Principal-component characterization of noise for infrared images

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Principal-component decomposition is applied to the analysis of noise for infrared images. It provides a set of eigenimages, the principal components, that represents spatial patterns associated with different types of noise. We provide a method to classify the principal components into processes that explain a given amount of the variance of the images under analysis. Each process can reconstruct the set of data, thus allowing a calculation of the weight of the given process in the total noise. The method is successfully applied to an actual set of infrared images. The extension of the method to images in the visible spectrum is possible and would provide similar results. © 2002 Optical Society of America

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1. Introduction

The images obtained by infrared systems have, in general, lower contrast than the corresponding images in the visible. This fact makes infrared images more vulnerable to any kind of noise. In scanning systems the main contribution to noise is due to temporal noise. For focal-plane-array systems (FPAs) the number of detectors is large, and they can show significant differences in responsivity, gain, and noise. However, their spatial distribution produces another type of noise that is usually called spatial noise. These facts make the noise characterization of an infrared image-forming system an important task to accomplish for a complete specification of those systems. The principal-component method explained in this paper can also be applied to a set of images in the visible spectrum.

In one approach, for a set of frames taken in a time sequence from the output of an image-forming system, it is possible to define two main types of noise: spatial noise and temporal noise. Spatial noise can be characterized by the fixed-pattern noise (FPN), which is usually defined as a fixed image imposed over the actual image. In contrast, temporal noise appears as a salt-and-pepper noise that changes from frame to frame. Summarizing, spatial noise is a constant image appearing in all the frames, and temporal noise is an image that changes from frame to frame.

Another description of the noise given by Mooney[1] itemizes the following four physical sources:

- Nonuniform pixel nonlinearities, which correspond to the nonuniformity of the response of detectors and their lack of linearity. It is usually compensated for by correction algorithms on the basis of measurements at two temperatures. Some residual noise remains uncorrected after the calibration. After some time the calibration usually has to be redone.
- Detector $1/f$ noise, which is the nonstationary noise of detectors. It produces a drift or blinking in the signal for each individual pixel.
- Array $1/f$, which is usually produced by the processing of the image after detection in the readout. It introduces a pattern in the image that changes with time.
- Spectral nonuniformities, which appear because of the differences among the spectral responsivities of the individual detectors of the array.

Because the time variations of these four contributions to the noise are slow, these four sources are usually included in the description of the spatial

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noise. An important consequence of this reasoning is that pure temporal noise and pure spatial noise (the FPN) are two limiting cases of a more general description of the noise structure of a set of frames.

The usual way to measure spatial noise and temporal noise is the averaged frame method. To apply this method, a set of frames is taken of a uniform object (a blackbody). An averaged frame is obtained by averaging the signal for each detector along the time. The standard deviation of this averaged frame is taken as the spatial noise. Then this averaged frame is removed from each frame. The standard deviation of the remaining frame is the value of the temporal noise. This method assumes that the correlation of the spatial noise image with every frame is strictly one, and the correlation of the temporal noise image with each frame is zero. That is, temporal noise is something that changes from frame to frame and spatial noise is something appearing in all images (the FPN). There are two problems when conceptions of characterization and measurement of noise in infrared cameras are considered:

- In general, the actual absolute values of the correlation between frames go from zero to one. Mooney[1] defines a correlation coefficient as

$$\rho(t_1, t_2) = \frac{\frac{1}{M}\sum_{i=1}^{M} (f_{i,t_1} - \langle f_{t_1}\rangle)(f_{i,t_2} - \langle f_{t_2}\rangle)}{{\sigma_{\mathrm{sf},t_1}}^2}, \qquad (1)$$

where f_{i,t_1} and f_{i,t_2} are the signals of the pixels spatially located by i of the frames F_{t_1} and F_{t_2}, respectively, M is the number of pixels, $\langle f_{t_1}\rangle$ and $\langle f_{t_2}\rangle$ are the mean of the pixels for the two frames, and ${\sigma_{\mathrm{sf},t_1}}^2$ is the total variance measured in a single frame at a time t_1. This equation can be rewritten as

$$\rho_{t_1,t_2} = \frac{F_{t_1} \star F_{t_2} - \langle f_{t_1}\rangle\langle f_{t_2}\rangle}{F_{t_1} \star F_{t_1} - \langle f_{t_1}\rangle}, \qquad (2)$$

where $\star$ means correlation. By use of this coefficient it is possible to find a continuous scale of values for the different sources of spatial noise. In increasing order (from 0 to 1), Mooney finds shot noise, $1/f$ noise, nonlinear nonuniformities, spectral nonuniformities, and additive nonuniformities. In accordance with the averaged frame method, pure spatial noise is the correlated portion of the total noise. He defines the temporal noise value as the uncorrelated portion of total noise. In the image this corresponds with frames that changes randomly from frame to frame.[1]

- The characterization of noise, specially spatial noise, as a single value (the standard deviation) is not enough when images are considered. For example, different FPNs can have the same standard deviation. Some other models have been developed to increase the noise description complexity, providing again a collection of numbers to describe the noise of an image.[2] It is important to remember that noise is studied to know its influence over a given image target that has to be detected. Therefore it would be useful to have a method for discriminating the different types of image produced for different types of noise according to the Mooney classification[1] or any other classification of interest.

In Section 2 of this paper we present a method to solve the previous problems. The mathematical tool used is the principal-component decomposition. This method takes the set of frames as a linear combination of eigenimages, each one associated with one principal component. When the method is applied to the analysis of noise of images, it calculates the contribution of different eigenimages to the noise. The noise information is embedded in the covariance between frames. At the same time, when the principal components are interpreted as images, it is possible to associate the noise with an image pattern. We will see how these eigenimages allow for reconstructing subsets of frames from the original set and for associating them with different processes that contribute to the total noise proportionally to the variance of the eigenimages. In this section we also present the operation and use of the principal-component decomposition for the analysis of the noise of the data provided by an infrared imaging system. Section 3 is devoted to the classification of the contributions of the noise by the proper grouping of the principal components into processes. This grouping and classification is based on the decomposition of the variance of the set of frames, as a sum of contributions that can be statistically distinguished. The identification of the processes with physical subsystems responsible for a given part of the noise requires the analysis of the whole image-forming system and the auxiliary subsystems that provide the set of frames. That is not the goal of this paper. This paper is mainly focused on the presentation of an alternative way of analyzing the noise structure of actual set of frames. An example of the operation of the method is presented in Section 4. The results justify the use of the principal-component characterization for the analysis of the noise of imaging systems. Finally, Section 5 presents the main conclusions of the paper.

