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ScienceDirect
Optics Communications 281 (2008) 756–759
OPTICS COMMUNICATIONS
www.elsevier.com/locate/optcom

Beam quality changes of radially and azimuthally polarized fields propagating through quartic phase plates

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Received 18 July 2007; received in revised form 11 September 2007; accepted 6 October 2007

Abstract

In terms of the so-called irradiance moments of a light field, the beam quality change, ΔQ, of radially and azimuthally polarized beams caused by propagation through a quartic phase plate (as occurs, for example, in strongly pumped laser rods used in high-power solid-state lasers) is studied. Analytical expressions for ΔQ are given, and a comparison between the scalar and vectorial regimes is also shown. The results are applied to several cases of interest.

Keywords: Beam quality; Beam propagation factor; Radially polarized beams; Thermal lensing; Solid-state lasers

1. Introduction

As is well known [1–8], in high-power Nd:YAG lasers the rod can be considered as a birefringent lens. In this kind of laser materials, inhomogeneous thermal effects combined with birefringent mechanical strains (also introduced by the temperature gradient) can generate, in practice, wavefront distortions with higher than quadratic dependence on radial position, the first of which corresponds to spherical aberration. As a consequence, both the power range of the stable operation and the beam propagation factor (also called beam quality) are negatively affected.

In the last years, radially polarized beams [9–17] in solid-state lasers have been used to reduce such thermally-induced effects. The aim of the present work is to analytically determine the beam quality change of a radially (or azimuthally) polarized field, caused by quartic phase distortions, in terms of the so-called irradiance moments of the beam [18–23].

The paper is arranged as follows: In the next section, the formalism and the key definitions are introduced. In Section 3, the beam quality change of a radially (or azimuthally) polarized field is analysed when the beam propagates through a quartic phase plate. A comparison between scalar and vectorial expressions is also given. In Section 4, the above results are applied to some illustrative cases of interest. Finally, Section 5 summarizes the main conclusions of this work.

2. Formalism and key definitions

Let us consider a paraxial vectorial beam whose electric field vector, transverse to the propagation direction z, is written as follows:

$$\mathbf{E}(x,y) = (E_s(x,y), E_p(x,y), 0), \tag{1}$$

where the subscripts s and p refer to the Cartesian components orthogonal to z, and x and y denote the Cartesian coordinates at a plane $z = \text{constant}$. To get a joint description of the focusing and collimation capabilities of a laser beam, it is useful to evaluate the beam quality parameter, Q, defined for the vectorial case in the form [2,24]

$$Q = \langle r^2\rangle\langle \eta^2\rangle - \langle \mathbf{r}\cdot\boldsymbol{\eta}\rangle^2, \tag{2}$$

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0030-4018/$ - see front matter © 2007 Elsevier B.V. All rights reserved.
doi:10.1016/j.optcom.2007.10.096

where

$$\langle r^2\rangle = \frac{P_s}{P}\langle r^2\rangle_s + \frac{P_p}{P}\langle r^2\rangle_p, \tag{3.a}$$

$$\langle \eta^2\rangle = \frac{P_s}{P}\langle \eta^2\rangle_s + \frac{P_p}{P}\langle \eta^2\rangle_p, \tag{3.b}$$

$$\langle \mathbf{r}\cdot\boldsymbol{\eta}\rangle = \frac{P_s}{P}\langle \mathbf{r}\cdot\boldsymbol{\eta}\rangle_s + \frac{P_p}{P}\langle \mathbf{r}\cdot\boldsymbol{\eta}\rangle_p, \tag{3.c}$$

with $k\boldsymbol{\eta} = (ku,kv) = (k_x,k_y)$ giving the wavevector components along the x- and y-axis (k being the wavenumber of the light field and $\boldsymbol{\eta} = (u,v)$). In Eqs. (3.a), (3.b), (3.c)

$$\langle\alpha\beta\rangle_i = \frac{1}{P_i}\frac{k^2}{4\pi^2}\int\int\int \alpha\beta\overline{E_i^*(\mathbf{r}+\mathbf{s}/2,z)E_i(\mathbf{r}-\mathbf{s}/2,z)} \times \exp(ik\mathbf{s}\cdot\boldsymbol{\eta})\mathrm{d}\mathbf{s}\mathrm{d}\mathbf{r}\mathrm{d}\boldsymbol{\eta},\quad i=s,p;\alpha,\beta=x,y,u,v, \tag{4}$$

