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Physical interpretation of the paraxial estimator

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ABSTRACT

The paraxial estimator (PE) is a parameter quantifying the paraxiality of a light beam. Even if some of its features were previously tackled, key details on its behavior were not fully presented. This paper robustly presents the physical meaning of the PE in a global way, enlarging its interpretation out of the paraxial region what permits to get a first view of the beam propagation dynamics from the value of this parameter. The physical interpretation is given in the spatial domain and in the spectral domain as well. In the first one, the value of PE is related to the competition between the fast oscillations and the remaining oscillations of a propagating field. Looking at spectral domain, the PE deals with the spectral dispersion (or width) of the plane waves forming the field. In this context, a negative value of PE concerns the effective contribution of the evanescent waves what only happens in a strong nonparaxial regime. The PE also accounts the geometric and physical features on the concept of the paraxial approximation in a natural way. An analysis performed for beams propagating through a spherical thin lens reveals that the loss of paraxiality is due to the geometric effect of ray bending by the lens and by another physical effect, concerning the nonideal collimation of the beam.

1. Introduction

The paraxial wave optics and derived branches have been developed on the hypothesis of the validity of the so-called paraxial approximation (PA) [1,2]. The PA has an ondulatory foundation and a spectral foundation as well. The first one is based on the assumption that the complex amplitude of the field slowly varies when compared with the typical oscillatory phase factor (often called "fast oscillatory term"), $\exp(i\phi_f)$, with $\phi_f = 2\pi z/\lambda$ being z the propagation direction and λ, the beam wavelength. The second one assumes that, if the beam is expanded in terms of plane waves propagating in different directions, the PA holds if and only if the dispersion of all these directions is small and the mean direction is close to the optical axis. In the case of a tightly focused light beam for instance [3], it could occur the failure of the PA validity. It is not a trivial task to accurately delimit the paraxial–nonparaxial limit such that a special care must be taken for not applying the PA to situations in which it is not longer valid, what could lead to a misleading interpretation of the beam propagation dynamic. A robust analysis of the nonparaxial propagation requires refined mathematical tools [4]. The task in quantifying the PA validity by means of a single parameter was firstly done in the interesting work of Seshadri [5]. At the same time, another work [6] introduces a parameter, the so-called "paraxial estimator" (PE), inspired on the idea of comparing propagation invariants associated to Helmholtz and paraxial wave equations. The paraxial equation is a first-order approximation of the exact wave equation [7,8] and its properties were widely study (see for example Refs. [9–11] among others). The PE was defined as [6]

$$\mathcal{P} = \frac{\int_{-\infty}^{\infty} \mathrm{Im}\{E^* \partial_z E\}\, dxdy}{\int_{-\infty}^{\infty} (2\pi/\lambda) E E^*\, dxdy}, \tag{1}$$

where $E(x,y,z,)$ is a paraxial field and $\partial_z \equiv \partial/\partial z$. To define the PE, it has thought the following argument: if the PA is fulfilled, then the numerator as well as the denominator in Eq. (1) both represent the true energy power of the beam crossing a transverse area. In this case, $\mathcal{P} \cong 1$. On the contrary, if the true field no longer is represented by the paraxial solution, the PA fails and the ratio in Eq. (1) moves away from the unit. The PE was applied to elucidate the paraxiality in free space of Hermite–Gauss, Laguerre Gauss [6] and Bessel–Gauss beams [6,8]. Also, a useful relationship between $\mathcal{P}$ and the fundamental parameter in determining the beam quality, the beam propagation factor, M^2 [12], was derived in Ref. [13]. Recently, the PE was used to analyze the validity of the PA under ABCD transformations [14]. From its definition, the range of PE lies between the interval $(-\infty,1)$ [6], but the limit $\mathcal{P} \to 1$ would apparently be the only that has physical meaning since it guarantees the PA validity. Moreover, as $\mathcal{P}$ can take

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negative values, its interpretation out of the paraxial limit could seem even more tangled. Even if some of its features were previously tackled as mentioned above, key details on its behavior were not fully presented. In this communication, we robustly presents the physical meaning of the PE in a global way, enlarging its interpretation out of the paraxial region what permits to get a first view of the beam propagation dynamics from the value of this parameter. This interpretation is related to an approach in the spatial domain and another in the spectral domains. In the spatial domain, it points out the competition between the fast oscillations of the propagating beam and the remaining ones. In the spectral domain, the PE leaks out the spectral dispersion (or width) of the plane wave spectrum of the field. This means that the qualitative dynamic of the beam propagation might be known from, scarcely, the value of $\mathcal{P}$. Besides, in addition to the free space propagation, the PE also accounts the geometric and physical features on the concept of the paraxial approximation in a natural way in a linear optical system. An analysis performed for beams propagating through a spherical thin lens reveals that the loss of paraxiality is due to the geometric effect of ray bending by the lens and by another physical effect, concerning the nonideal collimation of the beam. By ideal collimation, it must understood beam propagation with all rays parallel. As such, the PE naturally interconnects the geometric and the wave optics on the concept of the paraxial approximation.

