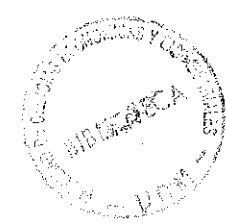


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Interest Rates in the U.S. Economy

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The Fed Controls Only One of the Two Interest Rates in the U.S. Economy

Ricardo Martín Manjón *
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ABSTRACT (keywords underlined)

Detailed analyses of a wide variety of monthly U.S. interest rates series for 1985-1995 are summarized. Short term rates (of term two years or less) are seen to operate in bivariate $CI(1,1)$ cointegration relationships with the Fed's Federal Funds Rate Target (\bar{R}^0), though rates of longer term than two years do not. The latter operate in trivariate $CI(1,1)$ cointegration relationships with \bar{R}^0 and a single longer term rate. Thus there are two and only two nonstationary common factors in the set of interest, one identified with both short term rates and monetary policy, the other not. (JEL C22, C32, E43, E52).

RESUMEN (palabras clave subrayadas)

Se presenta el resumen de un conjunto de análisis detallados de una amplia variedad de tipos de interés de Estados Unidos en el período 1985-1995. Observamos que cada tipo de interés a corto plazo (con plazo de dos años o menos) opera en una relación bivalente de cointegración $CI(1,1)$ con el tipo de interés objetivo que establece el Fed para el Federal Funds Rate (\bar{R}^0); esto no ocurre para los tipos a más largo plazo. Estos últimos operan en relaciones trivariantes de cointegración $CI(1,1)$ con \bar{R}^0 y cualquier otro tipo a plazo superior a dos años. Por tanto, solamente uno de los dos factores comunes no estacionarios de este conjunto de tipos de interés, el asociado con los tipos a corto plazo, puede ser identificado con la política monetaria.

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1. Introduction

This paper presents the results of empirical time series analyses, of monthly data on a wide variety of U.S. interest rates in 1985-1995, designed to answer the following questions of central relevance to macroeconomics and financial economics. Does monetary policy control all interest rates at all maturities regardless of the nature of the issuer of the interest-bearing asset? If not, what characterizes those interest rates that monetary policy does control and does monetary policy at least influence the remaining interest rates?

The concept of "the level of interest rates", sometimes stated as "the level of *the* interest rate", is widely employed in many branches of economic thought. However, the use of this concept implies that all interest rates are driven by a single Nonstationary Common Factor (NCF), that is, all pairs of interest rates operate in $CI(1,1)$ cointegration relationships. See Engle and Granger (1987) for the definition of $CI(1,1)$. We will say that a series is $I(1)$ when its first difference has a stationary, invertible representation. If n $I(1)$ time series satisfy $r < n$ $CI(1,1)$ relationships, then we say that they are driven by $n-r$ NCF's. Thus, one of the issues we address can be posed as: Is there only one NCF in interest rates? We find that the answer is: no, there are two NCF's.

However, even if the hypothesis of only one NCF were valid, i.e. even if it were empirically legitimate to speak of "the level of interest rates", the issue would still remain of whether or not this NCF is controlled by monetary policy. If two NCF's are found, then the question of whether or not one of them is controlled by monetary policy is still open. We find that one of the two NCF's is plausibly associated with monetary policy, but the other is not.

We find that all market interest rates of maturity two years or less, which we will call "short term interest rates" in the sequel, operate in bivariate $CI(1,1)$ relationships with the Federal Funds Rate Target, \bar{R}^0 in the sequel, which is set exactly by the Fed. However, \bar{R}^0 only influences longer term rates, a full accounting for the trends of which requiring the second NCF.

The highly relevant question as to the economic meaning of the second NCF is not resolved in this paper, but is the subject of ongoing research, though it is clear that it is not controlled by monetary policy and it can be represented by any market interest rate of maturity above two years. In this paper we represent it by the market yield on 30-year Treasury bonds, R^{30} in the sequel.

1.1 Representing Monetary Policy

Monetary policy is represented in this research by the midpoint of the Fed's target band, announced before the fact throughout the sample analyzed, for the Federal Funds Rate, the overnight interbank loan rate. The effective market Federal Funds Rate is denoted by R^0 in the following.

An important justification for the use of \bar{R}^0 as the main indicator of monetary policy in this study is that there is ample evidence that the Fed actually achieves this target in the sense that the error $R^0 - \bar{R}^0$ is stationary. Rudebusch (1995) offers such evidence for daily data in 1974-1979 and 1984-1992 and, in Section 4 below, we offer monthly results in 1985-1995 confirming his findings. Presumably the Fed employs open market operations to implement its target.

The reason that January 1985 is taken as the initial sample date for this research is that, from this date on, the Fed announces its target band before the fact and this band is fairly narrow. In the 1979-1982 period of so-called New Operating Procedures (NOP), when the Fed claimed to be targeting a monetary aggregate rather than \bar{R}^0 , an alternative formulation might well be needed. And in many non-NOP periods before 1985 the Federal Funds Rate Target was either not public or was presented as a band with such wide limits as to suggest its possible irrelevance to monetary policy.

The Fed discount rate and the reserves requirements it sets for depository institutions are also tools of monetary policy, of course, but both of these are changed so infrequently in this sample that they must be treated as deterministic. The discount rate was changed only 31 times in these 132 months. An analysis of the empirical relationships between \bar{R}^0 and the discount rate and reserves

requirements reveals that, though these relationships are quantitatively outstanding, indicating a plausibly high degree of coherence in Fed behavior, the trend properties of \bar{R}^0 are by no means explained by these other instruments of monetary policy.

In the empirical work reported, considerable care is exercised in looking for possible effects of the discount rate and reserves requirements on each of the interest rates studied. Such effects are, however, largely irrelevant, once the effects of \bar{R}^0 are taken into account. The effects of these other instruments are occasionally visible in residual outliers.

There are alternative ways of representing monetary policy to be found in the existing literature. One frequently cited recent paper with such an alternative is Bernanke and Blinder (1992), who use R^0 and $R^{10} - R^0$, where R^{10} refers to the 10-year T-bond yield, as alternative indicators of monetary policy. Our results indicate this to be erroneous. R^0 is in a $CI(1,1)$ relationship with \bar{R}^0 and hence can be somewhat justified, though \bar{R}^0 is preferable for samples in which it is available. But our findings indicate that $R^{10} - R^0$ does not have the same NCF and hence should not be taken as an indicator of monetary policy.

The bivariate analyses between \bar{R}^0 and each market yield, reported in Section 4, and the trivariate analyses between \bar{R}^0 , R^{30} and each other market yield of term above two years, reported in Section 6, place \bar{R}^0 at the center of a conceptual model (CM). A full exactly identified CM would, for each variable, present a single equation with this variable as dependent, the other variables as independent and with stochastic error processes independent across equations. The CM for this research states that the (average) Federal Funds Rate Target set by the Fed for a month receives no contemporary effects of other variables, that is, the target is varied as a function of strictly lagged variables, not contemporary values. This is enough in the bivariate case to exactly identify the CM with the empirical model. In the trivariate case, this hypothesis is insufficient for full identification, but it does enough for our purposes. The hypothesis is clearly useful and appears plausible enough as an approximation; research to support it would have to be based on reducing the length of the sample

interval from one month to some value more in accord with the Fed decision interval, a task beyond the limits of the present research. Results on cointegration, which are central to this paper, do not depend at all on this identification hypothesis.

1.2. Time Series Analysis Practices

Though this paper is motivated by a desire for better economic understanding, the research itself involves extensive applications of many Time Series Analysis (TSA) practices, some of which are not commonplace in the existing applied time series econometrics literature. In fact, the Current Consensus-Science Paradigm (CCSP) for econometric TSA is, in our view, extremely naive, inexpensive but ineffective. Specific differential aspects of TSA practices are described in conjunction with reports of each kind of results in Sections 3-6. Some general aspects are treated here.

We take for granted basic results in discrete time domain analysis as synthesized by Box, Jenkins and Reinsel (1994) and extended vastly since the initial appearance of this book in 1970 and the initial formulations of intervention analysis, Box and Tiao (1975), and Vector Autoregressive Moving Average (VARMA) analysis, in Jenkins and Alavi (1981) and Tiao and Box (1981).

The TSA practices followed here systematically employ ARMA forms rather than limiting them to pure AR, the CCSP practice. We allow MA forms because they often make for more economy in parameterization. A particular aspect of this arises in the treatment of differencing, where the practice we follow is to difference sufficiently to be sure of thus achieving stationarity in the differenced series and then detect overdifferencing by the use of specific MA(1) factors that signal overdifferencing when they turn out to be noninvertible under efficient estimation of a statistically adequate model with as few other parameters as possible.

Though the TSA practices employed here recognize the need for a conceptual model based on economic theory, initially wide enough to restrict no empirically detectable linear correlation

coefficient, but also suggestive of testable restrictions to be considered in the model-building process, we use data-based specification, diagnostic and reformulation procedures in a consciously iterative model-building approach along lines first suggested by Box and Jenkins. This is in marked contrast with the CCSP for econometric time series analysis, which tries to deal with specification through the estimation of overparameterized models, with redundant parameters that are often not even pruned out at the end of the analysis, and which scarcely pays any attention to data-based diagnostics, save for lip-service encountered in the extensive application of formal diagnostic test statistics without looking at, or at least without presenting, either graphs of data, residuals or sample correlograms. The present paper offers such information for the illustrative cases treated and an (unpublished) Appendix B is available from the authors on request, containing the 1-page univariate specification and diagnostic tools for each of the interest rates studied and diagnostic tools for each bivariate and trivariate model presented.

In the TSA approach followed here, univariate stochastic and intervention analyses are performed on each variable before consideration of relationships between variables, this for many reasons, including that: (1) this allows for systematic initial data screening, (2) establishes an initial knowledge of structures present in the data and (3) sets up an initial criterion for goodness of representation with which all more complex models can be compared. The CCSP for econometric time series analysis largely ignores univariate analysis, which seems to suffer from a kind of canonized undervaluation. Authors often write as if univariate analysis were too elementary for them to need but offer no alternative approach to the three objectives mentioned above.

One of the areas in which, compared with the CCSP, our procedures are very different and much more costly, though also much more effective, is in the treatment of extreme values and other potentially influential incidents in data. We take pains to evaluate the influence of such incidents on parameter estimates and other inferences, this by means of efficiently estimating alternative models with and without the corresponding intervention parameters, taken one at a time, in pairs, in triples, etc., this in an effort to avoid masking. This is, of course, applied only after a search for extraneous

information that can explain the presence of each such incident, because all intervention terms justified by such information are left in the model. We cannot report all of the models estimated for evaluating influence, because space does not permit it. The reader should, however, be aware that the only intervention terms reported are those justified by extraneous information or explicitly found to be influential. Substantial outliers may well arise in residuals, but the reader should note that all such incidents are of unknown origin and have been explicitly parameterized and checked for influence and have been found not to be influential in the model reported.

Influential extreme values are not, in fact, a major problem in the sample that we analyze here, though they very definitely are important in most of the samples treated in the existing literature, e.g. those including the NOP period, though authors do not appear to have made any attempt at all to detect or to evaluate their influences on findings.