2. Principal-Component Expansion

Generally speaking, the method of principal components analyzes the variance of different observations of a set of variables.[3] To apply the method to the spatial and temporal noise characterization of infrared cameras, we assume that the variables are the frames taken in a time sequence, and the pixels' signals are the observations of these variables. The frames are taken when the infrared system is staring at an uniform object, usually a blackbody. Then the object is assumed to have constant values of exitance, spatially and temporally. Therefore the lack of uniformity among the frames is produced by the noise introduced by the image-forming system and all the subsystems used to provide those frames. This set of data can be denoted as

$$F = \{F_1, F_2, \ldots, F_t, \ldots, F_N\}, \tag{3}$$

where N is the number of frames and F_t is the frame taken at a given time. The whole set comprises all the data we want to analyze.

To apply the principal-component analysis, each frame is considered a random variable. The realization of one of these variables are the signals obtained by each detector at a given frame. Then the analysis of the covariances between the elements of this set of variables, F, is equivalent to calculate the covariance matrix of the frames. The goal of the principal-component decomposition applied to infrared optical systems is to obtain the set of frames as a sum of several processes, showing a clear behavior of their covariances. These processes have to be uncorrelated. In the analysis of the frames, the temporal variable is discrete, and the covariance can be calculated by means of the covariance matrix between frames.[3]

To build this covariance matrix we first define the set of variables as in Eq. (3). Each one of these frames is composed of the signals obtained by the individual pixels. If the camera has R rows and C columns, it is possible to arrange the $M = R \times C$ signals from the two-dimensional pixels as a column vector. The index of the elements of the vector, k, is related with row, r, and column, c, characterizing the pixel's position as

$$k = (c - 1)R + r, \tag{4}$$

where r runs from 1 to R and c runs from 1 to C. Therefore k runs from 1 to M, where M is the total number of pixels in the array. With this arrangement the frame is given as $F_t^T = (f_{1,t}, f_{2,t}, \ldots, f_{k,t}, \ldots, f_{M,t})$, where $f_{k,t}$ is the signal of the pixel k at the frame t (T means transposition). It is important to note that by using this algorithm we do not lose the information about the location of the pixels in the image. By use of this method, the set of data is placed in a $M \times N$ matrix, F. The covariance of this set of data is defined by the following $N \times N$ matrix,

$$S = \frac{1}{M - 1} \bar{F}^T \bar{F}, \tag{5}$$

where $\bar{F}$ is a set of data having zero mean. This modified set of data is obtained from the original one by subtracting its mean from each frame. This is accomplished by the following relation,

$$\bar{F} = F - \frac{1}{M} U_M U_M{}^T F, \tag{6}$$

where U_M is a column vector with M elements equal to one. This transformation is equivalent to an offset correction that removes a dc level from the signal.

The diagonal elements of S represent the variance of the frames. Meanwhile, the nondiagonal elements are related to the covariance between pairs of frames. Our interest is focused in the covariance structure of the data. The principal-component expansion corresponds with new variables, obtained as a linear combination of the original ones, that do not present covariance among them.[3–6] In addition, the variance of these new variables is arranged in decreasing order. Mathematically, this expansion is obtained by the diagonalization of the S matrix that produces a set of eigenvalues, λ_α, and eigenvectors, E_α. The diagonalization relation is

$$(S - \lambda_\alpha I) E_\alpha = 0, \tag{7}$$

where I is the $N \times N$ unity matrix. The set of eigenvectors, $\{E_1, E_2, \ldots, E_\alpha, \ldots, E_N\}$, can be arranged as a $N \times N$ matrix, E, where the α column contains the elements of the vector $E_\alpha{}^T = (e_{1,\alpha}, e_{2,\alpha}, \ldots, e_{\beta,\alpha}, \ldots, e_{N,\alpha})$, obtained from the eigenvalue equation. With this matrix, the principal-component expansion is obtained as a $M \times N$ matrix, Y, as follows:

$$Y = \bar{F} E. \tag{8}$$

Each one of the principal components is given by the following relation:

$$Y_\alpha = \sum_t^N e_{t,\alpha} \bar{F}_t. \tag{9}$$

Even more interesting for the analysis of the noise in successive frames is the derivation of the original frames as an expansion of the principal components. These principal components can be taken as images. Therefore each one of the principal components is an image that is properly combined to produce the original set of frames. From the previous derivation of the principal-component expansion, it is possible to invert the process and obtain the frames as a combination of the principal components. This is done by the matrix relation

$$\bar{F} = Y E^T, \tag{10}$$

which provides the following equation:

$$\bar{F}_t = \sum_\alpha^N e_{t,\alpha} Y_\alpha. \tag{11}$$

The element $e_{t,\alpha}$ is the weight of the principal component Y_α in the frame $\bar{F}_t$. An exact reconstruction of the original frames with zero mean, $\bar{F}_t$, is obtained when the index α runs from 1 to N.

To have a clearer picture of the problem, we show in Figs. 1 and 2 an example with only three frames. These frames have 4756 pixels arranged in a 58×82 array. We restrict the example to three frames because it is the maximum number able to be represented with a three-dimensional (3D) plot in which the three orthogonal directions are associated with the frames. The signal given by each pixel, determined by its spatial index, k, is represented by a point in a 3D diagram whose coordinates are the three different values of that pixel in the three frames, $(f_{k,1}, f_{k,2}, f_{k,3})$. When all the pixels are located in this

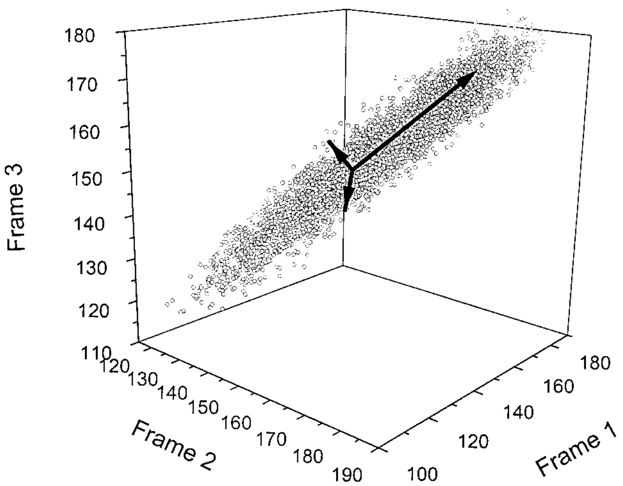

Fig. 1. Scatter plot of the values of 4756 pixels in three frames. The principal components correspond with the main directions of the 3D ellipsoid that contains the points. These directions are represented as three orthogonal arrows with lengths proportional to the eigenvalue. For the sake of clarity, the scale for the second and third values has been modified. The actual values of the eigenvalues are, in decreasing percentage, 95.82%, 2.21%, and 1.96%.

