Leptons: charged and uncharged (neutrinos). Inside a proposal for a geometrical model. Some considerations.

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We study the leptons in two different settings, first with the standard theory (sections I and II) and second in a geometrical model (sections III and IV). All the leptons with no null masses ($m \neq 0$).

The starting point in the Dirac equation for free charged leptons using the Pauli-Dirac representation of the $\gamma-$matrices. We examine in detail the roles of the spin, of the helicity ($\mathbf{p} \neq \mathbf{0}$) and of the chirality. The chiral eigenstates can not be eigenstates of the Dirac equation with $\mathbf{p} \neq \mathbf{0}$ and $m \neq 0$. We emphasize the non symmetrical aspect in the relation of the helicity with the chirality. This has an important implication in the statistical interpretation and it is most relevant in the dilemma matter − antimater for the uncharged leptons. We show a solution of the Dirac equation which seems appropriate for the neutrinos. We also pay attention to semantic questions.

In our studies, geometric structure of the leptons and the quarks, we distinguish a spin as a scalar (up − down) and as a vector-spin. They are intrinsic properties for them, as important as the electric charge.

For the charged leptons: the vector-spin is at every instant in a plane perpendicular to the vector that represents the momentum. The magnetic moment and the vector-spin are proportional.

For the uncharged leptons: the intrinsic properties driving to an electromagnetic interaction, even if it is of a very low intensity (absence of electric charge). There are minor modifications appearing as radiative corrections, perhaps the origin for magnetic and electric moments. We propose, tentatively, a mechanism for explaining the oscillations of the uncharged leptons. The non null masses of the neutrinos appear as a consequence.

Our presentation is not standard, suggesting further research at several points (still many possible questions).

Keywords: Spin. Chirality. Helicity. Leptons. Electrons. Positrons. Neutrinos. Dirac, Majorana. Geometry.

Two key (ideal) experiments in the conceptual development of the quantum mechanics and one comment. Briefly.

a) **The Stern-Gerlach experiment.** After passing through an inhomogeneous magnetic field, the 'electrons' (silver atoms) split in two different beams. Let us distinguish them: one with a character ′up′ and another one with a character ′down′. [1] [2]
Upon the selection of only one of the two beams and subjected to a second Stern-Gerlach (the magnets), identical to the previous one, with the same direction, it maintains this only one beam. The magnetic moment (a vector) is the central physical concept with an associated concept, the spin.

It deserves a reading the considerations of Stern, Einstein, Ehrenfest, Feynman and Schwinger in *The Stern-Gerlach Experiment Revisited* (pages 5, 6 and 8) [1]: there is not a classical explanation.

Is this permanency of this character ′up′ or ′down′ an inherent property representing two different types of electrons: the ones ′up′ and the ones ′down′? Completely polarized beans? Statistic.

* *A suggestion for a conservation of the ′up′ or the ′down′ of the free electrons. The spin, a scalar?* Later on we will look at $\mathbf{S}$ and $\mathbf{J} = \mathbf{L} + \mathbf{S}$, vectors.

b) **The double slit experiments.** [2] [3] [4]

With particles (bullets). After a large set of particles, either one by one or in a bunch, there is not an interference pattern.
With water. The presence of a large set of particles (like a continuum) at once, we observe an interference pattern.
With electrons, driven as a few individual events, we observe a behavior as particles, not any one with interference. Upon a collection of a large amount of them (even if it is one by one) an interference pattern shows up; but, a knowledge of the slit that each individual particle is going through do not let the appearance of such pattern. Statistic.

The free particle Dirac equation contains plane wave solutions (wavy aspects) which correspond to the charged leptons. The particles compounding the material building up the slits also satisfy the quantum field theory (also wavy aspects). At this level, 'the observation' alters 'the observed'.

Reminding to P. W. Anderson: *More is different.* [5]

** *How far can we go in implementing the statistical results in the interpretation of the properties and of the concepts of the 'individual' elementary particles?*

c) It is interesting to mention a few lines of **V. Weisskopf** written in the 0. Introduction of the Quantum Field Theory of David Tong: [6] " " "There are no real one-particle systems in nature, not even few-particle systems. The existence of virtual pairs and of pair fluctuations shows that the days of fixed particle numbers are over."

Viki Weisskopf " "

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A Preliminary.

I. ELECTRICALLY CHARGED LEPTONS: ELECTRONS AND POSITRONS (ANTI-ELECTRONS).

• 1 The electric charge. The positrons ↔ anti-electrons were defined as the antiparticles of the electrons. Shortly after their theoretical introduction (Dirac, 1928 - 1931), they were found experimentally (Anderson, 1932).

The antiparticle concept connoted, at that time, two aspects:

0− **opposite** electric charge and **annihilation** with the particles (electrons) in the form of γ rays (also pair production).
Besides:

i − two electrons with whatsoever spins undergo electric repulsion, and do not annihilate each other, (1)

ii− an electron can not become an anti-electron via a simple process, it requires two units of electric charge, but it can evolve to the one with the opposite spin via the emission or the capture of a photon. (2)

• 2 The spin (up − down , or a vector?) The word 'spin' suggest a relation with the rotations as continuous transformations with the time. And, therefore, of an axis of rotation and its defining vector.

With the classical physics we can establish a correspondence of a magnetic moment (an axial vector) with a mechanical angular moment (an axial vector), therefore with some kind of rotation around some axis of rotation which can be defined also via an axial vector. An Amperian model. [7] This works fine for the electrons in the orbitals in the atoms. It appears a first discretization in an angle with a direction which is in the usual space ($\mathbb{R}^3$). Quantum numbers in the old atomic theory.

The splittings in the anomalous Zeeman effect and in the Stern–Gerlach experiment due to magnetic fields tell us of some kind of additional magnetic moment in the materials used in these experiments, which could not be explained via the classical physics.

If we accept an Amperian model for the definition of this additional magnetic moment $\vec{\mu} \in \mathbb{R}^3$ (our ordinary space), we are driven, like Kronig (unpublished), Uhlenbeck and Goudsmit (1925) to a rotation over an indeterminate 'internal axis' ('internal space'), with a half unit of angular momentum, so that the spin is a vector $\vec{S}$,

such that: $\vec{\mu} \sim \vec{S}$. Where is it? What is an 'internal space' ? Somehow we do not know.

In 1927 Pauli in his *On the quantum mechanics of magnetic electrons* [8] writes: " … by a suitable choice of linear operators for the components s_x, s_y, s_z of the proper moment in a prescribed coordinate axis-cross … " and " … general connection between operator algebra and matrix algebra. … amplitudes for s_z (measured in units of $\frac{1}{2}\frac{h}{2\pi}$) to assume the value $+1$ or -1 … ". Remember the spin $1/2$. "…in the symbolic matrix form:

$$s_x(\Psi) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \cdot \Psi\,, \qquad s_y(\Psi) = \begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix} \cdot \Psi\,, \qquad s_z(\Psi) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \Psi\,. \text{"} \qquad \text{(Spin matrices).}$$

Afterwards σ^x, σ^y, σ^z. Their connection with the quaternions (imaginary geometrical objects) noted to Pauli by P. Jordan.

A feature in the usage of **SU(2)** is the necessity of doing a 4π rotation for recovering the identity, for spin 1/2 particles.

Meanwhile we can assimilate the scalars $+1$ or -1 (in the matrix s_z) with the 'up' and the 'down', respectively, but: where are the s_x and s_y components of the spin $\vec{S}$? With Feynman and Schwinger, 'this is quantum mechanics'. [1]

Therefore, in this sense, the 'up' and the 'down' are well obtained. But, quantum mechanics prohibits the definition with precision of the direction of a vector of rotation for massive elementary particles. We remind its origin: it is Amperian.

Associated to the 'up' and the 'down' the helicity concept in a reference frame $(\mathbf{p} \neq \mathbf{0})$ and the polarizations with the possible mixtures of the helicities, a statistical concept.

An electron and an anti-electron (a positron). Consider their annihilation. **The spin is there**, but it is secondary as it concerns the annihilation. It tells us about the way of the annihilation, at first (for the positronium) producing two or three photons (see Feynman, *Quantum Electrodynamics*. [9] , 21^{st}, 22^{sd} and 26^{th} Lectures, and in particular page 107 "…only the singlet state (spins antiparallel) can disintegrate into two photons. … into three photons (spins must be parallel).":

$$\overset{\uparrow}{e}{}^- + \overset{\downarrow}{e}{}^+ \longrightarrow \overset{\uparrow\uparrow}{\gamma}{}^0 + \overset{\downarrow\downarrow}{\gamma}{}^0, \qquad \overset{\uparrow}{e}{}^- + \overset{\uparrow}{e}{}^+ \longrightarrow \overset{\uparrow\uparrow}{\gamma}{}^0 + \overset{\uparrow\uparrow}{\gamma}{}^0 + \overset{\downarrow\downarrow}{\gamma}{}^0\,, \tag{3}$$

similarly inverting all the spins. Notation: one arrow for the spin 1/2, and two arrows for the spin 1 (with theirs 'up − down').

With the Dirac´s free particle Hamiltonian, $\mathcal{H}_D = \boldsymbol{\alpha} \cdot \mathbf{p} + m\,\boldsymbol{\beta}$ (see (5)), the total angular momentum is conserved:

$$[\,\mathcal{H}_D\;,\,\mathbf{J}\,] = [\,\mathcal{H}_D\,,\,(\mathbf{L}+\mathbf{S})\,] = \mathbf{0}.$$

For free charged leptons, we choose the z axis in the direction of the motion $(\mathbf{L} = \mathbf{0})$. We also consider processes with exclusively internal angular momentum (spins) $(\Delta\,\mathbf{L} = \mathbf{0})$. With the spin (up − down), we write for the charged leptons:

iii− *the spin (up − down) of a free charged lepton or the total spin before and after an interaction or process is conserved (relativistic limit).* (4)

In • 8 ((33)) we depict a Gamow-Teller decay, which include a mixing of the spins (up − down) of leptons and quarks.

• 3 The **solutions of the Dirac equation.**

See David Griffiths: section 7.2 (specially page 234 and the note †) [10] and Mark Thompson: chapter 4, subsection 6.2.2 and section 6.4 [11].

The γ - matrices are, in the Dirac-Pauli representation:

$$\boldsymbol{\beta} = \gamma^0 = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{-1} \end{pmatrix}, \quad \gamma^x = \begin{pmatrix} \mathbf{0} & \sigma^x \\ -\sigma^x & \mathbf{0} \end{pmatrix}, \quad \gamma^y = \begin{pmatrix} \mathbf{0} & \sigma^y \\ -\sigma^y & \mathbf{0} \end{pmatrix}, \quad \gamma^z = \begin{pmatrix} \mathbf{0} & \sigma^z \\ -\sigma^z & \mathbf{0} \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} \end{pmatrix}, \quad \boldsymbol{\alpha} = \boldsymbol{\beta}\,\boldsymbol{\gamma}. \tag{5}$$

We write the Dirac equation for a free particle: $(\,\mathrm{i}\,\gamma^\mu \partial_\mu - m\,\mathbf{I}\,)\,\Psi = \mathbf{0}$,
which for plane wave solutions, with $S = -E\,t + \mathbf{p}\,.\,\mathbf{x}$, drives to:
for electrons, $\Psi^+(\mathbf{x}.t) = \mathrm{e}^{\,\mathrm{i}\,S}\,\mathbf{u}(E,\mathbf{p})$, taking the form $(\,\gamma^\mu \mathbf{p}_\mu - m\,\mathbf{I}\,)\;\mathbf{u} = \mathbf{0}$ or $(\,\boldsymbol{\alpha}\,.\,\mathbf{p} - E\,\mathbf{I} + m\,\boldsymbol{\beta}\,)\,\mathbf{u} = \mathbf{0}$, and
for anti-electrons, $\Psi^-(\mathbf{x}.t) = \mathrm{e}^{-\mathrm{i}\,S}\,\mathbf{v}(E,\mathbf{p})$, taking the form $(\,\gamma^\mu \mathbf{p}_\mu + m\,\mathbf{I}\,)\;\mathbf{v} = \mathbf{0}$ or $(-\boldsymbol{\alpha}\,.\,\mathbf{p} + E\,\mathbf{I} + m\,\boldsymbol{\beta}\,)\,\mathbf{v} = \mathbf{0}$,
with the spinors $\mathbf{u}$ and $\mathbf{v}$.

We are able to write these solutions in various **algebraic** forms with different types of **u spinors (taken as vectors)**:

$$\Psi^+(\mathbf{x},t) \;=\; \mathrm{e}^{\,\mathrm{i}\,S} \begin{cases} \alpha_1\,\mathbf{u}_1 + \alpha_2\,\mathbf{u}_2 & \text{(unspecific for the helicity or the chirality , } \{\alpha_1\,,\,\alpha_2\} \in \mathbb{C}\text{).} \quad (a \\ \beta_-\mathbf{u}_{Lh} + \beta_+\mathbf{u}_{Rh} & \text{(helicity spinors, specific for the spin , } \{\beta_-\,,\,\beta_+\} \in \mathbb{C}\text{).} \quad (b \\ \sim \gamma_1\tilde{\mathbf{u}}_{Lch} + \gamma_2\tilde{\mathbf{u}}_{Rch} & \text{(chirality spinors, } \gamma_1 \text{ and } \gamma_2 \text{ restricted). See eq. (26) and bellow.} \quad (c \end{cases} \qquad - \;(6)$$

Any spinor of the form $\mathbf{u} = \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} = \frac{1}{2}\left\{\begin{pmatrix} \chi_1 - \chi_2 \\ -(\chi_1 - \chi_2) \end{pmatrix} + \begin{pmatrix} \chi_1 + \chi_2 \\ \chi_1 + \chi_2 \end{pmatrix}\right\}$, with $\begin{cases} \chi_1 - \chi_2 = 2\chi_a \\ \chi_1 + \chi_2 = 2\chi_b \end{cases}$, can be rewritten as:

$$\mathbf{u} = \left\{ \begin{pmatrix} \chi_a \\ -\chi_a \end{pmatrix} + \begin{pmatrix} \chi_b \\ \chi_b \end{pmatrix} \right\} = \left\{ -\gamma^5 \begin{pmatrix} \chi_a \\ -\chi_a \end{pmatrix} + \gamma^5 \begin{pmatrix} \chi_b \\ \chi_b \end{pmatrix} \right\} = \mathbf{P}_- \begin{pmatrix} \chi_a \\ -\chi_a \end{pmatrix} + \mathbf{P}_+ \begin{pmatrix} \chi_b \\ \chi_b \end{pmatrix} = \begin{pmatrix} \chi_a \\ -\chi_a \end{pmatrix} + \begin{pmatrix} \chi_b \\ \chi_b \end{pmatrix} \equiv \mathbf{u}_{Lch} + \mathbf{u}_{Rch}\,, \tag{7}$$

where we have defined the projection operators:

$$\mathbf{P}_- = \tfrac{1}{2}\,(\,\mathbf{I} - \gamma^5) \qquad \text{and} \qquad \mathbf{P}_+ = \tfrac{1}{2}\,(\,\mathbf{I} + \gamma^5). \qquad \mathbf{P}_- + \mathbf{P}_+ = \mathbf{I}, \qquad \mathbf{P}_-\mathbf{P}_+ = \mathbf{P}_+\mathbf{P}_- = \mathbf{0}\,. \tag{8}$$

Similarly with the $\mathbf{v}$ spinors (anti-electrons).

• 4 The **chirality.**

The word 'chiral' has its roots in the Greek word "$\chi\epsilon\iota\rho$" *cheir* (the palm of a hand, a left or a right).

The chirality is here too, with the γ^5 matrix and the projection operators $\mathbf{P}_-$ and $\mathbf{P}_+$.

The γ^5 matrix has a special role in the Dirac equation: it interchanges the two parts of the spinors, χ_1 and χ_2, in its solutions $\mathbf{u}$ and $\mathbf{v}$ (see later in equations (12) and (14)):

$$\underline{\mathbf{u} = \mathbf{u}_{Lch} + \mathbf{u}_{Rch}} : \begin{cases} \mathbf{u}_{Lch} = \mathbf{P}_-\,\mathbf{u} \\ \mathbf{u}_{Rch} = \mathbf{P}_+\,\mathbf{u} \end{cases} \text{for the electron} \quad \text{and} \quad \underline{\mathbf{v} = \mathbf{v}_{Rch} + \mathbf{v}_{Lch}} : \begin{cases} \mathbf{v}_{Rch} = \mathbf{P}_-\,\mathbf{v} \\ \mathbf{v}_{Lch} = \mathbf{P}_+\,\mathbf{v} \end{cases} \text{for the anti-electron,} \tag{9}$$

in the Dirac equation (Griffiths [10], Sections 7.2, 9.6 and 9.7). The chirality is a Lorentz invariant (frame independent), but it is not conserved. Both, the < left chiral , Lch, ← > and the < right chiral , Rch, → > parts are "at once" in the solutions of the Dirac equation, though in a flipping way due to the mass. A left chiral part evolves to a right chiral part, and vice versa. ("A particle that starts out left-handed will soon evolve a right-handed component." * in Griffiths page 342 [10]).

The weak interaction with its *"vector − axial vector"* structure ("V−A") in the form $\gamma^\mu - \gamma^\mu\gamma^5 = \gamma^\mu\,(\,\mathbf{I} - \gamma^5)$ stipulated the usage of a $(\mathbf{1} - \gamma^5)$ matrix over the solutions of the Dirac equation (the chirality). We write the following algebraic formulas:

$$\mathbf{P}_-\begin{Bmatrix} \mathbf{u} \\ \mathbf{v} \end{Bmatrix} = \frac{1}{2}\,(\mathbf{I} - \gamma^5)\begin{Bmatrix} \mathbf{u} \\ \mathbf{v} \end{Bmatrix} = \frac{1}{2}\begin{pmatrix} \mathbf{1} & \mathbf{-1} \\ \mathbf{-1} & \mathbf{1} \end{pmatrix}\begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} \chi_1 - \chi_2 \\ -(\chi_1 - \chi_2) \end{pmatrix} = \begin{pmatrix} \chi_a \\ -\chi_a \end{pmatrix} = \begin{Bmatrix} \mathbf{u}_{Lch} \\ \mathbf{v}_{Rch} \end{Bmatrix},$$
$$\mathbf{P}_-\begin{Bmatrix} \mathbf{u}_{Lch} \\ \mathbf{v}_{Rch} \end{Bmatrix} = \begin{Bmatrix} \mathbf{u}_{Lch} \\ \mathbf{v}_{Rch} \end{Bmatrix}, \qquad \mathbf{P}_+\begin{Bmatrix} \mathbf{u}_{Lch} \\ \mathbf{v}_{Rch} \end{Bmatrix} = \mathbf{0}\,. \tag{10}$$

$$\mathbf{P}_+\begin{Bmatrix} \mathbf{u} \\ \mathbf{v} \end{Bmatrix} = \frac{1}{2}\,(\mathbf{I} + \gamma^5)\begin{Bmatrix} \mathbf{u} \\ \mathbf{v} \end{Bmatrix} = \frac{1}{2}\begin{pmatrix} \mathbf{1} & \mathbf{1} \\ \mathbf{1} & \mathbf{1} \end{pmatrix}\begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} \chi_1 + \chi_2 \\ \chi_1 + \chi_2 \end{pmatrix} = \begin{pmatrix} \chi_b \\ \chi_b \end{pmatrix} = \begin{Bmatrix} \mathbf{u}_{Rch} \\ \mathbf{v}_{Lch} \end{Bmatrix},$$
$$\mathbf{P}_+\begin{Bmatrix} \mathbf{u}_{Rch} \\ \mathbf{v}_{Lch} \end{Bmatrix} = \begin{Bmatrix} \mathbf{u}_{Rch} \\ \mathbf{v}_{Lch} \end{Bmatrix}, \qquad \mathbf{P}_-\begin{Bmatrix} \mathbf{u}_{Rch} \\ \mathbf{v}_{Lch} \end{Bmatrix} = \mathbf{0}\,. \tag{11}$$

Therefore, the spinors $\mathbf{u}_{Lch}$, $\mathbf{u}_{Rch}$, $\mathbf{v}_{Rch}$ and $\mathbf{v}_{Lch}$ are **chiral eigenstates**: $\gamma^5\mathbf{u}_{Lch} = (-1)\,\mathbf{u}_{Lch}$, $\gamma^5\mathbf{v}_{Rch} = (-1)\,\mathbf{v}_{Rch}$ and $\gamma^5\mathbf{u}_{Rch} = (+1)\,\mathbf{u}_{Rch}$, $\gamma^5\mathbf{v}_{Lch} = (+1)\,\mathbf{v}_{Lch}$. **But, with $\mathbf{p} \neq \mathbf{0}$, they are not eigenstates of the massive Dirac equation.**

• 5 The **helicity** $(\mathbf{p} \neq \mathbf{0})$.