All estimated models reported in this paper are estimated under the Exact Maximum Likelihood (EML) criterion and implementation described by Mauricio (1995, 1996, 1997) with a program written by him which allows the analyst to check and impose a wide variety of parameter restrictions, both within and across the equations of the VARMA form. Unit MA parameters are legitimate under this criterion and implementation and this allows them to signal overdifferencing. We also occasionally employ the Generalized Likelihood Ratio test for unit MA(1) parameters of Davis, Chen and Dunsmuir (1995), to check cases in which the MA(1) parameter estimates are near but not literally equal to one.

The methods we employ for detecting cointegration are based on EML estimation of potentially overdifferenced models with potentially noninvertible MA(1) parameters. When cointegration is reliably detected by these methods, a reformulation of the model without overdifferencing or noninvertible MA parameters is estimated by EML. It can easily be shown that the resulting models are equivalent to the Phillips (1991) Triangular System Error Correction Model (TSECM), not to be confused with the Engle and Granger (1987) ECM. Phillips (1991) shows that

maximum likelihood estimates of the cointegration coefficients in this TSECM form have standard statistical properties, a result that thus carries over to the EML estimates in our model form.

Our procedures are equivalent to those proposed by Phillips (1991) and hence comply with his suggestions (p. 295) that "If unit roots are known to be present, then our results argue that they should be directly incorporated in model specification.", but are not subject to his criticisms (p. 295) of the single-equation approaches currently popular in the applied econometrics literature for dealing with CI, and are not subject to his critique (p. 302) that "... the use of unrestricted VAR's for inferential purposes about the cointegration subspace suffers drawbacks relative to system ECM estimation." However, he avoids (p. 297) "...the construction of the likelihood function for general ARMA systems", which Mauricio (1995, 1996, 1997) covers and we use, and he wants an approach (p. 299) that "...avoids the complications of explicit time series modelling", which we consciously undertake.

1.3 Recent Related Econometric Studies

There is a vast literature relating to one or more of the parts of this research, but apparently none that puts the parts together as we do here. Table 1 summarizes the characteristics of the main econometric studies of U.S. interest rates published recently.

Rudebusch (1995) is relevant for this paper, because it helps justify the use of \bar{R}^0 to represent monetary policy. It does not, however, treat any other interest rates than the Federal Funds Rate and the Fed target for it.

The effects of changes in \bar{R}^0 on changes in a number of T-bill and T-bond rates over a wide range of maturities for 1974-1979 are studied in Cook and Hahn (1989) and for 1974-1979 and 1987-1995 are studied by Roley and Sellon (1996) by similar methods. In both papers significant effects are found for all interest rates studied and these effects decline with term. The TSA methods employed in these two papers do not allow for cointegration testing. The 1974-1979 sample period is

also for a much earlier time than that studied here. Though these factors reduce their relevance for the present work, these are two of the few papers that consider the relationships between the Fed's target for the Federal Funds Rate and other interest rates.

Garfinkel and Thornton (1995) study R^0 instead of \bar{R}^0 and look for CI with other very short term rates, finding one NCF.

It is remarkable that a substantial number of recent papers study the question of how many NCF's are present in U.S. interest rates, though none of them explicitly relates one NCF with Fed activities to control a short term interest rate. Hall, Anderson and Granger (1992) is a frequently cited study of this kind. Though it only includes T-bills, i.e. Treasury issues of maturity one year or less, its coverage of these is rather exhaustive. Part of the authors' results consider a post-NOP sample and find only one NCF. This result is consistent with one of our results, though the authors do not interpret this NCF as reflecting monetary policy and their overall coverage of U.S. interest rates is much narrower than ours.

Zhang (1993) studies a wide spectrum of maturities, 19 in total, but he limits his interest rate series to Treasury issues and he treats the sample of monthly data for 1964-1986 with total disregard to the possible structural differences under the pre-NOP, NOP and post-NOP monetary policy regimes. His results do suggest, however, that our findings for a later sample may well characterize earlier periods as well. He claims to find three NCF's for the full set of rates and interprets them as the level, the slope and the curvature of the yield-to-maturity curve. However, he ignores any relationship of one of these NCF's with monetary policy and seems to have found one NCF too many, this due to an error. When he treats the 19 rates together, he finds three NCF's, probably because of the low power of the Johansen tests when the number of variables is large; see Johansen (1991b). When he examines the 12 T-bill rates alone, he finds one NCF, and when he studies the seven T-bond rates alone he finds two, then concluding too soon that his finding of three NCF's is confirmed. But he ignores the possibility that the one NCF in T-bills may also be one of the two NCF's in T-bonds,

which is one of our main results. This paper is also characterized by many TSA practices that are, in our view, misguided, though popular in the contemporary applied econometrics literature. The minimum number of parameters estimated in these models, given the VAR order of four, is 1634 for the model with 19 variables, 654 for the model with 12 variables and 224 for the model with 7 variables. This is a case of massive overparameterization. No attention appears to be given to the data anomalies and the only residual diagnostics employed is the usual "battery" of formal test statistics.

Johnson (1994) criticizes Zhang, claiming that the finding of more than one NCF is due to Zhang's use of a mixture of T-bills, which bear no coupon, and T-bonds, which do, and claiming that only zero-coupon equivalents should be used. For this reason Johnson (1994) uses the McCulloch (1990) data. We examine this issue for a subperiod of our sample in Section 5 and do not find this position to be empirically justified. At the same time, Johnson (1994) indulges in many of the same CCSP practices in applied econometrics that we find unacceptable and even treats a longer sample (1951-1987) than Zhang, thus raising the likelihood of structural differences due to different monetary policy regimes.

It is remarkable that three papers, using the same TSA practices and essentially the same data by McCulloch (1990), arrive at rather different results. Both Johnson (1994) and Engsted and Tangaard (1994) find only one NCF, but Shea (1992) suggests that this conclusion is in doubt once long term interest rates are included. This strongly suggests that certain details of the TSA practices actually used are different and important. Probably the generalized carelessness of all three papers, with respect to extreme values (in first differences) and structural changes corresponding to different monetary policy regimes, is responsible. There are common extreme values of more than seven standard deviations in size in the first differences of most interest rates, whenever the NOP period is included in a data set together with non-NOP periods; such anomalies, when ignored, are enough to mislead the researcher to a finding of cointegration where none exists. We find, in Section 5 below,

that the discussion over whether to use McCulloch-type data versus conventional market yields is empirically irrelevant in any case, at least for the issues of the present paper.

There are several articles, notably Engle and Granger (1987) and Stock and Watson (1988), which are essentially TSA methodological proposals that use U.S. interest rates to illustrate the proposals. No attention appears to be given to extreme or influential values, especially surprising in the case of Engle and Granger (1987) who use a sample including the NOP period, though carelessness as regards the data is patent in the case of this paper in which the data source is not even mentioned.

Both Engle and Granger (1987) and Campbell and Shiller (1987) analyze the yields on the one-month T-bill and the 20-year T-bond and find only one NCF, that is, they find these two yields to operate in a CI relationship. The sample periods analyzed in these papers are earlier than the one we analyze, but our findings are quite different. We do not find these two yields to be cointegrated in our sample.

[INSERT Table 1 near here or before]

The remainder of this paper is organized as follows. Section 2 describes data sources and manipulations. Section 3 summarizes the results of univariate stochastic and intervention analyses. Section 4 offers bivariate analyses of market yield series with the Federal Funds Rate Target. Section 5 presents bivariate analyses of Treasury yields and McCulloch rates of equal maturity. Section 6 summarizes trivariate analyses of \bar{R}^0 , R^{30} and each other market yield of maturity above two years. Concluding remarks are offered in Section 7.

2. Data Analyzed

With the exception of data on \bar{R}^0 , Corporate Bonds and the McCulloch and Kwon (1993) data employed in Sections 3 and 5, all of the data analyzed here comes from the same source, the Federal Reserve Economic Data (FRED) database, located at the Federal Reserve Bank of St. Louis

Internet page at the address www.stls.frb.org/fred/data/irates.html. Data on Corporate Bonds (Moody's AAA) are from the Federal Reserve Statistical Releases (H.15 Selected Interest Rates) offered at the Board of Governors' Internet page at the address www.bog.fed.us/releases/H15/data.htm. Data are monthly for 1/85-12/95 and are means of daily data. Included are the Federal Funds Rate (R^0), T-bills at 3, 6 and 12 month maturities (secondary market and auction averages), Commercial Paper and Certificates of Deposit at terms of 1, 3 and 6 months, T-bonds (Treasury Constant Maturity Rate) at terms of 1, 2, 3, 5, 7, 10 and 30 years, the Fixed Contracts for 30-year Mortgage Rate (FHA) and Corporate Bonds (Moody's AAA).

T-bills and Commercial Paper appear in the source "quoted on a discount basis" and for analysis were transformed to yields to maturity to have them on the same basis as the other series. All rates are analyzed as the logarithm of one plus the yield as a pure number so that our data are on a continuously compounded basis. For more details on how the original data is constructed, the reader should consult the source.

Data on \bar{R}^0 derive from the original source, the Federal Reserve Bank of New York Report of Open Market Operations and Money Market Conditions and was obtained on request to the Research Division of the Federal Reserve Bank of St Louis (webmaster@stls.frb.org). We employ the midpoint of the official band. Monthly data as averages of daily data are constructed with the same criteria as those used by the Fed to construct R^0 .

McCulloch and Kwon (1993) offer the data up to 2/91 that we employ in Section 5, univariate results on which appearing in Table 4 of Section 3. This data is the most recent available version of that found in McCulloch (1990). It attempts to estimate zero-coupon equivalents of yields for bonds with nonzero coupons, but also is adjusted for differences in fiscal treatment. For more details on how it is constructed, see McCulloch (1975, 1990). It also differs from our yield data in being end-of-month data rather than monthly average of daily data, though this implies no difference in

integration and cointegration properties. It appears in the source on a continuously compounded basis so transformation is not necessary.

Our work covers a number of interest rates on private sector issues that apparently have not so far been analyzed in the literature: Commercial Paper and Certificates of Deposit at different terms, Moody's AAA Corporate Bonds Rate and the 30-year Mortgage Rate.

There appear to be few rates covered in the literature that we do not cover. Some studies treat more maturities of T-bills, but this is irrelevant generality in practice, since all T-bill rates are clearly in CI(1,1) relationships with \bar{R}^0 . In any case, our coverage of T-bills is quite ample.

There are some interest rate series available that we do not study. We ignore bank deposit rates, all of which were fixed by law until 1986; demand deposit rates are still so fixed. We ignore the Prime Loan Rate, because it changes too infrequently to be regarded as relevant for statistical analysis. Analyses of quarterly data on average commercial bank interest rates on Personal Loans at two-year term and on New Car Loans at four-year term reveal that the former operates in a CI(1,1) relationship with \bar{R}^0 and that the latter does so with \bar{R}^0 and R^{30} , but these results are not reported here, since they add little to those reported. Rates of interest on State and Local Government bonds are ignored because they are too numerous to know how to choose them; we would expect them to operate in CI(1,1) relationships with \bar{R}^0 and R^{30} as do the yields on Federal Government bonds of equal term.