diagram, a cloud of dots is obtained; each dot is one pixel. The center of this cloud of dots is determined by the mean of each frame. The transformation of the frames to have a zero mean [Eq. (6)] is better adapted to the analysis of the dispersion of the data. In Fig. 1 the pixels are mainly aligned along a given direction. This direction corresponds with the first principal component, which explains most of the variance of the data. The second and third principal components are located in a plane perpendicular to the first one and explain the residual variance of the data. The directions of these three eigenvectors are mutually orthogonal to each other. In Fig. 1 we have plotted the directions of these three eigenvectors with arrows whose lengths are related to the values of the corresponding eigenvalues (Fig. 1 is not in scale; the actual contributions of the three principal components are, in decreasing order, 95.82%, 2.21%, and 1.96%). Figure 2 shows a pictogram of how the principal-component expansion works with these three frames. Each original frame, having zero mean, can be arranged as an image corresponding to the columns of the frame matrix, $\bar{F}$. This matrix has 4,756 rows and 3 columns. After applying Eq. (8), we find three principal components that can be interpreted as images and arranged in the matrix Y. The relation between these two matrices is given by the matrix E for transforming $\bar{F}$ into Y and by the matrix E^T for the inverse transformation. This inverse transformation [Eq. (10)] is the one presented in Fig. 2 rearranging the rows of $\bar{F}$ into an image pattern according to transformation (4). A given row of $\bar{F}$ is a triplet having the values of the corresponding pixel in each frame; the same row of Y is another triplet having the values of the same pixel in each principal-component direction. These two ways of representing a given individual pixel (and the whole image after the procedure is extended to every

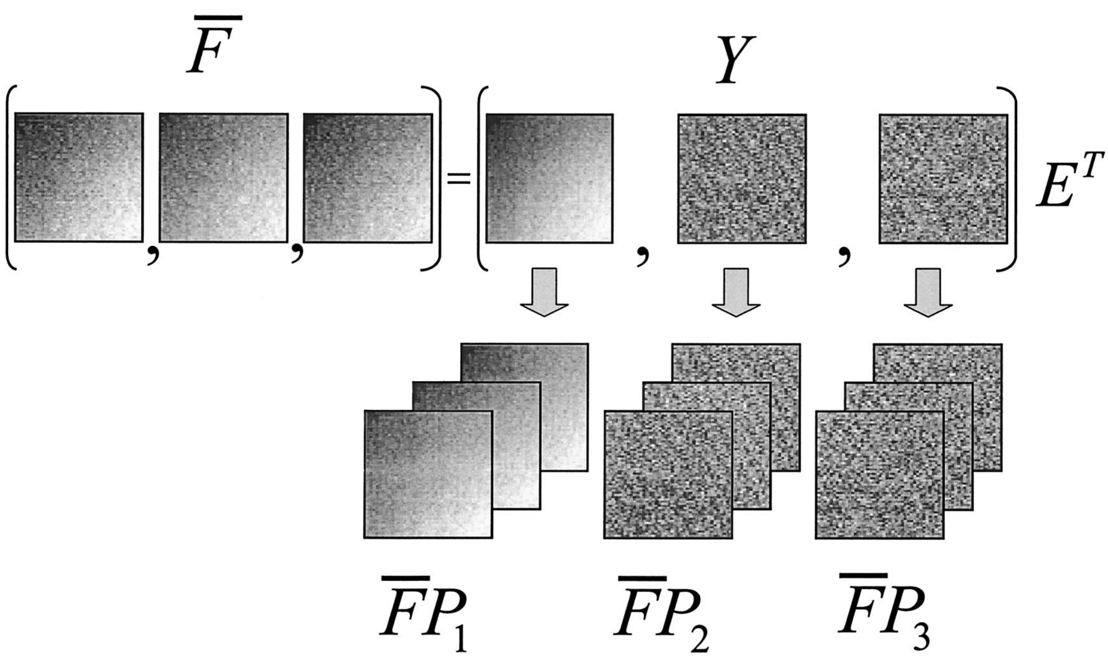

Fig. 2. The original frames (after subtraction of the mean) are related to the principal component through a rotation matrix. In this plot we show how the original frames ($\bar{F}_1$, $\bar{F}_2$, $\bar{F}_3$), whose pixels' values are represented in Fig. 1, are transformed into three principal components, (Y_1, Y_2, Y_3), that are presented as images. The 4756 pixels are actually arranged as a 58 × 82 array. Below each one of the principal components we plot the three frames generated by each principal component [see Eq. (14)]. To improve the clarity of the representation, we have maximized the range of the gray level to the maximum for each image (principal component, frames, and generated frames).

row) are linked by the 3×3 matrix containing the values of the eigenvectors, E [see Eqs. (8) and (10)]. Then it is clearly shown that E can be interpreted as a rigid rotation transformation.

A. Meaning of the Eigenvalues

The covariance matrix defined for the principal-component decomposition [calculated by Eq. (5), substituting $\bar{F}$ by Y] is related to the covariance matrix of the frames as follows:

$$S_Y = \frac{1}{M-1} Y^T Y = E^T S E. \tag{12}$$

This new covariance matrix, S_Y, is a diagonal matrix having the eigenvalues of Eq. (7) along the diagonal. The nondiagonal elements of S_Y are zero, showing that the new set of variables, the principal components, are mutually independent. Transformation (12) can also be seen to be derived from a rigid rotation of coordinates.[7] The total variance of the data is given by the sum of the eigenvalues. These eigenvalues are the variances of the corresponding principal components. The relative contribution to the total variance of the data of a selected principal component will be given as the quotient between the corresponding eigenvalue and the sum of all the eigenvalues. The principal components explain the variance of the original frames in decreasing order. Therefore, in some particular cases, a selection of the first principal components is able to describe the main contributions of the variance.

B. Fixed-Pattern Noise and Pure Temporal Noise

The FPN is defined as an image that appears in every frame with a fixed spatial pattern. If the set of data contains such an artifact, after applying the principal-component decomposition, we would find that the principal component associated with the FPN, Y_{FPN}, is contributing equally to every frame. We should recall that each frame represents an image at a given time. The FPN is invariant with time, and it appears as a constant image along the frames. The importance, or weight, of the FPN in the set of frames is given by the elements of the associated eigenvector [see Eqs. (10) and (11)], $E_{\mathrm{FPN}} = (e_{1,\mathrm{FPN}}, e_{2,\mathrm{FPN}}, \ldots, e_{t,\mathrm{FPN}}, \ldots, e_{N,\mathrm{FPN}})$. By definition of the FPN, these contributions must be equal. In addition, the eigenvectors are unitary vectors. Therefore $e_{1,\mathrm{FPN}} = e_{2,\mathrm{FPN}} = \cdots = e_{t,\mathrm{FPN}} = \cdots = e_{N,\mathrm{FPN}} = 1/\sqrt{N}$. This direction is the bisectrix of the coordinate system of the original frames. When pure spatial noise is the most important source of noise, this eigenvector corresponds with the first one, $E_{\mathrm{FPN}} = E_1$.