2. Physical interpretation of the paraxial estimator in free space

In the following, we analyze the physical interpretation of the paraxial estimator. The oscillatory interpretation consists of relating the beam paraxiality to the phase competition of the propagating beam in the real space. The spectral interpretation is concerned in linking the $\mathcal{P}$-value to the amount of plane waves constituting the beam in the conjugate space.

2.1. Oscillatory interpretation

A scalar beam E propagating along z can be written as composed by a complex amplitude A and the fast phase term $e^{i\phi_f}$:

$$E = A\exp(i\phi_f) = \Psi\exp[i(\phi_f - \phi_r)], \qquad (2)$$

where the complex amplitude, A, is in turn divided in a real amplitude, Ψ, and a phase, ϕ_r (that we call the remaining phase), containing all the other beam oscillations. The structure consists of a z-dependent part (associated with the Gouy phase shift) and another also dependent on transverse coordinates representing the radial phase factor indicating how a equiphase surface curves from the planar phase front $z = cons$. In this frame, the tying between $\mathcal{P}$ and ϕ_r is formalized in terms of the first moment or mean value of the variation of this latter, $\langle \partial_z\phi_r \rangle$. In fact, by replacing Eq. (2) into Eq. (1), the PE is then expressed as

$$\mathcal{P} = \frac{\partial_z\phi_f}{2\pi/\lambda} - \frac{\int(\partial_z\phi_r)\,\Psi^2 dxdy}{2\pi/\lambda\int\Psi^2\,dxdy} = 1 - \frac{\langle\partial_z\phi_r\rangle}{\partial_z\phi_f}. \qquad (3)$$

This is of key importance since it allows making the rigorous comparison between the fast and remaining phases giving rise to the oscillatory interpretation of the paraxial estimator. The first term in (3) accounts the fast phase variation with respect to itself. The second term compares the variations of the mean value of the remaining phase (numerator) and the fast oscillations (denominator). Hence, the larger the contribution of the remaining phase, the higher the loss of paraxiality of the beam. For a deeper analysis, a positive parameter ε is introduced by

$$\langle\partial_z\phi_r\rangle = (2\pi/\lambda)\varepsilon, \qquad (4)$$

so that $\mathcal{P} = 1 - \varepsilon$. Thereby, one is able to do an oscillatory interpretation of $\mathcal{P}$, as illustrated in Fig. 1 (a). From (4), the mean *remaining phase* will have an increment of $\langle\Delta\phi_r\rangle = 2\pi\varepsilon$ over a propagation distance $\Delta z = \lambda$, while the *fast phase* will have an increment of $\Delta\phi_f = 2\pi$. If $\mathcal{P}\to 1$, then $\varepsilon \ll 1$ and, thereby, $\langle\Delta\phi_r\rangle \ll \Delta\phi_f$. Therefore, the change in the mean value of the remaining phase (continuum curve) will be negligible with respect to the fast phase change (dashed curve) as Fig. 1 (a) illustrates. This is the usual hypothesis in wave optics for the paraxial approximation fulfilment. Hence, the remaining phase is often called the "slow phase" in this regime. Later on, $\mathcal{P}$ moving away from the unit ($\mathcal{P} \not\cong 1$) means a *loss of paraxiality* for the beam. If $0 < \mathcal{P} < 1$, then $\varepsilon < 1$ so that $\langle\Delta\phi_r\rangle < \Delta\phi_f$. In this frame, the remaining phase always varies less than the fast phase but, even so, it can be significative to break the PA validity as Fig. 1 (a) indicates. The wave propagation occurs out of the paraxial regime. Already within a strongly nonparaxial regime, the value $\mathcal{P} = 0$ implies $\varepsilon = 1$ and, hence, $\langle\Delta\phi_r\rangle = \Delta\phi_f$. The change in the mean remaining phase exactly equals to the change