We began analyses with the 20-year T-bond series published by the Fed. At a certain point in relationship analyses with this variable, we detected very unjustifiable extreme values. On requesting clarification from the Fed, we learned that, for some periods, this series is constructed by the Treasury as an interpolation between 10-year and 30-year market T-bond series. For this reason we dropped the series from analysis.

A complete numerical listing of all data analyzed in this paper is available from the authors on request as (unpublished) Appendix A.

3. Univariate Analyses

Data analysis begins with single-variable analyses for each interest rate included in the study.

Let B stand for the lag or backshift operator such that, for any time series X_t , $BX_t \equiv X_{t-1}$, and let $\nabla \equiv 1 - B$ be the difference operator such that $\nabla X_t \equiv X_t - X_{t-1}$. Findings are very similar for all interest rates studied: the series R is $I(1)$ and the first difference ∇R has zero mean and $AR(3)$ structure with one positive real root and two conjugate imaginary roots, giving rise to damped oscillations with a period of 3-4 months and damping factor around .5, and, in some cases, there is a small number of influential impulse interventions in level (R). In a few cases, the imaginary pair of roots does not arise. The generic univariate model for an interest rate can thus be written:

$$R_t = \xi_t + N_t, \quad \phi(B)\nabla N_t = a_t \text{ iid } N(0, \sigma_a^2)$$

$$\text{where } \phi(B) = (1 - \phi_{11}B)(1 - \phi_{21}B - \phi_{22}B^2)$$

$$\text{with } 0 < \phi_{11} < 1, \phi_{22} + \phi_{21} < 1, \phi_{22} - \phi_{21} < 1, |\phi_{22}| < 1,$$

$$\text{and } \phi_{21}^2 + 4\phi_{22} < 0$$

and where ξ_t represents a sum of intervention terms, each of the form $\omega_0 \xi_t^{1,t*}$ for a parameter ω_0 and

$$\xi_t^{1,t*} \equiv \begin{cases} 1.0 & t = t^* \\ 0.0 & t \neq t^* \end{cases}$$

Results are summarized in Table 2. In the following, we detail one case as illustration. The case considered is that of \bar{R}^0 , the Federal Funds Rate Target. Figure 1 presents the graphics and other tools used in the analysis. Note that the Q statistics presented below the autocorrelation function (acf) and cross correlation function (ccf) graphs in this paper are of the Ljung and Box (1978) type; degrees of freedom for the relevant χ^2 are given in ().

[INSERT Table 2 near here]

[INSERT Figure 1 near here]

The data graph of the level wanders, showing no affinity for a constant mean; the sample mean is crossed by the graph only three times in 132 observations. This strongly suggests that $\bar{R}^0 \sim I(d)$ for $d \geq 1$. The sample acf dies out only very slowly, confirming this impression.

The data graph of the first difference $\nabla \bar{R}^0$ appears to show fairly strong affinity for a constant mean near zero. This series is special in the sense that $\nabla \bar{R}^0$ has 40 zero values in 131 observations, 15 of them together in 1993-1994, which suggests it is somewhat questionable to treat it as stochastic; no other interest rate series in the study has this property, which is certainly due to the character of \bar{R}^0 as exactly set by the Fed. Though aware of this special property of \bar{R}^0 , we proceed to treat it as stochastic, and it seems plausible to take $\nabla \bar{R}^0$ as mean stationary. The sample acf of $\nabla \bar{R}^0$ seems to die out fast enough to support that $\nabla \bar{R}^0$ is mean stationary. The sample partial autocorrelation function (pacf) reveals a value at lag three that appears large enough to entertain an $AR(3)$ form. The $ARI(3,1)$ model specified for \bar{R}^0 as estimated is:

$$(1 - .46B - .002B^2 - .19B^3) \nabla \bar{R}_t^0 = a_t, \quad \hat{\sigma}_a^2 = .17\% \\ (.09) \quad (.09) \quad (.09)$$

The factored form of the $AR(3)$ operator is:

$$(1 - .78B)(1 + .32B^2 + .25B^2) \text{ with period} = 3.30 \text{ months and damping factor} = .50 \\ (.07) \quad (.10) \quad (.10) \quad (.20) \quad (.10)$$

where, for an $AR(2)$ operator with imaginary roots, $1 - \phi_1 B - \phi_2 B^2$, the damping factor is $\sqrt{-\phi_2}$ and the period is $2\pi / \cos^{-1}(\phi_1 / (2\sqrt{-\phi_2}))$. All numbers in () below parameter estimates are large sample standard error (s.e.) estimates.

The residuals for this estimated model seem to be well centered at zero mean and approximately homoskedastic, though there are 13 zeros in 1993-1994. Of 131 residuals, there are

seven of absolute value above two residual standard deviations; this number is not excessive for the normal distribution, nor are any of the largest residuals very extreme. The graphs of residual acf/pacf reveal no further structure. There is, furthermore, nothing in the estimated model or the diagnostic tools to suggest overdifferencing.

To check for the possible influence of extreme values in \bar{R}^0 , step intervention terms (in level) are added selectively to the model at dates for such extreme values. This is done one-by-one and in groups of two, three, etc. at once. In the case of \bar{R}^0 , estimated model parameters and diagnostic results do not change significantly; thus no influential anomalies are detected and none are reported in Table 2. There are seven interest rates, however, that do present one or more influential impulse interventions. Impulse intervention terms are detected when two consecutive step intervention terms of opposite sign are found to just compensate one another. Especially outstanding is the fact that the 1-month rates for Certificates of Deposit and Commercial paper have influential December effects in 1986-1992 and 1994 but not in 1985, 1993 and 1995. Table 3 summarizes extreme and moderate residual values at common dates, suggesting that contemporaneous relationships abound in this set of data.

[INSERT Table 3 near here]

Full analyses for the auction averages for T-bills at 3, 6 and 12-month maturities indicate that these have essentially the same univariate models as the secondary market series presented in Table 2. In all cases, the differentials between auction averages and secondary market series have small but significant positive means but otherwise appear to follow white noise processes with very small variances. We thus do not report further on the auction averages.

Table 4 summarizes univariate results for the McCulloch and Kwon (1993) data. For pairs of the same term, one from Table 2 (Treasury yields) and one from Table 4, the univariate models are seen to be similar.

[INSERT Table 4 near here]

4. Bivariate Analyses of Market Yields and Federal Funds Rate Target

The starting point for relationship analyses is a set of univariate analyses of the series to be related, summarized in a jointly estimated multivariate model with diagonal dynamics but general contemporaneous specification, and an Initial Conceptual Model (ICM) of the relationships. The ICM employed here uses the working hypothesis that the Federal Reserve, through open market operations to control the Federal Funds Rate, is capable of determining the levels of all market interest rates in the U.S. economy, except for stationary errors. The many papers that find only one NCF in the U.S. term structure of interest rates, together with the widespread idea that the Federal Reserve is at least able to so control very short term interest rates, lead us to formulate this ICM. In order to check this ICM with the data, bivariate analyses of all interest rates in the study, taken one at a time together with the Federal Funds Rate Target (\bar{R}^0), are performed. The statistical implication of the ICM is that every market interest rate is in a CI(1,1) relationship with the Federal Funds Rate Target.

In fact, our working hypotheses are confirmed only in part. It does seem to be the case, as Rudebusch (1995) has already observed, that the Fed's realized error at target, $R^0 - \bar{R}^0$, is stationary. It is also apparently the case that interest rates, on both public debt and different forms of private debt, operate in CI(1,1) relationships with the Federal Funds Rate Target for terms up to and including two years, though the cointegration coefficient α in $R^1 - \alpha\bar{R}^0$ appears to fall significantly below one for terms of one and two years. At terms of three years or higher, however, R^1 and \bar{R}^0 are not found to be in a CI(1,1) relationship at all, but are found to be jointly integrated of order one, I(1), though the first differences do present substantial positive contemporaneous correlation. If one assumes that \bar{R}^0 receives no contemporaneous effect from other interest rates, it is seen that the long run effect of \bar{R}^0 on interest rates falls roughly with term.

Two kinds of bivariate stochastic result can be described in a single model form as follows:

$$\begin{bmatrix} \phi_{11}(B) & 0 \\ 0 & \phi_{22}(B) \end{bmatrix} \begin{bmatrix} \nabla \bar{R}_t^0 \\ \nabla (R_t^i - \alpha \bar{R}_t^0) \end{bmatrix} = \begin{bmatrix} 1 & -\theta_{12}B \\ -\theta_{21}^*B(1-\theta B) & 1-\theta B \end{bmatrix} \begin{bmatrix} a_{1t} \\ a_{2t} \end{bmatrix} \quad (1)$$

CI(1,1) arises when $\alpha \neq 0$ and $\theta = 1$. CI(1,1) does not arise when $\alpha = 0$ and/or $\theta < 1$. The parameters θ_{12} and θ_{21}^* are included to cover lagged effects detected in the data; $\theta_{21}^* = 0$ arises in all cases without CI(1,1), in all cases of public debt and in the effective Federal Funds Rate. Of course, one assumes $\phi_{ii}(B) = 0 \Rightarrow |B| > 1$ for $i = 1, 2$, i.e. autoregressive operators are stationary. The vector $(a_{1t}, a_{2t})^T$ is assumed to follow a bivariate gaussian white noise process. The initial specifications for $\phi_{11}(B)$ and $\phi_{22}(B)$ are taken from the univariate models of \bar{R}^0 and R^i respectively.

Cases in which CI(1,1) is found are summarized in Table 5 and cases in which CI(1,1) is not found are summarized in Table 6. These tables do not report results for \bar{R}^0 , that is, for $\phi_{11}(B)$ and σ_1 , because these are virtually the same in all cases and the same as those appearing in Table 2 for the univariate analysis of \bar{R}^0 . The residual standard deviation presented in Tables 5 and 6 is that estimated for the other interest rate, $\hat{\sigma}_2$. The parameter ρ_0 refers to the contemporaneous correlation coefficient between a_1 and a_2 .

[INSERT Table 5 near here]

[INSERT Table 6 near here]

To clarify the analysis procedures employed, we proceed to exposit two illustrative cases, the first treating (\bar{R}^0, R^0) where CI(1,1) is found, the second treating (\bar{R}^0, R^{30}) , where JI(1) is found. These two cases reported in detail are extreme cases in terms of the maturity of the second yield, but they are nevertheless entirely typical of the two kinds of results obtained. The distinction between

yields on 2-year T-bonds, which are CI(1,1) with \bar{R}^0 , and on 3-year T-bonds, which are JI(1) with \bar{R}^0 , is just as clearcut as that between \bar{R}^0 and R^{30} .

4.1 Federal Funds Rate: Target (\bar{R}^0) and Effective (R^0)

The bivariate model jointly estimating the two univariate models has parameter estimates that differ substantially from the univariate estimates for some parameters and there are many cases of high correlations between parameter estimates, that is, the estimation situation is ill defined. Figure 2 presents the diagnostics for this model. One can appreciate that: (1) the quality of univariate representation has declined very markedly, the acf's and pacf's suggesting further structure to be present and (2) the residual ccf indicates the presence of relationships in both directions and virtually all values are positive. These are very typical symptoms of a case with CI(1,1).