The word 'helical': a curve advancing circularly around a central line, in two possible senses, which we relate with the two senses marked by using the thumbs of our two closed hands.

For a **massless fermion**, the helicity is a **measure** of the z**-component of the spin** (a vector, 'a rotation in an internal space') **over the (its)** z**-direction of motion** (a unit vector in 'our space'). It is a sign.

With a (two-) spinor χ, a two component wave function inside a solution of the Dirac equation, we write:

$$\hat{\mathbf{h}}\,\chi = \boldsymbol{\sigma}\boldsymbol{.}\,\frac{\mathbf{p}}{||\mathbf{p}||}\,\chi = h\,\chi\,,$$ with h related to one of the two possibilities: antiparallel or parallel.

(Perkins, page 67 in [12] and pages 19-20 in [13] ; Thomson, page 105 [11])

With the interpretation of the spin as a rotation, the helicity represents a left or right screw sense for a rotation, one or the other of the two hands **-handed-**.

Two possibilities with respect to a momentum $\mathbf{p}$: the < Left helicity, - (1), $Lh, \downarrow$ > and the < Right helicity, + (1), $Rh, \uparrow$ >.

Besides, the momentum $\mathbf{p}$ has to do with a specific reference frame. The helicity is not a Lorentz invariant.

The spin operator $\left(\Sigma = \begin{pmatrix} \vec{\sigma} & \mathbf{0} \\ \mathbf{0} & \vec{\sigma} \end{pmatrix}\right)$ does not commute with $\mathcal{H}_D$, but the helicity operators (in (13) , (15)) do so: $[\,\mathcal{H}_D\,,\,\hat{\mathbf{H}}\,] = \mathbf{0}$.

We write the Dirac equation for a free electron, with $\mathrm{e}^{\,\mathrm{i}\,S}$, $\mathbf{u} = \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix}$ and the spin operator $\Sigma_p = \begin{pmatrix} \vec{\sigma} & \mathbf{0} \\ \mathbf{0} & \vec{\sigma} \end{pmatrix}$:

$$\frac{1}{||\mathbf{p}||}\,(\,E\,\mathbf{I} + m\boldsymbol{\beta}\,)\,\{\gamma^5\,\mathbf{u}\,\} = \frac{1}{||\mathbf{p}||}\begin{pmatrix} E+m & 0 \\ 0 & E-m \end{pmatrix}\begin{pmatrix} \chi_2 \\ \chi_1 \end{pmatrix} \overset{\text{Dirac eq.}}{=} \frac{1}{||\mathbf{p}||}\begin{pmatrix} \boldsymbol{\sigma}\boldsymbol{.}\,\mathbf{p} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\sigma}\boldsymbol{.}\,\mathbf{p} \end{pmatrix}\begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix}. \tag{12}$$

We define the helicity operator for a particle (p), an electron, and we search for the helicity eigenstates ($*$):

$$\hat{\mathbf{H}}_p\,\mathbf{u} = \frac{1}{||\mathbf{p}||}\begin{pmatrix} \boldsymbol{\sigma}\boldsymbol{.}\,\mathbf{p} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\sigma}\boldsymbol{.}\,\mathbf{p} \end{pmatrix}\mathbf{u} = \frac{1}{||\mathbf{p}||}\begin{pmatrix} \boldsymbol{\sigma}\boldsymbol{.}\,\mathbf{p}\;\chi_1 \\ \boldsymbol{\sigma}\boldsymbol{.}\,\mathbf{p}\;\chi_2 \end{pmatrix} \overset{*}{=} h\begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} = h\,\mathbf{u}\,, \quad \text{helicity eigenstates.} \tag{13}$$

For their antiparticles, free anti-electrons, with $\mathrm{e}^{-\mathrm{i}\,S}$, $\mathbf{v} = \begin{pmatrix} \hat{\chi}_1 \\ \hat{\chi}_2 \end{pmatrix}$ and the spin operator $\Sigma_a = \begin{pmatrix} -\vec{\sigma} & \mathbf{0} \\ \mathbf{0} & -\vec{\sigma} \end{pmatrix} = -\Sigma_p$:

$$-\frac{1}{||\mathbf{p}||}\,(\,E\,\mathbf{I} - m\boldsymbol{\beta}\,)\,\{\gamma^5\,\mathbf{v}\,\} = -\frac{1}{||\mathbf{p}||}\begin{pmatrix} E-m & 0 \\ 0 & E+m \end{pmatrix}\begin{pmatrix} \hat{\chi}_2 \\ \hat{\chi}_1 \end{pmatrix} \overset{\text{Dirac eq.}}{=} \frac{1}{||\mathbf{p}||}\begin{pmatrix} -\boldsymbol{\sigma}\boldsymbol{.}\,\mathbf{p} & \mathbf{0} \\ \mathbf{0} & -\boldsymbol{\sigma}\boldsymbol{.}\,\mathbf{p} \end{pmatrix}\begin{pmatrix} \hat{\chi}_1 \\ \hat{\chi}_2 \end{pmatrix}. \tag{14}$$

And the helicity operator for an anti-electron (a), with the condition for a helicity eigenstate ($**$):

$$\hat{\mathbf{H}}_a\,\mathbf{v} = \frac{1}{||\mathbf{p}||}\begin{pmatrix} -\boldsymbol{\sigma}\boldsymbol{.}\,\mathbf{p} & \mathbf{0} \\ \mathbf{0} & -\boldsymbol{\sigma}\boldsymbol{.}\,\mathbf{p} \end{pmatrix}\mathbf{v} = \frac{1}{||\mathbf{p}||}\begin{pmatrix} -\boldsymbol{\sigma}\boldsymbol{.}\,\mathbf{p}\;\hat{\chi}_1 \\ -\boldsymbol{\sigma}\boldsymbol{.}\,\mathbf{p}\;\hat{\chi}_2 \end{pmatrix} \overset{**}{=} h\begin{pmatrix} \hat{\chi}_1 \\ \hat{\chi}_2 \end{pmatrix} = h\,\mathbf{v}\,, \quad \text{helicity eigenstates.} \tag{15}$$

Therefore, we are searching for spinor solutions $\mathbf{u}$ and $\mathbf{v}$ of the Dirac equation which are also helicity eigenstates.

We define:

$$A = \frac{||\mathbf{p}||}{E+m} = \frac{E-m}{||\mathbf{p}||} = \sqrt{\frac{E-m}{E+m}} = \sqrt{1-\frac{2m}{E+m}}\,, \quad 0 \le A \le 1 \quad , \quad \text{and} \quad \mathbb{A} = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & A\,\mathbf{1} \end{pmatrix} = A^{1/2}\begin{pmatrix} A^{-1/2}\,\mathbf{1} & \mathbf{0} \\ \mathbf{0} & A^{1/2}\,\mathbf{1} \end{pmatrix}.$$

With $E^2 = m^2 + ||p||^2 \quad (c = 1)$, we have for
$$\begin{cases} \text{massless particles } (E = ||p||): & A\,|_{m\,=\,0} = \dfrac{||\mathbf{p}||}{E+m}\,|_{m\,=\,0} = 1 \\ \text{relativistic particles:} & m \lll E \approx ||\mathbf{p}|| \quad \Rightarrow \quad \{\,A < 1 \text{ and } A \approx 1\,\} \end{cases}.$$

From (13) (the condition $*$) we can find χ_{1h} either as χ_- or as χ_+ with $h\;\boldsymbol{\sigma}\boldsymbol{.}\,\frac{\mathbf{p}}{||\mathbf{p}||}\;\chi_{1h} = \chi_{1h}$ $\quad(\,h \in \{-,+\}\,)$.

And from (12): $\chi_{2h} = A\;\boldsymbol{\sigma}\boldsymbol{.}\,\frac{\mathbf{p}}{||\mathbf{p}||}\;\chi_{1h} = h\,A\;\chi_{1h}\,.$

We denote: $\mathbf{u}_- = \mathbf{u}_{Lh}$ and $\mathbf{u}_+ = \mathbf{u}_{Rh}$; and afterwards, in (b-(6), for the helicity solutions we can choose β_- and β_+ as normalizing constants. And, in particular it could be $\beta_- = \beta_+$ or $\beta_+ = 0$ or $\beta_- = 0$ in:

$$\begin{aligned} \Psi^+(\mathbf{x},t) &= \Psi^+_- + \Psi^+_+ = \mathrm{e}^{\,\mathrm{i}\,S}\left(\beta_-\,\mathbf{u}_{Lh} + \beta_+\,\mathbf{u}_{Rh}\right) = \mathrm{e}^{\,\mathrm{i}\,S}\left(\beta_-\,\mathbf{u}_- + \beta_+\,\mathbf{u}_+\right) = \mathrm{e}^{\,\mathrm{i}\,S}\sum_{h\,=\,\{-,+\}}\beta_h\,\mathbf{u}_h = \\ &= \mathrm{e}^{\,\mathrm{i}\,S}\sum_{h\,=\,\{-,+\}}\beta_h\begin{pmatrix} \chi_h \\ h\,A\,\chi_h \end{pmatrix} = \mathrm{e}^{\,\mathrm{i}\,S}\,\mathbb{A}\,\left\{\sum_{h\,=\,\{-,+\}}\beta_h\begin{pmatrix} \chi_h \\ h\,\chi_h \end{pmatrix}\right\}. \end{aligned} \tag{16}$$

Similarly for the $\mathbf{v}$ spinors (anti-electrons). From (15) (the condition **) we can find $\hat{\chi}_{2h}$ either as $\hat{\chi}_+$ or as $\hat{\chi}_-$ with $\hat{\chi}_{2h} = -\,h\,\boldsymbol{\sigma}.\frac{\mathbf{p}}{\|\mathbf{p}\|}\,\hat{\chi}_{2h}$ ($h \in \{+,-\}$). The change of sign in this assignment (antiparticles) justified with the definition in (15).

And from (14): $\hat{\chi}_{1h} = -A\,(-\boldsymbol{\sigma}.\frac{\mathbf{p}}{\|\mathbf{p}\|})\,\hat{\chi}_{2h} = -h\,A\,\hat{\chi}_{2h}$. With $\mathbb{B} = \begin{pmatrix} A\,\mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix} = A^{1/2}\begin{pmatrix} A^{1/2}\,\mathbf{1} & \mathbf{0} \\ \mathbf{0} & A^{-1/2}\,\mathbf{1} \end{pmatrix}$.

We write: $\mathbf{v}_+ = \mathbf{v}_{Rh}$ and $\mathbf{v}_- = \mathbf{v}_{Lh}$; so that in a similar way as with the particle solutions:

$$\Psi^-(\mathbf{x},t) = \Psi^-_+ + \Psi^-_- = e^{-iS}\big(\hat{\beta}_+\,\mathbf{v}_{Rh} + \hat{\beta}_-\,\mathbf{v}_{Lh}\big) = e^{-iS}\big(\hat{\beta}_+\,\mathbf{v}_+ + \hat{\beta}_-\,\mathbf{v}_-\big) = e^{-iS}\sum_{h=\{+,-\}}\hat{\beta}_h\,\mathbf{v}_h = $$
$$= e^{-iS}\sum_{h=\{+,-\}}\hat{\beta}_h\begin{pmatrix} -h\,A\,\hat{\chi}_h \\ \hat{\chi}_h \end{pmatrix} = e^{-iS}\,\mathbb{B}\Big\{\sum_{h=\{+,-\}}\hat{\beta}_h\begin{pmatrix} -h\,\hat{\chi}_h \\ \hat{\chi}_h \end{pmatrix}\Big\}. \tag{17}$$

• An alternative. Towards a solution for the uncharged leptons?

Customarily: $\chi_- = \hat{\chi}_+ = \begin{pmatrix} -\sin\frac{\theta}{2} \\ \cos\frac{\theta}{2}\,e^{i\phi} \end{pmatrix}$, $\chi_+ = \hat{\chi}_- = \begin{pmatrix} \cos\frac{\theta}{2} \\ \sin\frac{\theta}{2}\,e^{i\phi} \end{pmatrix}$ and $\sqrt{E+m} = \sqrt{\|\mathbf{p}\|}\,A^{-1/2}$, the four solutions with the same normalizing constant (see Thomson, page 107 [11]).

Let us now ignore: the interpretation of the signs in the exponential in relation to the signs of the electric charge, also the interpretation in relation to the spin ($-\Sigma_a = \Sigma_p = \Sigma$), and finally of the chirality in relation to the weak interaction.

We only assume that we have a set with four independent solutions.

Departing from $S = -E\,t + \mathbf{p}\,.\,\mathbf{x}$, $E^2 = m^2 + \|p\|^2$ (eventually with $m \lll E$) and $\Delta = \ln A^{1/2} = \frac{1}{4}\ln\frac{E-m}{E+m}$, we define:

$$S^{\pm}(E,\mathbf{p}) = S \pm i\,\Delta \overset{m\lll E}{\longrightarrow} S_{(m=0)}\,, \qquad \text{so that} \qquad \begin{cases} \cosh(\pm\Delta) \overset{m\lll E}{\longrightarrow} 1 + \frac{5}{8}(\frac{m}{E})^2 \\ \sinh(\pm\Delta) \qquad 0 \pm \frac{1}{2}(\frac{m}{E}) \end{cases}.$$

We redefine the normalizing constants: $\beta_\pm = \sqrt{E+m}\;\beta'_\pm$, $\hat{\beta}_\pm = \sqrt{E+m}\;\hat{\beta}'_\pm$ ($\beta'_\pm \in \mathbb{C}$, $\hat{\beta}'_\pm \in \mathbb{C}$).

With these expressions, we rewrite a general solution of the Dirac equation, using (16) and (17) :

$$\Psi(\mathbf{x},t) = \sqrt{\|\mathbf{p}\|}\left\{\left[\beta'_-\begin{pmatrix} e^{\,iS^+}\chi_- \\ -\,e^{\,iS^-}\chi_- \end{pmatrix} + \beta'_+\begin{pmatrix} e^{\,iS^+}\chi_+ \\ e^{\,iS^-}\chi_+ \end{pmatrix}\right] + \left[\hat{\beta}'_+\begin{pmatrix} -e^{-iS^+}\chi_- \\ e^{-iS^-}\chi_- \end{pmatrix} + \hat{\beta}'_-\begin{pmatrix} e^{-iS^+}\chi_+ \\ e^{-iS^-}\chi_+ \end{pmatrix}\right]\right\}, \tag{18}$$

It is suggestive the partial additions of the following terms: the first with the third and the second with the fourth. We can rewrite the solution in the form $\Psi(\mathbf{x},t) = \Psi_1 + \Psi_2$:

$$\Psi_1 = \sqrt{\|\mathbf{p}\|}\,\Big\{\big(\beta'_- e^{\,iS} - \hat{\beta}'_+ e^{-iS}\big)\cosh\Delta\begin{pmatrix} \chi_- \\ -\chi_- \end{pmatrix} - \big(\beta'_- e^{\,iS} + \hat{\beta}'_+ e^{-iS}\big)\sinh\Delta\begin{pmatrix} \chi_- \\ \chi_- \end{pmatrix}\Big\} \equiv \Psi_\downarrow$$

$$\Psi_2 = \sqrt{\|\mathbf{p}\|}\,\Big\{\big(\beta'_+ e^{\,iS} + \hat{\beta}'_- e^{-iS}\big)\cosh\Delta\begin{pmatrix} \chi_+ \\ \chi_+ \end{pmatrix} - \big(\beta'_+ e^{\,iS} - \hat{\beta}'_- e^{-iS}\big)\sinh\Delta\begin{pmatrix} \chi_+ \\ -\chi_+ \end{pmatrix}\Big\} \equiv \Psi_\uparrow$$

Without the aid of the electric charge, we only have at our disposal a definition for the spin (only Σ): the two helicities (this has motivated (19)). In particular, we can choose: $\beta'_- = N_1\,e^{i\alpha_1}\,\frac{\sqrt{2}}{2}e^{-i\frac{\pi}{4}} = i\,\hat{\beta}'_+$ and $\beta'_+ = N_2\,e^{i\alpha_2}\,\frac{\sqrt{2}}{2}e^{-i\frac{\pi}{4}} = -i\,\hat{\beta}'_-$, and we write $\Psi(\mathbf{x},t)$ in terns of trigonometric functions (in S), wavy aspect, and hyperbolic functions (afterwards with the A). These functions are **even** or **odd**, and besides with imaginary factors:

$$\boxed{\begin{aligned} \Psi_1 &= (N_1\,e^{i\alpha_1})\,\sqrt{\|\mathbf{p}\|}\,\big(\cos(-Et+\mathbf{p.x}) + \sin(-Et+\mathbf{p.x})\big)\Big\{\cosh\Delta\begin{pmatrix} \chi_- \\ -\chi_- \end{pmatrix} + i\,\sinh\Delta\begin{pmatrix} \chi_- \\ \chi_- \end{pmatrix}\Big\} \equiv \Psi_\downarrow \\ \Psi_2 &= (N_2\,e^{i\alpha_2})\,\sqrt{\|\mathbf{p}\|}\,\big(\cos(-Et+\mathbf{p.x}) + \sin(-Et+\mathbf{p.x})\big)\Big\{\cosh\Delta\begin{pmatrix} \chi_+ \\ \chi_+ \end{pmatrix} + i\,\sinh\Delta\begin{pmatrix} \chi_+ \\ -\chi_+ \end{pmatrix}\Big\} \equiv \Psi_\uparrow \end{aligned}} \tag{19}$$

Could we speak of only two solutions due the previous fixings of the $\beta' s$? Perhaps as double solutions (once imposed $\beta'_- = i\,\hat{\beta}'_+$ and $\beta'_+ = -i\,\hat{\beta}'_-$). Remember the double zeros in an algebraic equation (see later in our model, at the beginning of page 22).

Could this be relevant for the uncharged leptons? Now, the interpretation in relation to the chirality has to be reexamined. Should also be revised the attributions for the chirality and the predominance of the particles over the anti-particles? Perhaps, prevalence of the neutrinos (linked to this chirality): $\Psi_\uparrow\Big|_{\text{Weak}} \overset{m\lll E}{\longrightarrow} \mathbf{0}$?

The treatment of the chirality has to be completed after introducing the virtual processes with the QED (see section V).

• 6 $m \neq 0$. The **helicity** $(\mathbf{p} \neq \mathbf{0})$ and the **chirality**.

A) *The helicity eigenstates are Dirac eigenstates (eigenstates of $\mathcal{H}_D$). We have solutions of the free particle Dirac equation $\mathbf{u}_{Lh}$ or $\mathbf{u}_{Rh}$; for them it could be either $\beta_+ = 0$ (only $\mathbf{u}_{Lh}$) or $\beta_- = 0$ (only $\mathbf{u}_{Rh}$).* Similarly for the anti-electrons (antiparticles).

We choose a +z semi-axis in relation to the direction of the motion ("spin along its direction of flight", Thomson, in page 105 in [11]), and *we have specific individual values of the spin (the scalar),*

the down for the left helicity and the up for the right helicity.

B) *The chiral eigenstates are not Dirac eigenstates. As different with the helicity eigenstates, the chiral parts (chiral eigenstates) $\mathbf{u}_{Lch}$ and $\mathbf{u}_{Rch}$ can not be individual solutions of the Dirac equation ($\mathbf{p} \neq \mathbf{0}$), i.e. it could not be $\gamma_1 = 0$ or $\gamma_2 = 0$* (see after equation (11), and * in Griffiths, page 342 [10]) :

$$(\boldsymbol{\alpha}\cdot\mathbf{p} + m\boldsymbol{\beta})\begin{cases}\mathbf{u}_{Lch}\\ \mathbf{u}_{Rch}\end{cases} = E\begin{cases}\mathbf{u}_{Lch}\\ \mathbf{u}_{Rch}\end{cases} \quad \underset{\mathbf{p}\neq\mathbf{0}}{\overset{m\neq 0}{\Longrightarrow}} \quad \mathbf{u}_{Lch} = \mathbf{u}_{Rch} = \mathbf{0} = \mathbf{u}\,.$$

$$\left[\begin{pmatrix}(E-m)\,\mathbf{1} & -\boldsymbol{\sigma}\cdot\mathbf{p}\\ -\boldsymbol{\sigma}\cdot\mathbf{p} & (E+m)\,\mathbf{1}\end{pmatrix}\begin{pmatrix}\chi_1\\ \chi_2\end{pmatrix} = \begin{pmatrix}\mathbf{0}\\ \mathbf{0}\end{pmatrix} \Longrightarrow \begin{matrix}(E-m)\,\mathbf{1}\,\chi_1 - \boldsymbol{\sigma}\cdot\mathbf{p}\,\chi_2 = \mathbf{0}\\ (E+m)\,\mathbf{1}\,\chi_2 - \boldsymbol{\sigma}\cdot\mathbf{p}\,\chi_1 = \mathbf{0}\end{matrix}\right\} \underset{\mathbf{u}_{Lch},\,\mathbf{u}_{Rch}}{\overset{\chi_2 = \mp\chi_1}{\Longrightarrow}} \; m\,\chi_1 = \mathbf{0} \Longrightarrow \chi_1 = \mathbf{0} = \chi_2 \right]$$

Similar results for the anti-electrons.