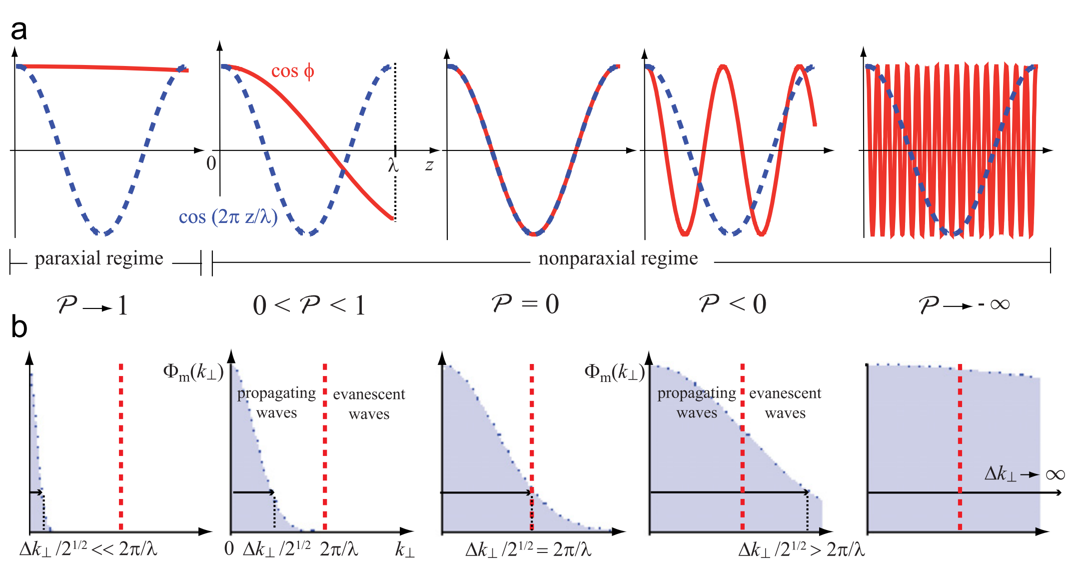

Fig. 1. Oscillatory and spectral interpretation of the paraxial estimator. Part (a) compares the mean slow oscillations ($\langle\phi_s\rangle$) and fast oscillations (ϕ_f) of a wave field. Part (b) depicts the spectral wide of the field.

acquired by the fast phase. Finally, $\mathcal{P}<0$ implies that $\varepsilon>1$ and, so, $\langle\Delta\phi_r\rangle>\Delta\phi_f$. As Fig. 1 (a) shows, this means a major increment of the mean remaining phase with respect to what happens in the fast phase. In particular, when $\mathcal{P}\to-\infty$, there is a tendency to infinite oscillations of the mean remaining phase in a distance equal to the wavelength ($\varepsilon\to\infty$).

Let us exemplify with a Laguerre–Gauss (LG) mode. The transverse size parameter is w_0 [1] and the minimum spot size is $w_m=\sqrt{N+1}w_0$ [15] with N giving the mode of the beam. Only the fundamental Gaussian beam ($N=0$) fulfills $w_m=w_0$. The PE for this beam is then given by $\mathcal{P}=1-(N+1)^2/[(2\pi/\lambda)w_m]^2$ [6]. The scale setting for the PE follows the criterion for the validity limits of the PA for the fundamental Gaussian mode ($N=0$), as in Ref. [1]. The PA may be a questionable hypotheses for $\mathcal{P}$-values of the order of and lower than 0.94 [14]. We choose as an example a beam with $N=10$ that is currently obtained in experiments. From the w_m-value, one can derive the $\mathcal{P}$-value and, thereby, to infer the influence of the remaining oscillations in the beam propagation dynamic. For example, the calculus shows that the PA begins to be critical from $w_m\approx 7.15\lambda$. For $w_m>7.15\lambda$, one can assert that the remaining phase practically does not vary in a length equal to λ being negligible against the fast oscillations. For the beam waist ranged between $1.75\lambda<w_m<7.15\lambda$, one has a considerable variation in the remaining phase in a wavelength distance so that the PA fails. The remaining oscillations are significative but still lesser than the fast oscillations. For $w_m=1.75\lambda$, the PE is null and both oscillations are equal. Already, for $w_m<1.75\lambda$ (in practice, a unusual spot size) the remaining oscillations overcome the fast ones and the regime is extremely nonparaxial.