[INSERT Figure 2 near here]

A model of form (1) but with $\theta_{12} = \theta_{21}^* = 0$ is estimated. The result is very different from the previous experiment. Here, at the same time that α is estimated to be very significantly different from zero and positive and θ very close to one, the univariate parameters for \bar{R}^0 return to values very near those found in univariate estimation, two of the three parameters in $\phi_{22}(B)$ do not differ significantly from zero, the estimation situation is well defined, with no relevant correlation between parameters, and diagnostics show the univariate representations to be acceptable and the ccf shows a large positive correlation between a_{2t-1} and a_{1t} . A check is then carried out to see if the latter correlation is due merely to a few coincidental extreme values; it is not. The next action taken is to add θ_{12} to the model; nothing notable arises and the full representation seems adequate. We proceed to prune out the two parameters in $\phi_{22}(B)$ that are not needed.

When CI(1,1) is found, as in all cases reported in Table 5, a final estimation is carried out in which: (1) the ∇ factor is removed from the second row of the MA matrix and from the second

variable $R^i - \hat{\alpha} \bar{R}^0$ and (2) a constant (mean) parameter μ is subtracted from the second variable $R^i - \hat{\alpha} \bar{R}^0$. This estimation yields all of the parameter estimates given in Table 5, except for that for θ , which arises in the previous estimation, and also provides a diagnostic operation, since erroneously specified cointegration should lead to signs of nonstationarity in the second variable: changes in parameter estimates relative to the first estimation, especially a rise in the positive $\phi_{22}(B)$ parameters, and loss in the quality of univariate representation as seen in acf/pacf. In the case of R^0 , Table 5 gives these final estimates and diagnostics are in Figure 3.

[INSERT Figure 3 near here]

The whole analysis with R^0 proceeds with the impulse intervention term (in level) in 12/86 that is found to be influential in the univariate analysis. See the Federal Reserve Bulletin of June 1987 (p. 444) for a plausible explanation of this incident. Further checks for influence are performed with impulse effects in level at 7/85, 12/85, 1/87 and 4/87, which are the incidents that stand out most in the residuals a_2 in Figure 3. These terms are added to the model one-at-a-time, two-at-a-time, etc., but the model appears to be robust with respect to these exercises.

Note that the impulse intervention effects in 12/87 in Table 2 for the univariate analyses of the yields on 3-month and 6-month Certificates of Deposit and Commercial Paper do not appear in Table 5. This is because these effects are not influential in the bivariate analyses though they are influential in the univariate analyses.

4.2 30-year Treasury Bond Rate (R^{30}) and Federal Funds Rate Target (\bar{R}^0)

The joint estimation of the univariate models for \bar{R}^0 and R^{30} yields parameter estimates close to those found in univariate estimation, there is no evidence of indefiniteness in the estimation situation, diagnostics presented in Figure 4 show the univariate representation to be adequate and the residual ccf merely suggests contemporaneous correlation and correlation between a_{2t-1} and a_{1t} .

These are symptoms suggesting $II(1)$, not $CI(1,1)$. When the feedback effect is incorporated with θ_{12} , the final model reported in Table 6 and Figure 5 is obtained. It seems adequate.

[INSERT Figure 4 here]

[INSERT Figure 5 here]

Given our working hypothesis, however, we expect $CI(1,1)$. To double check for this, the model of type (1) with $\theta_{12} = \theta_{21}^* = 0$ is estimated. Very unsatisfactory results arise. The CI coefficient α is estimated to be not significantly different from zero and the estimate is highly correlated with the estimate of ρ_0 , both results being typical of $II(1)$, not $CI(1,1)$. The θ parameter is estimated to not differ significantly from zero and is also highly correlated with parameters in $\phi_{22}(B)$. When the feedback coefficient θ_{12} is introduced, as suggested by the residual ccf, the previous very unsatisfactory results are repeated. Checks for influence do not alter the conclusions. R^{30} and \bar{R}^0 are apparently $II(1)$ rather than $CI(1,1)$, which violates our ICM.

4.3 Comment

The outstanding conclusion of the bivariate analyses with \bar{R}^0 is that yields for maturities of 3 years or more are not in $CI(1,1)$ relationships with \bar{R}^0 , though yields for maturities of two years or less are.

Johnson (1994), in a critique of Zhang (1993), claims that mixing yields on pure discount bonds, such as T-bills, with yields on coupon bonds, such as T-bonds, will lead to the rejection of the $CI(1,1)$ property, implied by the expectations theory of the term structure of interest rates, which is framed in terms of pure discount bonds. This point, if valid, would be applicable to the results reported in this section. We therefore examine the relevant empirical issue in the next section.

5. Bivariate Analyses of Treasury Yields and McCulloch Rates of Equal Maturity

In this section we summarize the results of eight bivariate analyses, each for a given maturity as shown in Table 7, which presents the findings. Only Treasury securities are considered in this section and the two variables analyzed for each maturity are the conventional yield, R^y , and the McCulloch and Kwon (1993) rate, R^m ; the sample period is that available for the latter and starting at the same date as our general sample, i.e. 1/85-2/91. The general bivariate model form employed is the same as form (1) of Section 4, but with ∇R^y taking the first position, instead of $\nabla \bar{R}^0$, and $\nabla(R^m - \alpha R^y)$ taking the second position, instead of $\nabla(R_t^i - \alpha \bar{R}_t^0)$. Table 7 is analogous to Table 5, because we find that the two series are in a CI(1,1) relationship in all cases. We do not report the AR(3) parameter estimates for $\phi_{11}(B)$ nor the residual standard deviation $\hat{\sigma}_1$, because they do not differ significantly from the univariate results shown in Table 2. $\phi_{22}(B)$ is of order one in all cases; Table 7 reports the estimate of ϕ_{22} in $\phi_{22}(B) = 1 - \phi_{22}B$. No intervention terms are reported in Table 7, because none of the potentially influential incidents is, in fact, found to be influential in the bivariate model.

[INSERT Table 7 near here]

The conclusion of most relevance for present purposes is that, at a given maturity, the McCulloch series operate in a CI(1,1) relationship with the conventional Treasury yield series analyzed in the present paper and this with cointegration coefficient very close to one. This means that, at least for the objectives of the present paper and the sample examined here, the kind of critique leveled by Johnson (1994) at Zhang (1993) is not empirically relevant.

6. Trivariate Analyses

The bivariate analyses of Section 4 indicate that yields with maturities of three years or more are not in bivariate CI(1,1) relationships with the Federal Funds Rate Target, \bar{R}^0 . The analyses of Section 5 indicate that this is not due to differences between pure discount and coupon bonds, or to

other differences between conventional yield series and McCulloch series. The questions that naturally follow are: (1) what is the number of I(1) common factors in the set of yields with maturity of three years or more? and (2) is \bar{R}^0 associated with one of them? It turns out that: (1) there are only two I(1) common factors in the set of yields at issue and (2) \bar{R}^0 is associated with one of them. Given this result, one can associate the second I(1) factor with any linear combination of yields of maturity three years or more that one chooses. In this section we present results that associate this second I(1) common factor with the 30-year T-bond yield, R^{30} .

Several kinds of trivariate stochastic result can be described in a single model form as follows:

$$\begin{aligned}
 & \left[\begin{array}{cc|cc} \phi_{11}(B) & 0 & 0 & 0 \\ 0 & \phi_{22}(B) & 0 & 0 \\ \hline 0 & 0 & \phi_{33}(B) & 0 \end{array} \right] \left[\begin{array}{c} \nabla \bar{R}_t^0 \\ \nabla R_t^{30} \\ \nabla(R_t^i - \alpha'_1 \bar{R}_t^0 - \alpha_2 R_t^{30}) \end{array} \right] \\
 & = \left[\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \hline 0 & 0 & (1-\theta_{33}B)(1-\theta B) & 0 \end{array} \right] \left[\begin{array}{ccc|c} 1 & -\theta_{12}B & -\theta_{13}B & a_{1t} \\ 0 & 1 & 0 & a_{2t} \\ \hline -\theta_{31}B & \theta'_{32}(B) & 1 & a_{3t} \end{array} \right] \quad (2)
 \end{aligned}$$

where $\theta'_{32}(B) = -\theta'_{321}B - \theta'_{322}B^2$ and trivariate CI(1,1) arises when $\alpha'_1 \neq 0$, $\alpha_2 \neq 0$ and $\theta = 1$. If $\alpha'_1 = 0$, $\alpha_2 \neq 0$ and $\theta = 1$, then bivariate CI(1,1) between R^{30} and R^i characterizes the case; this does not occur in this study. If $\alpha'_1 = \alpha_2 = 0$ and $\theta < 1$, then no CI(1,1) relation is found; this case does not occur in this study either. The bivariate results of Table 6 show that the case of $\alpha'_1 \neq 0$, $\alpha_2 = 0$ and $\theta = 1$ does not arise.

The initial specifications for $\phi_{11}(B)$, $\phi_{22}(B)$ and θ_{12} arise from the bivariate analysis of \bar{R}^0 and R^{30} and none of the trivariate analyses require these to be modified. The initial specification for $\phi_{33}(B)$ is taken from the univariate model of R^1 . One assumes stationarity, i.e., $\phi_{1i}(B) = 0 \Rightarrow |B| > 1$ for $i = 1, 2, 3$. The initial specification of the presence of the θ_{13} parameter arises from the bivariate analysis of \bar{R}^0 and R^1 . The θ parameter signals overdifferencing. The $\theta'_{32}(B)$ parameter(s) and the θ_{33} parameter arise from typical reformulations. The θ_{31} parameter arises in the reformulation for the 30-year mortgage rate only.

The results of these analyses are summarized in Table 8, which does not report on $\phi_{11}(B)$, $\phi_{22}(B)$, θ_{12} , σ_1 and σ_2 , because these are virtually the same in all cases and the same as those appearing in Table 6 for the bivariate analyses of \bar{R}^0 and R^{30} . The residual standard deviation presented in Table 8 is that estimated for R^1 , $\hat{\sigma}_3$. The parameters ρ_{13} and ρ_{23} refer to the contemporaneous correlations between a_1 and a_3 and between a_2 and a_3 respectively. Note that $\phi_{33}(B) = I - \phi_{33}B$ arises in all cases, the initial AR(3) structure simplifying in the analysis process. The CI coefficient α_1 reported in Table 8 is defined as $\alpha_1 \equiv \alpha'_1 + 0.39\alpha_2$, which is useful in making comparisons with the bivariate analyses of Section 4. The two I(1) series \bar{R}^0 and $R^{30} - 0.39\bar{R}^0$ are contemporaneously uncorrelated, where .39 is the estimate of α in (1) with R^{30} under $\theta = \rho_0 = 0$.

Trivariate CI(1,1) is found in all six cases and in each a final estimation is carried out in which: (1) the ∇ factor is removed from the third row of the MA matrix and from the third variable $R^1 - \alpha_1\bar{R}^0 - \alpha_2(R^{30} - 0.39\bar{R}^0)$ and (2) a constant (mean) parameter μ is subtracted from the third variable. This estimation yields all of the parameter estimates given in Table 8, except for that for θ , which arises in the previous estimation, and also provides a diagnostic operation as in the bivariate CI(1,1) case. Note that, for T-bonds, $\hat{\alpha}_1$ seems to fall with term and $\hat{\alpha}_2$ rises.