Within the "averaged frame method" the temporal noise is assumed to be uncorrelated along the frames.[7] This assumption, along with the independence between the principal components, suggests that the eigenvectors that correspond to the principal components associated with pure temporal noise lie on a hyperspace perpendicular to the bisectrix of the coordinate system, i.e., perpendicular to the eigenvector associated with the FPN.

These two limiting cases of noise structures are easily found with the principal-component analysis. In addition to them, other kinds of noise structure that show covariances at an intermediate time scale can also be treated with principal-component decomposition.

C. Contribution of the Principal Components to the Frame Set. Rectification

A type of noise involving only one eigenimage is a single image whose weight changes with the frames (if this weight is constant along the frames, it corresponds with the FPN). A more complex type of noise may involve a selected set of principal components, not only one. Then we wonder how this set determines the structure of the noise in the original frames. The operational method for answering this question is called "rectification" or "principal component filtering."[5] It works by building a rectification matrix obtained by the following equation,

$$P = E' E'^T, \tag{13}$$

where E' is a matrix containing as many columns as the number of principal components involved in the rectification. These columns are the elements of the eigenvectors selected for the analysis. Another possible point of view takes P as a projection matrix that extracts the influence of the selected principal components on the original data. The corresponding filtered data are given as

$$F' = \bar{F} P + \frac{1}{M} U_M U_M{}^T F. \tag{14}$$

This rectification, or filtering, allows a representation of the portion of the original frames that include, or not include, a given noise structure. Figure 2 shows how the rectification process works. The three stacks of three images below the principal components represent the portions of the original set of frames obtained after this set was filtered with the three projection matrices corresponding to each principal component. The original set is obtained by adding up the three filtered sets. To improve the graphical representation, we have maximized the range of values of the pixels' variations. Actually, the first principal component, Y_1, and therefore the first filtered set $\bar{F}P_1$, contributes 95.82% to the total noise of the frames. By using Eq. (14), it is possible to extract from the original data those contributions that do not contain, for example, the FPN.

Summarizing the application of the principal-component decomposition to the analysis of the noise of a set of frames, we find some clear advantages. The total variance of the data is sectioned into uncorrelated parts: the eigenvalues of the decomposition. Each eigenvalue is associated with its principal component. In addition, each principal component represents an eigenimage. The importance of an eigenimage in each frame is properly obtained by the components of the associated eigenvector. In Section 3 we will show how some of the principal components can be grouped into a process that explains a given amount of the total variance. The frames obtained by the principal-component filtering that uses the subset of principal components grouped into a process will not be correlated with the frames obtained after rectification with the rest of the principal components that do not belong to the subset associated with the process.

The next step is to relate the subset of principal components with different physical mechanisms contributing to the noise of the system. This task needs not only the application of the method as it is expressed in this paper but also the knowledge of the specifications of the subsystems integrated in the infrared optical system (lenses, focal-plane array, electronics, frame grabber, digitizers, etc.) Besides the physical sources, the classification of the noise proposed by Mooney[1] and presented in Section 1, establishes a given dependence of the noise structure with respect to the frequency (for example, the $1/f$ noise). The analysis and identification of such sources or types of noise is beyond the objectives of this paper. Our processes are related with a clear relation between a subset of principal components (the eigenimages) and a portion of the total variance of the set of data (their associated eigenvalues). Therefore criteria to build the right subset of principal components will be necessary to practically apply the method to the analysis of noise in a temporal series of frames. This is the objective of Section 3.

3. Classification and Grouping of the Principal Components as Processes

In Section 2 we have seen the meaning and importance of the principal component Y_{FPN} that was identified with the FPN. In that case the process that generates the FPN involves only one principal component. In the general case the number of principal components associated with a given process can be greater than one. In this section we analyze more in depth the method of grouping a subset of principal components in such a way that the whole subset is associated with a given portion of the total variance of the original data. To do this, we will validate the assumption, by means of statistical criteria, that several eigenvalues are undistinguishable.[8,9] As we have seen, the principal-component method can be applied to a given set of frames that is a sample of the actual phenomena. Our goal is to characterize the noise of the original set. We will need criteria to extract the variation of the signals that are due to identifiable noise process from any existing experimental uncertainty. Then the obtained eigenvalues will have an associated error for a given confidence level. The grouping of a subset of eigenvalues (and therefore eigenimages and eigenvectors) will be based on this concept. A detailed calculation is presented in Appendix A.

In other fields in which the principal-component decomposition is applied, after the principal components are obtained, it is useful to know which ones are the most significant. The context, the meaning, and the nature of the data help in making this analysis.[10–12] The significance test most commonly used is the Anderson test.[7] It analyzes the degeneracy of a set of eigenvalues, λ_k, of the covariance matrix of data obtained with M samples and estimated with values l_k. Although the estimators do not need to be equal, the degeneracy condition is expressed as the following hypothesis,

$$H_o : \lambda_{k_1} = \lambda_{k_2} = \cdots = \lambda_{k_n};\; k_i \in K, \tag{15}$$

where K is a set of n subindices. The problem is that the estimators are obtained from an actual set of data that is a sample. Therefore another sampling will produce a different value of estimators that should be, under the previous hypothesis, statistically undistinguishable from the actual eigenvalues that we assume are equal. The eigenimages belonging to the subset under test can be different (different spatial patterns) from sample to sample, but all of them, by hypothesis, have the same variance. If this is the case, the associated eigenvectors of the two samples can be related by means of rotations with arbitrary angles. In that case they form a degenerated multiplet, and they generate the same subspace. The associated principal components can be seen as a rotation of the actual ones.[12] This hypothesis can be analyzed by several methods. In Appendix A we show a way of calculating the expected distribution of probability of the eigenvalues. With that method the hypothesis can be validated and used.

As an example of the previous assessments, let us take two eigenvalues (a duplet), λ_1, λ_2, that are assumed to be equal to λ. Their estimators, l_1, l_2, are undistinguishable within the uncertainty that is due to the sampling procedure. The associated eigenvectors, e_1, e_2, define a plane. Now these eigenvectors are rotated with respect to an axis perpendicular to that plane by an angle $\theta_{1,2}$.