2.2. Spectral interpretation

The paraxial estimator can be expressed in the spectral domain by using the angular spectrum formalism [16] to give a physical interpretation even out of the paraxial region. The partial waves forming the beam can be divided into two parts, the first one contains only propagating waves while the second part consists of evanescent waves. Such a division straightforwardly relates the competition of both kinds of partial waves to the values of the paraxial estimator. A finite extension of the beam in the transverse directions x and y implies a non-null distribution of transverse wavenumbers, k_x and k_y. This spectral distribution is directly related to the beam spreading. Following Ref. [13], the PE can be expressed as

$$\mathcal{P}=1-(\lambda/2\pi)^2(\Delta k_x^2+\Delta k_y^2)/2. \tag{5}$$

where Δk_i^2 is the variance or dispersion of k_i, with $i=x,y$ that equals the second-order moments

$$(\Delta k_i)^2\equiv\langle k_i^2\rangle=\frac{\int(k_i-0)^2|\mathcal{E}_0(k_x,k_y)|^2\,dk_x\,dk_y}{\int|\mathcal{E}_0(k_x,k_y)|^2\,dk_x\,dk_y}, \tag{6}$$

where $\mathcal{E}_0(k_x,k_y)$ is the angular spectrum of the beam at the waist given by the Fourier transform of $E(x,y,0)$ and it fulfils that $\langle k_i\rangle=0$. For circularly symmetric beams, $\mathcal{P}$ assumes the form

$$\mathcal{P}=1-\frac{1}{2}\frac{\int k_\perp^2|\tilde{\Phi}_m(k_\perp,0)|^2\,dk_x\,dk_y}{(2\pi/\lambda)^2\int\tilde{\Phi}_m(k_\perp,0)|^2\,dk_\perp}=1-\frac{1}{2}\frac{\Delta k_\perp^2}{(2\pi/\lambda)^2}, \tag{7}$$

where $\int k_\perp^2|\tilde{\Phi}_m(k_\perp,0)|^2\,dk_x\,dk_y/\int|\tilde{\Phi}_m(k_\perp,0)|^2dk_\perp=\langle k_\perp^2\rangle$ can be addressed to as the dispersion of the transverse wave number $\Delta k_\perp^2$ (under the occurrence of a corresponding null mean value) for azimuthal symmetry. The function $\tilde{\Phi}_m(k_\perp,0)$ represents the Hankel transform of the radial function $\Phi_m(\rho,0)$ at the initial plane $z=0$, This function directly relates to the field complex amplitude $A(x,y,z)$ according to the usual adopted factorization $A(x,y,z)=\Phi_m(\rho,z)e^{im\varphi}$, with (ρ,φ) addressing polar coordinates in the transverse spatial domain as $(k_\perp,\theta)$ those in the corresponding spatial frequency domain.

Eqs. (5) and (7) are the main result to give the spectral interpretation since they straightforwardly relate the PE to the angular dispersion of the plane wave spectrum constituting the propagating beam. Let us analyze the PE in terms of the spectral contribution of the beam. In Fig. 1(b) the spectral distribution of the field, $\tilde{\Phi}_m(k_\perp)$, is represented by the shadow region. As $\Delta k_\perp$ represents the standard deviation of a such distribution, a continuous number of transverse spatial frequencies contributes to the field up the value given by $k_\perp=\Delta k_\perp/\sqrt{2}$. It is well known that the angular spectrum can be constituted by both, propagating plane waves for $k_\perp/\sqrt{2}\le 2\pi/\lambda$ as well as evanescent waves for $k_\perp/\sqrt{2}>2\pi/\lambda$ [16]. The existence of these waves in free space is strongly supported both theoretically and experimentally [17] even for vectorial fields [18]. The dashed line in Fig. 1(b) divides both kinds of waves. In the paraxial regime, $\mathcal{P}\to 1$ implies that $\Delta k_\perp\ll 2\pi/\lambda$. Only a very narrow distribution of spatial frequencies close to $k_\perp=0$ contributes to the field as Fig. 1(b) shows. One has a slightly spreading beam. As $\mathcal{P}$ moves away from the unit but even being positive, the width $\Delta k_\perp$ increases and a greater number of spatial frequencies contribute to the field so that the PA could fail, increasing the beam spreading. While $\Delta k_\perp/\sqrt{2}<2\pi/\lambda$, only propagating plane waves contribute to the beam. In a strong nonparaxial regime, $\mathcal{P}=0$ means that $\Delta k_\perp/\sqrt{2}=2\pi/\lambda$. The full spectrum of propagating waves significatively contributes to the beam as the Fig 1(b) shows. On the other hand, $\mathcal{P}<0$ implies $\Delta k_\perp/\sqrt{2}>2\pi/\lambda$. One has a significative contribution of wavenumbers corresponding to evanescent waves in addition to the contribution of propagating waves. Hence, negative values of the PE point out the effective contribution of evanescent waves to the field. In the example of the Laguerre–Gauss mode with $N=10$, the effective contribution of evanescent waves might only occur for a beam spot size less than 1.24λ. As this values is in practice unusual, one can state the angular spectrum for a LG_{10} beam is only constituted for propagating waves even for a strong nonparaxial case. Finally, the limit $\mathcal{P}\to-\infty$ implies that $\Delta k_\perp\to\infty$. All the spatial frequencies corresponding to propagating as well as evanescent waves significatively contributes to the beam.