[INSERT Table 8 near here]

To clarify the analysis procedures employed, we proceed to expost the illustrative case of the 3-year T-bond.

The analysis process begins with the estimation of a trivariate model with block diagonal dynamics, composed of a bivariate block in $(\nabla\bar{R}^0, \nabla R^{30})$ as specified in Section 4 and a univariate block in ∇R^3 as specified in Section 3; contemporaneous correlations are not restricted. In the case of R^3 (and of all yields at higher maturities), this trivariate estimation involves a loss in the adequacy of univariate representation for the three variables and residual ccf's show substantial contemporaneous correlations between R^3 and each of the other rates. See Figure 6.

[INSERT Figure 6 here]

The analysis proceeds with the estimation of a trivariate model of form (2) but with $\theta_{31} = \theta'_{32}(B) = \theta_{33} = 0$. Both α_1 and α_2 are estimated to be very significantly different from zero and positive, θ is estimated to be one, the estimation situation is very well defined except for evidence of overparameterization in $\phi_{33}(B)$, diagnostics show the univariate representations to be acceptable and the residual ccf of a_2 with a_3 reveals θ'_{321} .

Estimation is repeated with θ'_{321} included; this parameter is negative and significantly different from zero. The $\phi_{33}(B)$ operator degenerates at the first step to an AR(2) with two real factors, both with positive parameters, but these parameters are poorly estimated, i.e. estimates are highly correlated and estimated standard errors are suspiciously high, and one parameter is much smaller than the other, suggesting an MA(1) reformulation, leading to $1 - \theta_{33}B$. The final model, with the reformulations and with differences removed and μ added, is reported in Table 8 and Figure 7.

[INSERT Figure 7 near here]

As in the earlier univariate and bivariate analyses, a large variety of checks for influence of extreme values shows none to be important in these trivariate models.

A conclusion from Sections 4 and 5 is that there are at least two $JI(1)$ factors driving U.S. interest rates at maturities of three years or more. The conclusion of this section is that there are only two $JI(1)$ factors driving U.S. interest rates at maturities of three years or more and that one of them is the same factor driving all short term rates, i.e. the one associated with monetary policy (\bar{R}^0).

7. Concluding Remarks

A very simple initial CM is stated in Section 1.1 and Section 3 offers the details of the univariate analyses of all variables. The bivariate analyses of Section 4 show that there is but one NCF, associated with \bar{R}^0 and hence with monetary policy (Fed behavior), covering all yields of maturity two years or less, but that yields of maturity above two years involve at least one further NCF.

The latter result in itself rejects the expectations theory of the term structure of interest rates in a specific way and, even for terms at two years or less, it appears likely that a further facet of rejection of this hypothesis arises in the apparent differences in the estimated CI coefficients for rates with similar risk characteristics other than term. Our work is not oriented toward testing the expectations theory of the term structure. But this rejection and the Johnson (1994) critique of Zhang (1993) for a similar rejection leads us to the bivariate analyses of Section 5, between Treasury yields and McCulloch and Kwon (1993) series of equal maturity. The finding is roundly that these pairs of alternative measures (and concepts) of the interest rate at a given maturity are, in fact, operating in $CI(1,1)$ relationships. The Johnson (1994) critique is thus not empirically relevant, at least for the purposes of this paper and in this sample.

Section 6 goes on to ask and answer the question of how many NCF's other than \bar{R}^0 are present in interest rates of maturity above two years. There is one, which we represent with the

longest term T-bond rate in the data, R^{30} . This second NCF in U.S. interest rates is not associated with monetary policy. Hence the Fed does not control all interest rates, but only the short term rates.

The CM plays a central role in this paper, because it requires that \bar{R}^0 be in every model and tells how to identify its contemporaneous correlations with other variables in a plausible way. There are several alternative purely statistical common factor approaches that one can apply to the levels of interest rates to get a first guess as to the number of NCF's present; see Escribano and Peña (1994) for a survey of these. However, it is not wise to forego building full multivariate stochastic VARMA models to check out this number efficiently. Furthermore, the purely statistical common factor approaches fail to face the relevance of the CM. For example, they might well suggest, in the bivariate case for short term rates, that some average of \bar{R}^0 and the other rate should be taken to represent the NCF rather than \bar{R}^0 itself.

Note that positive feedback to \bar{R}^0 from almost all cointegration vectors is detected at a one month lag, that is, $\hat{\theta}_{12} < 0$ in all cases in Table 5 and $\hat{\theta}_{13} < 0$ in all cases in Table 8, except Corporate bonds and 30-year mortgages. This means that the Fed does follow the market to some extent when setting its target for the Federal Funds Rate.

There are several directions for further research in which we hope to obtain results in the near future. What is the economic meaning of the non- \bar{R}^0 NCF in longer term interest rates? Trivariate (or larger) models with \bar{R}^0 and R^{30} present and with third variables in a $CI(1,1)$ relationship with them would shed light on this. Foreign interest rates are a possibility as are a number of domestic variables. The Fed reaction function already has some structure from the results in Sections 4 and 6 where feedback to \bar{R}^0 from the other variables is detected, but the addition of further variables can enrich this relation; see Khoury (1990) for a survey on this subject. Hopefully research for periods before 1985 can establish what kinds of structural changes, if any, occurred between the several

different monetary policy regimes and thus make possible effective model building for longer series of interest rates.

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Table 1. Characteristics of econometric works cited

	Variables (number)	Sample / Sampling Method *	Data Source	Econometric Methods Used
Engle & Granger (1987)	1-month T-bill 20-year T-bond (2)	1952-1982 monthly / NA	Not mentioned	Proposed in the paper
Campbell & Shiller (1987)	1-month T-bill 20-year T-bond (2)	1959-1983 monthly / NA	Salomon Brothers Treasury Bulletin Wall Street Journal	Engle & Granger (1987), Phillips & Ouliaris (1986)
Stock & Watson (1988)	Fed Funds Rate, 3-month and 1-year T-bills (3)	1960-1979 monthly / AD	Citibase	Proposed in the paper
Cook & Hahn (1989)	Fed Funds Rate Target, X-month T-bill, X=3, 6, 12 and Y-year T-bond, Y=3, 5, 7, 10, 20 (8)	1974-1979 Incidents of change in Fed Funds Rate Target	Federal Reserve Macro Data Library	Linear OLS regression on first differences
Shea (1992)	X-month, X=1, 3, 6, and Y-year, Y=3, 5, 10, 15, 20, 25 McCulloch rates (9)	1952-1987 monthly and quarterly / EP	McCulloch (1990)	Johansen (1988, 1991a)
Hall, Anderson & Granger (1992)	X-month T-bill X=1, 2, 3,..., 11 (11)	1970-1988 monthly / EP	Centre for Research in Security Prices (Univ. of Chicago)	Johansen (1988, 1991a)
Zhang (1993)	X-month T-bill, X=1,2,...,12 and Y-year T-bond, Y=2, 3, 4, 5, 10, 20, 30 (19)	1964-1986 monthly / BP	Salomon Brothers Mishkin (1990)	Johansen (1988, 1991a)
Engsted & Tanggaard (1994)	X-month, X=1, 2,...,12 and Y-year, Y=2, 5, 10 McCulloch rates (15)	1952-1987 monthly / EP	McCulloch (1990)	Johansen (1988, 1991a)
Johnson (1994)	X-month, X=1, 2, 3, 4, 5, 6, 9, 12 and Y-year, Y=2, 3, 4, 5, 10 McCulloch rates (13)	1951-1987 monthly / EP	McCulloch (1990)	Johansen (1988, 1991a)
Garfinkel & Thornton (1995)	Fed Funds Rate, Repurchase agreements, 3-month T-bill (3)	1972-1993 daily and weekly / AD	Federal Reserve Bank of New York	Johansen (1988, 1991a)
Rudebusch (1995)	Fed Funds Rate; target and effective rates (2)	1974-1979, 1984-1992 daily (midpoint of the official target band)	Federal Reserve Bank of New York	Robust regression techniques
Roley & Sellon (1996)	Fed Funds Rate Target, X-month T-bill, X=3, 6 and Y-year T-bond, Y=1, 3, 5, 7, 10 (8)	1974-1979, 1987-1995 Incidents of change in Fed Funds Rate Target	Federal Reserve Bank of New York, Federal Reserve data files	Linear OLS regression on first differences

* AD stands for Averages of Daily figures; EP and BP stand for End of Period and Beginning of Period figures respectively; NA stands for Not Available.

Table 2. Estimated univariate models for first differences of monthly averages of yields: 1/1985 - 12/1995

Variable	$\phi(B) = (1 - \phi_{11}B - \phi_{21}B^2 - \phi_{22}B^3)$			Estimated Properties AR(2) Factor		Influential Impulse Intervention Terms (in Level)		Resid. St.Dev. (%)
	$\hat{\phi}_{11}$ (s.e.)	$\hat{\phi}_{21}$ (s.e.)	$\hat{\phi}_{22}$ (s.e.)	Period (Months) (s.e.)	Damping Factor (s.e.)	Value (%) (s.e.)	Dates	
Federal Funds Target	.78 (.07)	-.32 (.10)	-.25 (.10)	3.3 (.2)	.50 (.10)	-	-	.17
Federal Funds (Effective Rate)	.72 (.08)	-.28 (.11)	-.23 (.10)	3.4 (.1)	.48 (.10)	.57 (.11)	12/86	.19
1-month Certificates of Deposit	.71 (.08)	-.14 (.11)	-.33 (.11)	3.7 (.2)	.57 (.10)	* X=86-92, 94	12/X X=86-92, 94	.19
1-Month Commercial Paper	.71 (.08)	-.14 (.11)	-.35 (.10)	3.7 (.2)	.59 (.08)	+ X=86-92, 94	12/X X=86-92, 94	.22
3-Month T-Bills (Secondary Market)	.70 (.09)	-.12 (.11)	-.32 (.10)	3.8 (.2)	.57 (.09)	-	-	.20
3-Month Certificates of Deposit	.57 (.13)	-.03 (.14)	-.27 (.11)	3.9 (.3)	.52 (.11)	.70 (.12)	12/87	.23
3-Month Commercial Paper	.58 (.12)	-.06 (.13)	-.30 (.11)	3.9 (.3)	.55 (.10)	.69 (.11)	12/87	.26
6-Month T-Bills (Secondary Market)	.58 (.14)	-.04 (.16)	-.21 (.12)	3.9 (.4)	.46 (.13)	-	-	.23
6-Month Certificates of Deposit	.51 (.16)	-.02 (.17)	-.27 (.11)	4.0 (.4)	.52 (.11)	.57 (.13)	12/87	.26
6-Month Commercial Paper	.52 (.15)	-.03 (.16)	-.28 (.11)	4.1 (.4)	.53 (.10)	.55 (.09)	12/87	.28
1-Year T-Bills (Secondary Market)	.58 (.12)	-.04 (.13)	-.32 (.11)	3.9 (.3)	.57 (.10)	-	-	.25
1-Year T-Bonds	.59 (.12)	-.04 (.12)	-.33 (.10)	3.9 (.2)	.57 (.09)	-	-	.24
2-Year T-Bonds	.52 (.14)	.05 (.14)	-.32 (.10)	4.1 (.4)	.57 (.09)	-	-	.26
3-Year T-Bonds	.51 (.14)	.06 (.14)	-.33 (.10)	4.1 (.3)	.57 (.09)	-	-	.26
5-Year T-Bonds	.52 (.14)	.07 (.14)	-.34 (.10)	4.2 (.4)	.58 (.09)	-	-	.25
7-Year T-Bonds	.55 (.12)	.04 (.12)	-.36 (.10)	4.1 (.3)	.60 (.08)	-	-	.24
10-Year T-Bonds	.56 (.12)	-.01 (.12)	-.36 (.10)	4.0 (.3)	.60 (.08)	-	-	.23
30-Year T-Bonds	.60 (.11)	-.08 (.12)	-.36 (.10)	3.8 (.2)	.60 (.08)	-	-	.21
Corporate Bonds (Moody's AAA)	.37 (.09)	-.16 (.09)	-	5.8 (.8)	.40 (.11)	-	-	.23
30-Year Fixed Mortgage (FHA)	.16 (.09)	-	-	-	-	-	-	.29

* Estimated values (%) for 86-92, 94 are .82, 1.18, .57, .31, .73, .49, .47 and .43; s.e. is .10% in all cases.
 + Estimated values (%) for 86-92, 94 are .77, 1.08, .49, .32, .53, .39, .33, and .32; s.e. is .10% in all cases.