For the electrons. With the conditions and definitions in (12)-(13) (16) we have:

$$\left\{ \mathbf{u}_{Lh} = \mathbf{u}_- = \begin{pmatrix}\chi_-\\ -A\,\chi_-\end{pmatrix}, \quad \mathbf{u}_{Rh} = \mathbf{u}_+ = \begin{pmatrix}\chi_+\\ A\,\chi_+\end{pmatrix} \right\}, \quad \overset{h\,\in\,\{-,+\}}{\longrightarrow} \quad \mathbf{u}_h = \begin{pmatrix}\chi_h\\ h\,A\,\chi_h\end{pmatrix}. \tag{20}$$

And also:

$$\mathbf{u}_h = \left\{\tfrac{1}{2}\,(\mathbf{I}+h\,\gamma^5) + \tfrac{1}{2}\,(\mathbf{I}-h\,\gamma^5)\right\}\mathbf{u}_h = \tfrac{1}{2}\,(1+A)\begin{pmatrix}\chi_h\\ h\,\chi_h\end{pmatrix} + \tfrac{1}{2}\,(1-A)\begin{pmatrix}\chi_h\\ -h\,\chi_h\end{pmatrix}, \tag{21}$$

in detail

$$\begin{cases} 1)\; \mathbf{u}_{Lh}\big|_{\chi_-} = \begin{pmatrix}\chi_-\\ -A\,\chi_-\end{pmatrix} = \tfrac{1}{2}(1{+}A)\begin{pmatrix}\chi_-\\ -\chi_-\end{pmatrix} + \tfrac{1}{2}(1{-}A)\begin{pmatrix}\chi_-\\ \chi_-\end{pmatrix} = \tfrac{1}{2}(1{+}A)\,\mathbf{u}_{Lch}\big|_{\chi_-} + \tfrac{1}{2}(1{-}A)\,\mathbf{u}_{Rch}\big|_{\chi_-}\\ 2)\; \mathbf{u}_{Rh}\big|_{\chi_+} = \begin{pmatrix}\chi_+\\ A\,\chi_+\end{pmatrix} = \tfrac{1}{2}(1{+}A)\begin{pmatrix}\chi_+\\ \chi_+\end{pmatrix} + \tfrac{1}{2}(1{-}A)\begin{pmatrix}\chi_+\\ -\chi_+\end{pmatrix} = \tfrac{1}{2}(1{-}A)\,\mathbf{u}_{Lch}\big|_{\chi_+} + \tfrac{1}{2}(1{+}A)\,\mathbf{u}_{Rch}\big|_{\chi_+}\end{cases}. \tag{22}$$

From these equations, we **can not** obtain $\mathbf{u}_{Lch}\big|_{\chi_-}$ or $\mathbf{u}_{Lch}\big|_{\chi_+}$ as a linear combination of the helical eigenstates.

In brief, **if** a chiral eigenstate could be written as a linear combination of the helicity eigenstates ($\mathbf{p} \neq \mathbf{0}$),

then it would be also an eigenstate of the Dirac equation, which it is not possible ($m \neq 0$, $\mathbf{p} \neq \mathbf{0}$).

For the anti-electrons. We have similar developments. With the definitions in (15):

$$\left\{ \mathbf{v}_{Rh} = \mathbf{v}_+ = \begin{pmatrix}-A\,\hat{\chi}_+\\ \hat{\chi}_+\end{pmatrix}, \quad \mathbf{v}_{Lh} = \mathbf{v}_- = \begin{pmatrix}A\,\hat{\chi}_-\\ \hat{\chi}_-\end{pmatrix} \right\}, \quad \overset{h\,\in\,\{+,-\}}{\longrightarrow} \quad \mathbf{v}_h = \begin{pmatrix}-h\,A\,\hat{\chi}_h\\ \hat{\chi}_h\end{pmatrix}. \tag{23}$$

And also:

$$\mathbf{v}_h = \left\{\tfrac{1}{2}\,(\mathbf{I}-h\,\gamma^5) + \tfrac{1}{2}\,(\mathbf{I}+h\,\gamma^5)\right\}\mathbf{v}_h = \tfrac{1}{2}(1+A)\begin{pmatrix}-h\,\hat{\chi}_h\\ \hat{\chi}_h\end{pmatrix} + \tfrac{1}{2}(1-A)\begin{pmatrix}h\,\hat{\chi}_h\\ \hat{\chi}_h\end{pmatrix}, \tag{24}$$

in detail

$$\begin{cases} 1)\; \mathbf{v}_{Rh}\big|_{\hat{\chi}_+} = \tfrac{1}{2}(1+A)\,\mathbf{v}_{Rch}\big|_{\hat{\chi}_+} + \tfrac{1}{2}(1-A)\,\mathbf{v}_{Lch}\big|_{\hat{\chi}_+}\\ 2)\; \mathbf{v}_{Lh}\big|_{\hat{\chi}_-} = \tfrac{1}{2}(1-A)\,\mathbf{v}_{Rch}\big|_{\hat{\chi}_-} + \tfrac{1}{2}(1+A)\,\mathbf{v}_{Lch}\big|_{\hat{\chi}_-}\end{cases}. \tag{25}$$

We have related the helical eigenstates with the chiralities parts for the electrons, and in a similar way with the anti-electrons.

The substitution of the relations (22) in the equations in (b-(6) gives a general algebraic relation between the heclicities and the chiralities:

$$\beta_- \, \mathbf{u}_{Lh}\big|_{\chi_-} + \beta_+ \, \mathbf{u}_{Rh}\big|_{\chi_+} = \beta_- \, \tfrac{1}{2}(1{+}A)\, \mathbf{u}_{Lch}\big|_{\chi_-} + \beta_+ \, \tfrac{1}{2}(1{-}A)\, \mathbf{u}_{Lch}\big|_{\chi_+} + \beta_- \, \tfrac{1}{2}(1{-}A)\, \mathbf{u}_{Rch}\big|_{\chi_-} + \beta_+ \, \tfrac{1}{2}(1{+}A)\, \mathbf{u}_{Rch}\big|_{\chi_+} \, , \tag{26}$$

so that we write, symbolically:

$$\beta_- \, \mathbf{u}_{Lh}\big|_{\chi_-} + \beta_+ \, \mathbf{u}_{Rh}\big|_{\chi_+} \overset{\text{Dirac sols.}}{\sim} \gamma_1(A) \;\; \tilde{\mathbf{u}}_{Lch} + \gamma_2(A) \;\; \tilde{\mathbf{u}}_{Rch} \overset{A\longrightarrow 1}{\longrightarrow} \beta_- \, \mathbf{u}_{Lch}\big|_{\chi_-} + \beta_+ \, \mathbf{u}_{Rch}\big|_{\chi_+} \overset{\text{Weak}}{\longrightarrow} \beta_- \, \mathbf{u}_{Lch}\big|_{\chi_-} \, . \tag{27}$$

For a massless particle $(m = 0)$ it is $E = ||\mathbf{p}||$ and therefore $A = 1$. The algebra of the equations (22) and (25) get reduced to:

$$\left.\begin{aligned} \mathbf{u}_{Lh}\big|_{\chi_-} &= \mathbf{u}_{Lch}\big|_{\chi_-} \\ \mathbf{u}_{Rh}\big|_{\chi_+} &= \mathbf{u}_{Rch}\big|_{\chi_+} \end{aligned}\right\} \quad \text{and} \quad \left.\begin{aligned} \mathbf{v}_{Rh}\big|_{\hat{\chi}_+} &= \mathbf{v}_{Rch}\big|_{\hat{\chi}_+} \\ \mathbf{v}_{Lh}\big|_{\hat{\chi}_-} &= \mathbf{v}_{Rch}\big|_{\hat{\chi}_-} \end{aligned}\right\} . \tag{28}$$

We can also apply these results to relativistic electrons and positrons, with: $m \lll E \approx ||\mathbf{p}|| \quad (A < 1, \; A \approx 1)$.

This previous algebra let us identify the expressions of the chiralities with the ones of the helicities:

$$\mathbf{u}_{Lh} \approx \mathbf{u}_{Lch} \, , \qquad \mathbf{u}_{Rh} \approx \mathbf{u}_{Rch} \qquad\qquad \mathbf{v}_{Rh} \approx \mathbf{v}_{Rch} \, , \qquad \mathbf{v}_{Lh} \approx \mathbf{v}_{Lch} \, . \tag{29}$$

With the $\mathbf{P}_-$ **(weak interaction)**, it is for them:

$$\begin{cases} \text{if } \; h = -1 \; (Lh) \, , & \text{then} \quad \mathbf{u}_{Lh} \approx \mathbf{u}_{Lch} = \begin{pmatrix} \chi_- \\ -\chi_- \end{pmatrix}, \;\; (Lch \longleftarrow) \; : \;\; \text{left helicty} \overset{\text{particle}}{\approx} \text{left chiralitty} \\ \text{if } \; h = +1 \; (Rh) \, , & \text{then} \quad \mathbf{v}_{Rh} \approx \mathbf{v}_{Rch} = \begin{pmatrix} -\hat{\chi}_+ \\ \hat{\chi}_+ \end{pmatrix}, \;\; (Rch \longrightarrow) \; : \;\; \text{right helicty} \overset{\text{antiparticle}}{\approx} \text{right chiralitty} \end{cases} . \tag{30}$$

And, with $\mathbf{P}_+$ (**outside of the weak interaction**):

$$\begin{cases} \text{if } \; h = +1 \; (Rh) \, , & \text{then} \quad \mathbf{u}_{Rh} \approx \mathbf{u}_{Rch} = \begin{pmatrix} \chi_+ \\ \chi_+ \end{pmatrix}, \;\; (Rch \longleftarrow) \; : \;\; \text{right helicty} \overset{\text{particle}}{\approx} \text{right chiralitty} \\ \text{if } \; h = -1 \; (Lh) \, , & \text{then} \quad \mathbf{v}_{Lh} \approx \mathbf{v}_{Lch} = \begin{pmatrix} \hat{\chi}_- \\ \hat{\chi}_- \end{pmatrix}, \;\; (Lch \longrightarrow) \; : \;\; \text{left helicty} \overset{\text{antiparticle}}{\approx} \text{left chiralitty} \end{cases} . \tag{31}$$

The **weak charged current** exclusively with left chiral particle parts and right chiral antiparticle parts (with $\mathbf{P}_-$) .

Handedness: a term with a misleading usage. Sometimes referring the helicity, some other times referring the chirality.

• 7 The **particle – antiparticle.** Electric charge conjugation and parity: $\mathbf{C} = \mathrm{i}\,\gamma^y, \quad \mathbf{P} = \gamma^0$,

$$\hat{\mathbf{C}} \, \psi_{\text{particle}} = \mathrm{i}\,\gamma^y \, (\psi_{\text{particle}})^* \, , \qquad \psi_{\text{particle, k}} = \mathrm{e}^{-\mathrm{i}\,S} \, \mathbf{u}_{\mathrm{k}} \overset{\hat{\mathbf{C}}\hat{\mathbf{P}}}{\longrightarrow} \psi_{\text{antiparticle, k}} = \mathrm{e}^{\mathrm{i}\,S} \, \mathbf{v}_{\mathrm{k}} \, , \quad \mathrm{k} \in \{1, 2\} \, . \tag{32}$$

(1 and 2 chosen in the appropriate way). Prevalence of the "y" in relation to this opposition (and the "t" ? $\hat{C}\hat{P}\hat{T}$).

• 8 Free charged leptons and some cases with processes, interactions.

(Towards a conservation of the spin up – down).

We summarize some results under the Dirac Hamiltonian (free massive particles, $m \neq 0$).

The chirality: $[\mathcal{H}_D, \gamma^5] = 2m\begin{pmatrix} \mathbf{0} & \mathbf{1} \\ \mathbf{-1} & \mathbf{0} \end{pmatrix}$ $\Longrightarrow$ the chirality is not conserved, but it is Lorentz invariant.

The helicity: $[\mathcal{H}_D, \hat{\mathbf{H}}] = \mathbf{0}$ $\Longrightarrow$ the helicity is conserved, but it is not Lorentz invariant.

The spin: $[\mathcal{H}_D, \mathbf{S}] \neq \mathbf{0}$ $\Longrightarrow$ the spin is not conserved.

But, in a reference frame with coincident z axis and direction of the motion ($p_x = p_y = 0$) we have (Griffiths, † in page 234 [10]):

$$\hat{\mathbf{H}}\,\mathbf{u}_h\Big|_{\sigma^z} = \frac{h}{\|\mathbf{p}\|}\begin{pmatrix} \boldsymbol{\sigma}.\mathbf{p} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\sigma}.\mathbf{p} \end{pmatrix}\mathbf{u}_h\Big|_{\sigma^z} = h\begin{pmatrix} \sigma^z & \mathbf{0} \\ \mathbf{0} & \sigma^z \end{pmatrix}\mathbf{u}_h\Big|_{\sigma^z} = h\,\Sigma^{(z)}\,\mathbf{u}_h\Big|_{\sigma^z},$$ which is easy to cast for a free particle.

We write: $[\mathcal{H}_D, \hat{\mathbf{H}}]\Big|_{\sigma^z} = [\mathcal{H}_D, h\,\Sigma^{(z)}]\Big|_{\sigma^z} = \mathbf{0}$.

It also can be shown that: " " helicity is effectively "conserved" in high-energy interactions " " (Thomson, page 144) [11].

With the weak theory – chirality, the scheme for the positronium with two or three photons was widen for the collision of electrons and anti-electrons (different energies involved), with the inclusion of the $\mathbf{Z}$ boson, among others. We represent this in the following way:

$$\overset{\{\uparrow,\downarrow\}}{e}{}^- + \overset{\{\uparrow,\downarrow\}}{e}{}^+ \longrightarrow \big\{\{\gamma\},\, Z^0_{\dots},\, \{\mu^- + \mu^+\},\ \dots,\big\}^{\{\uparrow\uparrow,\,0,\,\downarrow\downarrow\}}. \tag{33}$$

Some processes (decays).

– The β **decays:** *neutron* $\longrightarrow$ *proton* $+\, e^- + \bar{\nu}$. We consider two ways:

Fermi decay

$$\begin{cases} \overset{\uparrow}{d} \longrightarrow \overset{\uparrow}{u}\ \overset{0}{(W^-)}_{+}\ \overset{\downarrow}{e}{}^- + \overset{\uparrow}{\bar{\nu}} \\ \overset{\downarrow}{d} \longrightarrow \overset{\downarrow}{u}\ \overset{0}{(W^-)}_{+}\ \overset{\downarrow}{e}{}^- + \overset{\uparrow}{\bar{\nu}} \end{cases} \qquad \Sigma \text{ spins (leptons)} = (\downarrow + \uparrow) = 0 \ \text{(the leptons in a singlet state).} \tag{34}$$

Gamow–Teller decay

$$\begin{cases} \text{(like the previous one, but the leptons in a triplet state).} \\ \overset{\uparrow}{d} \longrightarrow \overset{\downarrow}{u}\ \overset{\uparrow\uparrow}{(W^-)}_{+}\ \overset{\uparrow}{e}{}^- + \overset{\uparrow}{\bar{\nu}} \end{cases} \qquad \begin{cases} \Sigma \text{ spins (leptons)} = (\uparrow + \uparrow) = +1 \\ \text{spin flips (with the quarks)} \quad -1 \end{cases} \tag{35}$$

– The **pion decay:**

$$\overset{0}{\pi}{}^-\Big|_{\text{rest frame}} \begin{Bmatrix} \overset{\uparrow}{d}\ \overset{\downarrow}{\bar{u}} \\ \overset{\downarrow}{d}\ \overset{\uparrow}{\bar{u}} \end{Bmatrix} \overset{\overset{0}{(W^-)}}{\longrightarrow} \Big\{ \overset{\downarrow}{\mu}{}^-_{\begin{cases} Lch \\ Rh \end{cases}} + \overset{\uparrow}{\bar{\nu}}_{\mu\,\begin{cases} Rch \\ Rh \end{cases}} \Big\}\Big|_{\text{cms (frame)}}. \tag{36}$$

The pion decay thanks to the equation 2) in (22). Though, as A is close to 1 (E is much larger than m), this is a small part (the left chiral part $\frac{1}{2}(1-A)\,\mathbf{u}_{Lch}\big|_{\chi_+}$) of the right helical spinor $\mathbf{u}_{Rh}\big|_{\chi_+}$ (the particles in a specific reference frame).

Let us mention a few sentences from several authors:

- D. Griffiths in *Introduction to Elementary Particles* (page 340) [10] :
"We call these various spinors ... 'chiral' fermion states ... I emphasize that this is nothing but notation; it is useful ... ".

- W. Greiner in *Gauge Theory of Weak Interactions* (page 214) [14] wrote for the pion decay:
"... Weak interactions, however, couple only to the left-handed chiral component... contains a small part of positive helicity...".

- M. Thomson in *Modern Particle Physic* [11] :
(page 299): "... and the weak decay to a RH helicity particle state can occur,... ", (page 163): "... in this limit the helicity eigenstates no longer correspond to the chiral eigenstates and helicity is not conserved in the interaction ..." ($A < 1$, lower energies).

- D. H. Perkins wrote in his *Particle Astrophysics* (page 67) [12] : " ... particles with finite mass such as electrons cannot exist in pure helicity eigenstates; they are mixtures of positive and negative helicity states. ...
... For interactions involving vector or axial–vector fields, helicity is conserved in the relativistic limit. ... ".
It is interesting to point out that Perkins in this book never wrote the words "chiral" and "chirality".

This point of view, with $m \neq 0$: a 'chiral' fermion part for a particle (which **is not** a solution of the Dirac equation), has "at once" a part with positive helicity and a part with negative helicity, each one of them a solution of the Dirac equation (an eigenstate). Wide usage of $A \approx 1$; $(1-A \approx \frac{m}{E},\ m \lll E)$. This $(m \lll E)$ is specially truthful for the uncharged leptons.

In another study [15] we examine the above processes under a geometrical (and algebraic) model.

• 9 Statistic. The concept of polarization for the charged leptons has a very special role in checking the adjustment of the experimental and the theoretical results in the nuclear beta decay. Its definition is for electrons:

$$P = \frac{N^{Rh} - N^{Lh}}{N^{Rh} + N^{Lh}} \qquad \begin{cases} N^{Rh}: & \text{number of electrons with right helicity} \\ N^{Lh}: & \text{number of electrons with left helicity} \end{cases} . \tag{37}$$

See Sections 3.5 and 3.6 with the formula (3.9) in Perkins [12]. Therefore, we have ensembles of particles.

As far as we consider the polarization (experiments), with **ensembles** of particles, we do split them in two sub-ensembles in terms of one or the other kind of (predominant?) helicity for each particle: the spin is involved.

Let us go back to the first form of the solutions of the Dirac equation in (a-(6). It is simple to write the following ones:

$$\Psi_1^{+}(\mathbf{x},t) = \mathrm{e}^{\,\mathrm{i}S}\,\alpha_1\,\mathbf{u}_1 = \mathrm{e}^{\,\mathrm{i}S}\,\frac{\alpha_1}{E+m}\begin{pmatrix} E+m \\ 0 \\ p_z \\ p_x+\mathrm{i}\,p_x \end{pmatrix}, \qquad \Psi_2^{+}(\mathbf{x},t) = \mathrm{e}^{\,\mathrm{i}S}\,\alpha_2\,\mathbf{u}_2 = \mathrm{e}^{\,\mathrm{i}S}\,\frac{\alpha_2}{E+m}\begin{pmatrix} 0 \\ E+m \\ p_x-\mathrm{i}\,p_x \\ -p_z \end{pmatrix}. \tag{38}$$

In a similar form for the antiparticles. In total four vectors for a complete set of solutions.

Griffiths wrote in a note in page 234 [10] :

> “ It is possible to construct eigenstates of helicity . . . , but these are rather cumbersome , . . .
> . . . and in practice it is easier to work with the spinors in 7.46 and 7.47, even though their physical interpretation is not so clean. All that really matters is that we have a complete set of solutions. ”

The referenced ones (7.46 and 7.47) are the ones in (38). The helicity eigenstates are in (16) and (17).