represent the second-order moments associated to the respective field components (the overbar symbolizes an ensemble average), $P_i = \int\overline{|E_i(\mathbf{r})|^2}\mathrm{d}\mathbf{r}, i=s,p$, denotes the power of the i-component, and $P = P_s + P_p$. Note that the presence of the factors P_i/P, $i = s, p$, in the right-hand side of Eqs. (3.a), (3.b), (3.c) arises from the different normalization constants, P and P_i, of the moments associated to the global beam and to the field components, respectively. In Eqs. (2) and (3), the moment $\langle r^2\rangle$ represents the overall beam width (squared) at a transverse plane z = constant (i.e., the spatial size of the cross-section where the irradiance takes significant values); $\langle\eta^2\rangle$ is a measure of the (squared) beam divergence at the far field; $\langle\mathbf{r}\cdot\boldsymbol{\eta}\rangle$ gives the position of the beam waist, i.e., the plane where the beam width takes its minimum value (at this plane, $\langle\mathbf{r}\cdot\boldsymbol{\eta}\rangle$ vanishes). Taking these into account, small values of parameter Q mean high focusability (small beam width) along with low beam divergence at the far field, which are the required conditions in a number of laser applications.

For convenience, we will use in the present work planar polar coordinates, r and θ. Accordingly, the transverse field would read

$$\mathbf{E}(r,\theta) = \left(E_s(r,\theta),E_p(r,\theta),0\right), \tag{5}$$

and, for a coherent laser beam, the second-order irradiance moments that appear in Eq. (2) become

$$\langle r^2\rangle_i = \frac{1}{P_i}\int_0^\infty\int_0^{2\pi} r^2|E_i(r,\theta)|^2 r\mathrm{d}r\mathrm{d}\theta, \tag{6.a}$$

$$\langle \eta^2\rangle_i = \frac{1}{k^2P_i}\int_0^\infty\int_0^{2\pi}\left(\left|\frac{\partial E_i}{\partial r}\right|^2 + \frac{1}{r^2}\left|\frac{\partial E_i}{\partial\theta}\right|^2\right) r\mathrm{d}r\mathrm{d}\theta, \tag{6.b}$$

$$\langle \mathbf{r}\cdot\boldsymbol{\eta}\rangle_i = \frac{1}{2ikP_i}\int_0^\infty\int_0^{2\pi} r\left(E_i^*\frac{\partial E_i}{\partial r} - E_i\frac{\partial E_i^*}{\partial r}\right) r\mathrm{d}r\mathrm{d}\theta. \tag{6.c}$$

We will next apply these general expressions to radially and azimuthally polarized fields.

3. Beam quality changes

To begin with, let us write the electric field amplitude of a coherent radially and azimuthally polarized field:

$$\mathbf{E}_R(r,\theta) = f(r)(\cos\theta,\sin\theta,0), \tag{7.a}$$

$$\mathbf{E}_\theta(r,\theta) = f(r)(-\sin\theta,\cos\theta,0), \tag{7.b}$$

where the subscripts R and θ stand for radial and azimuthal polarization, respectively. For both types of beams we have

$$P_s = P_p. \tag{8}$$

Furthermore, the global second-order moments (see Eqs. (3.a), (3.b), (3.c)) are now given by

$$\langle r^2\rangle = \frac{1}{P'}\int_0^\infty r^2|f(r)|^2 r\mathrm{d}r \tag{9.a}$$

$$\langle \eta^2\rangle = \frac{1}{k^2P'}\int_0^\infty\left(\left|\frac{\mathrm{d}f}{\mathrm{d}r}\right|^2 + \frac{1}{r^2}|f(r)|^2\right) r\mathrm{d}r \tag{9.b}$$

$$\langle \mathbf{r}\cdot\boldsymbol{\eta}\rangle = \frac{1}{2ikP'}\int_0^\infty r\left(f^*\frac{\mathrm{d}f}{\mathrm{d}r} - f\frac{\mathrm{d}f^*}{\mathrm{d}r}\right) r\mathrm{d}r, \tag{9.c}$$

where

$$P' = \frac{P}{2\pi} = \int_0^\infty |f(r)|^2 r\mathrm{d}r. \tag{10}$$

Note that the above expressions do not depend on the coordinate θ.

It is interesting to compare these formulae (calculated by using Eq. (2) along with the definitions of the *vectorial* fields given by Eq. (7)) with the corresponding expressions for *scalar* field rotationally-symmetric around the z-axis. It should be noted that, in the *scalar* case, we would have to apply the scalar definition of the beam quality parameter (see, for example, Refs. [18–23]). After some calculations, we get that the only but important difference between both cases arises from the term $|f|^2/r^2$ in Eq. (9.b): In other words, it can formally be written

$$Q_{\mathrm{vectorial}} = Q_{\mathrm{scalar}} + \langle r^2\rangle g, \tag{11}$$

where

$$g = \frac{1}{k^2P'}\int_0^\infty \frac{|f(r)|^2}{r^2} r\mathrm{d}r. \tag{12}$$

The second term on the right-hand side of Eq. (11) does not appear in the scalar regime. Since this term is positive, it then follows that the value of the vectorial beam quality parameter is always greater than the corresponding value of the scalar parameter.