3. Paraxial estimator in the context of a linear optical system

The paraxial estimator can also be interpreted when the beam passes through a linear optical element. What we want to highlight, in this sense, is that the paraxial estimator includes geometric and physical features on concept of the paraxial approximation in a natural way. To see this, let us consider, for instance, a LG beam (or also an Hermite–Gauss (HG) beam) passing through a thin spherical lens. We take this particular ABCD system composed by free space and a thin spherical lens as a key example, since the most important optical transformations, such as Optical Fourier Transform, Optical Imaging Systems and more complex ones, Gyrator and Rotator systems, as well as Fractional Fourier transforms can be implemented using lenses and free space. Hence, we assert that this system is the most emblematic ABCD systems since it acts as a basis system of more complex composed systems. Besides, this system is sufficient to show the potentiality of $\mathcal{P}$ in the convergence of the geometric and physical optics. Before the lens, the PE in free space propagation for these modes is [6]

$$\mathcal{P}=1-(N+1)\lambda^2/(4\pi^2w_0^2). \tag{8}$$

When the beam is modified by the lens, the paraxiality of the transformed beam, $\mathcal{P}'$, will be given by (8) but with a new waist parameter, w'_0, instead w_0. Necessarily, w'_0 and, therefore $\mathcal{P}'$, will depend on the parameters characterizing the optical element, in this case, on the focal distance f. Behind the lens, $\mathcal{P}'$ can be calculated by two ways: by one hand, having into account the change in amplitude and phase distributions, and on the other hand, by using the ABCD theory transforming the beam second order moments as done in [14]. Both ways must necessarily give identical results for $\mathcal{P}'$. First, let us see if this key fact is fulfilled. Only then the paraxial estimator after a lens transformation can be physically interpreted. The action of the lens modifies the beam phase by introducing an additional quadratic term. Hence, it is well known (see for example Ref. [2]) that the relationship between w_0 and w'_0 is

$$w'_0 = w_0 |f/(z-f)| [1 + z_R^2/(z-f)^2]^{-1/2}, \tag{9}$$

where z_R is the Rayleigh distance and z is the distance between the waist plane and the lens plane. Introducing (9) in (8), one has that the beam paraxiality after passing by the lens is given by

$$\mathcal{P}' = 1-(N+1)\frac{\lambda^2 (z/f-1)^2}{4\pi^2 w_0^2} - (N+1)\frac{w_0^2}{4f^2}. \tag{10}$$

The above expression is identical to the derived in Ref. [14] by using the ABCD theory what supports the robustness of the PE. Let us now interpret the two terms that decrease the beam paraxiality in Eq. (10). With this aim, we first analyze the ray bending by a spherical thin lens (a phenomenon arising from geometric optic). In this frame, the angles of the refracted and incident paraxial rays are related by $\theta' = \theta - y/f = \theta - z \tan\theta/f$ [2] that in paraxial approximation can be written as

$$\theta' \approx \theta(1-z/f). \tag{11}$$

The divergence angle (or far field angle) for HG and LG modes before and after the lens is well known [2] and given by $\theta = \lambda/(\pi w_0)$ and $\theta' = \lambda/(\pi w'_0)$, respectively. Replacing both angles into (11), it obtains $\lambda/(\pi w'_0) \approx \lambda/(\pi w_0)(1-z/f)$. Solving for w'_0 and replacing in Eq. (8) (with w'_0 instead of w_0), one finally obtains the *geometric paraxial estimator*