We have two possible interpretations for the relation of the helicity and the chirality for the massive particles:

helicities mixtures of chiralities and **chiralities mixtures of helicities**.

The equations in (22) and (26) let the following correspondences (with the factors $(1 \pm A)$) in the statistical interpretation (in the equation (37)) of the results under the weak interaction (see previous comments by Perkins):

$$N^{Lh} = \sum \text{particle}_{Rh\cup Lh} \leftrightarrow \sum \text{particle}_{Rch\cup Lch}, \qquad N^{Rh} = \sum \text{particle}_{Rh\cup Lh} \leftrightarrow \sum \text{particle}_{Rch\cup Lch}. \tag{39}$$

The correspondences in the equations in (22), (26) and (27) tell us that there are differences between the two considerations. We advertise of the statements in • 6 A) and B):

once we look at the Dirac equation we observe that the previous interpretations are not completely symmetrical, and also:

$$\left\{ \beta_- \mathbf{u}_{Lh}\big|_{\chi_-} + \beta_+ \mathbf{u}_{Rh}\big|_{\chi_+} \right\}\Big|_{\text{Weak}} = \beta_- \tfrac{1}{2}\,(1{+}A)\,\mathbf{u}_{Lch}\big|_{\chi_-} + \beta_+ \tfrac{1}{2}\,(1{-}A)\,\mathbf{u}_{Lch}\big|_{\chi_+} \sim \gamma_1(A)\,\tilde{\mathbf{u}}_{Lch} \overset{A\longrightarrow 1}{\longrightarrow} \beta_- \,\mathbf{u}_{Lch}\big|_{\chi_-} \;!$$

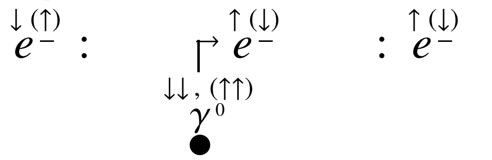
These points will be very relevant in the treatment of the neutrinos.

• 10 More with spins (up , down). See the above mentioned cite in Perkins (page 67) [12]

Could an electron with **a spin** (↑ , ↓) become an electron with **the opposite spin** via **one photon** (an elementary boson)?

We depict some sketches stressing for them the role of the spin, the up and the down (not as a vector), as a conserved quantity:

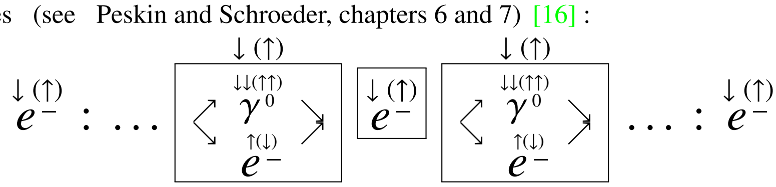
emission or absorption of a photon by an electron in an electrical field.
The $\lambda_0 = 21\text{cm}$ of the atomic hydrogen



(40)

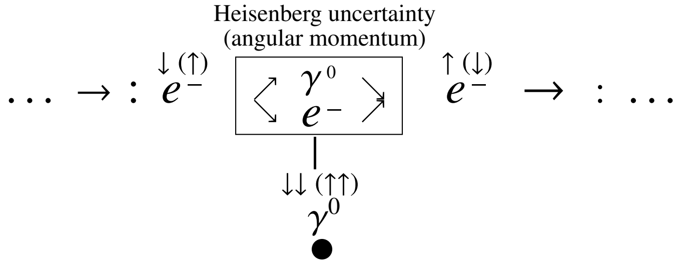
Radiative corrections, virtual processes (see Peskin and Schroeder, chapters 6 and 7) [16] :

self-energy:



(41)

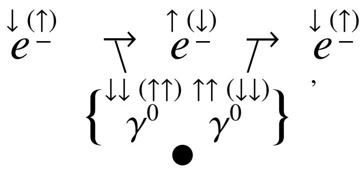
anomalous magnetic moment:



(42)

Finally:

bremsstramhlung:



(photons: one radiated, the other one with an external field)

(43)

II. ELECTRICALLY UNCHARGED LEPTONS: NEUTRINOS – ANTINEUTRINOS. MATTER $\overset{?}{=}$ ANTIMATTER.

At first, as particles without electric charge, they were taken as exclusively influenced by the weak interaction and besides with a null mass assignation.

Experimental results, up to the present day:

1) all the **neutrinos** are **left helical**, therefore with **spin down** (antiparallel to **its** direction of motion),

2) all the **antineutrinos** are **right helical**, therefore with **spin up** (parallel to **its** direction of motion),

the reference frame behind, related to the emitting particles or to any actual observer or laboratory and,

3) with the discovery of the oscillation of the solar neutrinos we have to take account for their non null masses.

This last one suggests two considerations:

it opens their possible sensitiveness to the electromagnetic force, even if it is of a very small order, and

the application of the second equation in (22) and in (25), like in the pion decay case ((36)).

With a possible electromagnetic interaction, and thanks to the zero electric charge,

the type of spin (up – down, a vector) becomes more relevant for the uncharged leptons than for the charged leptons

for the assignments of the concepts of particle and antiparticle (see comment previous to equation (3)).

There are experiments looking for a magnetic moment for the uncharged leptons (electromagnetic response), unsuccessfully. [17]

The uncharged leptons,

$$\mathscr{O} \equiv \textit{neutrinos and antineutrinos} \overset{?}{\equiv} \{ \overset{\downarrow(\uparrow?)}{\mathscr{O}} \cup \overset{\uparrow(\downarrow?)}{\mathscr{O}} \} \overset{?}{\equiv} \{ \overset{\downarrow,\uparrow}{\mathscr{O}} \}, \quad \text{or exclusively} \quad \{ \overset{\downarrow}{\mathscr{O}} \cup \overset{\uparrow}{\mathscr{O}} \},$$

as an statistical ensemble, have spin down and spin up, in a not so similar way to the charged leptons.

Neutrinos (as referring to particles) with spin down and antinueutrinos (as referring to antiparticles) with spin up. Up to the present time, this is the way it is. Exclusively? What does it happen to them as individual particles?

The original **semantic** significance. The uncharged leptons appeared in the beta decays in two different types:

the ones associated with the anti-electrons and the ones associated with the electrons.

The lepton number conservation behind. The electrons have spin up and spin down, and the anti-electrons also have spin up and spin down. In principle, this does not provide information about the **anti** in the uncharged leptons and the **anti** with the type of matter, as it concern their possible mutual annihilation. Therefore, main question,

are the antineutrinos the actual (annihilation) antiparticles of the neutrinos?

With the original semantic for the *matter – antimatter* question (0) , i) and ii)), considering $'' + 0 = - 0''$ (for the electric charge), under their electro-magneto-weak interaction, as a weaker interaction than the electric one with the charged leptons, and also with the Pauli´s exclusion principle (fermions), we ask ourselves:

0 ′– do a neutrino (spin down) and an antineutrino (spin up) attract each other and annihilate, like an electron and an anti-electron (for these last ones the spins are less relevant)? If so, they are particle – antiparticle. Or,

i ′ – do they repel like two electrons (at any energy), so that they would not annihilate each other (for the charged leptons the spins are not so relevant)? If so, they are not particle – antiparticle,

ii ′– *iii ′*– could the spin (up or down) <u>flip</u> or <u>mix</u>? (See later, (44) - (45)). A new setting. Another question. In this case, could they be taken as a single type of particle (Majorana) with two sub-states?

The **lepton number conservation** involved, and therefore a Dirac or a Majorana type of particle.

In 1937 Majorana opened a different setting. Could a particle and an antiparticle be the same particle ($'' + 0 = - 0''$)? It is possible to obtain a simpler wave equation in such a way that

a) we get real equations (in the coefficients, with a σ^y), and

b) two solutions instead of four? In relation to (32): $\psi_{\text{antiparticle}} = \psi_{\text{particle}}$.

This condition, $\psi^C = \mathrm{i}\,\gamma^y\,\psi^* = \psi$ (in (32)) , in the standard formulation (in equations (6), (7)) drives to $\chi_a = \mathrm{i}\,\sigma^y\,\chi_b^*$.

Could it/they be the uncharged lepton/s? There were known the beta decays and the existence of two types of neutrinos. In a first step with the broken parity for the weak interactions. And, with the $V - A$ theory, 1) and 2) are properly asserted in relation to the chirality. The weak interaction tied up to the chirality.

With these ingredients: 'is / are' the uncharged lepton(/s) a Majorana particle or Dirac particles?

– Dirac particles: the neutrino and the antineutrino are different particles. The "two" neutrinos are under the Dirac equation. We should consider four solutions of the Dirac equation. Are there sterile neutrinos? The lepton number is conserved.

– A Majorana particle. A neutrino coincides with 'its' antiparticle, the antineutrino. The lepton number is not conserved. What does it happen with the two spins, the ′up′ and the ′down′?

Is it possible to consider only one particle with spin $\frac{1}{2}$, but two 'subs'-states- (+1 - up and −1 - down), with the two helicities?

$$\nu_{Lh} \left(\equiv \overline{\nu}_{Lh} \right) \overset{?}{\equiv} \overset{\downarrow(\uparrow)}{\mathscr{O}} \quad \text{and} \quad \overline{\nu}_{Rh} \left(\equiv \nu_{Rh} \right) \overset{?}{\equiv} \overset{\uparrow(\downarrow)}{\mathscr{O}}, \quad \text{which we actually name as "neutrino" and "antineutrino"?}$$

The helicity is **conserved** but it is a **frame dependent** concept for the massive particles, not an intrinsic one.

Although, due to their very small masses, they would be at (almost?) any experiment ultrarelativistic particles and therefore the left and the right characters of the helicity and the chirality would (almost) correspond each other.

Remember, the chirality is a **frame independent** concept and it is **not conserved**.

But, there is not experience of neutrinos with a right helicity and neither of antineutrinos with a left helicity. Is it an exclusive result of the almost null masses and therefore their ultra relativistic nature, and once considered to be subject exclusively to the weak interaction (chirality)? Besides, could there be an electromagnetic part of a force that reveal them (ν_{Rh} and $\overline{\nu}_{Lh}$) ?

Some questions have to be clarified:

the definition of the mass and

the possible existence of a type of sterile neutrino(s) which is (are) not under the weak interaction (ν_{Rh} , $\overline{\nu}_{Lh}$).

Again semantic?

The nucleon: with Heisenberg the proton and the neutron as its ' two states '. (Different charges, equal or different spins).

Is a pion a particle with three ' subs ' $\{\overset{0}{\pi}{}^{-}, \overset{0}{\pi}{}^{0}, \overset{0}{\pi}{}^{+}\}$, or are they three different particles? (Equal spin, different charges). Pion, anti-pion (complex fields each one) and pion zero (real scalar field) with different degrees of freedom, three particles.

Is an electron a particle with two ' subs ' $\{\overset{\uparrow}{e}{}^{-}, \overset{\downarrow}{e}{}^{-}\}$, or also $\{\overset{\uparrow,\downarrow}{e}{}^{-}, \overset{\downarrow,\uparrow}{e}{}^{-}\}$, or are they two different particles? (Equal charge, different spins).

Or, is a lepton (families apart) a particle with six ' subs ' $\{\overset{\uparrow}{e}{}^{-}, \overset{\downarrow}{e}{}^{-}, \overset{\uparrow}{\nu}{}^{0}, \overset{\downarrow}{\nu}{}^{0}, \overset{\uparrow}{e}{}^{+}, \overset{\downarrow}{e}{}^{+}\}$, or are they six different particles?

Similarly with a particle (s) quark.

The neutrino(s) oscillates(.). Is it just one particle with six (two, or three, or twelve) ' subs ' $\{\overset{\uparrow}{\nu}{}^{0}_{e}, \overset{\downarrow}{\nu}{}^{0}_{e}, \overset{\uparrow}{\nu}{}^{0}_{\mu}, \overset{\downarrow}{\nu}{}^{0}_{\mu}, \overset{\uparrow}{\nu}{}^{0}_{\tau}, \overset{\downarrow}{\nu}{}^{0}_{\tau}\}$, (also, $\{\overset{\uparrow}{\nu}{}^{0}_{1}, \overset{\downarrow}{\nu}{}^{0}_{1}, \overset{\uparrow}{\nu}{}^{0}_{2}, \overset{\downarrow}{\nu}{}^{0}_{2}, \overset{\uparrow}{\nu}{}^{0}_{3}, \overset{\downarrow}{\nu}{}^{0}_{3}\}$) ? or, are they six (two, or three, or twelve) different particles?

Customarily, we do mind for the electric charge and we do not mind as much for the spin; behind the electric part of the force with the conservation of the electric charge, and the non conservation of the spin under the Dirac´s Hamiltonian.

Careless language, or playing with the language:

' if I am my antiparticle, could I annihilate myself (as an individual particle)? '

Perhaps it would be more appropriate to consider the Majorana mathematical description of the neutrinos, obliterating the annihilation character of the '*anti* '. Again, the named *neutrinos* and *antineutrinos* were associated with the two types of beta decays and a prescribed **lepton number conservation**. The '*anti* ' suggest us, in an actual way, that we need to know if the neutrinos and the antineutrinos annihilate each other (see 0– in our preliminary). Also, do the neutrino(s) solutions have two or four degrees of freedom?

Therefore, looking for a conciliation of the Majorana idea and the two spin (subs) states, we can think in:

1) the flip of the spin, "spontaneously" or in a process (like in (44)). "Spontaneous" or due to other fields (neutrinos are massive)? In this last case, where is the ΔS (ΔJ)? Like in a Gamow-Teller decay? If "spontaneous", why not a flipping (once and once more) in a kind of discrete oscillatory mechanism (like with the mass and the families)? Or

2) whatsoever spin (like in (45)), with the main point in the partial correspondences of helicities and chiralities.

In order to decide the Dirac or the Majorana character for the neutrinos there is an experiment that represents more than semantic for them. This is the *neutrinoless double beta decay*: if the process exists, then the neutrino-antineutrino is considered a Majorana particle.

There has been observed in some atoms two beta decays at once: $2d \longrightarrow 2u + 2e^- + 2\bar{\nu}_{Rh}$.

Would it be possible a process without the emission of neutrinos? $2d \longrightarrow 2u + 2e^-$ (still it is not assured).

One way to visualize this with:

$$\text{①}\quad d_1 \longrightarrow u_1 + [\, e_1^- + \underset{(Lch)}{Rch}\} \overset{\{\uparrow,\downarrow\}}{\bar{\nu}}\underset{(Lh)}{Rh} \,]$$

$$\text{②}\quad \nmid$$

$$\text{③}\quad \underset{Lh}{(Rh)}\overset{\{\uparrow,\downarrow\}}{\nu}\,\{ \underset{Lch}{(Rch)} + d_2 \longrightarrow e_2^- + u_2$$

$$\left(\text{Majorana:}\ \begin{cases} \bar{\nu}_{Rh} \equiv {}_{(Rh)}\nu \equiv \overset{\uparrow}{\mathcal{O}} \\ {}_{Lh}\nu \equiv \bar{\nu}_{(Lh)} \equiv \overset{\downarrow}{\mathcal{O}} \end{cases}\right) \tag{44}$$

① is a standard beta decay. ② represents a **flip** in the spin (quantum electrodynamics). ③ is a beta interaction.

One other way with:

Or:

$$\text{①}\quad d_1 \longrightarrow u_1 + [\, \overset{\downarrow}{e}{}_1^- + \underset{Lch}{Rch}\} \overset{\uparrow}{\mathcal{O}}_{Rh} \,] \quad \text{and} \quad \text{②}\quad {}_{Rh}\overset{\uparrow}{\mathcal{O}}\,\{ \underset{Rch}{Lch} + d_2 \longrightarrow \overset{\uparrow}{e}{}_2^- + u_2$$

$$\text{①}\quad d_1 \longrightarrow u_1 + [\, \overset{\uparrow}{e}{}_1^- + \underset{Rch}{Lch}\} \overset{\downarrow}{\mathcal{O}}_{Lh} \,] \quad \text{and} \quad \text{②}\quad {}_{Lh}\overset{\downarrow}{\mathcal{O}}\,\{ \underset{Lch}{Rch} + d_2 \longrightarrow \overset{\downarrow}{e}{}_2^- + u_2 \tag{45}$$

This treatment likens (resembles) the one of the pion decay. Although Majorana, as we have a non conservation of the lepton number. With Dirac (but, if two solutions!) it would have to be a mixing in the equations in (22) and (25) $(\mathbf{v}_{Rh}|_{\hat{\chi}_+} \overset{!}{\doteq} \mathbf{u}_{Rh}|_{\chi_+})$.

Or, with D. H. Perkins in his *Introduction to High Energy Physics* (page 286) [13] :

"...a tiny LH helicity component will also be present..." (for the antineutrino).

An important point behind in the interpretation of an statistical result as a property of an individual particle; remember the polarization (equation (37)).

Finally, one more aspect, in relation to the neutrinos coming from a supernova, specifically SN1987A (see Mohapatra and Pal, pages 345-348 [18]). There, it is assumed that neutrinos with a magnetic moment can be scattered by charged particles in a plasma, with a photon exchange, leading to the "flipping of the neutrino spin". Schematically:

$$\nu_{Lh\downarrow} + \gamma_{\uparrow\uparrow} \longrightarrow \nu_{Rh\uparrow}\,.$$

In our framework, the neutrinos are exclusively spin **down** and the antineutrinos only spin **up**, therefore the scheme would be:

$$\nu_{Lh\downarrow} + \gamma_{\uparrow\uparrow} \longrightarrow \bar{\nu}_{Rh\uparrow}\,.$$

As it concerns to the spin, neutrinos and antineutrinos are different particles. Perhaps the word "flipping" is not the most appropriate, we better write 'transforming in an antineutrino'.

It would be more meaningful to write of the **spin conservation** (the scalar) than to impose a lepton number conservation. Bosons in between. It also would be interesting to bring here the (constancy?) spin one half (up and down, any axis – vector?) of the protons or neutrons, which are particles composed of elementary fermions (quarks) and bosons (gluons). [19]

Generalizing this way of description, for the electric charge, we could write for the quarks (again bosons in between):

a **d**-quark "to flip to" (to transform in) a **u**-quark with a $\mathbf{W}^-$-boson.

Something similar for the charged leptons with the neutrinos.

A brief and clear exposition of the neutrinoless double beta decay in the article of B. Jones: *The Physics of Neutrinoless Double Beta Decay: A Primer.* [20]

A very extensive exposition in the *Massive Neutrinos in Physics and Astrophysics* of R. N. Mohapatra and P. B. Pal. [18] We are interested in the chapters 4 and 14. It also contains other various treatments of the neutrinoless double beta decay.

Very interesting expositions with: C. Giunti, A. Studenikin, S. Bilenky, J. E. Kim. [21] [22] [23] [24] [17]

B The geometrical model. [25]

Jordan and Wigner found, in 1927, a form for obtaining operators for the fermions: an anti-commuting structure. [26] [27] Essentially an **algebraic** method. They defined generic creation and annihilation operators:

$$\mathbf{a}^\dagger \equiv \Big(\prod_{k=1}^{m-1} \sigma^z_k\Big)\ \sigma^+_m \quad \Big[\text{or}\ \ \sigma^+_m \ \Big(\prod_{k=m-1}^{1} \sigma^z_k\Big)\Big], \qquad\qquad \mathbf{a} \equiv \Big(\prod_{k=1}^{m-1} \sigma^z_k\Big)\ \sigma^-_m \quad \Big[\text{or}\ \ \sigma^-_m \ \Big(\prod_{k=m-1}^{1} \sigma^z_k\Big)\Big].$$

Afterwards, in relation to σ^- or σ^+, if we assign $\mathbf{a}^\dagger$ to the creation of a particle, $\mathbf{a}$ can represent two different operators: its annihilation or also the creation of an antiparticle. With this second case, $\mathbf{a}^\dagger$ also as the annihilation of the antiparticle.

In a previous study [25] we have proposed a generalization of the method developed by Jordan and Wigner, obtaining distinct creation and annihilation operators for every lepton and quark (families apart). This time with a deep **geometrical** meaning. These creation and annihilation operators are constructed in relation to four-dimensional rotations (inside $\mathbb{C}^4$), reminding the way in which Rodrigues and Hamilton defined the three-dimensional rotations.