Figure 1. Specification and diagnostic tools for the Federal Funds Rate Target, \bar{R}^0 (data graphs standardized)
 Level

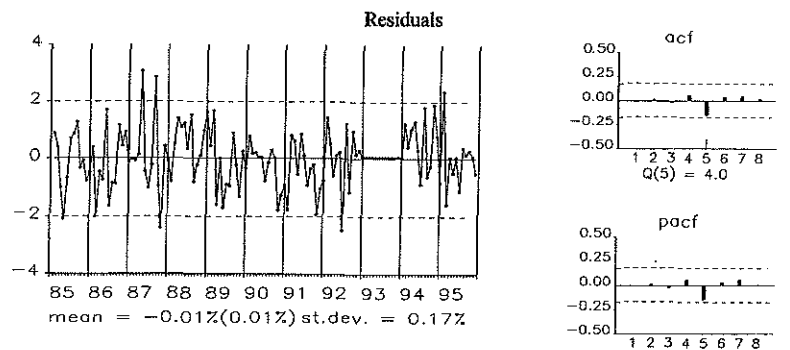
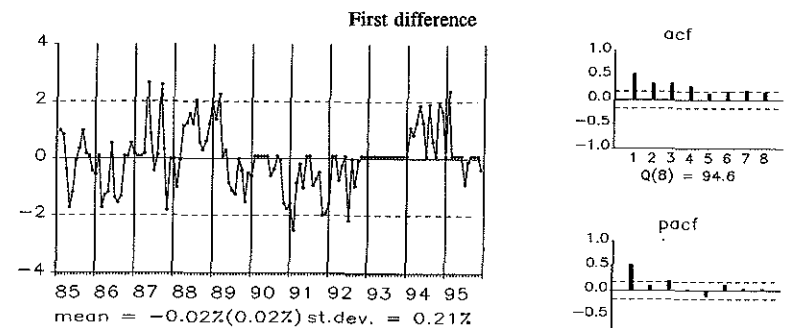
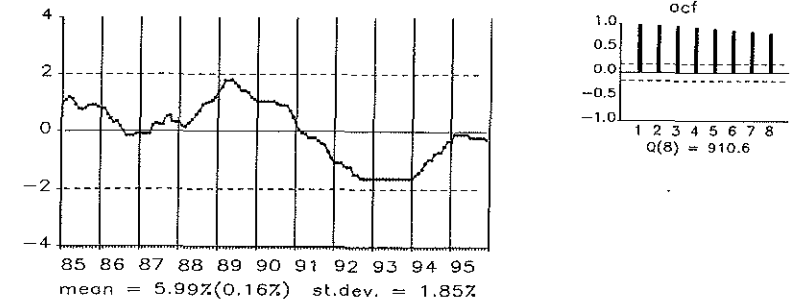


Table 3. Summary of common univariate residual extreme values

Variable	2/85	4/85	5/85	6/85	7/85	2/86	3/86	5/86	12/86	4/87	5/87	9/87	11/87	12/87	12/88	6/89	1/91	2/91	7/92	5/94	11/94	12/94	2/95
F. Funds (Target)			N				N				P	P	N			-	-	-	N	+	P		P
F. Funds (Eff.)		-	-	-	P		-		P		P	P	N	+	+	-	-	N	N	+	+		+
1-M C. Deposit	+	N	-	-	P		-	+	P*		+	P	N	P*	P*		N	-	N	P	+	+	P*
1-M C. Paper	P	N	-	-	P		-	+	P*		+	P	N	P*	P*		-	-	N	+	+	+	
3-M T-Bill	P	N		N	P		N	+			+	N	+			-	-	-	N	+			
3-M C. Deposit	+	N	-	-	P		-	+	+	+	+	P	N	P*	+		N	-	-	+			+
3-M C. Paper	+	N	-	-	+		-	+	+	+	+	P	N	P*	+		N	-	-	+			+
6-M T-Bill	+	N		N	+		N	+		+	+	N	+			N			N	+			+
6-M C. Deposit	P	N	-	-	+		-	+	+	+	+	P	N	P*	+		-	-	-	+			P
6-M C. Paper	P	N	-	-	+		-	+	+	+	+	P	N	P*	+		-	-	-	+			P
12-M T-Bill	+	N		N	+		-	+		P	+	+	-	+	+		-	-	N	+		+	+
1-Y T-Bond	+	N		N	+		-	+		P	+	+	-	+	+		-	-	N	+		+	+
2-Y T-Bond	+	N		N	+		N	+		P	+	+	N	+					N	+			
3-Y T-Bond		N	-	-	+		-	+		P	+	+	N	+					N	+			
5-Y T-Bond		N		N	+		N	+		P		+	N	+					-	+			
7-Y T-Bond		N		N	+		N	P		P		P	N	+					-	+			
10-Y T-Bond		N		N	+	N	N	+		P		P	N	+					-	+			
30-Y T-Bond		-		N	+		N	+		P		P	N	+		N			-	+			
Corporate Bond		N	-	-		N	N	+		P	+	+	N	+					-	+			
30-Y Mortgage			-	-						P	+	+	P	-		+							

P, + stand for positive residuals with absolute value greater than two and between one and two standard deviations respectively (N, - for negative values). This table is constructed in two steps: 1) including all dates where there is a value of more than +/- 2 st.dev. in any one of the univariate residual series and 2) checking whether at the same dates there is a large value (> +/- 1 st.dev.) in the other residual series. The first step provides a benchmark and the second searches for coincidences in extreme values that might reveal how interest rates are related. Some extreme values that are not common are not reported; all those in the Federal Funds Rate Target are reported.

* There is a positive influential impulse intervention term at this date in this series. Therefore there is a positive extreme value followed by a negative value of approximately the same size in the first difference of the series at these dates. For the 1-month Certificates of Deposit and Commercial Paper rates, there are additional influential impulse intervention terms at 12/89, 12/90, 12/91 and 12/92, but they are not reported in the table because these incidents only appear in these two variables.

Table 4. Estimated univariate models for first differences of McCulloch and Kwon (1993) rates: 1/1985-2/1991

McCulloch & Kwon (1993) Rates	$\phi(B) = (1 - \phi_{11}B - \phi_{21}B^2 - \phi_{22}B^3)$			Estimated AR(2) Properties (Months)	Damping Factor (s.e.)	Influential Impulse Terms (in Level)		Resid. St.Dev. (%)	
	$\hat{\phi}_{11}$ (s.e.)	$\hat{\phi}_{21}$ (s.e.)	$\hat{\phi}_{22}$ (s.e.)			Value (%)	Dates		
3-Month	.55 (.19)	-.21 (.20)	-.25 (.16)	3.5 (.3)	.50 (.16)	.31 (.16)	.84 (.16)	2/85 9/87	.27
6-Month	.53 (.17)	-.25 (.19)	-.30 (.15)	3.5 (.3)	.55 (.14)	.50 (.17)	.83 (.18)	2/85 9/87	.29
12-Month	.47 (.09)	-.19 (.21)	-.29 (.15)	3.6 (.4)	.54 (.14)	.47 (.20)	.74 (.18)	2/85 9/87	.31
2-Year	.35 (.27)	-.10 (.26)	-.29 (.15)	3.8 (.5)	.54 (.14)	.44 (.22)	.67 (.20)	2/85 9/87	.31
3-Year		.20 (.11)	-.21 (.11)	4.6 (.4)	.46 (.12)	.49 (.23)	.69 (.23)	2/85 9/87	.35
5-Year		.13 (.12)	-.16 (.11)	4.5 (.5)	.40 (.14)	.49 (.25)	.65 (.24)	2/85 9/87	.36
7-Year		.17 (.11)	-.16 (.12)	4.7 (.4)	.46 (.12)	.55 (.24)	.61 (.23)	2/85 9/87	.35
10-Year		.08 (.11)	-.14 (.11)	4.3 (.5)	.37 (.15)	.55 (.25)	.63 (.24)	2/85 9/87	.36

Table 5. Summary of bivariate models of each yield, R^i , with the Federal Funds Rate Target, \bar{R}^0 (cases where CI(1,1) is found)

Variable (R^i)	$\hat{\alpha}$ (s.e.)	$\hat{\theta}$ (s.e.)	$\hat{\mu}$ (%) (s.e.)	$\hat{\phi}_{22}(B)$ (s.e.)	$\hat{\theta}_{12}$ (s.e.)	$\hat{\theta}_{21}^*$ (s.e.)	Influential Impulse Intervention Terms (in Level)		$\hat{\rho}_0$ (s.e.)	Resid. St.Dev. (%)
							Value (%) (s.e.)	Dates		
Federal Funds (Effective Rate)	1.01 (.01)	1.00 (.01)	-.01+ (.04)	(1-.47B) (.08)	-.68 (.17)	-	.65 (.07)	12/86	.05 (.08)	.09
1-Month Certificates of Deposit	.97 (.01)	1.00 (.01)	.21 (.07)	(1-.67B) (.06)	-.63 (.16)	-.11 (.05)	*	12/X X=86-92, 94	.11 (.08)	.09
1-Month Commercial Paper	1.08 (.01)	1.00 (.02)	-.01 (.09)	(1-.69B) (.06)	-.53 (.14)	-.13 (.06)	*	12/X X=86-92, 94	.16 (.08)	.10
3-Month T-Bills (Secondary Market)	.97 (.04)	1.00 (.03)	.15 (.27)	(1-.86B) (.04)	-.58 (.08)	-	-	-	-.06 (.08)	.15
3-Month Certificates of Deposit	.97 (.02)	1.00 (.03)	.34 (.14)	(1-.70B) (.06)	-.32 (.09)	-.15 (.08)	-	-	.14 (.08)	.16
3-Month Commercial Paper	1.06 (.02)	1.00 (.03)	.21 (.15)	(1-.68B) (.06)	-.27 (.08)	-.17 (.09)	-	-	.16 (.08)	.18
6-Month T-Bills (Secondary Market)	.94 (.04)	1.00 (.01)	.47 (.25)	(1-.79B)(1-.19B) (.08) (.12)	-.52 (.08)	-	-	-	.07 (.08)	.17
6-Month Certificates of Deposit	.95 (.04)	1.00 (.01)	.52 (.20)	(1-.80B) (.05)	-.37 (.07)	-.25 (.11)	-	-	.25 (.08)	.19
6-Month Commercial Paper	1.02 (.03)	1.00 (.01)	.48 (.21)	(1-.80B) (.05)	-.33 (.07)	-.29 (.11)	-	-	.25 (.08)	.20
1-Year T-Bills (Secondary Market)	.89 (.05)	1.00 (.01)	.86 (.29)	(1-.77B)(1-.34B) (.09) (.13)	-.49 (.06)	-	-	-	.15 (.08)	.19
1-Year T-Bonds	.87 (.05)	1.00 (.01)	.96 (.29)	(1-.77B)(1-.34B) (.09) (.13)	-.50 (.07)	-	-	-	.15 (.08)	.19
2-Year T-Bonds	.82 (.07)	.96** (.05)	1.77 (.45)	(1-.81B)(1-.43B) (.09) (.13)	-.43 (.06)	-	-	-	.13 (.12)	.22

* If the cointegration coefficient is restricted to be one, a hypothesis that cannot be rejected, the estimated mean is .05% (.01%). This means that the errors at target have a positive but very small estimated mean.