The new eigenvectors, e_1', e_2', are related to the old ones by the following transformation:

$$\begin{pmatrix} e_1' \\ e_2' \end{pmatrix} = \begin{pmatrix} \cos\theta_{1,2} & \sin\theta_{1,2} \\ -\sin\theta_{1,2} & \cos\theta_{1,2} \end{pmatrix} \begin{pmatrix} e_1 \\ e_2 \end{pmatrix}. \tag{16}$$

According to Eqs. (9) and (16), the new principal components are

$$Y_1' = \sum_t e_{1,t}' F_t = Y_1 \cos\theta_{1,2} + Y_2 \sin\theta_{1,2},$$
$$Y_2' = \sum_t e_{2,t}' F_t = -Y_1 \sin\theta_{1,2} + Y_2 \cos\theta_{1,2}, \tag{17}$$

and the variance of the new principal components are

$$l_1' = l_1 \cos^2\theta_{1,2} + l_2 \sin^2\theta_{1,2},$$
$$l_2' = l_1 \sin^2\theta_{1,2} + l_2 \cos^2\theta_{1,2}, \tag{18}$$

where we can apply the fact that l_1 and l_2 are undistinguishable (within the sampling uncertainties) to conclude that the new estimators of the eigenvalues, l_1', l_2' are also undistinguishable for any value of $\theta_{1,2}$. Then the new principal components are equivalent to the old ones because they explain the same amount of variance. This reasoning produces the following result for the covariance between Y_1' and Y_2' with Eqs. (17),

$$\mathrm{cov}(Y_1', Y_2') \simeq \frac{l_2 - l_1}{2} \sin 2\theta_{1,2}, \tag{19}$$

where, for a given sample, λ_1 and λ_2 have been estimated by l_1 and l_2 with an uncertainty that is due to the sample. If hypothesis (15) is correct, then the covariance is a variable having zero mean and a variance $\sigma^2 = (5/M)\lambda^2(\sin^2 2\theta_{1,2}/4)$ when we take different sets of frames [see Eq. (A16) with $r = 2$ and M is the number of pixels]. Then the frames reconstructed by Y_1' and Y_2' cannot be taken as actually independent because there exists an uncertainty in their covariance when another set of frames is taken. These uncertainties caused by the sample of the frames and by the number of detectors make it impossible to ensure that the frames reconstructed by Y_1' are strictly uncorrelated with those coming from Y_2'. Therefore it is necessary to take the frames generated with both degenerated eigenvalues because they show an inner connection in their covariance.

The same reasoning that worked for two eigenvalues can be extended to the set of eigenvalues that comply with the degeneracy condition. Now let us take a set of $s + 1$ eigenvalues with their corresponding eigenvectors and principal components. The decomposition method ordinates the eigenvalues in such a form that obeys $\lambda_r > \lambda_{r+1} \ldots > \ldots > \lambda_{r+s}$. We assume that all the consecutive pairs of eigenvalues form a degenerate duplet [they obey hypothesis (15) with two consecutive indices]. If we sample again and take another set of frames, the new calculated principal components will change in form, maintaining the value of the variance within the uncertainty associated with the sampling procedure. The principal component associated with the smallest eigenvalue has changed from Y_{r+s} to $Y_{r+s} + \Delta Y_{r+s}$. This principal component belongs to the degenerate duplet (Y_{r+s-1}, Y_{r+s}). By rotating these principal components we can obtain a new pair of principal components. The change induced by the new sampling is related by means of the rotation. If we assume, for simplicity, that only Y_{r+s} has changed, then after the rotation the induced change in the pair (Y_{r+s-1}', Y_{r+s}) is

$$\Delta Y_{r+s-1}' = \Delta Y_{r+s} \sin \theta_{r+s,\, r+s-1},$$

$$\Delta Y_{r+s}' = \Delta Y_{r+s} \cos \theta_{r+s,\, r+s-1}. \tag{20}$$

Therefore the change induced by the new sampling in Y_{r+s} is forwarded to Y_{r+s-1}' by the rotation. In addition, the new principal component Y_{r+s-1}' explains the same amount of variance as Y_{r+s-1}. Besides, we have assumed that all the consecutive pairs of principal components, from r to $r + s$, are degenerate diplets. In this case a new rotation of angle $\theta_{r+s-1,r+s-2}$ can transform the pair (Y_{r+s-2}, Y_{r+s-1}) into a new pair (Y_{r+s-2}', Y_{r+s-1}'). By use of this rotation it is possible to transfer the change ΔY_{r+s} consecutively from Y_{r+s} to Y_r. The above reasoning explains that a change in one of the principal components propagates to all the principal components degenerated in consecutive pairs.

At this point it is possible to present a clear definition of process. By use of the principal-component decomposition to analyze the noise of a set of frames, a process is defined as a filtered set of frames generated by a subset of principal components (eigenimages); besides, their corresponding eigenvalues are consecutive degenerate pairs. The frames generated by the principal components belonging to one process will show, between them, a nonnegligible degree of correlation because of the inner connections established by the existence of consecutive degenerate pairs. At the same time, two frame sets generated by the principal components associated with two different processes will show a negligible value of covariance. For a given image-forming system under test, this behavior remains stable if we take another set of data with the same of number of frames and detectors. In other words, there is not a simpler, and more stable, way of sectioning different contributions of the noise having null covariance.

Once the concept of process is defined, it is necessary to establish a procedure to group the principal components into processes. This procedure is explained more in depth in Appendix A by use of statistics concepts. This appendix is an application of the rule of thumb given by North *et al.*[9] It says that two eigenvalues can be considered as belonging to a degenerated multiplet when their values are within the sampling error of each of them; i.e., the uncertainty ranges overlap. In the appendix we derive the distribution of the values of the eigenvalues for our case.

4. Example: Classification of Noise for an Infrared Focal-Plane-Array Camera

In this section we have tested the method with actual values obtained from a set of frames taken from the video output of an infrared camera. The camera operates in the 3–5-μm band, having a lens with $f' = 250$-mm lens. Twenty-one frames from the video signal of the camera were digitized at 8 bits with a frame grabber. The scene viewed for the camera is a blackbody extended source at several temperatures (15 °C, 20 °C, 25 °C, 30 °C, 35 °C, and 40 °C) and projected with a reflective collimator ($f' = 1778$ mm). The camera is calibrated with a two-point internal method and one-point external calibration at 20.8 °C and 39% relative humidity. The analysis is made in a small region of the field of view containing 131 × 167 points from a total of 512 × 512 pixels given by the video output of the camera. A portion of the image was selected to reduce the computing time. The original image produces a F matrix having

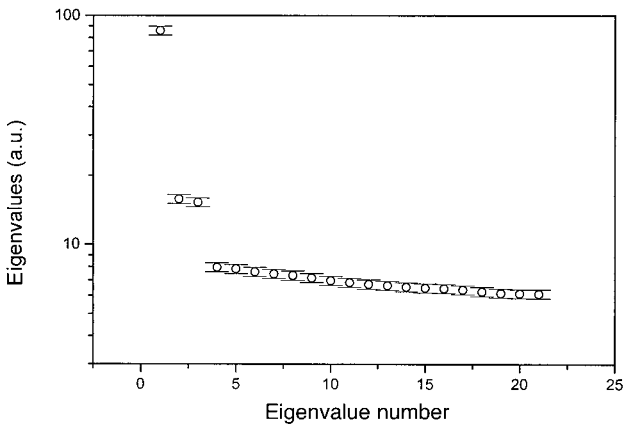

Fig. 3. Plot of the 21 eigenvectors for a set of 21 experimental frames analyzed by the principal-component method. The scree test clearly identifies three groups of eigenvalues. The method to group eigenvalues described in Section 3 also produces the same grouping. The values are also plotted with the error bars obtained after calculation by use of the operational method described in Appendix A. The grouping appears when the eigenvalues overlap their error bars continuously.

