility function. They all are independent of the Government financing strategy.

We will alternatively examine two deficit measures: the real deficit, as

difference between Government expenditures and direct tax collections:  $G-T_1$ , which is actually negative (surplus) with our parameter values, as shown in Table 3 (column 3), and the more global deficit measure, which adds interest liabilities to the previous one (column 4). The latter one is, in steady state, equal to the inflation tax (column 5), so that the Government budget stays balanced. Columns 6 and 7 show the deficit measures as percentages of output.

The variances were chosen:  $\sigma_y = .005$ , and  $\sigma_c = .0005$ . With them, the coefficient of variation of output and consumption were 1.35 and 1.27 for all the simulations; this output variability is not far from what is obtained in actual filtered data while, as it is usually the case with frictionless models, consumption is more volatile than in actual data (see table 3 in Christiano(1991), for instance). These statistics do not play any relevant role in the qualitative results we present.

### 7.3 Empirical results

The equilibrium time series obtained following the procedures described above, were used to compute: 1) the sample mean and the coefficient of variation of prices, as well as those of real balances and real debt holdings under the two alternative policies, 2) autoregressive models that could be used to forecast prices, and hence obtain the one step ahead forecast error, and 3) the value of the time aggregate utility function.

In the cases when the inflation rate is not zero, the price series are clearly non stationary. The exponential trends, which are known by construction, were taken out of the price series before fitting the forecasting models.

The estimated price autoregressions (not shown in the paper) were very stable for different parameter vectors. The price series that emerges from a money supply control policy could always be represented by an AR(1) model with an estimated coefficient around .98. We needed a longer lag length for the prices series obtained under a nominal interest rate control policy.

The main results that were obtained in the simulations are summarized in Tables 4 and 5. When examining them, we should bear in mind that each panel going down refers to a smaller value of the Government deficit, corresponding to values of  $T_1/G^* = 1.05, 1.20, 2.0, 5.0$ . Going down inside each of those panels, each row corresponds to a lower inflation rate, from 6% to -2%.

- 1) the sample coefficient of variation of prices under a money growth control policy is independent of the rate of inflation and the size of the government surplus (column 3 in table 4). This observation suggests a sort of neutrality in second order moments: increases in money growth increase the price level, but also its standard deviation, and by the same factor.

According to this measure, a nominal rate policy produces a more stable price series the larger is the Government surplus (column 4). Would the monetary authority care about this sample statistic, nominal rate policies should be preferred to money growth rules, except for small Government surpluses.

- 2) the standard error of estimate in the univariate price autoregressions, taken as the size of the one step ahead prediction error, changes with different policy parameters (columns 5 and 6). The figures in brackets are the empirical standard deviations, computed over the set of simulations.

For a given inflation rate, the price forecast error under a nominal interest rate policy falls when fiscal policy becomes more strict (i.e., for a bigger Government surplus), which is consistent with the previous observation on the sample variance. The one step ahead forecast error seems to be bigger under a nominal rate policy only when the Government surplus is small, although it is then estimated with very low precision.

- 3) given a particular deficit size, price forecast errors are smaller for low than for high inflation under both policies, as should be expected. But once these estimated standard errors are normalized to become conditional coefficients of variation (columns 7 and 8), they turn out to be independent of the inflation rate.

Under a nominal rate policy, the conditional variation of prices falls again when fiscal policy gets tighter; prices under a money growth rule are more predictable than under a nominal rate policy for small Government surpluses, the reverse being true when the surplus gets large. The break-even point seems to be just above tax collections of 20% in excess of Government consumption, which correspond to a surplus (not including interest payments) of about 4% of output.

Therefore, the three sample criteria give a similar picture about the implied price volatility.

- 4) under an interest rate pegging policy, there is enough variability in prices which is in

consonance with the money supply, that real balances turned out to always be more volatile under the comparable money growth pegging policy. Under the interest rate policy, the coefficient of variation of real balances again declines for more strict fiscal policies (columns 9 and 10).

That means that if the monetary authority was to care about the volatility of real balances, then it would prefer a nominal rate policy to the money growth policy that produces the same rate of inflation, since it uniformly gives better results. It also suggests that if we would introduce the volatility of real balances with a negative weight in the utility of the representative agent, nominal interest rates might be preferable. However, we would then have a different model, whose implications would have to be derived.