* Estimated values are virtually the same as those shown in notes to Table 2.

** Though noninvertibility seems clear, an additional check is carried out in this case: the estimated linear combination is taken as a single time series and an ARIMA(2,1,1) is estimated. By the Generalized Likelihood Ratio Test of Davis, Chen and Dunsuir (1995) we cannot reject the null hypothesis of noninvertibility, since the likelihood ratio is .002, much smaller than the cut-off values of 4.41, 1.94 and 1.00, which are those for confidence levels of 99, 95 and 90% respectively. CI(1,1) is thus confirmed.

Table 6. Summary of bivariate models of each yield, R^i , with the Federal Funds Rate Target, \bar{R}^0 (cases where CI(1,1) is not found)

Variable (R^i)	$\phi_{22}(B) = (1 - \phi_{11} B) / (1 - \phi_{21} B - \phi_{22} B^2)$			$\hat{\theta}_{12}$ (s.e.)	$\hat{\theta}_{21}^*$ (s.e.)	Estimated Properties AR(2) Factor		$\hat{\rho}_0$ (s.e.)	Resid. St.Dev. (%)
	$\hat{\phi}_{11}$ (s.e.)	$\hat{\phi}_{21}$ (s.e.)	$\hat{\phi}_{22}$ (s.e.)			Period (Months) (s.e.)	Damping Factor (s.e.)		
3-Year T-Bonds	.47 (.13)	.04 (.13)	-.28 (.09)	-.36 (.05)	-	4.1 (.3)	.53 (.09)	.51 (.08)	.26
5-Year T-Bonds	.52 (.11)	-.01 (.12)	-.29 (.09)	-.36 (.05)	-	4.0 (.3)	.54 (.08)	.45 (.08)	.25
7-Year T-Bonds	.56 (.09)	-.03 (.10)	-.33 (.08)	-.39 (.05)	-	3.9 (.2)	.57 (.07)	.41 (.08)	.24
10-Year T-Bonds	.58 (.10)	-.08 (.10)	-.34 (.09)	-.38 (.06)	-	3.8 (.2)	.58 (.08)	.41 (.08)	.23
30-Year T-Bonds	.60 (.09)	-.11 (.10)	-.32 (.09)	-.36 (.07)	-	3.8 (.2)	.60 (.08)	.37 (.08)	.21
Corporate Bonds (Moody's AAA)	-	.37 (.08)	-.12 (.08)	-.30 (.06)	-	6.2 (1.3)	.35 (.12)	.42 (.08)	.23
30-Year Fixed Mortgage (FHA)	-	-	-	-	-.71 (.13)	-	-	.35 (.08)	.26

Figure 2. Diagnostic tools for the jointly estimated univariate models of the Federal Funds Rate, target \bar{R}^0 and effective R^0 (data graphs standardized)

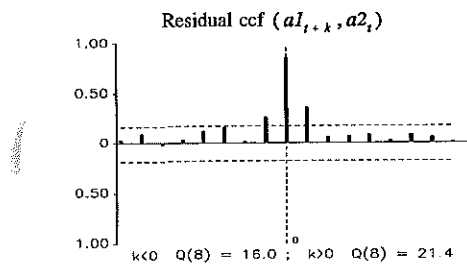
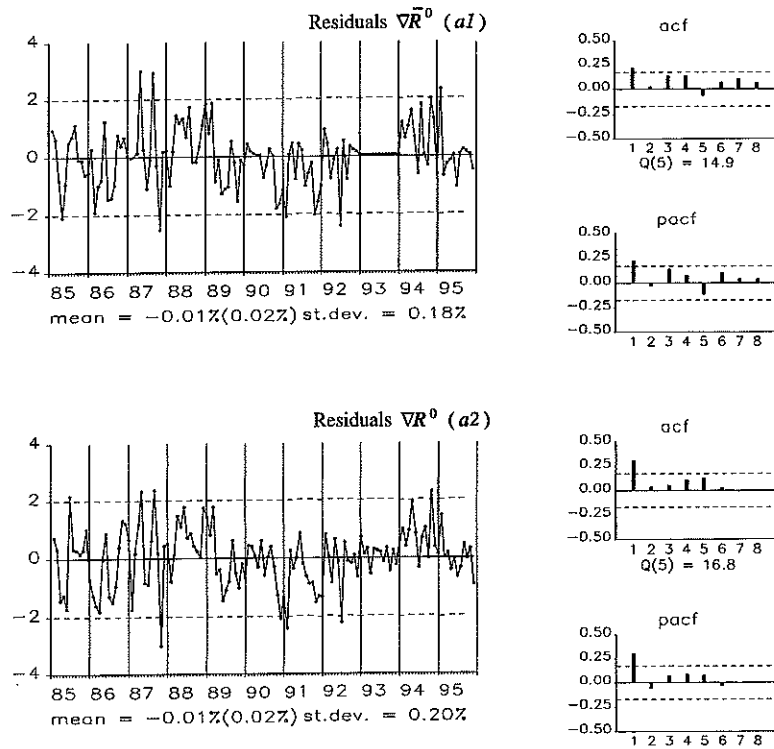


Figure 3. Diagnostic tools for the final bivariate model of the Federal Funds Rate, target \bar{R}^0 and effective R^0 (data graphs standardized)

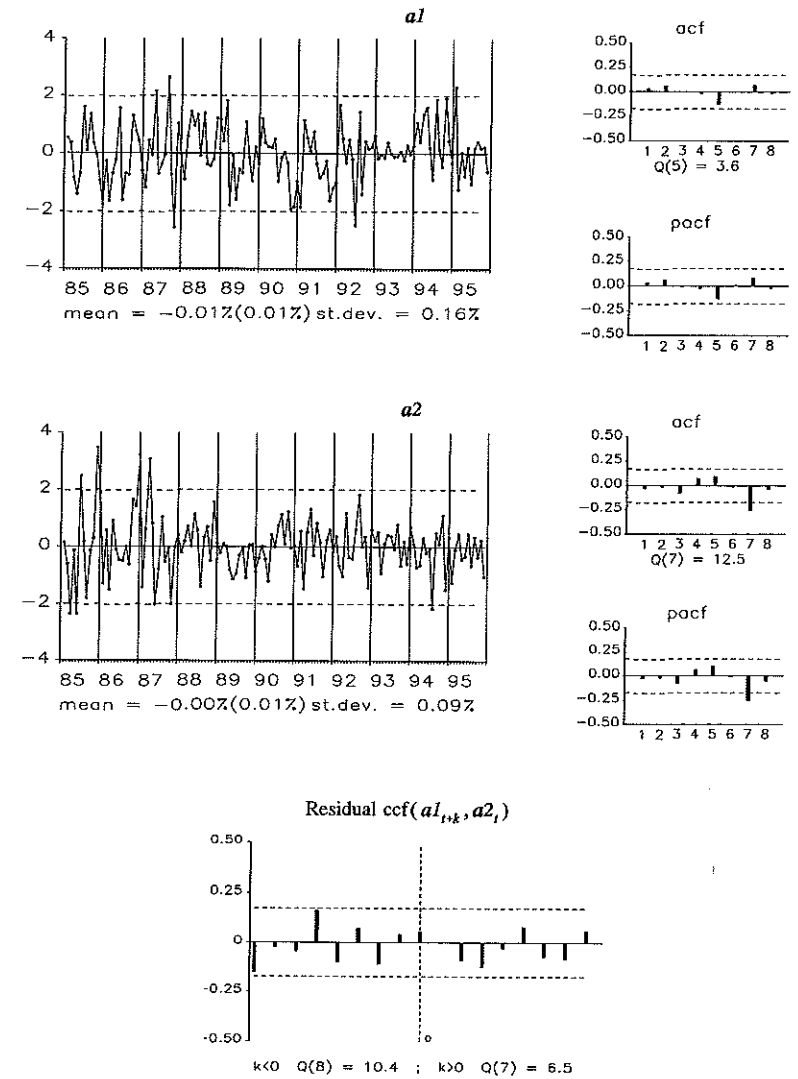


Figure 4. Diagnostic tools for the jointly estimated univariate models of the Federal Funds Rate \bar{R}^0 and the 30-Year T-Bond rate R^{30} (data graphs standardized)

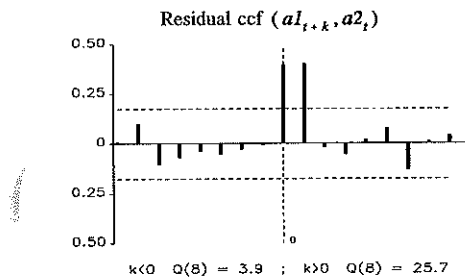
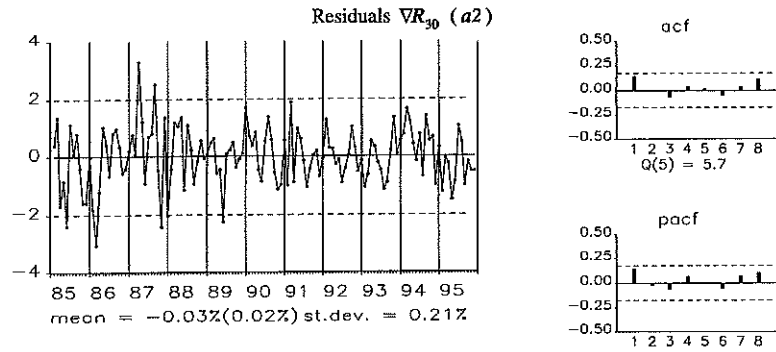
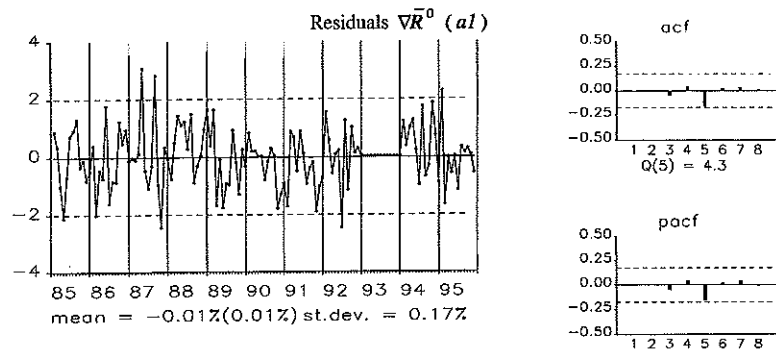