Let us now consider that the radially (or azimuthally) polarized beam propagates through a strongly pumped laser rod. For simplicity, it will be assumed in the calculations that the beam diameter does not change, in a significant way, along the rod. This approach will be accurate for small values of the ratio L/z_R, where L denotes the length of the rod and z_R the Rayleigh length of the beam.

As was pointed out in Section 1, at high pump power, combined thermal lensing and birefringence results in a spherically aberrated lens, or, equivalently, in an induced phase plate, $\exp[\mathrm{i}\,k\Delta(r)]$, where

$$\Delta(r) = \alpha\frac{r^2}{2} + \beta\frac{r^4}{4}, \tag{13}$$

where α and β are, respectively, the quadratic and quartic coefficients containing the physical characteristics of the overall process [7] (thermal conductivity, heat load, shape of pumping distribution, etc.). The values of α and β also depend on the polarization of the incident beam (radial or azimuthal). The possibility of vanishing the low-order spherical aberration (SA) by using a non-homogeneous pumping distribution is described in detail in Ref. [7]. However, it is interesting to remark [25] that when low-order SA vanishes high-order spherical aberration could rapidly increase. Consequently, in terms of beam quality preservation, a compromise should be established between low- and high-order SA.

Here we are interested on the beam quality changes generated by the quartic term of Eq. (13). After the application of Eqs. (2) and (9), together with the field definitions (7.a) and (7.b), it is not difficult to show that the beam quality change, ΔQ, induced by spherical aberration, is given by

$$\begin{aligned}\Delta Q &= Q_{\text{out}} - Q_{\text{inp}} \\ &= 2\beta(A\langle r^2\rangle - \langle r^4\rangle\langle \mathbf{r}\cdot\boldsymbol{\eta}\rangle) + \beta^2(\langle r^2\rangle\langle r^6\rangle - \langle r^4\rangle^2),\end{aligned} \tag{14}$$

where the subscripts "inp" and "out" refer to the input and output values of Q before and after crossing the rod, and

$$\langle r^n\rangle = \frac{1}{P'}\int_0^\infty r^n|f(r)|^2 r\mathrm{d}r, \quad n = 2, 4, 6 \tag{15}$$

$$A = \frac{i}{2kP'}\int_0^\infty r^3\left(f\frac{\mathrm{d}f^*}{\mathrm{d}r} - f^*\frac{\mathrm{d}f}{\mathrm{d}r}\right)r\mathrm{d}r. \tag{16}$$

A general consequence can be inferred from Eq. (14): the sign of ΔQ (in other words, degradation or improvement of the beam quality) depends on the spatial shape of the irradiance profile of the propagating beam. In this sense, it should be remarked that the second term of the right-hand side of Eq. (14) is positive for every $f(r)$, whereas the sign of the first term on depends on both the transverse irradiance and the phase distribution of the output field.

Let us finally compare Eq. (14) with the beam quality change in the scalar case [22,23]. It can be seen that ΔQ is the same for both rotationally-symmetric scalar beams and radially (or azimuthally) polarized fields. It should, however, be noted that the ratio $\Delta Q/Q$ differs: In fact,

$$\left(\frac{\Delta Q}{Q}\right)_{\text{vectorial}} < \left(\frac{\Delta Q}{Q}\right)_{\text{scalar}}, \tag{17}$$

so that, for the fields we are comparing, the relative beam quality change is lower in the vectorial case.

4. Application to particular cases

As an illustrative example of interest, let us consider a radially or azimuthally polarized beam whose input amplitude $f(r)$ is written in the form

$$f(r) = f_0(r)\exp(\mathrm{i}kbr^2), \tag{18}$$

where b and f_0 are real quantities. In particular, beams whose amplitude reads [26,27]

$$f_{0n}(r) = rL_n^1\left(\frac{2r^2}{\omega_0^2}\right)\exp\left(-\frac{r^2}{\omega_0^2}\right), \quad n = 1, 2, \ldots, \tag{19}$$

belong to this family (L_n^1 denoting generalized Laguerre polynomials [28]). For such fields we get

$$\langle \mathbf{r}\cdot\boldsymbol{\eta}\rangle = 2b\langle r^2\rangle, \tag{20.a}$$

$$A = 2b\langle r^4\rangle, \tag{20.b}$$

so that

$$\Delta Q = \beta^2(\langle r^2\rangle\langle r^6\rangle - \langle r^4\rangle^2). \tag{21}$$

Then $\Delta Q > 0$, and we conclude that, in this case, the beam quality is always degradated after passing through highly pumped laser rods. Note also that, for this kind of radially (or azimuthally) polarized fields, the beam quality degradation only depends on the shape of the irradiance profile. Moreover, ΔQ is independent of the position of the waist plane of the incident beam. Fig. 1 shows ΔQ (a proportionality factor apart) for the set of beams whose amplitudes are given by Eq. (19). We see that the beam quality degradation strongly increases for high values of the order n.