- 5) bond holdings under a nominal rate policy also become more stable when the Government surplus increases (columns 11 and 12). They are more volatile under a money supply control policy for small surpluses, the result reversing for large ones. Again, a real surplus of about 4% seems to be the turning point for this ordering.
- 6) even though the price volatility properties, as well as those of real bond and money holdings, are different under the two policy schemes, they lead to an equal value of the time aggregate utility function, already shown in Table 2. Consumers' welfare declines with inflation, i.e., with large nominal rates of interest, but is independent from fiscal policy. Hence, so long as the representative private agent just cares about the level of real balances in a monotonic way, he/she is indifferent between the way monetary policy is implemented in order to achieve a given consumption standard.

The results we just mentioned are obtained maintaining constant the variance of the technology shock as well as that of the expectation error  $u_{1t}$ . When we estimated short autoregressions for the relative size of the Government deficit:  $(G_t - T_t)/C_t$ , which enters in the determination of the price series under interest rate policies, its forecast error was essentially constant for the different parameterizations, around .12. Hence, although the  $G_t$  volatility changes with the policy parameters, the ratio deficit/consumption, which is what matters to determine the equilibrium prices and allocations, displays fluctuations whose size is independent of the parameter values. So, our results are not based on just different relative volatilities for  $G_t$  across simulations.

On the contrary, a careful examination of the model provides us with an

interpretation for our results: under a money pegging policy, any increase in the demand for real balances coming from a high technology shock, must be accommodated by the price level. Under an interest rate policy, the higher demand can be in part accommodated by an increase in the money supply. The way to implement that increase is through open market operations, retiring (or adding) some debt from consumers. But real bonds are essentially the expected present value of future deficits so, for a given fiscal policy, if the stock of nominal bonds changes, prices must adjust, too. The relative importance of these operations is lower for large amounts of real bonds which, as we know from Table 1, correspond to higher surpluses, i.e., higher values of the ratio:  $T/G^*$ . Hence, the relative size of the price distortion induced by the initial increase in the demand for real balances will also be smaller for higher values of that ratio, so prices will become more stable for the more strict fiscal policies.

As a final comment on robustness, we experimented with different specification changes: The results obtained under a purely transitory technology shock (i.e.,  $\theta$ , being white noise), available upon request, are essentially identical to those presented here. Changes in the variances of the expectation errors led to corresponding proportional changes in the coefficients of variation shown in tables 4 and 5, so the qualitative results stay the same. Changing the sensitivity of taxes to debt under money growth policies,  $a_m$ , does not introduce any change in the results; making tax collections under an interest rate policy react to real debt, i.e.  $a$ , changes the volatility estimates, but still leads to the same qualitative results we have presented.

## 8. CONCLUSIONS

The mechanisms by which monetary policy is implemented have sometimes been blamed for producing an unnecessarily high price volatility. This paper has examined whether a monetary policy of controlling the money supply, versus one of controlling nominal interest rates, is more likely to produce undesired price fluctuations.

Although the inability of the private sector to forecast future price movements may lead to an inefficient resource allocation, it is far from clear which specific characteristics of price fluctuations are to be considered undesirable from the point of view of choosing a more preferred monetary policy. With that in mind, we have considered in this paper different loss functions for the monetary authority. The aim of the paper is then to give some insights into the mapping between the authority's loss functions and the more preferred monetary policy implementation mechanisms.

The main result derived from our money in the utility function model is that the effect on price volatility of the alternative monetary policies is not independent of the fiscal policy in effect: money supply control policies produce more stable prices when fiscal policy is somewhat loose, Government financing being almost balanced every period. As fiscal policy gets tougher and the Government runs bigger surpluses, nominal interest rate policies generate more price stability. With small differences, the result arises with independence of whether the monetary authority worries about full sample volatility statistics or about the uncertainty induced on just the future behavior of prices. If the monetary authority worries about the stability of real balances, then nominal rate policies seem to always be preferred.

On the other hand, the intuition behind a social distaste for price fluctuations is linked to the idea that price unpredictability may distort the decision making process in the private sector. But if that is the case, then the time aggregate utility function of the representative agent in the economy will take a lower value than under certainty. From that point of view, all the decision maker needs to worry about is private agents welfare. According to this criterium, both ways of implementing monetary policy are equivalent to each other.