Figure 5. Diagnostic tools for the final bivariate model of \bar{R}^0 and R^{30} (data graphs standardized)

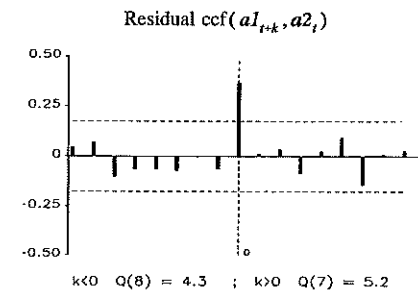
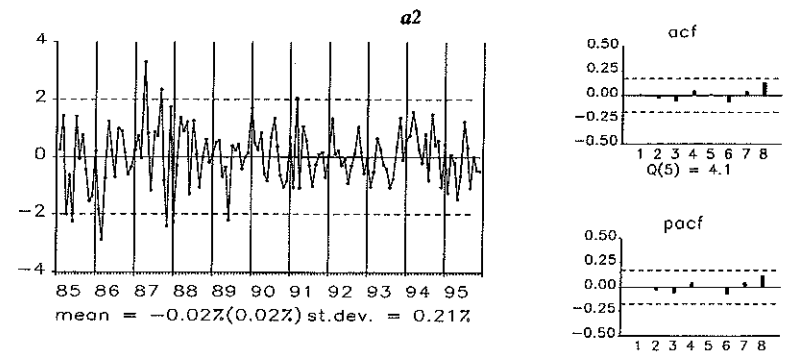
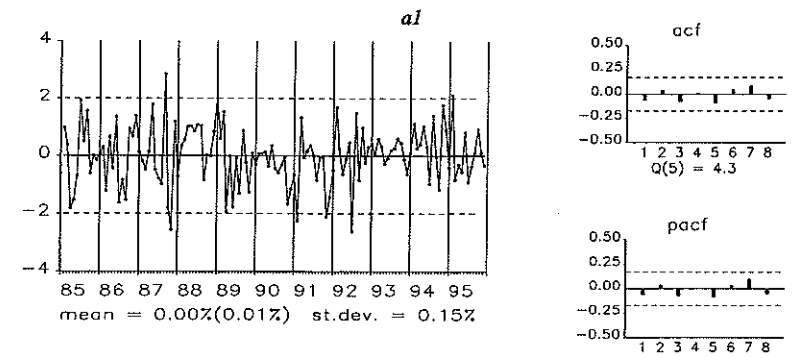


Table 7. Summary of bivariate models of Treasury yields with the McCulloch rate of the same maturity

McCulloch Rates	$\hat{\alpha}$ (s.e.)	$\hat{\theta}$ (s.e.)	$\hat{\mu}$ (%) (s.e.)	$\hat{\phi}_{22}$ (s.e.)	$\hat{\theta}_{12}$ (s.e.)	$\hat{\theta}_{21}^*$ (s.e.)	$\hat{\rho}_0$ (s.e.)	Resid. St.dev. (%)
3-Month	.98 (.02)	1.00 (.01)	.02 (.02)	.24 (.10)	-.70 (.14)	-	.48 (.08)	.17
6-Month	.98 (.01)	1.00 (.01)	.10 (.02)	.14 (.10)	-.94 (.11)	-	.42 (.08)	.19
12-Month *	1.01 (.01)	1.00 (.02)	.10 (.02)	.09 (.09)	-1.00 (.12)	-	.52 (.08)	.19
2-Year	1.03 (.01)	1.00 (.03)	-.06 (.02)	.15 (.09)	-.97 (.14)	-	.59 (.07)	.19
3-Year	1.04 (.02)	1.00 (.03)	-.09 (.02)	.28 (.10)	-.87 (.15)	-	.52 (.08)	.20
5-Year	1.07 (.02)	1.00 (.03)	-.27 (.03)	.32 (.09)	-.81 (.13)	.29 (.10)	.53 (.08)	.19
7-Year	1.07 (.02)	1.00 (.01)	-.29 (.03)	.35 (.08)	-.86 (.12)	.30 (.09)	.57 (.08)	.19
10-Year	1.05 (.03)	1.00 (.01)	-.07 (.03)	.46 (.08)	-.89 (.09)	.49 (.09)	.64 (.08)	.21

* The Treasury yield is on the T-bill; this yield operates in a CI(1,1) relationship with the T-bond with unit CI coefficient.

Figure 6. Diagnostic tools for the initial block diagonal trivariate model of ∇R^0 , ∇R^{30} and ∇R^3 (data graph standardized)

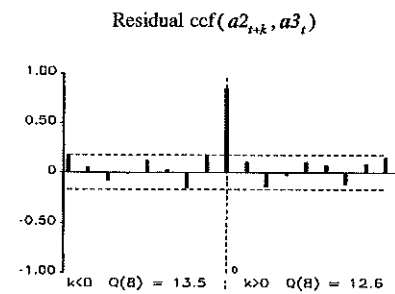
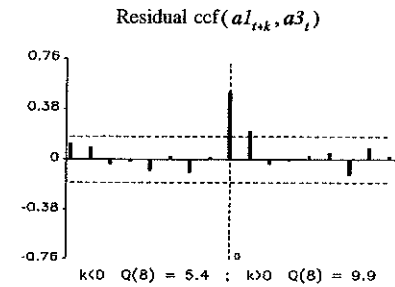
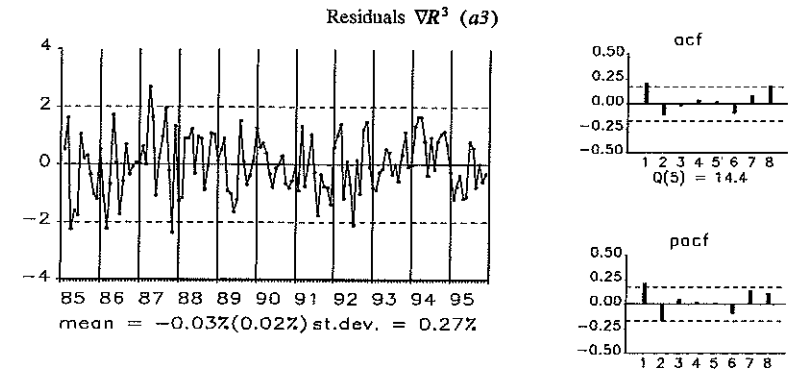


Figure 7. Diagnostic tools for the final trivariate model of

∇R^0 , ∇R^{30} and ∇R^3 (data graph standardized)

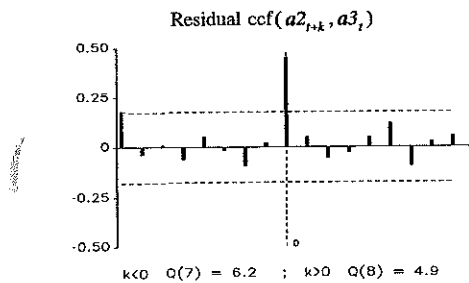
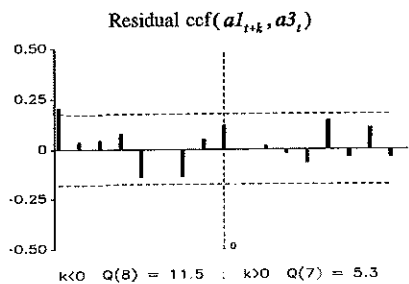
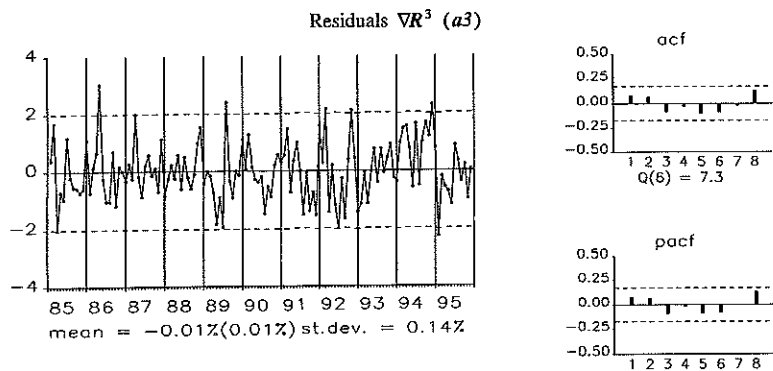


Table 8. Summary of trivariate models containing the two nonstationary common factors

Variable	$\hat{\alpha}_1$ (s.e.)	$\hat{\alpha}_2$ (s.e.)	$\hat{\theta}^2$ (s.e.)	$\hat{\mu}(\%)$ (s.e.)	$\hat{\phi}_{33}$ (s.e.)	$\hat{\theta}_{33}$ (s.e.)	$\hat{\theta}_{13}$ (s.e.)	$\hat{\theta}_{31}$ (s.e.)	$\hat{\theta}_{32}(B)$ (s.e.)	$\hat{\rho}_{13}$ (s.e.)	$\hat{\rho}_{23}$ (s.e.)	Residual St. Dev. (%)
3-Year T-Bonds	.73 (.03)	.63 (.07)	1.00 (.01)	-.89 (.40)	.79 (.05)	-.29 (.08)	-.48 (.10)	-	-.12B (.06)	-.06 (.10)	.47 (.19)	.14
5-Year T-Bonds	.61 (.03)	.81 (.05)	1.00 (.01)	-.88 (.32)	.77 (.04)	-.34 (.07)	-.56 (.12)	-	-.10B (.04)	-.07 (.11)	.39 (.14)	.11
7-Year T-Bonds	.56 (.02)	.91 (.04)	1.00 (.01)	-.92 (.24)	.76 (.05)	-.30 (.07)	-.75 (.15)	-	-.06B (.03)	.12 (.10)	.31 (.13)	.08
10-Year T-Bonds	.51 (.01)	.96 (.03)	1.00 (.01)	-.71 (.19)	.76 (.05)	-.40 (.07)	-.94 (.24)	-	-	-.16 (.10)	.36 (.15)	.05
Corporate Bonds (Moody's AAA)	.45 (.03)	.87 (.06)	1.00 (.01)	1.25 (.40)	.87 (.05)	-	-	-	-	-.08 (.25)	.20 (.35)	.09
30-Year Fixed Mortgage (FHA)	.53 (.03)	.87 (.07)	1.00 (.01)	1.18 (.40)	.74 (.05)	-	-	-.18 (.10)	-.18B -29B ² (.09) (.08)	-.15 (.09)	-.53 (.09)	.22