262,144 rows and the selected region has "only" 21,877 rows.

Figure 3 shows the classification of eigenvalues for the set of frames at 20 °C. Each one is shown with the error bars deduced with the method described in Appendix A. Accordingly, with this method of classification three different processes are found. The first one is governed by the first principal component, the second one involves two, and the third is obtained combining the remaining eighteen eigenvalues. A simple way, named the scree method,[8] to group the eigenvalues is clearly applicable in this case. The foundations and the application of the intuitive scree method to data representing images have been presented in Section 3 and Appendix A. This classification relates these three sets of eigenvalues to three processes that produce a given type of noise represented by the associated subsets of principal components.

One of the reasons to apply the principal-component decomposition to the analysis of the noise is that it provides a set of eigenimages (the principal components). In Fig. 4 we show these eigenimages obtained for two different temperatures (20 °C and 40 °C). The spatial noise is related in this case to the first principal component. The second group of eigenvalues, or the second process, is associated with the second and third principal components. These two components clearly differ from the rest of eigenimages, and they are associated with a running fringe pattern that crosses the image periodically. The third group of eigenvalues (from the 4th to the 21st) corresponds to a process that is better associated with temporal noise. At this point, a detailed analysis of the whole system used to produce the frames can identify these three processes, specially the first and second, with an actual subsystem responsible for the noise.

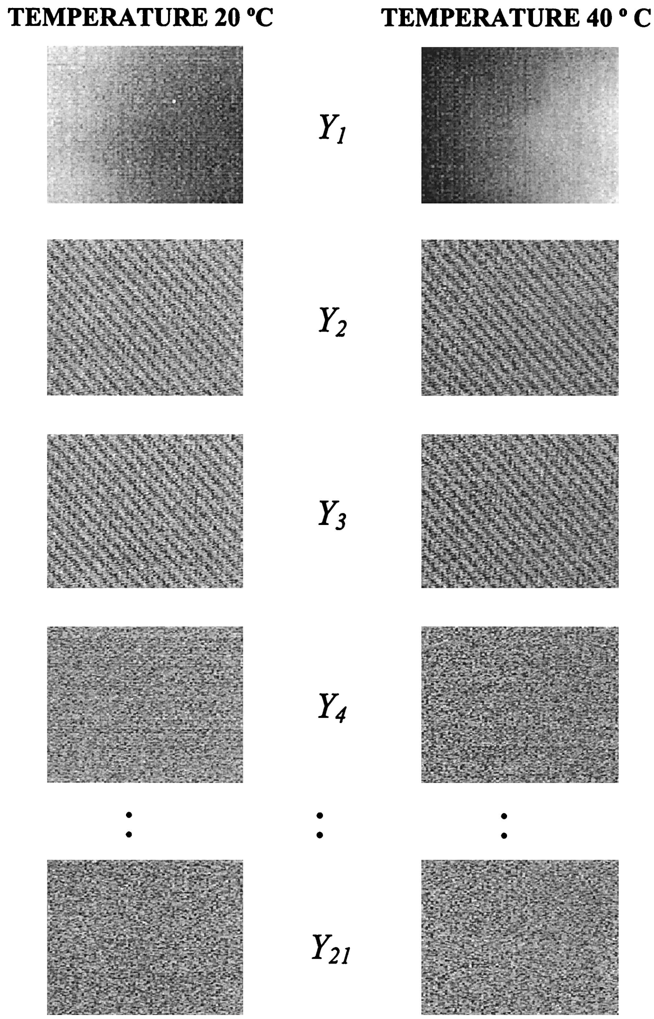

Fig. 4. Eigenimages corresponding to the principal components obtained at two different temperatures. The first principal component can be associated with spatial noise. The second and the third correspond with a process clearly different from the other two. It represents a fringe pattern crossing the scene periodically. The rest of eigenimages can be associated with temporal noise.

In Fig. 5 the eigenvalues are represented for several temperatures of the blackbody. Again, at each temperature, the same three processes are found. As mentioned in Section 2, the first principal component and eigenvalue are usually associated with the spatial noise. This is the only one whose contribution changes significantly with the background temperature supporting such association. This behavior is due to the calibration procedure previously described that performs a better compensation at 25 °C than at other temperatures.

After classifying the principal components into processes, it is possible to reconstruct a subset of 21 frames for each process by the rectification method. One possible analysis is to study the standard deviation of the frames related to each process. In Fig. 6 we plot the evolution with temperature of the three mean standard deviations associated with the three processes. After rectifying the original data set with

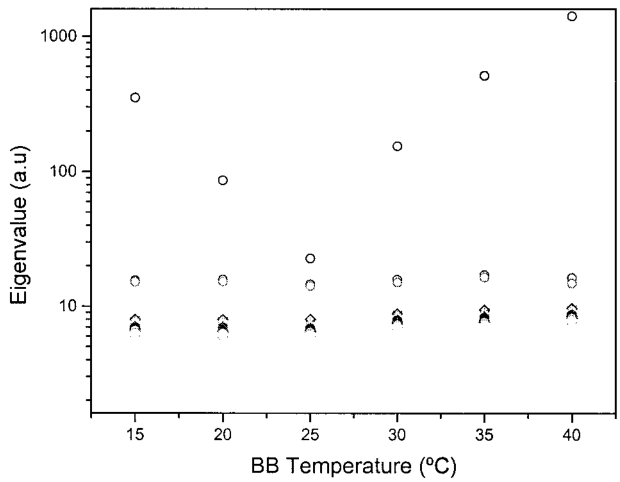

Fig. 5. Plot of the 21 eigenvalues at several temperatures of the blackbody. The same three processes shown in Fig. 3 are found. The first process, associated with spatial noise, changes in accordance with the expected evolution of spatial noise with temperature, reaching a minimum around the calibration point.

the three subsets of principal components, we calculate the standard deviation of the 21 frames and plot their mean value along with their uncertainty as error bars. We can see that the second process represents a lower value than the third process. However, it has been clearly identified and extracted from the data set by the principal-component decomposition. This means that the principal-component analysis has been able to enhance a contribution to the variance below the pure temporal noise contribution. If the intent is to remove the effect of a given portion of the noise, then we need only to reconstruct, or filter, the original set of frames by using those principal components that do not belong to that portion of noise being removed [see Eq. (14)].