Although this latter one is a way of analyzing the price volatility issue from a general equilibrium point of view, references to the undesirability of price volatility are still so widespread in the economic policy literature, that the discussion based on price statistics is needed. From that approach, our results suggest that a money supply control policy is

not always the best way to achieve price stability.

Some extensions of this analysis should complement this work: a money in the utility function specification imposes extensive monetary neutrality, which might condition the results. An alternative way to rationalize money demand like a transaction cost technology in which money is no longer neutral, might produce implications different from the ones we have presented. In addition, although it is a fairly standard assumption, the separability of the consumption good and real balances that we have incorporated in the utility function might be too stringent an assumption. A Government expenditures process with feedback from the real variables in the economy should be another interesting situation in which to discuss the optimal choice of monetary instrument.

Table 1: STEADY STATES AND STABILITY PARAMETERS

$\beta$	T/G*	aM	Prices	b*	m*	TOTAL TAXES	tau_1	tau_3	tau_4
6%	1.05	0.80	5344.0	269.6	184.4	1.21	40.1	0.9756	97.2
4%	1.05	0.80	4320.5	227.0	229.2	1.21	49.9	0.9803	97.6
2%	1.05	0.80	3272.0	154.3	304.1	1.21	66.2	0.9851	98.1
0%	1.05	0.80	2197.4	5.7	455.0	1.21	99.0	0.9900	98.6
-2%	1.05	0.80	1096.9	-453.9	916.9	1.21	199.5	0.9950	99.1
6%	1.20	0.80	5344.0	286.4	184.4	1.38	40.1	0.9757	98.8
4%	1.20	0.80	4320.5	243.9	229.2	1.38	49.9	0.9803	99.3
2%	1.20	0.80	3272.0	171.3	304.1	1.38	66.2	0.9851	99.8
0%	1.20	0.80	2197.4	22.7	455.0	1.38	99.0	0.9900	100.3
-2%	1.20	0.80	1096.9	-436.8	916.9	1.38	199.5	0.9950	100.8
6%	2.00	0.80	5344.0	376.1	184.4	2.30	40.1	0.9757	107.8
4%	2.00	0.80	4320.5	334.0	229.2	2.30	49.9	0.9803	108.3
2%	2.00	0.80	3272.0	261.8	304.1	2.30	66.2	0.9851	108.8
0%	2.00	0.80	2197.4	113.7	455.0	2.30	99.0	0.9900	109.4
-2%	2.00	0.80	1096.9	-345.3	916.9	2.30	199.5	0.9950	109.9
6%	5.00	0.80	5344.0	712.4	184.4	5.74	40.1	0.9757	141.4
4%	5.00	0.80	4320.5	671.9	229.2	5.74	49.9	0.9803	142.1
2%	5.00	0.80	3272.0	601.4	304.1	5.74	66.2	0.9851	142.8
0%	5.00	0.80	2197.4	455.0	455.0	5.74	99.0	0.9900	143.5
-2%	5.00	0.80	1096.9	-2.4	916.9	5.74	199.5	0.9950	144.2

Note:  $\beta$ ,  $m^*$  and  $b^*$  denote steady state inflation, real money and bond holdings.

T/G\* is the ratio of total taxes (column 7) to Government expenditures.

aM is the coefficient on real bonds in the fiscal rule under a money growth policy.

Table 2

INFLATION RATE	NOMINAL RATES		PRICES	WELFARE
	Quarterly	Annual		
6%	2.49	10.35	5344.0	514.5
4%	2.01	8.27	4320.5	535.6
2%	1.51	6.18	3272.0	563.4
0%	1.01	4.10	2197.4	603.2
-2%	0.50	2.02	1096.9	672.3