Note that ΔQ, given by Eq. (21), can also be written in the form

$$\Delta Q = \frac{16}{k^2}\left(\frac{k\beta\langle r^2\rangle^2}{4}\right)^2\left[\frac{\langle r^6\rangle}{\langle r^2\rangle^3} - \left(\frac{\langle r^4\rangle}{\langle r^2\rangle^2}\right)^2\right]. \tag{22}$$

After comparation with Eq. (13), we see that the factor $(k\beta\langle r^2\rangle^2/4)$ closely resembles the quartic phase term. Thus this factor explicitly exhibits the influence of the spherical aberration on the beam quality degradation.

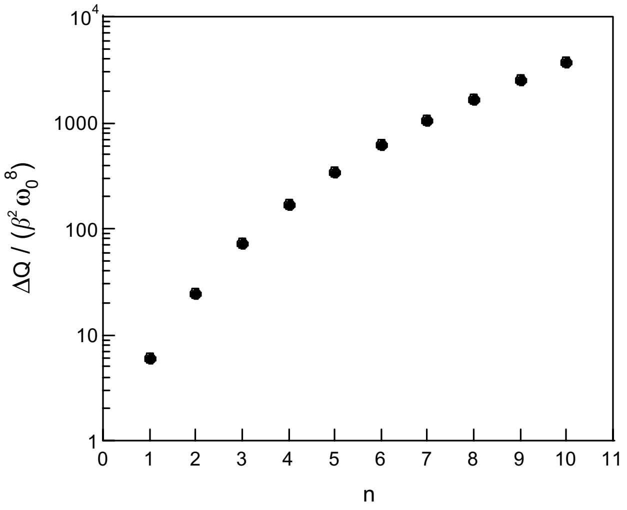

Fig. 1. Plot of the dimensionless ratio $(\Delta Q/\beta^2\omega_0^8)$ for the set of radially (or azimuthally) polarized beams whose amplitude is defined by Eq. (19). The abscisses give the order n of the generalized Laguerre polynomials L_n^1.

Let us finally consider a radially (or azimuthally) polarized beam whose incident amplitude $f(r)$ (see Eq. (7)) takes the form

$$f(r) = [I(r)]^{1/2} \exp[\mathrm{i}k\varphi(r)], \tag{23}$$

where

$$\varphi(r) = \frac{ar^4}{4} + \frac{br^2}{2}, \tag{24}$$

and $I(r)$ represents the irradiance. Thus Eq. (16) now reads

$$A = \frac{1}{P'} \int_0^\infty r^3 \frac{\mathrm{d}\varphi(r)}{\mathrm{d}r} I(r) r \mathrm{d}r, \tag{25}$$

along with

$$\langle \mathbf{r} \cdot \boldsymbol{\eta} \rangle = \frac{1}{P'} \int_0^\infty r \frac{\mathrm{d}\varphi(r)}{\mathrm{d}r} I(r) r \mathrm{d}r. \tag{26}$$

After simple calculations, Eq. (14) becomes

$$\Delta Q = (\langle r^6 \rangle \langle r^2 \rangle - \langle r^4 \rangle^2)(2\beta a + \beta^2). \tag{27}$$

Two special cases deserve attention:

- $a = 0$ (as occurs in the above Laguerre–Gauss example). In this case we have $\Delta Q > 0$, for any b and $I(r)$.
- $a = -\beta$. From Eq. (27) we then get $\Delta Q < 0$, for any b and $I(r)$. In other words, for this kind of polarized fields, a quartic phase plate could improve the beam quality.

5. Conclusions

The effect of a quartic phase plate (or, equivalently, a spherically aberrated lens) on the overall spatial structure of a radially (or azimuthally) polarized laser beam has been evaluated by using the (measurable) beam quality parameter Q. The analytical expressions obtained in the present work are valid for both radial and azimuthal polarization and can be of use to know how the beam is altered when passing through a strongly pumped rod in high-power Nd:YAG lasers. It has been shown that the ratio $\Delta Q/Q$ takes lower values in the vectorial case compared with the scalar regime. In the particular but interesting case of the family of generalized Laguerre–Gaussian beams, a simple formula for ΔQ has been provided which reveals that the position of the beam waist has no influence on ΔQ. It can also be concluded that the beam quality drastically degradates for high-order modes of this family. Finally, an analytical example has been provided in which the beam quality improves after propagation through a quarter phase plate.

Acknowledgement

This work has been supported by the Ministerio de Educación y Ciencia of Spain, Project FIS2004-1900, and by CM-UCM, Research Group 910335 (2007).

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