$$\mathcal{P}'_g = 1-(N+1)\lambda^2 (z/f-1)^2/(4\pi^2 w_0)^2, \tag{12}$$

that justly coincides with (10) with exception of the third term. From this analysis, it can be asserted that the second term in (10) is a pure geometric term accounting the loss of paraxiality by the ray bending effect. Where does then the third term in (10)? This reflects the loss of paraxiality as a consequence that a real beam cannot be perfectly collimated (all rays being parallels). To a lesser collimation, a greater loss of paraxiality. In order to analyze this feature, we assume the waist of the incident beam placed at the lens focal plane, what means to perform the beam collimation process. In this case, $z=f$ and Eq. (10) becomes the so-called *collimation paraxial estimator*

$$\mathcal{P}'_c = 1-(N+1)w_0^2/(4f^2). \tag{13}$$

In this frame, the transmitted beam becomes in a collimated beam having its waist located at the lens plane. In geometric optics, the ideal collimation implies a null spot size. But for real beams, $w_0 \neq 0$. This waist is related to w'_0 by $w_0 \approx \lambda f/(\pi w'_0)$ [2]. Replacing in (13), one obtains $\mathcal{P}'_c$ once the beam collimation process have been performed:

$$\mathcal{P}'_c = 1-(N+1)\lambda^2/(4\pi w_0'^2). \tag{14}$$

This is the free space paraxial estimator with waist parameter w'_0. Hence, the third term in (10) has taken into account the loss of paraxiality due to the nonideal collimation of a paraxial beam. In fact, only if $w_0 = 0$ and/or $f = \infty$, then $\mathcal{P}' = 1$ and the beam is perfectly collimated (all rays parallel). But $\mathcal{P}' = 1$ is not possible showing that the perfect collimation is unrealizable for real beams. As the spot size increases and the focal length decreases, the third term in (10) increases indicating a major loss of paraxiality since the collimation turns more poor (the beam spreading, in this case, increases). Therefore, this term arisen from ondulatory optics does not have analog in geometric optics. For this reason it does not depend on the wavelength since $\lambda \to 0$ is the limit from wave optics to geometric optics. The third term in (10) is also peculiar because it is opposed to the intuitive fact that the PA only fails when $w_0 \to 0$ as in free space propagation. Therefore, it is clear from above that both terms of PE have different origin, one a physical origin and the other, a geometric one. Besides, the perfect collimation of the beam (in the geometric optics sense) is not possible for a real beam. It will imply necessarily $\mathcal{P} = 1$, a limit that cannot be reached.

For a beam passing through a spherical thin lens, the PA becomes questionable for great values of w_0 since a large spot size produces a less collimated beam at the exit of the lens. For example, in many practical applications such as laser scanning, laser printing and laser fusion, it is desirable to generate the smallest spot size at the exit of the lens. This may be achieved by use of the thickest incident beam and the shortest focal length for a given wavelength. In experiments, it is not surprising to have $w_0 \approx 10$ mm for a lens with focal $f = 10$ mm for such a purpose [2]. In such case, a spherical thin lens system crossed by a Gaussian beam produces a strongly uncollimated beam giving $\mathcal{P} = 0.75$ and the validity of the paraxial approximation is clearly broken for a large value of the beam spot size ($w_0/\lambda = 2 \times 10^4$ for $\lambda = 0.5$ μm).

4. Conclusions

The physical interpretation of the paraxial estimator was given. Even if some of its features were previously tackled, key details on its behavior were not fully presented. Hence, the PE was robustly presented in a global way, enlarging its interpretation out of the paraxial region. The physical interpretation was given in the spatial domain and in the spectral representation as well. In the first one, the value of PE was related to the competition between the fast oscillations and the remaining oscillations of a propagating field. Looking at spectral domain, the PE deals with the spectral dispersion (or width) of the plane waves forming the field. In this context, a negative value of PE was related to the effective contribution of the evanescent waves to the constitution of the optical beam what only happens in a strong nonparaxial regime. The PE has also accounted the geometric and physical features on the concept of the paraxial approximation in a natural way. An analysis performed for beams propagating through a spherical thin lens reveals that the loss of paraxiality is due to the geometric effect of ray bending by the lens and by another physical effect, concerning the nonideal collimation of the beam. In summary, the obtained results permits to get a first view of the beam propagation dynamics by only knowing the value of this parameter.

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