Table 3: GOVERNMENT DEFICIT

R	T/G*	DEFICIT DEFICIT+ INFLATION		DEFICIT + INTEREST as %PIB	
		(Surplus) INTEREST TAX	as %PIB		
6%	1.05	-0.06	2.71	-1.00	47.10
4%	1.05	-0.06	2.26	-1.00	39.31
2%	1.05	-0.06	1.51	-1.00	26.27
0%	1.05	-0.06	0.00	-1.00	0.00
-2%	1.05	-0.06	-4.62	-1.00	-80.41
6%	1.20	-0.23	2.71	-4.00	47.10
4%	1.20	-0.23	2.26	-4.00	39.31
2%	1.20	-0.23	1.51	-4.00	26.27
0%	1.20	-0.23	0.00	-4.00	0.00
-2%	1.20	-0.23	-4.62	-4.00	-80.41
6%	2.00	-1.15	2.71	-20.00	47.10
4%	2.00	-1.15	2.26	-20.00	39.31
2%	2.00	-1.15	1.51	-20.00	26.27
0%	2.00	-1.15	0.00	-20.00	0.00
-2%	2.00	-1.15	-4.62	-20.00	-80.41
6%	5.00	-4.60	2.71	-80.00	47.10
4%	5.00	-4.60	2.26	-80.00	39.31
2%	5.00	-4.60	1.51	-80.00	26.27
0%	5.00	-4.60	0.00	-80.00	0.00
-2%	5.00	-4.60	-4.62	-80.00	-80.41

Table 4: PRICE VOLATILITY STATISTICS

INFLATION RATE	T/G*	PRICES				REAL BALANCES		REAL BONDS			
		Coefficients of Variation		One step ahead Forecast Errors	Conditional coeffs. of variation		Sample coefficients of variation		Sample coefficients of variation		
		M	i		M	i	M	i	M	i	
6%	1.05	1.18	1.47	12.4 (.46)	50.15 (34.77)	0.23	0.94	1.37	1.16	0.29	1.16
4%	1.05	1.18	1.47	9.97 (.37)	40.24 (27.91)	0.23	0.93	1.37	1.15	0.31	1.15
2%	1.05	1.17	1.46	7.52 (.28)	30.56 (21.18)	0.23	0.93	1.36	1.16	0.41	1.16
0%	1.05	1.17	1.47	5.02 (.18)	20.47 (14.20)	0.23	0.93	1.35	1.15	0.48	1.16
-2%	1.05	1.16	1.47	2.49 (.09)	10.21 (7.08)	0.23	0.93	1.35	1.15	0.29	1.15
6%	1.20	1.18	0.79	12.4 (.46)	14.41 (8.43)	0.23	0.27	1.36	0.29	0.28	0.29
4%	1.20	1.18	0.79	9.97 (.37)	11.05 (3.20)	0.23	0.26	1.36	0.29	0.30	0.29
2%	1.20	1.17	0.79	7.52 (.28)	8.81 (5.16)	0.23	0.27	1.36	0.29	0.38	0.29
0%	1.20	1.17	0.80	5.02 (.18)	5.90 (3.48)	0.23	0.27	1.35	0.29	0.34	0.32
-2%	1.20	1.16	0.80	2.49 (.09)	2.94 (1.73)	0.23	0.27	1.35	0.29	0.29	0.29

Note: The symbols 'M' and 'i' denote the alternative monetary policies.

Table 5: PRICE VOLATILITY STATISTICS

INFLATION RATE	T/G*	PRICES				REAL BALANCES		REAL BONDS			
		Coefficients of Variation		One step ahead Forecast Errors	Conditional coeffs. of variation		Sample coefficients of variation		Sample coefficients of variation		
		M	i		M	i	M	i	M	i	
6%	2.0	1.18	0.55	12.4 (.46)	5.41 (1.97)	0.23	0.10	1.37	0.10	0.26	0.10
4%	2.0	1.18	0.55	9.97 (.37)	4.37 (1.59)	0.23	0.10	1.37	0.10	0.27	0.11
2%	2.0	1.17	0.55	7.52 (.28)	3.31 (1.21)	0.23	0.10	1.36	0.10	0.29	0.11
0%	2.0	1.17	0.56	5.02 (.18)	2.23 (.81)	0.23	0.10	1.35	0.10	0.32	0.11
-2%	2.0	1.16	0.56	2.49 (.09)	1.11 (.41)	0.23	0.10	1.35	0.10	0.32	0.11
6%	5.0	1.18	0.36	12.4 (.46)	3.19 (.55)	0.23	0.06	1.36	0.06	0.24	0.06
4%	5.0	1.18	0.36	9.97 (.37)	2.58 (.44)	0.23	0.06	1.36	0.06	0.24	0.06
2%	5.0	1.17	0.36	7.52 (.28)	1.95 (.34)	0.23	0.06	1.36	0.06	0.24	0.06
0%	5.0	1.17	0.37	5.02 (.19)	1.31 (.23)	0.23	0.06	1.35	0.06	0.24	0.06
-2%	5.0	1.16	0.37	2.49 (.09)	0.65 (.11)	0.23	0.06	1.35	0.06	0.25	0.08

Note: The symbols 'M' and 'i' denote the alternative monetary policies.