III. A GEOMETRICAL METHOD FOR THE CHARGED LEPTONS. (FIG. 1 AND APPENDIX B).

We are going to use the following notations:

electrons and anti-positrons parts $\{e, \overline{\underline{e}}\ ;\ \overline{p}, \underline{p}\}$, anti-electrons and positrons parts $\{\overline{e}, \underline{e}\ ;\ p, \overline{\underline{p}}\}$,

the spin: $[\{\text{up}, \uparrow, +1\}$, $\{\text{down}, \downarrow, -1\}]$,

the chirality: $[\{\text{left}, \leftarrow\}$, $\{\text{right}, \rightarrow\}]$,

the vector-spin: $[\{<\mathbf{R}_e^{\frac{\pi}{2}}>, (n_z = 0\,,\ \tau = 1)\}$, $\{<\mathbf{R}_e^{-\frac{\pi}{2}}>\ (n_z = 0\,,\ \tau = -1)\}]$,

the "overbars" and the "underbars" in order to account for the symmetries in the time and the space.

We take from $\mathcal{PART\ I}$ in [25] the creation operators ("daga"). [28]

We defined the *creation operators* for the charged leptons (see Figure 1 in the Appendix A) in the following form:

$$\text{electrons} \updownarrow \text{antipositrons}\quad \left\{\begin{array}{lll}
\overset{\downarrow}{e} \left\{\begin{array}{lll}
(\ 1) & \overset{\downarrow}{e}\overset{\frac{\pi}{2}}{\leftarrow} & l^\dagger_{1,\mathcal{M}} \equiv \Big[\prod_{k=1}^{m-1}\big(\ \mathbf{R}_{e\,k}^{\frac{\pi}{2}}\big)\Big]\Big(\mathbf{R}_o^{\frac{\pi}{4}}\,(-\mathrm{m}^+\sigma^-)\,\mathbf{R}_o^{-\frac{\pi}{4}}\Big)_m \\
(\ 7) & \overset{\downarrow}{\underline{p}}\overset{-\frac{\pi}{2}}{\rightarrow} & l^\dagger_{7,\mathcal{M}} \equiv \Big[\prod_{k=1}^{m-1}\big(-\mathbf{R}_{e\,k}^{\frac{\pi}{2}}\big)\Big]\Big(\mathbf{R}_o^{\frac{\pi}{4}}\,(\ \mathrm{m}^+\sigma^-)\,\mathbf{R}_o^{-\frac{\pi}{4}}\Big)_m \quad \Big(\overset{\text{algebra}}{=} l^\dagger_{-5,\mathcal{M}}\Big)
\end{array}\right. \\
\overset{\uparrow}{\overline{\underline{e}}} \left\{\begin{array}{lll}
(-5) & \overset{\uparrow}{\overline{\underline{e}}}\overset{-\frac{\pi}{2}}{\leftarrow} & l^\dagger_{-5,\mathcal{M}} \equiv \Big[\prod_{k=1}^{m-1}\big(-\mathbf{R}_{e\,k}^{\frac{\pi}{2}}\big)\Big]\Big(\mathbf{R}_o^{\frac{\pi}{4}}\,(\ \mathrm{m}^+\sigma^-)\,\mathbf{R}_o^{-\frac{\pi}{4}}\Big)_m \\
(-3) & \overset{\uparrow}{\overline{p}}\overset{\frac{\pi}{2}}{\rightarrow} & l^\dagger_{-3,\mathcal{M}} \equiv \Big[\prod_{k=1}^{m-1}\big(\ \mathbf{R}_{e\,k}^{\frac{\pi}{2}}\big)\Big]\Big(\mathbf{R}_o^{\frac{\pi}{4}}\,(-\mathrm{m}^+\sigma^-)\,\mathbf{R}_o^{-\frac{\pi}{4}}\Big)_m \quad \Big(\overset{\text{algebra}}{=} l^\dagger_{1,\mathcal{M}}\Big)
\end{array}\right.
\end{array}\right. , \qquad (46)$$

$$\text{antielectrons} \updownarrow \text{positrons}\quad \left\{\begin{array}{lll}
\overset{\uparrow}{\overline{e}} \left\{\begin{array}{lll}
(\ 5) & \overset{\uparrow}{\overline{e}}\overset{\frac{\pi}{2}}{\rightarrow} & l^\dagger_{5,\mathcal{M}} \equiv \Big[\prod_{k=1}^{m-1}\big(\ \mathbf{R}_{e\,k}^{\frac{\pi}{2}}\big)\Big]\Big(\mathbf{R}_o^{\frac{\pi}{4}}\,(\ \mathrm{m}^-\sigma^+)\,\mathbf{R}_o^{-\frac{\pi}{4}}\Big)_m \\
(\ 3) & \overset{\uparrow}{\overline{\underline{p}}}\overset{-\frac{\pi}{2}}{\leftarrow} & l^\dagger_{3,\mathcal{M}} \equiv \Big[\prod_{k=1}^{m-1}\big(-\mathbf{R}_{e\,k}^{\frac{\pi}{2}}\big)\Big]\Big(\mathbf{R}_o^{\frac{\pi}{4}}\,(-\mathrm{m}^-\sigma^+)\,\mathbf{R}_o^{-\frac{\pi}{4}}\Big)_m \quad \Big(\overset{\text{algebra}}{=} l^\dagger_{-1,\mathcal{M}}\Big)
\end{array}\right. \\
\overset{\downarrow}{\underline{e}} \left\{\begin{array}{lll}
(-1) & \overset{\downarrow}{\underline{e}}\overset{-\frac{\pi}{2}}{\rightarrow} & l^\dagger_{-1,\mathcal{M}} \equiv \Big[\prod_{k=1}^{m-1}\big(-\mathbf{R}_{e\,k}^{\frac{\pi}{2}}\big)\Big]\Big(\mathbf{R}_o^{\frac{\pi}{4}}\,(-\mathrm{m}^-\sigma^+)\,\mathbf{R}_o^{-\frac{\pi}{4}}\Big)_m \\
(-7) & \overset{\downarrow}{p}\overset{\frac{\pi}{2}}{\leftarrow} & l^\dagger_{-7,\mathcal{M}} \equiv \Big[\prod_{k=1}^{m-1}\big(\ \mathbf{R}_{e\,k}^{\frac{\pi}{2}}\big)\Big]\Big(\mathbf{R}_o^{\frac{\pi}{4}}\,(\ \mathrm{m}^-\sigma^+)\,\mathbf{R}_o^{-\frac{\pi}{4}}\Big)_m \quad \Big(\overset{\text{algebra}}{=} l^\dagger_{5,\mathcal{M}}\Big)
\end{array}\right.
\end{array}\right. . \qquad (47)$$

The direct matrix products, in the appropriate space, underlies these products. [27] With:

$$\begin{cases} \mathbf{R}_o^{\varphi}(\phi) \equiv \begin{pmatrix} \cos\varphi & -\sin\varphi\, e^{-i\phi} \\ \sin\varphi\, e^{i\phi} & \cos\varphi \end{pmatrix} = \cos\varphi\, \mathbb{1} + \sin\varphi\, \mathbf{R}_o^{\frac{\pi}{2}}(\phi) \\ \mathbf{R}_e^{\varphi}(\phi) \equiv \begin{pmatrix} \cos\varphi & \sin\varphi\, e^{-i\phi} \\ \sin\varphi\, e^{i\phi} & -\cos\varphi \end{pmatrix} = \cos\varphi\, \sigma^{Z} + \sin\varphi\, \mathbf{R}_e^{\frac{\pi}{2}}(\phi) \end{cases} , \quad \begin{cases} n_z = \cos(2\varphi)\,, \quad \tau = \sin(2\varphi) \\ \varphi = \pm\frac{\pi}{4} \Rightarrow \begin{cases} n_z = 0 \\ \tau = \pm 1 \end{cases}, \quad \phi \in [0\,,\pi) \\ (\, n_x = \tau\cos\phi\,, \qquad n_y = \tau\sin\phi\,) \end{cases} .$$

$$\mathfrak{m}^{+} \equiv e^{\,i\,(\phi-\frac{\pi}{2})} = -i\, e^{\,i\,\phi}\,, \qquad \mathfrak{m}^{-} \equiv e^{-i\,(\phi-\frac{\pi}{2})} = i\, e^{-i\,\phi} = \overline{\mathfrak{m}^{+}} = \mathfrak{m}^{+\,-1}$$

$$\left(\mathbf{R}_o^{\epsilon\frac{\pi}{2}}(\pm\mathfrak{m}^{-}\sigma^{+})\,\mathbf{R}_o^{-\epsilon\frac{\pi}{2}}\right) = (\pm\mathfrak{m}^{+}\sigma^{-}) \quad (^{*})\,, \qquad \pm\mathbf{R}_e^{\epsilon\frac{\pi}{2}} = \mp\mathbf{R}_e^{-\epsilon\frac{\pi}{2}} = \left(\mathbf{R}_o^{\epsilon\frac{\pi}{4}}(\pm\sigma^{Z})\,\mathbf{R}_o^{-\epsilon\frac{\pi}{4}}\right), \quad \epsilon \in \{+,-\} \tag{48}$$

There are two important questions:

first, the assignation of the creation of a particle or/and the annihilation of 'its' antiparticle with a matrix σ^{+}, and therefore in their corresponding ways with σ^{-}. This is related with the above equality (*). And also,

second, the usage of the algebraic equality $\overset{\text{algebra}}{=}$. Beneath this algebraic equality there are the assignations of opposite spins and chiralities (geometry – physics). For a discussion on this question see [25] and our *Study II.1)*.

With our geometrical construction, we can synthesize the particle – antiparticle scheme for the charged leptons:

$$\begin{matrix} \left\{\left\langle (1)\, \overset{\downarrow\,\frac{\pi}{2}}{e\leftarrow} \;,\; (7)\, \overset{\downarrow\,-\frac{\pi}{2}}{\underline{p}\rightarrow} \right\rangle ,\; \left\langle (-5)\, \overset{\uparrow\,-\frac{\pi}{2}}{\overline{\underline{e}}\leftarrow} \;,\; (-3)\, \overset{\uparrow\,\frac{\pi}{2}}{p\rightarrow} \right\rangle\right\} & \text{Electrons} & (\,\langle\, \text{electron , anti-positron} \,\rangle\,) & \downarrow,\uparrow \\ \left\{\left\langle (5)\, \overset{\uparrow\,\frac{\pi}{2}}{\overline{e}\rightarrow} \;,\; (3)\, \overset{\uparrow\,-\frac{\pi}{2}}{\underline{p}\leftarrow} \right\rangle ,\; \left\langle (-1)\, \overset{\downarrow\,-\frac{\pi}{2}}{\underline{e}\rightarrow} \;,\; (-7)\, \overset{\downarrow\,\frac{\pi}{2}}{p\leftarrow} \right\rangle\right\} & \text{Anti-electrons} & (\,\langle\, \text{anti-electron , positron} \,\rangle\,) & \uparrow,\downarrow \end{matrix} , \tag{49}$$

$\langle\quad,\quad\rangle$ denotes an electron or an anti-electron. The anti-positron and positron parts are hidden as it concerns the weak interaction.

In total, two electrons (one spin down, one spin up) and two anti-electrons (one spin up, one spin down).

Inside each $\langle\quad,\quad\rangle$ (a particle) with:

<u>the same</u> $\begin{cases} \textit{A) electric charge} & (\; - : \{\langle e, \underline{p}\rangle, \langle \overline{\underline{e}}, \overline{p}\rangle\}\,; \quad + : \{\langle \overline{e}, \underline{p}\rangle, \langle \underline{e}, p\rangle\}\,) \\ \text{and } \textit{B) spin} & (\text{ down}, \downarrow : \{\langle e, \underline{p}\rangle, \langle \underline{e}, p\rangle\}\,; \quad \text{up}, \uparrow : \{\langle \overline{\underline{e}}, \overline{p}\rangle, \langle \overline{e}, \underline{p}\rangle\}\,) \end{cases}$,

fixed for every particle, but the e's and the p's (≡ anti-e's) are

<u>different</u> in the $\begin{cases} \textit{C) chirality} \;(\text{ left}, \leftarrow \;;\; \text{right}, \rightarrow) \\ \text{and } \textit{D) vector-spin with } \varphi = (-\frac{\pi}{2}\,,\,\frac{\pi}{2}) \end{cases}$.

For the magnetic moments (axial vectors). A relation for the $\{\, e\,, \overline{\underline{e}}\,, \overline{e}\,, \underline{e}\,\}$ parts:

Sign(charge) × Sign(spin (up , down)) = Sign(2φ (vector-spin)) ,

	Sign(charge)	×	Sign(spin)	=	Sign(2φ)	
for e	−	×	−	=	+	(left chiral, '(left helicity)')
for $\overline{\underline{e}}$	−	×	+	=	−	(left chiral, '(right helicity)')
for $\overline{e}$	+	×	+	=	+	(right chiral, '(right helicity)')
for $\underline{e}$	+	×	−	=	−	(right chiral, '(left helicity)')

(50)

Sign(2φ(vector-spin)) = Sign(Projection$_{\mathbf{R}_e^{\frac{\pi}{2}}}$ $\mathbf{R}_e^{\{\epsilon\}\frac{\pi}{2}}$) .

'(Helicity)' in the appropriate reference frame, which we denote as 'its own frame'. These charged leptons has a momentum $\vec{\mathbf{p}}$ in the rest frame of the producing particles as they are created. With our construction we impose: $\vec{\mathbf{p}} \Rightarrow \sigma^{Z}$ and $\sigma^{Z} \Rightarrow (\vec{\mathbf{p}})\big|_{\text{(own frame)}}$ <u>locally</u>, in the reference frame of the charged lepton, although $\|(\vec{\mathbf{p}})\| = 0$. This is 'its own frame'.

Let us recede with the quotation of these two sentences in the Thomson's *Modern Particle Physics* (page 105) [11]:

"*. . . its spin along its direction of flight, . . .* ", and also

"*. . . For a spin-half particle, the component of spin measured along any axis is quantised to be either* ±1/2.. . . "; relate this with the Stern-Gerlach experiment.

Curiously, the spin, an axial vector in an "internal space", represented by the Pauli matrices with three real components, is treated as a vector in $\mathbb{R}^3$ (see the Fig. 4.4 in [11] with the helicity $h = \frac{\mathbf{S}\cdot\mathbf{p}}{\mathbf{p}}$).

Also, recall here the spin as a scalar (↑, ↓). See figures 2 and 4 in our study: *Geometry and Physics of the Elementary Fermions.* **2.** *(Interactions). Towards a boson structure* $\{\, \gamma\,, Z^0\,, W^{\pm}\,, g\,\}$. [15].

In our model, the spins (up − down) and the vector-spins, with the relations expressed in (50), are also charges. They are the sources of a magnetic moment. This means that this magnetic moment is neither an Amperian concept (a specific underlying type of rotation) and neither a Gilbertian one (a magnetic dipole). We have studied these questions in *Axial vector magnetic charge and magnetic moment. Maxwell's equations and Lorentz force law.* [29].

An important feature of the model is: $\underline{(\vec{\mu} \sim \vec{S}) \perp \vec{\mathbf{p}}}$, where we have identified: $\vec{S}(\epsilon) = \mathbf{R}_e^{\{\epsilon\}\frac{\pi}{2}} \in \mathbb{R}^3 \subset \mathbb{C}^4$,

the vectors-spin of the charged leptons $(2\varphi = \pm\frac{\pi}{2})$ **are perpendicular to the direction of movement (above $\vec{\mathbf{p}}$), at every instant and evolving in** $\mathbb{C}^4$ (we probe this elsewhere).

The Stern-Gerlach experiment. Shortly after the experiment, Einstein and Ehrenfest [30] [31] showed the impossibility for a classical interpretation of the measurement. Main point in the Larmor precession around the direction of the magnetic field of a magnetic moment, starting in any arbitrary direction in $\mathbb{R}^3$. With the classical treatment, the energies involved and consequently, the amount of time for the alignment would be larger in many orders of magnitude than the actual one.

Some **qualitative considerations on the Stern-Gerlach experiment** at the light of our geometrical model.

At first, entering the magnetic field (outside the magnets the field is weak) we have to consider three different actions:

the couple tending to align the magnetic moment with the field (like in the static case), a very weak effect,

the beginning of a torque of the magnetic moment (it is not static) with the production of a Larmor precession, which frequency is independent of the angle. It should dominate preventing the alignment in the period of time of the experiment. [30]

and, let us now add our model by implementing the orthogonality of the vector-spin and the momentum $\vec{S} \perp \vec{\mathbf{p}}$ (see bellow (50)), which with the Larmor precession would imply a continuum with a drastic change in the momentum, so that we would need to add energy for changing the direction of the particle. This opens the possibility for the alignment hindering a complete Larmor precession.

In a further step we include QED with the virtual processes (self energy). As a result there could be a kind of **Zitterbewegung**. [32] [33].

We summarize.

The particles.

$\left\langle (\ 1)\, e\overset{\downarrow\frac{\pi}{2}}{\leftarrow},\ (\ 7)\, \underline{p}\overset{\downarrow-\frac{\pi}{2}}{\rightarrow} \right\rangle$ is the left helical electron and $\left\langle (\ 5)\, \bar{e}\overset{\uparrow\frac{\pi}{2}}{\rightarrow},\ (\ 3)\, \underline{\bar{p}}\overset{\uparrow-\frac{\pi}{2}}{\leftarrow} \right\rangle$ is the right helical anti-electron,

in 'their own frames', both participating in the weak interaction for the negative and positive pion decays, as respectively right helical electron (muon) and left helical anti-electron (anti-muon) in the frame of the pions, with the left chiral part (1 − electron) and the right chiral part (5 − anti-electron).

$\left\langle (-5)\, \underline{\bar{e}}\overset{\uparrow-\frac{\pi}{2}}{\leftarrow},(-3)\, \bar{p}\overset{\uparrow\frac{\pi}{2}}{\rightarrow} \right\rangle$ is the right helical electron and $\left\langle (-1)\, \underline{e}\overset{\downarrow-\frac{\pi}{2}}{\rightarrow},(-7)\, p\overset{\downarrow\frac{\pi}{2}}{\leftarrow} \right\rangle$ is the left helical anti-electron, (51)

in 'their own frames', both participating in the beta decays, as respectively left helical electron and right helical anti-electron in the frame of the laboratory, with the left chiral part (-5 − electron) and the right chiral part (-1 − anti-electron).

The role of the spin (up , down).

We also consider as different particles to the two electrons with different spins. But they are not particle − antiparticle, as they would repel each other, due to the strength of the electric force. Apply here the Pauli's exclusion principle. Similarly for the anti-electrons.

For the weak interaction:

the anti-electron and electron parts $(\{\bar{e}, \underline{e}, e, \underline{\bar{e}}\})$, are subjected to the weak interaction,

the positron and anti-positron parts $(\{p, \underline{\bar{p}}, \bar{p}, \underline{p}\})$ are not subjected to the weak interaction,

they are hidden parts with respect to the electron anti-electron parts. For all of them $n_z = 0$ $(2\varphi = \pm\frac{\pi}{2})$.

Symmetries.

If we depart from $(1)\, e\overset{\downarrow\frac{\pi}{2}}{\leftarrow}$ (part of an ⟨electron , anti-positron⟩ with spin down) we have "2(x2)" possible different "antis", **as it concern with opposite electric charge**. And looking at the different time and space symmetries we have:

$$\left\langle\ (-1)\, \underline{e}\overset{\downarrow-\frac{\pi}{2}}{\rightarrow},\ \text{after } \tau \text{ opposition alone },\ (-7)\, p\overset{\downarrow\frac{\pi}{2}}{\leftarrow},\ \text{after } n_t \text{ (and } n_z\text{) opposition} \right\rangle,$$

also spin down, both vectors-spin and both chiralities,

$$\left\langle\ (\ 5)\, \bar{e}\overset{\uparrow\frac{\pi}{2}}{\rightarrow},\ \text{after } n_t \text{ opposition alone },\ (\ 3)\, \underline{\bar{p}}\overset{\uparrow-\frac{\pi}{2}}{\leftarrow},\ \text{after } \tau \text{ (and } n_z\text{) opposition} \right\rangle,$$

now spin up, both vectors-spin and both chiralities. And similarly with the others.

This shows that we have opposite sign for the electric charge under **four different time and space symmetries**, in two different particles, the positive charged leptons, one with spin up another one with spin down.

But only **two different symmetries** for the opposite charge with opposite spin:

the one with only n_t opposition (with (5)), and the one with τ (and n_z) opposition (with (3)), in one particle.

These two are the remaining symmetries for the uncharged leptons (see Fig. 2).

The Fig. 1 depicts the previous paragraphs for the charged leptons.

IV. A GEOMETRICAL METHOD FOR THE UNCHARGED LEPTONS. (FIG. 2 AND APPENDIX B).

In our formulation, we have seen the set of particles represented with $\{\langle 1,7\rangle\,,\ \langle -5,-3\rangle\}$ to have a certain electric charge, one with spin down the other one spin up, and with opposite electric charge the ones with $\{\langle 5,\ 3\rangle\,,\ \langle -1,-7\rangle\}$ and they are denoted as the set of their antiparticles, also one with spin up the other with spin down. As it concerns the electric charge, these two sets are characterized by having either opposite time n_t either opposite $\tau\,(n_y)$, but n_z is not as relevant $(n_z = 0)$.

