

# Bulk Viscosity of a Pion Gas

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**Abstract.** We calculate the bulk viscosity of a pion gas at low energies within the kinetic theory approach and show the importance of dealing properly with the zero modes of this transport coefficient.

**Keywords:** Bulk viscosity, pion gas, hadronic matter at finite temperature

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The bulk viscosity  $\zeta$  is a transport coefficient that controls the rate of equilibrium restoration when the perturbation out of equilibrium is purely compressive (correction of pressure deficit/excess). The relation between  $\zeta$  and the trace of the energy-momentum tensor is

$$-\zeta \nabla \cdot \mathbf{V} = T^\mu_\mu - T_{eq}^\mu_\mu. \quad (1)$$

This coefficient is zero for conformal fluids (e.g. an ultrarelativistic gas) and for monoatomic nonrelativistic gases.

We calculate the bulk viscosity for a gas of low energy pions through the Boltzmann-Uehling-Uhlenbeck (BUU) equation. The interaction of low energy pions ( $p \ll \Lambda_{QCD} \sim 1$  GeV) is given by Chiral Perturbation Theory. The elastic partial amplitudes are unitarized via the SU(2) Inverse Amplitude Method.

The departure out of equilibrium is parametrized by the perturbation function  $A(E_p)$ , ( $E_p = \sqrt{p^2 + m_\pi^2}$ ):

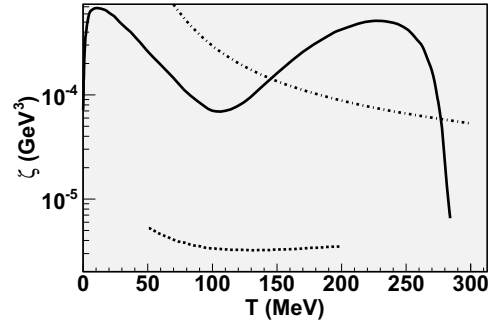
$$f(t, \mathbf{x}, \mathbf{p}) = f_{eq} - f_{eq}(1 + f_{eq})\beta \nabla \cdot \mathbf{V} A(E_p). \quad (2)$$

The linearized BE is a functional of  $A(E_p)$ , that enters in the collisional operator. This operator has two zero modes, corresponding to energy and particle number conservation.

$$\frac{df(t, \mathbf{x}, \mathbf{p})}{dt} = C[A_p] = 0 \text{ for } A_p \propto 1, E_p. \quad (3)$$

The collisional operator has no inverse due to these zero modes. It is important to understand that in order to solve the kinetic equation, these zero modes must be identified and properly separated from the rest of the solution Hilbert space. This can be done by solving the BE in the subspace perpendicular to these zero modes. If one uses a different ansatz for the perturbation function, the two zero modes would seem to disappear. However, they would be hidden within the solution's expansion and the result wouldn't converge to a finite value.

In the Landau reference frame it is enough to consider the BUU equation in the orthogonal space to one of these



**FIGURE 1.** Bulk viscosity for a pion gas at low temperature. Dotted line: Result from [1] in kinetic approach; Solid line: Result from [2] based on Kubo's formalism; Dashed-dotted line: Our preliminary result in the quasiparticle approximation.

zero modes. The other zero mode, proportional to the energy, can be fixed by the Landau condition and its value does not influence the bulk viscosity.

Our preliminary result [4] in the quasiparticle approximation is shown in Fig. 1. Previous approaches to this transport coefficient are not compatible between them, therefore a more detailed study of the bulk viscosity is still needed.

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## REFERENCES

1. D. Davesne, Phys. Rev. C **53**, 3069 (1996).
2. D. Fernandez-Fraile and A. G. Nicola, Phys. Rev. Lett. **102**, 121601 (2009)
3. P. Chakraborty and J. I. Kapusta, arXiv:1006.0257 [nucl-th].
4. A. Dobado, F. J. Llanes-Estrada and J. M. Torres-Rincon, arXiv:1010.0013 [hep-ph].

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