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Appendix 1: Convergence of the  $W_t$ -sequence under money growth.

Case 1:  $g > \beta$

In the case when the rate of money growth  $g$  exceeds the rate of time preference  $\beta$ , we can write:

$$\begin{aligned} E_t W_{t+k} &= \frac{1}{\beta^k} W_t - \gamma \frac{1}{M_0 g^{t+1}} \left[ \frac{1}{\beta^{k-1}} + \frac{1}{\beta^{k-2}g} + \frac{1}{\beta^{k-3}g^2} + \dots + \frac{1}{g^{k-1}} \right] = \\ &= \frac{1}{\beta^k} W_t - \gamma \frac{1}{M_0 g^{t+1}} \frac{1}{g^{k-1}} \left[ \frac{g^{k-1}}{\beta^{k-1}} + \frac{g^{k-2}}{\beta^{k-2}} + \dots + 1 \right] \end{aligned}$$

and the square bracket is, for each finite value of  $k$ , the sum of a sequence that starts at  $(g/\beta)^{k-1}$ , and decreases at a rate  $\beta/g$ , which is less than 1. Therefore:

$$\begin{aligned} E_t W_{t+k} &= \frac{1}{\beta^k} W_t - \gamma \frac{1}{M_0 g^{t+1}} \frac{1}{g^{k-1}} \frac{g^{k-1}}{\beta^{k-1}} \frac{1 - (\beta/g)^k}{1 - \beta/g} = \\ &= \frac{1}{\beta^k} \left[ W_t - \gamma \frac{1}{M_0 g^{t+1}} \beta \frac{1 - (\beta/g)^k}{1 - \beta/g} \right] \end{aligned}$$

As  $k$  goes to infinity, the first factor goes to plus infinity, whereas the square bracket goes to zero. Looking at this expression as the ratio of the square bracket to the  $k$ -th power of  $\beta$  and using the l'Hopital rule, it is not hard to see that  $E_t W_{t+k}$  converges to plus infinity when  $\beta < g < 1$ , and goes to zero when  $\beta < 1 < g$ . The first one is the case of a contractionary policy, under which prices are expected to go to zero. In the second case, the money supply actually expands over time, and prices are expected to go to plus infinity.

Case 2:  $g = \beta$

When the rate of money growth is equal to the rate of time preference, we have:

$$E_t W_{t+k} = \frac{1}{\beta^k} W_t - \gamma \frac{1}{M_0 g^{t+1}} \frac{k}{\beta^{k-1}} = \frac{1}{\beta^k} \left[ W_t - \gamma \frac{\beta k}{M_0 g^{t+1}} \right]$$

in which the first factor goes to infinity, whereas the square bracket goes to minus infinity, and so does the product. Consequently, there is no monetary equilibrium.

Case 3:  $g < \beta$

For money growth rates above  $\beta$ , we have:

$$E_t W_{t+k} = \frac{1}{\beta^k} W_t - \gamma \frac{1}{M_0 g^{t+1}} \frac{1}{g^{k-1}} \left[ \frac{g^{k-1}}{\beta^{k-1}} + \frac{g^{k-2}}{\beta^{k-2}} + \dots + 1 \right]$$

where the square bracket is a decreasing sequence, at a rate  $g/\beta$  lower than 1, starting from an initial value of 1, so that:

$$\begin{aligned} E_t W_{t+k} &= \frac{1}{\beta^k} \left[ W_t - \gamma \frac{1}{M_0 g^{t+1}} g \frac{\beta^k}{g^k} \frac{1 - g^k/\beta^k}{1 - g/\beta} \right] = \\ &= \frac{1}{\beta^k} \left[ \left[ W_t + \gamma \frac{1}{M_0 g^t} \frac{1}{1 - g/\beta} \right] - \gamma \frac{1}{M_0 g^t} \frac{1}{1 - g/\beta} \left( \frac{\beta}{g} \right)^k \right] \end{aligned}$$

where the first factor goes to infinity with  $k$ . Inside the square bracket, the first term remains bounded when  $k$  increases, whereas the second one explodes. The square bracket hence goes to minus infinity, and so does the product. Again, there is no monetary equilibrium.