We define the uncharged leptons after an angle value $\varphi_R = \varphi_\nu = 0$ or $\tau_\nu = 0$ and $n_{z_\nu} = \pm 1$. The non equality in the τ has disappeared. This means that we now have for the sub-indexes: $''-1 \equiv 1''$, $''-7 \equiv 7''$, $''-5 \equiv 5''$ and $''-3 \equiv 3''$. Therefore, we only have two particles:

$$\Big\{\Big\langle (1)\,\overset{\downarrow}{\nu}\overset{0}{\leftarrow}\ ,\ (7)\,\overline{\underline{\overset{\downarrow}{\nu}}}\overset{\pi}{\rightarrow}\Big\rangle\Big\} \qquad \text{Neutrino} \quad \big(\big\langle \overset{\downarrow}{\nu}\,,\ \overline{\underline{\overset{\downarrow}{\nu}}}\big\rangle\big),$$
$$\Big\{\Big\langle (5)\,\overset{\uparrow}{\nu}\overset{0}{\rightarrow}\ ,\ (3)\,\overline{\underline{\overset{\uparrow}{\nu}}}\overset{\pi}{\leftarrow}\Big\rangle\Big\} \qquad \text{Antineutrino} \quad \big(\big\langle \overline{\overset{\uparrow}{\nu}}\,,\ \underline{\overset{\uparrow}{\nu}}\big\rangle\big). \tag{52}$$

Fixed the electric charge (0) for both and fixed spin for each one: down for the neutrino, up for the antineutrino, but, every one of the two have $'$at once$'$ both chiralities and vectors-spin, and they do so in a different way:

$$\Big\langle \overset{0}{\leftarrow}\ ,\ \overset{\pi}{\rightarrow}\Big\rangle \ \text{for the neutrino} \qquad\qquad \text{and} \qquad\qquad \Big\langle \overset{0}{\rightarrow}\ ,\ \overset{\pi}{\leftarrow}\Big\rangle \ \text{for the antineutrino.}$$

For the uncharged leptons there is not a relation like the one expressed in (50) for the charged leptons. The vector-spin at $\varphi_\nu = 0$ is: $\{\,\mathbf{R}_e^0 = \sigma^z\,,\ \ (\tau = 0\,,\ n_z = 1\,)\}$. This vector is related with both the spin down for the neutrino and the spin up for the antineutrino, without changing the charge as it is zero. At $\varphi = \pm\frac{\pi}{2}$ it is: $\{\,\mathbf{R}_e^\pi = -\sigma^z\,,\ \ (\tau = 0\,,\ n_z = -1\,)\,\}$.

We define the *creation operators* for the uncharged leptons in the following form:

neutrino	spin $\downarrow$ charge 0	$*\ \overset{\downarrow}{\nu}\overset{0}{\leftarrow}$	$\nu^\dagger_{1,\mathcal{M}}(\varphi_R) \equiv \Big[\prod_{k=1}^{m-1}\big(\ \ \sigma^z_k\big)\Big]\big(-\mathrm{m}^+\sigma^-\big)_m$	(53)
		$\underline{\overset{\downarrow}{\nu}}\overset{\pi}{\rightarrow}$	$\nu^\dagger_{7,\mathcal{M}}(\varphi_R) \equiv \Big[\prod_{k=1}^{m-1}\big(-\sigma^z_k\big)\Big]\big(\ \ \mathrm{m}^+\sigma^-\big)_m$	
antineutrino	spin $\uparrow$ charge 0	$*\ \overline{\overset{\uparrow}{\nu}}\overset{0}{\rightarrow}$	$\nu^\dagger_{5,\mathcal{M}}(\varphi_R) \equiv \Big[\prod_{k=1}^{m-1}\big(\ \ \sigma^z_k\big)\Big]\big(\ \ \mathrm{m}^-\sigma^+\big)_m$	(54)
		$\underline{\overset{\uparrow}{\nu}}\overset{\pi}{\leftarrow}$	$\nu^\dagger_{3,\mathcal{M}}(\varphi_R) \equiv \Big[\prod_{k=1}^{m-1}\big(-\sigma^z_k\big)\Big]\big(-\mathrm{m}^-\sigma^+\big)_m$	

The $*$ indicates that this is the part of the particle which participates in the weak interaction $(n_z > 0)$.

These definitions resemble the original Jordan Wigner transformation, where we have added the novelty of some phases:
$\pm\mathrm{m}^+ = \mp\mathrm{i}\,\mathrm{e}^{\,\mathrm{i}\,\phi}$ and $\pm\mathrm{m}^- = \pm\mathrm{i}\,\mathrm{e}^{-\mathrm{i}\,\phi}$, they involve the angle ϕ, although it is $\tau = 0$!
See at the end in this page and in the pages 17 and 20. We also have included the two semi-directions of the z coordinate axis.

These definitions are the ones in (44) - (45), if we substitute the angles $\varphi \in \{\frac{\pi}{4}, -\frac{\pi}{4}\}$ (in the $\big(\mathbf{R}_o^{\varphi=\pm\frac{\pi}{4}}\big)_k$) by $\varphi \in \{0, \pm\frac{\pi}{2}\}$, so that it is $\big(\mathbf{R}_o^{\varphi=\pm 0}\big)_k = \mathbb{1}$, and $\Big\{\big(\mathbf{R}_e^{2\varphi=0}\big)_k = \sigma^z\ ,\ \ \big(\mathbf{R}_e^{2\varphi=\pm\pi}\big)_k = -\sigma^z\Big\}$. Besides: $\sigma^\pm = \frac{1}{2}(\sigma^x \pm \mathrm{i}\sigma^y)$.

We write some remarks. In our geometric framework:

the sign of the electrical charge is not meaningful for the uncharged lepton families, it is just 0 $(\tau = 0 = \varphi)$,
the neutrinos are exclusively spin down particles, and the antineutinos are exclusively spin up particles,
for them, "anti" represents opposite spin (opposite n_t alone or opposite n_z alone), not opposite charge,
therefore

the neutrino and the antineutrino are different particles,

(in a first sense, with the n_t or n_z oppositions they are not Majorana particles),
each of them have both chiralities and both vectors-spin.

This part of their construction maintains the previous opposition in n_t of the charged leptons for the opposite electric charge. The charged leptons have another kind of opposition in the electric charge with τ, but now τ is reduced to 0. It is in this sense that we have written $''-1 \equiv 1''$, $''-7 \equiv 7''$, $''-5 \equiv 5''$ and $''-3 \equiv 3''$; it is in this sense that we can speak with $\tau = 0$ of a Majorana particle (the not n_y opposition), but this is not a neutrino=antineutrino (previous sense).

We go further on these points in the section V, where we suggest the possibility for tiny values of τ.

The evolution of the uncharged leptons. Generalized rotations.

A type of oscillation, not among ν_e, ν_μ, ν_τ.

We propose generalized discrete rotations in $\mathbb{C}^4$, with the Minkowski metric behind, composed of:

three dimensional rotations (in their own axes σ_k^z) in the way of Rodrigues and Hamilton and

'rotations of the time' (one dimension), formulated as the phases in the complex plane.

This last one is responsible for the chirality as a left or a right rotation (flipping) of the $\pm\sigma_k^z$ (the vectors-spin) in the $\{1, \ldots, m-1\}$ positions.

We write the generalized discrete rotations in the following form:

$$\mathbb{w}' = \mathcal{R}_{s_t,s_z}[\,\widetilde{\mathbb{w}}\,] = e^{i\frac{\pi}{2}\mathbb{D}_{s_t,s_z}}\ \widetilde{\mathbb{w}}\ e^{i\frac{\pi}{2}\mathbb{D}_{s_t,-s_z}}, \qquad \widetilde{\mathbb{w}} \in \{\pm\sigma^z,\ \pm i\sigma^z,\ \pm\sigma^-,\ \pm\sigma^+\}. \tag{55}$$

With:

$$\mathbb{D}_{s_t,s_z} = \tfrac{1}{2}[\,s_t\mathbb{1} + s\,\sigma^z\,] = \tfrac{1}{2}\,s_t\,[\,\mathbb{1} + s_z\,\sigma^z\,] = \tfrac{1}{2}\,s_t\begin{pmatrix}1+s_z & 0\\ 0 & 1-s_z\end{pmatrix} \quad \begin{matrix} s_t \in \{-1,+1\}\\ s_z \in \{-1,+1\}\end{matrix} \Longrightarrow \left\{ \begin{matrix} \mathbb{D}_{++} = -\,\mathbb{D}_{-+} = \begin{pmatrix}1&0\\0&0\end{pmatrix} \\ \mathbb{D}_{+-} = -\,\mathbb{D}_{--} = \begin{pmatrix}0&0\\0&1\end{pmatrix}\end{matrix} \right\}. \tag{56}$$

$$e^{i\frac{\pi}{2}\mathbb{D}_{++}} = \begin{pmatrix}i&0\\0&1\end{pmatrix}, \quad e^{i\frac{\pi}{2}\mathbb{D}_{-+}} = \begin{pmatrix}-i&0\\0&1\end{pmatrix}, \quad e^{i\frac{\pi}{2}\mathbb{D}_{+-}} = \begin{pmatrix}1&0\\0&i\end{pmatrix}, \quad e^{i\frac{\pi}{2}\mathbb{D}_{--}} = \begin{pmatrix}1&0\\0&-i\end{pmatrix}$$

The explicit forms for the uncharged leptons:

Neutrino spin $\downarrow$ $\langle 1 \Updownarrow, \Updownarrow 7\rangle$ (57)

$$\ast\ 1)\quad \begin{matrix} \sigma^z & \longrightarrow & -i\sigma^z & \longrightarrow & -\sigma^z & \longrightarrow & i\sigma^z & \longrightarrow & \sigma^z & \ldots & \ast \\ & \mathcal{R}_{-+} & & \mathcal{R}_{-+} & & \mathcal{R}_{-+} & & \mathcal{R}_{-+} & & & \text{left chilarlity} \\ -\sigma^- & \longrightarrow & -\sigma^- & \longrightarrow & -\sigma^- & \longrightarrow & -\sigma^- & \longrightarrow & -\sigma^- & \ldots & -\sigma^-\ \text{fixed} \end{matrix}$$

$$7)\quad \begin{matrix} -\sigma^z & \longrightarrow & -i\sigma^z & \longrightarrow & \sigma^z & \longrightarrow & i\sigma^z & \longrightarrow & -\sigma^z & \ldots & \ast\ast \\ & \mathcal{R}_{+-} & & \mathcal{R}_{+-} & & \mathcal{R}_{+-} & & \mathcal{R}_{+-} & & & \text{right chilarlity} \\ \sigma^- & \longrightarrow & -\sigma^- & \longrightarrow & \sigma^- & \longrightarrow & -\sigma^- & \longrightarrow & \sigma^- & \ldots & \sigma^-\ \text{flips} \end{matrix}$$

Antineutrino spin $\uparrow$ $\langle 5 \Updownarrow, \Updownarrow 3\rangle$ (58)

$$\ast\ 5)\quad \begin{matrix} \sigma^z & \longrightarrow & i\sigma^z & \longrightarrow & -\sigma^z & \longrightarrow & -i\sigma^z & \longrightarrow & \sigma^z & \ldots & \ast\ast \\ & \mathcal{R}_{++} & & \mathcal{R}_{++} & & \mathcal{R}_{++} & & \mathcal{R}_{++} & & & \text{right chilarlity} \\ \sigma^+ & \longrightarrow & -\sigma^+ & \longrightarrow & \sigma^+ & \longrightarrow & -\sigma^+ & \longrightarrow & \sigma^+ & \ldots & \sigma^+\ \text{flips} \end{matrix}$$

$$3)\quad \begin{matrix} -\sigma^z & \longrightarrow & i\sigma^z & \longrightarrow & \sigma^z & \longrightarrow & -i\sigma^z & \longrightarrow & -\sigma^z & \ldots & \ast \\ & \mathcal{R}_{--} & & \mathcal{R}_{--} & & \mathcal{R}_{--} & & \mathcal{R}_{--} & & & \text{left chilarlity} \\ -\sigma^+ & \longrightarrow & -\sigma^+ & \longrightarrow & -\sigma^+ & \longrightarrow & -\sigma^+ & \longrightarrow & -\sigma^+ & \ldots & -\sigma^+\ \text{fixed} \end{matrix}$$

* and ** : fippings of the vectors-spin due to the discrete jumpings (time underneath) of a time type variable n_t, in $\mathbb{C}^{4(M-1)}$.

Look that these jumpings represents for the σ^z vectors an alternative settlement in a respective time region or space region types with respect to the corresponding time-space point: $\left\{\text{time region} \overset{i}{\leftrightarrow} \text{space region}\right\}\Big|_{\text{at every time-space point}}$. [34]

Could these rotations represent a vortex structure of the time and space geometry? [35] (See the Appendix B).

We present these results in a detailed form in other parts of our set of studies (see the Appendix C); a final (different) interpretation in the *Addenda*.

Is there a possibility for explaining the oscillation of the neutrinos?

Suggestions for tentative future researches.

The geometrical relations equivalent to the ones written in (48) are, for the uncharged leptons:

$$\left\{ n_z = r\cos(2\varphi) = \pm 1\,,\ \ \tau = r\sin(2\varphi) = 0 \right\}\Big|^{(r=1)}_{2\varphi = \left\{ \begin{smallmatrix} 0 \\ \pm\pi \end{smallmatrix} \right.} \ , \qquad \tau \xrightarrow{\ \phi\ } \left\{ n_x = \tau\cos\phi = 0\,,\ \ n_y = \tau\sin\phi = 0 \right\}\Big|^{\tau = 0}_{\forall\phi} . \tag{59}$$

The polar transformation of coordinates $(n_x, n_y) \longleftrightarrow (\tau, \phi)$ has a periodic character in ϕ (infinite values), where we make an election and, overall it can not be inverted in the point $(0, 0) \longleftrightarrow (0, \forall\phi)$. Although this, we are going to conjecture the possibility of specific values of ϕ in our definition of the creation and annihilation operators of the uncharged leptons, even with a value $\tau = 0$. Look that the value of ϕ appears apart of $\tau\ (= 0)$ in our construction, and also the considerations at the end of this page.

Perhaps, it is better to think in the transformation $(\tau, \phi) \longleftrightarrow (n_x, n_y)$, where we make an election of values for the angle ϕ:

$$\left\{ (\tau, \phi) \longleftrightarrow (n_x, n_y) \left\langle \begin{matrix} n_x = \tau\cos\phi = 0 \\ n_y = \tau\sin\phi = 0 \end{matrix} \right\} \right|^{\tau = 0}_{\phi_k} \ , \qquad \left\{ (r, \varphi) \longleftrightarrow (n_z, \tau) \left\langle \begin{matrix} n_z = r\cos(2\varphi) = \pm 1 \\ \tau\ = r\sin(2\varphi) = \ 0 \end{matrix} \right\} \right|^{(r=1)}_{2\varphi = \left\{ \begin{smallmatrix} 0 \\ \pm\pi \end{smallmatrix} \right.} . \tag{60}$$

Up to this point with the algebra and the geometry. Could we go further with the physics?

We conjecture three discrete values for the angle ϕ (ϕ_0, ϕ_1, ϕ_2), establishing with them relationships for the uncharged leptons, for example in the form of the following equations:

$$\left\{ \begin{array}{l}
(\overset{\downarrow}{\mathbf{v}})_l \left\{ \begin{array}{llll}
1) & (\overset{\downarrow}{\mathbf{v}})_l \overset{0}{\leftarrow} & \longleftrightarrow & \mathbf{v}^{\dagger}_{l\,1} = \prod_{k=1}^{m-1} (\ \sigma^z)_k (\ \ \mathrm{i}\, \mathrm{e}^{\mathrm{i}(\phi + l\,\delta_l)}\, \sigma^-)_m = \mathrm{e}^{\,\mathrm{i}\,l\,\delta_l}\, \mathbf{v}^{\dagger}_{e\,1} \\
7) & (\underline{\overset{\downarrow}{\mathbf{v}}})_l \overset{\pi}{\rightarrow} & \longleftrightarrow & \mathbf{v}^{\dagger}_{l\,7} = \prod_{k=1}^{m-1} (-\sigma^z)_k (-\mathrm{i}\, \mathrm{e}^{\mathrm{i}(\phi + l\,\delta_l)}\, \sigma^-)_m = \mathrm{e}^{\,\mathrm{i}\,l\,\delta_l}\, \mathbf{v}^{\dagger}_{e\,7}
\end{array} \right. \\
(\underline{\overset{\uparrow}{\mathbf{v}}})_l \left\{ \begin{array}{llll}
5) & (\underline{\overset{\uparrow}{\mathbf{v}}})_l \overset{0}{\rightarrow} & \longleftrightarrow & \mathbf{v}^{\dagger}_{l\,5} = \prod_{k=1}^{m-1} (\ \sigma^z)_k (\ \ \mathrm{i}\, \mathrm{e}^{-\mathrm{i}(\phi + l\,\delta_l)}\, \sigma^+)_m = \mathrm{e}^{-\mathrm{i}\,l\,\delta_l}\, \mathbf{v}^{\dagger}_{e\,5} \\
3) & (\underline{\overset{\uparrow}{\mathbf{v}}})_l \overset{0}{\leftarrow} & \longleftrightarrow & \mathbf{v}^{\dagger}_{l\,3} = \prod_{k=1}^{m-1} (-\sigma^z)_k (-\mathrm{i}\, \mathrm{e}^{-\mathrm{i}(\phi + l\,\delta_l)}\, \sigma^+)_m = \mathrm{e}^{-\mathrm{i}\,l\,\delta_l}\, \mathbf{v}^{\dagger}_{e\,3}
\end{array} \right. \\
\{\, l = 0 \ \text{ for } \ e^-, e^+ \,\}, \quad \{\, l = 1 \ \text{ for } \ \mu^-, \mu^+ \,\}, \quad \{\, l = 2 \ \text{ for } \ \tau^-, \tau^+ \,\} \\
\mp\mathrm{i}\, \mathrm{e}^{\pm\mathrm{i}(\phi + l\,\delta_l)} = \{ \mp\mathrm{i}\, \mathrm{e}^{\pm\mathrm{i}\phi} \}\, \mathrm{e}^{\pm\mathrm{i}\,l\,\delta_l} = \mathfrak{m}^{\pm}(\phi)\, \mathrm{e}^{\pm\mathrm{i}\,l\,\delta_l} = \mathfrak{m}^{\pm}(\phi + l\,\delta_l)
\end{array} \right. . \tag{61}$$

These are the defining equations (53)-(54), where we have chosen three specific values for the angle:

$$\phi_l = \phi_0 + l\,\delta_l\ , \qquad l \in \{0, 1, 2\}\ , \qquad (l = 0,\ \phi = \phi_0)\ .$$

We are going to write arrows to emphasize the vector character of some magnitudes, even if we do not specify the space $(\mathbb{C}^4)$. Under the assumption of the proportionality of the momentum of the uncharged leptons $\vec{\mathbf{p}}$ and their $(\vec{\sigma^z})_k$, we still would not obtain the oscillatory character; the plane of the **x** and **y** coordinates (with τ and ϕ) is orthogonal to the **z** coordinate line. In the m position we also have: $\vec{\sigma^{\pm}} = \frac{1}{2}\,(\vec{\sigma^x} \pm \mathrm{i}\vec{\sigma^y})$.

Let us bring here a couple of lines from Griffiths´s *Introduction to Elementary Particles* (in a note in the page 165): [10]

"... there are minute corrections introduced by quantum electrodynamics (QED)
that were first calculated by Schwinger in the late 1940s."

We need to ask for one more conjecture. The electromagnetic interaction: QED. In another study [29] we suggest the nullity of the magnetic moment of the neutrino, with the possibility of a very tiny anomalous magnetic moment. Even if in average the implied direction is still coincident with $(\vec{\sigma^z})$, we now suggest that at every discrete step of the evolution we have new directions forming a very small angle with the $(\vec{\sigma^z})$, and therefore with the momentum $\vec{\mathbf{p}}$ (an average). This opens the possibility of developing $\vec{\tau}$ components with angles $\phi_0 + l\,\delta_l$, in a non perpendicular plane to the direction $\vec{\mathbf{p}}$ (the averaged), and therefore with a non null projection over the direction of $\vec{\mathbf{p}}$. We show three possible consequences:

different behavior under the Lorentz transformations (accessibility to the oscillations, with an extension of (55)), also with the justification of the equations (22)-1 and of (25)-1 (the mass $m \neq 0 \implies A \neq 1$);

up to the present time, the experimental facts in agreement with $\mathbf{u}_{Rh}\big|_{\chi_+} = 0$ and $\mathbf{v}_{Lh}\big|_{\hat{\chi}_-} = 0$,

the development of very **tiny amounts of electric charge** and of tiny variations in the value of the vector-spin (see page 20).