Another type of analysis, for example, the 3D model of the noise, can be applied to every subset of 21 frames.[2] The results are plotted in Fig. 7 at two temperatures (20 °C and 40 °C). The first process is clearly related to spatial noise because it has almost

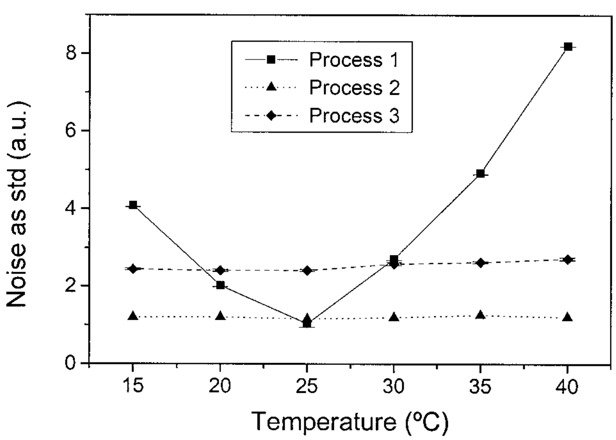

Fig. 6. Mean standard deviation and uncertainty for the frames generated for the three processes at different temperatures.

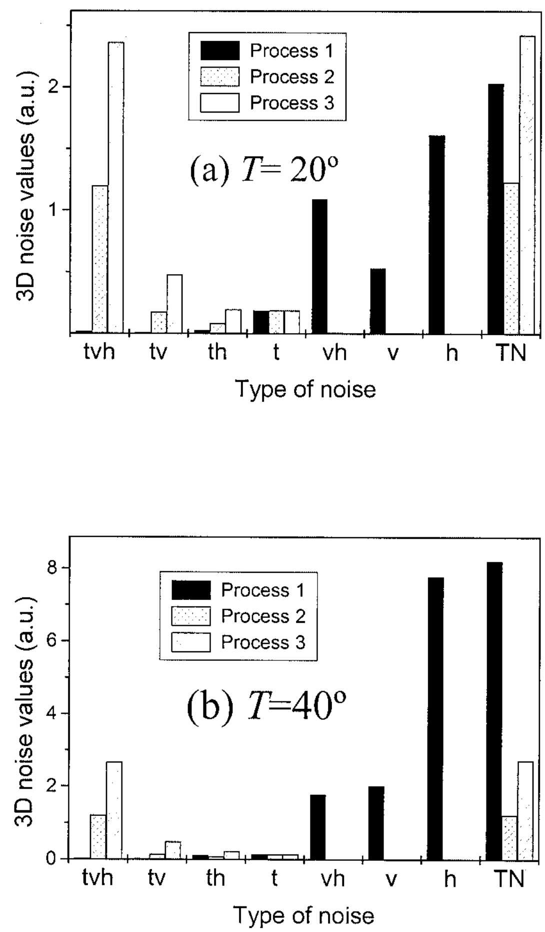

Fig. 7. Values of the noise for the three processes within the 3D model of noise. (a) is for a blackbody temperature of 20 °C, and (b) is for a blackbody temperature of 40 °C. The original set of data is analyzed along three directions: horizontal (h), vertical (v), and temporal (t) (see Ref. 2). Each subset of columns represents the rms value of the data along the labeled directions. TN is the total noise of the data obtained by adding in the quadrature all the other contributions.

no temporal components and its importance increases as the temperature increases. The other two processes are not much different within this 3D noise model. The principal-component decomposition clearly distinguishes these two processes by means of the associated eigenimages.

The covariance matrix for the whole set of frames at $T = 40$ °C can be seen as the sum of the covariance matrices obtained for each process. All these three matrices are shown in Fig. 8. The total covariance is the covariance of the original set of frames. It can be clearly seen how the covariance matrix is decomposed in three different parts, each one describing the covariance structure introduced by each process.

As can be seen in the previous figures, the second and third processes, specially, are involved with temporal behavior. The first process is mainly related to spatial noise, and it changes its weight with the temperature.

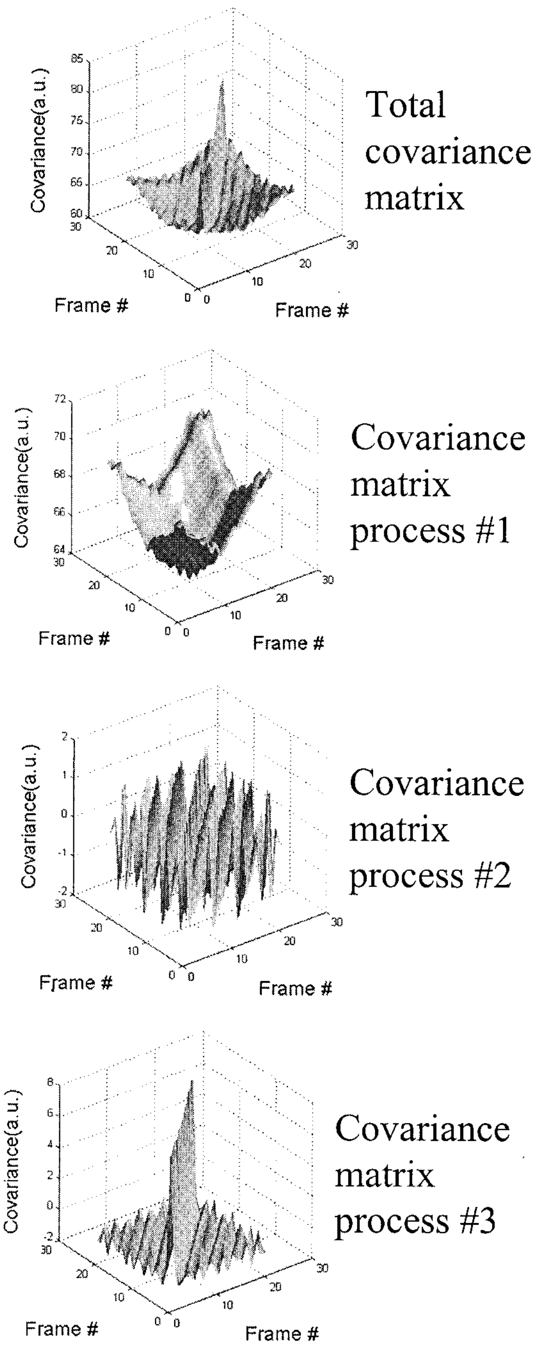

Fig. 8. Plot of the variance matrix of the whole set of frames obtained at $T = 40$ °C, and its decomposition into the three processes derived by the principal-component expansion.