Tiny amounts of the τ coordinate do not change the spins (up , down) of the uncharged leptons,
but do change: the electrical charge (previously zero), with both signs, and the vectors-spin (previously σ^z).

This could represent the appearance of anomalous electric and magnetic dipole moments (tiny).

V. DISCUSSION. PERSPECTIVES. SOME QUALITATIVE CONSIDERATIONS.

Departing point: the study of the leptons, all of them with no null masses, ruled by the Dirac equation which permits various combination of four four-spinors.

Charged leptons. Although we have in mind the initial quotation by Weisskopf, which we put into play later, let us start considering specific solutions of the Dirac equation with the charged leptons.

It is standard to separate the positive energy solutions from the negative ones (+ and − in the exponentials) in the Dirac equation, with an assignment to different type of particles: the positive and the negative electrically charged. In order to work with one lepton, we can not mix the positive with the negative energy solutions. Could their mixing represent a zero electric charge (a neutrino?!). Besides, would it be a fermion? It does not seem to be a particle showing up, a statistical mixing of a $+1$ and a -1 electric charge, in a similar way as the usual theory does with the mixing of the helicities (therefore the spin parts). We also could try to consider an analogous process to the one presented in (40) with the spins, but we do not know of a process changing a ±1 to a ∓1 electric charge via a simple boson, formally it is necessary to put into play two **W** bosons (with the weak interaction). We have not studied a mixing of the positive and negative energy solutions with the hole theory and the zitterbewegung. [32] Our treatment with (19) is not standard.

Let us handle the question of the spin for them.

First. Under the weak processes. A mixing of the helicities. We look at ensembles of particles.

A single helicity can be associated to an individual solution of the Dirac equation, meanwhile we can not associate a single chirality.

The experiments involving the weak interaction tell us about the appearance of ensembles of leptons with both types of helicities. We exemplify with high energy electrons. The usual researches, with the polarization, transfer the statistical results to the treatment of the individual particles, assuming for them a "tiny right handed component"; and a "tiny left handed component" for the antiparticles. Remind the paragraphs of Perkins after (45) and of Griffiths after (38). Using the equations in (22) we write in a weak process ($\gamma^\mu\,(\mathbf{I}-\gamma^5) = 2\gamma^\mu\,\mathbf{P}\_$, and $\mathbf{P}_-\mathbf{u}_{Rch} = 0$):

For the electrons:

$$\left\{ \overset{\{\downarrow,\uparrow\}}{e} \right\}\Big|_{\text{Weak}} \left\{ \begin{array}{ll} 1) & \mathbf{u}_{Lh}\big|_{\chi_-} = \tfrac{1}{2}(1{+}A)\,\mathbf{u}_{Lch}\big|_{\chi_-} + \tfrac12(1{-}A)\,\overset{0}{\cancel{\mathbf{u}_{Rch}}}\big|_{\chi_-} \\ 2) & \mathbf{u}_{Rh}\big|_{\chi_+} = \tfrac12(1{-}A)\,\mathbf{u}_{Lch}\big|_{\chi_+} + \tfrac12(1{+}A)\,\overset{0}{\cancel{\mathbf{u}_{Rch}}}\big|_{\chi_+}\end{array}\right\}\Big|_{\text{Weak}} \longrightarrow \tag{62}$$

$$\longrightarrow \left\{\Psi^+(\mathbf{x},t) = e^{iS}\left(\beta_-\mathbf{u}_{Lh}\big|_{\chi_-} + \beta_+\mathbf{u}_{Rh}\big|_{\chi_+}\right)\right\}\Big|_{\text{Weak}} = e^{iS}\left(\beta_-\frac{(1+A)}{2}\,\mathbf{u}_{Lch}\big|_{\chi_-} + \beta_+\frac{(1-A)}{2}\,\mathbf{u}_{Lch}\big|_{\chi_+}\right) \longrightarrow \begin{array}{l} \tfrac12(1{+}A)\nearrow\ N^{Lh} \\ \tfrac12(1{-}A)\searrow\ N^{Rh}\end{array} .$$

A charged lepton has at once the two sub states of the spin, the up and the down, in different proportion ('as an ensemble'). Besides, $A = A(E)$.

Against this previous usual form, and taking into account the contents in the PRELIMINARY Section I, we suggest: each charged lepton have specific values of the **electric charge** (+ , −) and of the **spin (up , down)**, and therefore of the helicity in a given reference frame, after their production and while not interacting.

For the electrons:

$$\left\{ \begin{array}{c} \overset{\downarrow}{e} \\ \\ \overset{\uparrow}{e} \end{array} \right\}\Big|_{\text{Weak}} \begin{array}{ll} 1) & \mathbf{u}_{Lh}\big|_{\chi_-} = \dfrac{1}{2}(1{+}A)\,\mathbf{u}_{Lch}\big|_{\chi_-} + \dfrac12(1{-}A)\,\overset{0}{\cancel{\mathbf{u}_{Rch}}}\big|_{\chi_-} \\ 2) & \mathbf{u}_{Rh}\big|_{\chi_+} = \dfrac12(1{-}A)\,\mathbf{u}_{Lch}\big|_{\chi_+} + \dfrac12(1{+}A)\,\overset{0}{\cancel{\mathbf{u}_{Rch}}}\big|_{\chi_+}\end{array}\Bigg|_{\text{Weak}} \begin{array}{l} \xrightarrow{\frac12(1+A)}\ N^{Lh} \\ \xrightarrow{\frac12(1-A)}\ N^{Rh}\end{array} . \tag{63}$$

As we can see, we get the same results for the charged leptons (considered as sets), with both interpretations.

The question is: in what way in a specific process we obtain more $\overset{\downarrow}{e}$ or $\overset{\uparrow}{e}$, due to the various possibilities for each form of such processes. Weak interaction (the such processes) 'favors' the production of the left helical electrons (with the right helical antineutrinos); even more with higher energy ($A \to 1$). See Fermi decays versus Gamow-Teller decays (in (34)-(35)). We are paying attention to the spin as a scalar (up − down), and not to the spin as a vector (in $\mathbb{R}^3$).

Second. QED. The "minute corrections", above mentioned by Griffiths. Also the quotation by Weisskopf.

We have to add to the previous schema the virtual processes, like the pair creation and annihilation (vacuum polarization), the self energy, the anomalous magnetic moment and also the bremsstramhlung. Some of them represented in the sketches (41)-(43). In all of them participate the photons, either as emission or as capture or as both, driving to ±1 units of spin, related to the $\pm\frac12$ units of spin of the charged leptons. We also have to take account of "minute" changes in the directions (the momentum, $\mathbf{p}$).

In addition, we would like to introduce one more intriguing concept obtained with an algebraic treatment of a complete orthonormal set and not experimentally observable: the **Zitterbewegung**. [32] [33] In old formulations related to the Dirac´s hole theory the usage of positive and negative energy plane-wave solutions produce interference between them, driving to rapid oscillations over a rectilinear motion.

Could we relate the virtual processes with the emission and the capture of a photon by an electron (self energy), represented in (41), with rapid oscillations, with some kind of Zitterbewegung in their evolution, the previous "minute" changes in the directions (the momentum, $\mathbf{p}$)?

Third. In our geometrical model the charged leptons already appear as different particles when having different spins (up or down). The electric charge and the spin (up or down) are intrinsic properties differentiating each type of charged lepton. Besides, the statements in (50) provide relations for the electric charge, the spin, the vectors-spin and afterwards the magnetic moment. In the page 14 and in another study [29] , we suggest an explanation for the results of the Stern-Gerlach experiment relating the magnetic moment with the vectors-spin thanks to their geometrical form (the $\varphi \in \{\pm\frac{\pi}{4}, \pm\frac{3\pi}{4}\}$ or $\{n_t = \pm 1,\ n_z = 0,\ \tau = \pm 1\}$), their orthogonality with the trajectory of the particle.

We have defined the physical properties (the charge and the spin) in terms of the coordinates. We remind (see figure 1):

opposition with the coordinates $\{n_t,\ \tau\ (n_y)\}$ $\leftrightarrow$ opposition of the electric charges.
opposition with the coordinates $\{n_t,\ n_z\}$ $\leftrightarrow$ opposition of the spins (up , down).

We suggest that the "minute" changes with the momentum alter the defining coordinates and therefore the physical properties. In this way, it is possible to interpret these minor changes in the following way:

$$\left.\begin{array}{l} (n_z = 0) \rightarrow \quad \pm\delta_1 \\ (\tau = \pm 1) \rightarrow \pm 1 \mp \delta_2 \\ \text{without a change in the } n_t \text{ coordinate} \end{array}\right\} \longrightarrow \begin{array}{l} (q = \pm 1) \rightarrow \quad \pm 1 \mp \tilde{\delta}_1 \\ (\pm \mathbf{R}_e^{\frac{\pi}{2}}) \rightarrow \mathbf{R}_e^{\frac{\pi}{2}} (\pm 1 \mp \tilde{\delta}_2) \\ \text{- vectors-spin -} \end{array} \quad \text{and} \quad \left\{\begin{array}{l} (\text{spin up}) \rightarrow \begin{array}{c}\text{spin down}\\ \text{spin up}\end{array} \\ (\text{spin down}) \rightarrow \begin{array}{c}\text{spin up}\\ \text{spin down}\end{array} \end{array}\right. .$$

The minor changes in the geometry can drive, for the charged leptons, to a drastic change of the spin (up $\leftrightarrow$ down), minor changes in the electric charge and minor changes in the vectors-spin.

Could these minor changes be associated to the evolution of the charged lepton (in a similar way as in (57)-(58))?

Could they be associated with the self energy (the virtual photons)? (Sketch (41)).

Or, with sketch (42) representing an anomalous magnetic moment. And in a parallel way,
could they give rise to an anomalous dipole electric moment (over the electric charges $(-1, -1 + \tilde{\delta}_1)$, $(1, 1 - \tilde{\delta}_1)$?

Uncharged leptons.

We start with the standard construction, which is different to the one presented in (19).

First. Under the weak processes. We still do not have positive results for the neutrinoless double β-decay. Therefore, we base our interpretation in the actual experimental results (actual reference frames) and in our model: neutrinos are exclusively left handed (spin down) and antineutrinos are exclusively right handed (spin up). The equations (22) and (25) get reduced to:

$$\begin{array}{lll} 1) & \mathbf{u}_{Lh}\big|_{\chi_-} = \tfrac{1}{2}(1{+}A)\,\mathbf{u}_{Lch}\big|_{\chi_-} + \tfrac{1}{2}(1{-}A)\,\overset{0}{\cancel{\mathbf{u}_{Rch}}}\big|_{\chi_-} & \Leftarrow \text{ our model} \quad \overset{\downarrow}{\mathbf{v}} \;\Big|\text{Weak} \quad \xrightarrow{A<1,\,A\approx 1} \quad N^{Lh} \\ 2) & \mathbf{u}_{Rh}\big|_{\chi_+} = 0 \quad \text{———} & \Rightarrow \text{ actual exp.} \quad \cancel{\overset{\uparrow}{\mathbf{v}}} \qquad \longrightarrow \quad N^{Rh} = 0 \end{array} \quad . \qquad (64)$$

$$\begin{array}{lll} 1) & \mathbf{v}_{Rh}\big|_{\hat{\chi}_+} = \tfrac{1}{2}(1{+}A)\,\mathbf{v}_{Rch}\big|_{\hat{\chi}_+} + \tfrac{1}{2}(1{-}A)\,\overset{0}{\cancel{\mathbf{v}_{Lch}}}\big|_{\hat{\chi}_-} & \Leftarrow \text{ our model} \quad \overset{\uparrow}{\mathbf{v}} \;\Big|\text{Weak} \quad \xrightarrow{A<1,\,A\approx 1} \quad \tilde{N}^{Rh} \\ 2) & \mathbf{v}_{Lh}\big|_{\hat{\chi}_-} = 0 \quad \text{———} & \Rightarrow \text{ actual exp.} \quad \cancel{\overset{\downarrow}{\mathbf{v}}} \qquad \longrightarrow \quad \tilde{N}^{Lh} = 0 \end{array} \quad . \qquad (65)$$

There are tiny chiral components in (64)-(65) not participating in the weak interaction. A process like the one sketched in (44) (the included tiny helicity components) is forbidden as it concern to two subs for the spin: there is not the other helicity. Look that (44) would be possible in a process similar to the one in (40) (including an interaction), as we already wrote:

$$\nu_{Lh\downarrow} + \gamma_{\uparrow\uparrow} \longrightarrow \bar{\nu}_{Rh\uparrow} \,.$$

One way to relate the other helicity is by changing the reference frame (opposite direction), which seems to be unrealistic from an actual experimental point of view for ultrarelativistic particles (the neutrinos).

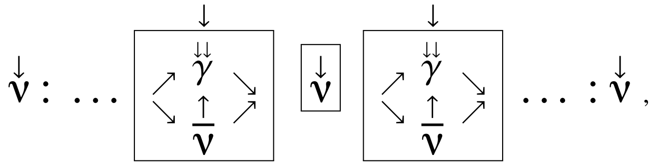
Compare formula 1) in (64) with $\mathbf{p} = -\mathbf{p}'$ and the second formula in (22) with $\mathbf{p}'$, which is for spin up.

These helicities are related with the frames of the producing particles and of the frames of the laboratories (the interactions). It does not seem to have experimental significance to write about the helicity of a particle in its own reference frame, as it is $\mathbf{p}'' = \mathbf{0}$ (see after (50)).

Other way, with the interactions; see later the possible neutrinoless double beta decay.

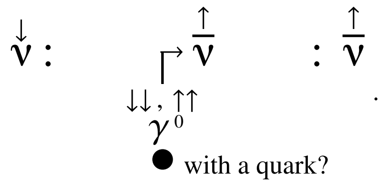
Second. QED. They can interact electromagnetically. Could they undergo processes similar to the ones in (41) and (40)?

Self-energy:



(66)

like in the emission or absorption of a photon by an electron in an electrical field. (The $\lambda_0 = 21\text{cm}$ of the atomic hydrogen). (Mohapatra and Pal, pages 345-348 [18]).



(67)

Third. (Figure 2) In our geometrical model the neutrinos have exclusively spin down and antineutrinos exclusively spin up.

We have the following relation of the geometric settings (axes, coordinates) and the physical properties (charge, spin):

opposition with the coordinates $\{n_t\,,\ \tau = 0\}$ $\leftrightarrow$ there is no opposition of the electric charges $(q = 0)$.

opposition with the coordinates $\{n_t\,,\ n_z\}$ $\leftrightarrow$ opposition of the spins (up , down).

As different to the charged leptons which satisfy the relations (50) for the vectors-spin, the uncharged leptons have the same vectors-spin: Vector-Spin(ν)=Vector-Spin($\bar{\nu}$)= σ^z. But Spin (ν)≠Spin ($\bar{\nu}$). This is important for the magnetic moments.

With some possible "minute" corrections (QED):

$$\left.\begin{array}{l} (\tau\ =\ 0) \rightarrow\ \ \pm\,\delta_1 \\ (n_z = \pm 1) \rightarrow \pm 1 \mp \delta_2 \\ \text{without a change in the } n_t \text{ coordinate} \end{array}\right\} \longrightarrow \begin{array}{l} (q = 0) \rightarrow\ \ \pm\tilde{\delta}_1 \\ (\pm\sigma^z) \rightarrow \sigma^z(\pm 1 \mp \tilde{\delta}_2) \\ \text{- vectors-spin -} \end{array} \ \text{and}\ \left\{\begin{array}{l} (\text{spin up}) \rightarrow \text{spin up} \\ (\text{spin down}) \rightarrow \text{spin down} \end{array}\right. .$$

For the uncharged leptons, the minor changes in the geometry drive to minor changes in the electric charge and minor changes in the vectors-spin. The spins (up , down) do not change. This could forbid a virtual process like the one depicted in (66) (the self-energy), unless we admit a non small change in n_t (with the evolution?). The process in (67) would justify the possibility for a double beta decay with a non conservation of the lepton number, but without Majorana.
Besides, could these minor changes be associated with anomalous moments (magnetic and electric)?

If we depart from an anti-electron $(\varphi = \frac{\pi}{4})$ and an electron $(\varphi = -\frac{\pi}{4})$ both with spins up, and we make $\varphi \to 0$ then $\tau \to 0$ $(n_y \to 0)$, so that we obtain the antineutrino. Consider the matrix γ^y and relate it with the definition of a y-axis. Now, the matrix γ^y is an ashlar in the Majorana construction. This reminds us of the double zeros in an algebraic equation.
In this sense the antineutrino is just one particle (there are not two particles), **Majorana 1**.
Similarly for the neutrino, with spin down, around the value $'\,\varphi = -\pi = +\pi\,'$. **Majorana 2**.

But, they are not only one particle. This is due to their different spins: one only 'up', the other one only 'down'. The difference in the geometry with the symmetries in n_t (the two values, $n_t = \pm 1$) and n_z (the two values, $n_z = \pm 1$), **Dirac**.

We summarize.

Again, **does a neutrino particle coincide with its antineutrino particle? With its "antiparticle"?**
And, **are they Dirac or are they Majorana particles?** In some way, it could be both and neither.

Usual interpretation.

As Dirac particles, at first, there would be only two spinor solutions which we can relate to the spin (the up and the down). We can not duplicate the number of solutions via the charge $(q = 0)$, which has been associated in the exponentials with a positive or a negative sign for the energy. We can look for the other two solutions in terms of opposite helicities (with a **p**!).
It is interesting to look at the interpretation with positive and negative energies and reversed helicities and chiralities for massless neutrinos; see Greiner´s *Relativistic Quantum Mechanics. Wave equations* (page 277) [36].
The other helicities do not appear in any actual experiment. Could there be two other solutions of the of the Dirac equation? Are there sterile neutrinos and sterile antineutrinos? They have not been found. Perhaps, **they are not Dirac particles.**

Or, is there a positive answer to the neutrinoless double beta decay (then Majorana)? Their non null masses permits the mixing of helicities and their expression in term of a mixing of the chiralities. The expectation in a mixing of the two sub-states of the spin 1/2, the up and the down. The experimental measurements are related to the helicities but not to the chiralities. The chirality is a condition for the weak interaction. The chirality is interpreted as a flipping property, always with a mixture of both types. There is not a positive result for the neutrinoless double beta decay. Perhaps, **they are not Majorana particles.**

With **our geometrical model** (see the final part of Third in the previous page and the figure 2).

They are Majorana particles as it correspond to the 'convergence' of the electrons and of the anti-electrons ($'q \to 0'$):
with $\varphi \to 0$ (for the antineutrino, only spin up), and with $\varphi \to \pm\pi$ (for the neutrino, only spin down).
This responds to the τ − symmetry, the n_y − symmetry, γ^y in (32).
Similarly with the anti-positrons and the positrons parts ($\varphi \to \pm\frac{\pi}{2}$).
Therefore in this sense we should not name them with $\varphi = 0$ as neutrino − antineutrino. Similarly with $\varphi = \pm\pi$.

But, they are not Majorana particles as they have different spins (the n_t and the n_z - symmetries), in this sense they could be Dirac particles. It is in this sense that the named neutrino − antineutrino can be appropriate.

Once established these points trying to clarify the first note in $0-$ (the oppositions), let us look for the second note: the annihilation. We do not know.

Therefore, we still need to clarity two important aspects:

a) do they actually annihilate each other? And,

b) why we do not see right helicity neutrinos and neither left helicity antineutrinos?

In order to answer these two questions we would need to analyze the processes of the interactions. A first step in the study of the kinetics of the particle as a geometrical entity with structure. The dynamic, the process of interaction changes the kinetics and could affect the geometry of the particles. (See Figure 3 C).

a): Could a neutrino annihilate with an antineutrino? In our treatment this correspond to oppositions in the time, i.e. 5 with 1 (see figure 2), in a similar way as an electron with an anti-electron producing two photons, −1 with +5 and −5 with +1; but not with three photons, −1 with +1 and −5 with +5 (see figure 1). This last would represent, for the uncharged leptons, a 'self-annihilation'. Perhaps, also with the space opposition (in n_z), 1 with 3 and 5 with 7; this is outside the weak interaction and it is related to point b) (see figure 3 C)) with a possible inversion of the helicities.

b): For the helicity, the answer is, in our treatment: there is not the other helicity, up to uninteresting Lorentz transformations. Each uncharged lepton has only one type of spin, the neutrino down and the antineutrino up, and a local direction of propagation, 'theirs' $+\sigma^z$, obtained after their production. So that we can not see the part of the neutrinos and antineutrinos that goes with $n_z = -1$ ('theirs' $-\sigma^z$). This establishes a relation of the structure of the time-space and of the weak interaction. Hypothetically, the $n_z = -1$ ('theirs' $-\sigma^z$) parts of the uncharged leptons would be obtained by their captures by very fast laboratories, but even for this, we propose a change in the geometry of the uncharged leptons in the detection − interaction (see figure 3 C).

We have obtained for the charged leptons the equations (50), relating the electrical charge $\{-, +\}$ the spin {down , up} and the vectors-spin $\{\mathbf{R}_e^{\frac{\pi}{2}}, \mathbf{R}_e^{-\frac{\pi}{2}}\}$. The weak interaction selects $\langle$ down $\leftrightarrow \mathbf{R}_e^{\frac{\pi}{2}} \rangle$ and $\langle$ up $\leftrightarrow \mathbf{R}_e^{-\frac{\pi}{2}} \rangle$ for the electrons and $\langle$ up $\leftrightarrow \mathbf{R}_e^{\frac{\pi}{2}} \rangle$ and $\langle$ down $\leftrightarrow \mathbf{R}_e^{-\frac{\pi}{2}} \rangle$ for the anti-electrons.
Look that the vectors $\mathbf{R}_e^{\pm\frac{\pi}{2}}$ are in a plane perpendicular to n_z . Also the related magnetic moments.

In sharp contrast with the charged leptons, we now have for both the neutrino and the antineutrino the same vector-spin, exclusively $\mathbf{R}_e^0 = +\sigma^z$ (an orientation in a three dimensional space), this is so as it concerns the weak interaction ($n_z \geq 0$). An important consequence in the treatment of the magnetic moment, once we do not have an electrical part. Or perhaps, as an analogy: "currents in same directions attract".

Taking account of the experimental results (at present time) the magnetic moment of the uncharged leptons is null. [17]
In another of our studies [29], we propose a modification of the Maxwell and of the Lorentz equations driving to this result.

Up to this point we have been dealing, essentially, with the weak interaction. Remember, the weak interaction already differentiate *a 'left' and a 'right' for specific particles*. Here, for the weak interaction with a forward direction in the appropriate reference system, that is, the $\vec{\mathbf{p}}$ after the creation of the particle in the reference frame of the creation particles.
The physical properties (electric charge, spin, vector-spin and chirality) suggest privileged directions (geometry) in the local time-space of the particle. We can summarize with:

we have particles because we have local discrete geometries with symmetries, and,
the physical properties tell us of the local discrete geometries of the particles.
The interactions (the forces) with the adjustments of the discrete geometries.

Besides the relation with the electric charge, the QED introduces some minor modifications, in a different manner for the uncharged leptons than for the charged leptons, main point in the spin (up , down) and the vector-spin.

It is interesting to continue the research concerning the possibility of explaining the QED minor effects in relation to the postulated generalized rotations (equations (57) and (58)).

VI. APPENDIX A: FIGURES.

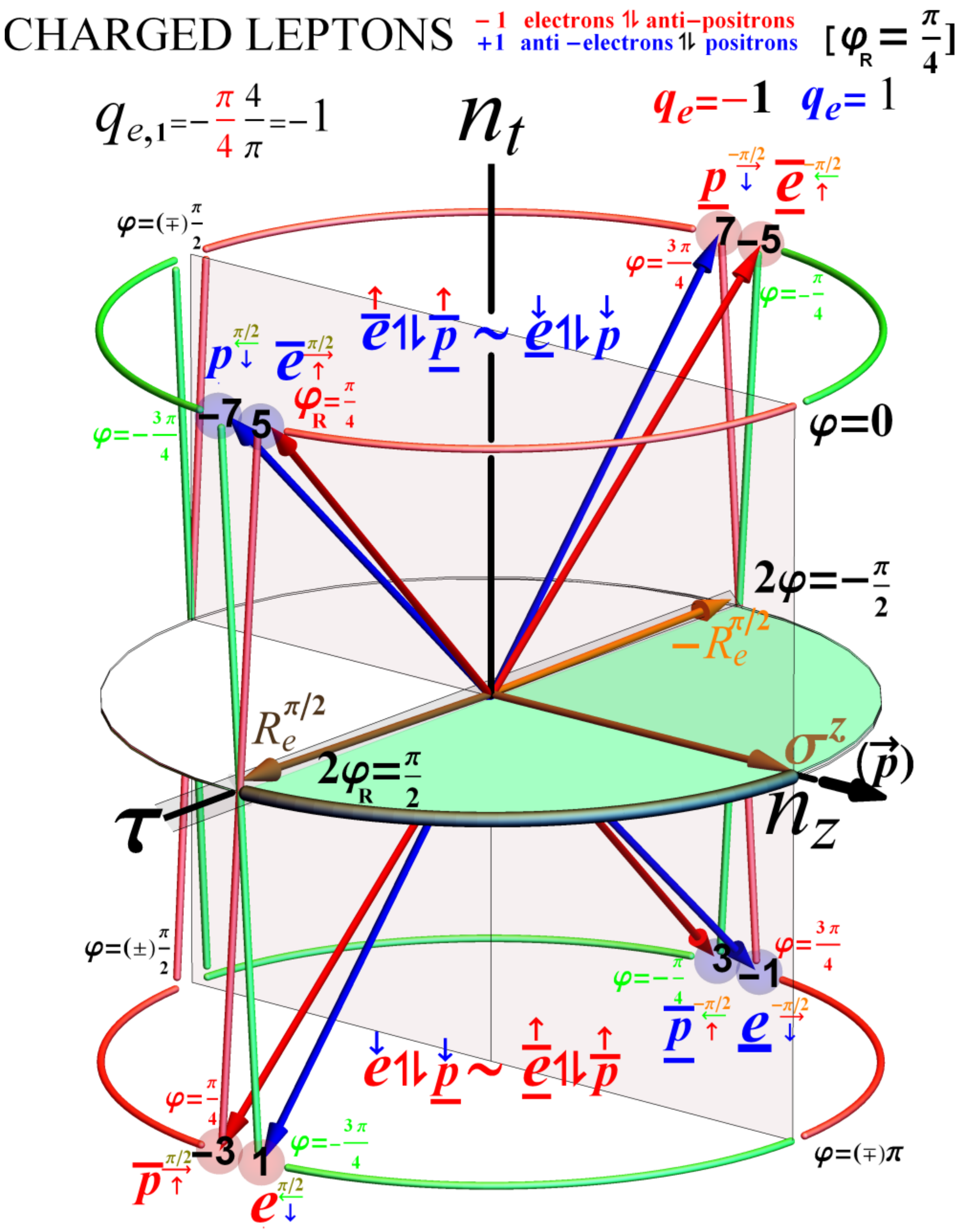


FIG. 1. Charged leptons ($n_z = 0$).

{ Negative electric charge with red color $q_e = -1$; Positive electric charge with blue color $q_e = +1$ }.

{ Spin down with a down arrow and blue color ↓ ; Spin up with an up arrow and red color ↑ }.

{ Left chirality with the green quarters of the circles ← ; Right chirality with the red quarters of the circles → }.

{ Oppositions in n_t alone and in τ alone or in the 3d-space alone change the sign of the electric charge ; Oppositions in n_t alone and in n_z alone or in the 3d-space alone change the spin (φ behind) ; Opposition in τ do not change the spin, and opposition in n_z do not change the electric charge }.

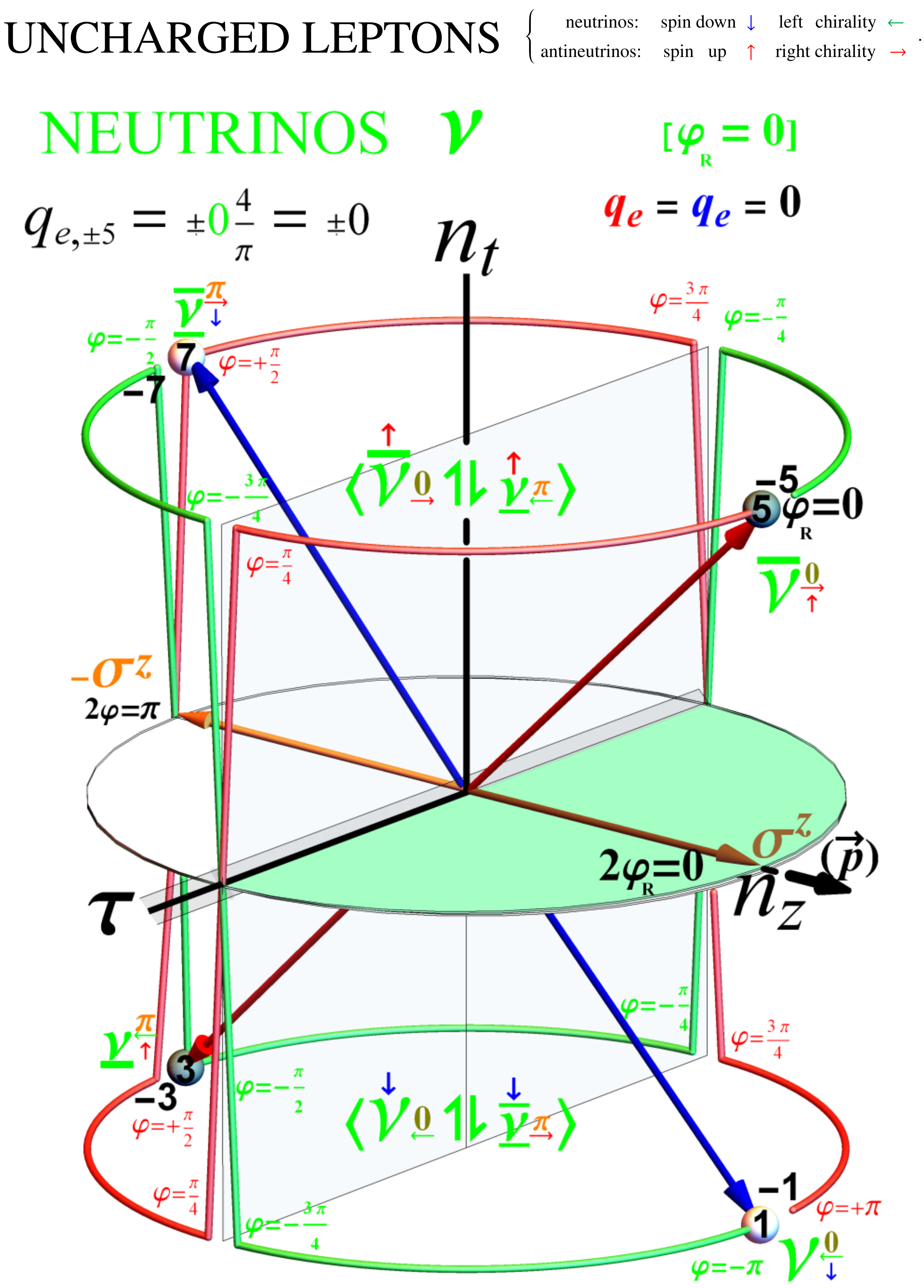


FIG. 2. Uncharged leptons ($\tau = 0$).

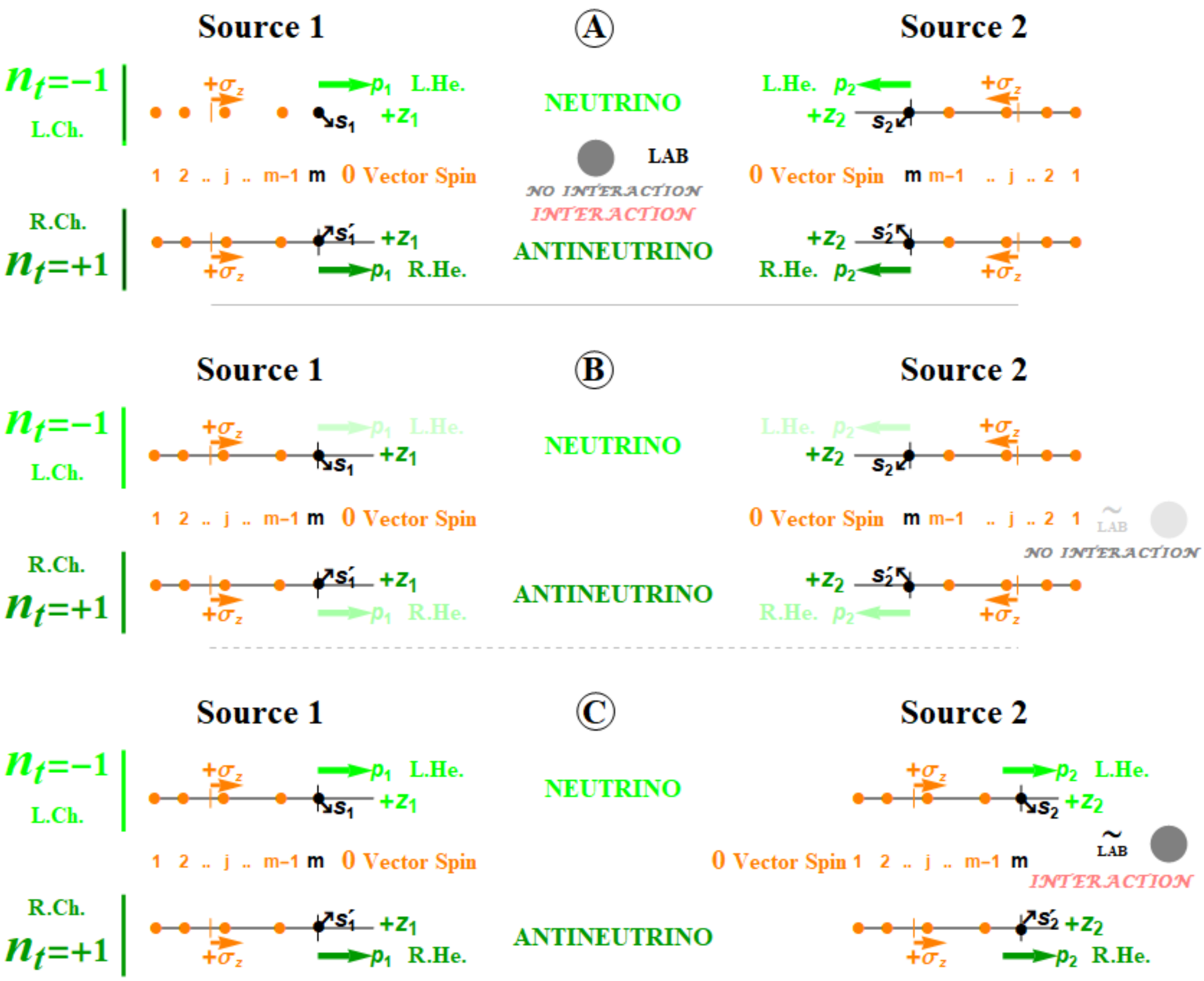


FIG. 3. Different settlements with uncharged leptons.

Ⓐ Assumed: produced and moving neutrinos and antineutrinos towards a laboratory (the observer, static).

And, perhaps, concerning the experiment of a ′moving laboratory′ faster than the uncharged leptons (the ones at source 2), chasing them:

Ⓑ Previous neutrinos and antineutrinos as ′presumptuously not seen′ (prior to the interaction),

in a ′fast moving $\widetilde{\text{lab}}$′ towards the uncharged leptons (or the uncharged leptons towards this $\widetilde{\text{lab}}$).
The **p′s** have not experimental meaning, and therefore neither the helecities (we are not measuring), which are assumed to have conserved values ′in $\widetilde{\text{lab}}$′ (actually for them ′there is no $\widetilde{\text{lab}}$′).

Ⓒ Previous neutrinos and antineutrinos as ′seen in a presumptuously fast moving $\widetilde{\text{lab}}$′, with the modifications **due to the interaction (the detection in the laboratory).** The interaction changes their local geometries.

VII. APPENDIX B: ANALOGIES. IMAGERY.

An analogy can be helpful but also could be misleading or a nonsense. Even though, we write:

- the leptons and quarks as *arrows*,
  the fletching (the feathers) for the positions 1 to $m-1$, and the head for the m position.
  In any case not as balls, not as spheres. More like dressed (the rotations) strings in ($\mathbb{C}^4$ with $\widetilde{\mathfrak{M}})^m$. Vortexes.

Their space is not $\mathbb{R}^4$, neither $\mathfrak{M}$ (Minkowski). $\widetilde{\mathfrak{M}}$ a modified Minkowski metric (for $\mathbb{C}$). [37] The algebraic dimensions over the real numbers are: with $m=2$, the dimension is 16; and with $m=3$, it is 24 (geometrically too?).

And,
- like an iceberg, we 'see' only the part over the water ($n_z \geq 0$), the underwater part is 'hidden' ($n_z \leq 0$),
- straight ahead from the earth we never 'see' the dark side of the moon, but
  the dark side of the moon is there, which, at times faces the sun and, at times 'hides' itself from the sun,
- each position in the leptons and quarks as *rotating discobolus* (the double disk in the children game),
- like vortexes of discrete geometry. See *Beautiful Losers: Kelvin's Vortex Atoms* by Frank Wilczek, 2011, [35]

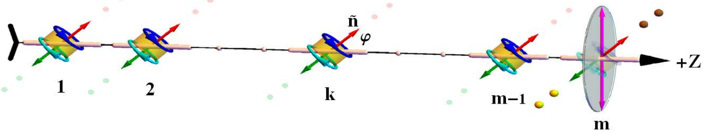


FIG. 4. "2001: A SPACE ODYSSEY" in the absolute solitude of an absolute local time-space
(a small tribute to the DISCOVERY ONE with "several tiny" SPACE STATION V),

- at each instant a jumping over a point with different placements, in an unfinished rotation; "perpetual mobile"; different points of time-space. Like the dancers with stilts (https://www.youtube.com/watch?v=Ls0BbQhddQs).

It is instructive to remember the double slit experiment. *Things are not always the way they look*, or
again with Feynman and Schwinger: 'this is quantum mechanics'. [1]

VIII. APPENDIX C: PROGRAM OF THE *STUDIES* CONTAINING THIS RESEARCH.

On the fermionization of the XYZ spin Heisenberg chain (algebra). (5*pages*).
(2022) https://hdl.handle.net/20.500.14352/71645 *Study -2)*

The JordanWigner transformations and the fermionization of the XYZ spin Heisenberg chain. (14*pages*).
Algebra, geometry and physics? (2022) https://hdl.handle.net/20.500.14352/71970 *Study -1)*

A tentative model of creation and annihilation operators for neutrinos. (7*pages*).
(2021) https://hdl.handle.net/20.500.14352/8066 *Study 0)*

Geometry of the time and the space. (Pending of a final wording). *Study I)*

Expression of the 3- and 4-dimensional vectors in total polar exponential form. (17*pages*).
(2021) https://hdl.handle.net/20.500.14352/8183 *Study I,1)*

Vectors. Dimensions 4 and 8. (31*pages*).
(2023) https://hdl.handle.net/20.500.14352/72871 *Study I,2)*

Geometry of the symmetries in dimension 4=(1+[1)+"2"], and general Time-Space-Spin vectors. (21*pages*, 6*fig.*).
(2023) https://hdl.handle.net/20.500.14352/72872 *Study I.3)*

Geometry and Physics of the Elementary Fermions. (On pride of Jordan Wigner Pauli Weyl Dirac).**1**. (33*pages*, 15*figs.*).
(2021) https://hdl.handle.net/20.500.14352/4587 *Study II)*

Geometry and Physics of the Fermions. **2.** (Interactions). Towards a boson structure $\{\gamma, Z^0, W^{\pm}, g\}$. (17*pages*, 4*fig.*).
(2024) https://hdl.handle.net/20.500.14352/**** *Study II.1)*

Geometry and Physics of the Fermions. **2.** (Pending of a final wording). *Study II.2)*

Axial vector magnetic charge and magnetic moment. Maxwell's equations and Lorentz force law. (23*pages*, 4*fig.*).
(2021) https://hdl.handle.net/20.500.14352/4586 *Study III)*

Leptons: charged and uncharged (neutrinos). Inside a proposal for a geometrical model. (27 pages, 4 fig.).
Some considerations. THIS STUDY (2024) https://hdl.handle.net/20.500.14352/ **** Study IV)

Addenda. (Pending of a final wording). *Study II.3)*

[1] H. Schmidt-Böking et al. *The Stern-Gerlach experiment revisited.* The Eur. Phys. J. H, 41, 4, 327-364 (2016). arXiv:1609.09311v1 [physics.hist-ph]. (Particularly pages 5-9 and Sections 3 and 4).

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