5. Conclusions

We have presented the principal-component decomposition as a method of characterizing the noise of infrared images. The method can also be applied to the visible image-forming devices. However, we have focused on the infrared spectrum here because the signal-to-noise ratio is lower and the knowledge of the noise structure is more important than in the visible. The principal-component analysis allows us to picture the noise as real independent images that are overimposed on the scene. It also sections the variance of the set of frames analyzed by the method. The process is defined as a robust and stable covariance structure with respect to the sample. The method is applied to an actual set of data and allows us to distinguish several processes involved in the noise. With this method, the different parts of noise can be identified as being in between pure spatial noise and pure temporal noise. The identification of the processes obtained by the principal-component expansion with physical sources of noise needs more detailed information about the physical characteristics and specifications of the optical systems, the detector array, the electronics, the digital or analog treatment of the signals, and any other subsystem influencing the final stream of data that is captured and treated. However, some classical noise types, as the $1/f$ noise, require a detailed connection of the importance of the noise with their temporal scales. That approach is beyond the goal of this paper, which has been mainly devoted to the presentation of the principal-component expansion to the analysis of the noise structure of a given set of images.

The results of the method are given as images. Therefore the analysis of the spatial patterns associated with each type of noise is easily derived. The previous methods for characterizing the noise (for example, the 3D noise) on the basis of single-value parameters cannot accomplish this kind of analysis. The application of the method to an actual set of frames taken from a given infrared camera has been able to show noise structures that were not described in previous models of noise characterization. In addition, the two limiting cases defined in the averaged frame method (spatial noise and pure temporal noise) have appeared from the analysis. A comparison of the principal-component analysis and the 3D noise description has been done, showing that the principal-component expansion provides additional information.

Appendix A: Test for the Grouping of the Principal Components

In this appendix we describe a test to find if r estimations of the eigenvalues, $(l_i, l_{i+1}, \ldots, l_{i+r-1})$, can be considered equal within the uncertainties that are due to the sample.[9] The hypothesis is that these r eigenvalues are equal. The estimators, l_i, of the eigenvalues of a variance–covariance matrix, calculated from a sample with M realizations of the variables, follows a multinormal distribution with variance–covariance matrix[13] given by

$$C_{i,j} = \frac{1}{M}\,(2\lambda_i^{\,2}\delta_{i,j} + c_4^{\,i}\delta_{i,j} + c_{2,2}^{\,i,j}), \qquad \text{(A1)}$$

where λ_i is the actual eigenvalue and $c_4^{\,i}$, $c_{2,2}^{\,i,j}$ are the cumulants of the principal components of orders 4 and 2, 2 respectively. The cumulant $c_{2,2}{}^{i,j}$ is defined as

$$c_{2,2}{}^{i,j} = \frac{1}{M}\sum_{k=1}^{M} (Y_i^k)^2 (Y_j^k)^2 - \left[\frac{1}{M}\sum_{k=1}^{M} (Y_i^k)^2\right] \times \left[\frac{1}{M}\sum_{k=1}^{M} (Y_j^k)^2\right], \tag{A2}$$

where Y_i^k is the realization k of the principal component i. This expression, owing to the finite character of the sample, is a random variable. A good estimator for this cumulant is its averaged value. This averaging is done by taking other samplings of the data and is denoted as $\langle\ \rangle$. At this point we assume that the process is ergodic, and therefore the averaging along the different samplings is the same as the averaging across the r different principal components. Then the average along the different samples can be written as $\langle (Y_i^k)^2\rangle$. By definition, the mean value of Y_i^k is zero. Then the previous quantity is the variance of the variable Y_i^k. Again assuming ergodicity, we can calculate this as

$$\langle (Y_i^k)^2\rangle \simeq \frac{1}{r-1}\sum_{j=0}^{r-1} (Y_{i+j}{}^k)^2 = s_k{}^2. \tag{A3}$$

This mean value can be calculated assuming that the realizations of the principal components correspond with independent random variables having zero mean and the same variance, $\langle (Y_i^k)^2\rangle = \langle (Y_{i+1}{}^k)^2\rangle = \ldots = \langle (Y_{i+r-1})^2\rangle = s_k{}^2$, where $s_k{}^2$ is the estimator of the variance.

The estimator of the $c_{2,2}{}^{i,j}$ cumulant becomes

$$\langle c_{2,2}{}^{i,j}\rangle = \frac{1}{M}\sum_{k=1}^{M} (s_k{}^2)^2 - \left(\frac{1}{M}\sum_{k=1}^{M} s_k{}^2\right)^2. \tag{A4}$$

Therefore this cumulant has the meaning of the variance of the variance distribution of the realizations of the variables Y_i^k. The distribution of each principal component is a normal distribution, then the estimation of the variance obeys the following relation,

$$s_k{}^2 = \frac{\chi_{r-1}{}^2}{r-1}\lambda, \tag{A5}$$

and then the variance of this distribution of variances is the estimator for $c_{2,2}{}^{i,j}$ that we were looking for

$$\langle c_{2,2}{}^{i,j}\rangle = c_{2,2} = \frac{2\lambda^2}{r-1}, \tag{A6}$$

where the dependence i, j is dropped because the value is the same for all of them.

The cumulants of the fourth order are given as

$$c_4{}^i = \frac{1}{M}\sum_{k=1}^{M} (Y_i^k)^4 - 3\left[\frac{1}{M}\sum_{k=1}^{M} (Y_i^k)^2\right]^2. \tag{A7}$$

Again, now the best estimator for this cumulant is provided by its mean value. To calculate this averaged value we again use the independence of the principal components and the normal distribution of their values to find the estimator as

$$\langle c_4{}^i\rangle \simeq \left\langle \frac{1}{M}\sum_{k=1}^{M} (Y_i^k)^4\right\rangle - 3\left\langle\left[\frac{1}{M}\sum_{k=1}^{M} (Y_i^k)^2\right]^2\right\rangle. \tag{A8}$$

Y_i^k follows a normal distribution with zero mean. The first term of Eq. (A8) is the centered fourth moment of the variable. Therefore it obeys the following relation,

$$\langle (Y_i^k)^4\rangle = 3\sigma_k{}^4 \simeq 3s_k{}^4, \tag{A9}$$

where $\sigma_k{}^2$ is the variance of Y_i^k and $s_k{}^2$ is an estimator of this variance. The second term of $\langle c_4{}^i\rangle$ can be calculated by analyzing the variable $y = \frac{1}{M}\sum_{k=1}^{M}(Y_i^k)^2 = \frac{1}{M}\sum_{k=1}^{M}\sigma_k{}^2\,[(Y_i^k)/\sigma_k{}^2]$. The quantity $(Y_i^k)^2/\sigma_k{}^2$ follows a χ^2 distribution with one degree of freedom. The variable y is the sum of M independent random variables. Its distribution, by the central limit theorem, is a normal distribution with mean and variance given by

$$\mu_y = \frac{1}{M}\sum_{k=1}^{M}\sigma_k{}^2 \simeq \frac{1}{M}\sum_{k=1}^{M} s_k{}^2, \tag{A10}$$

$$\sigma_y{}^2 = \frac{2}{M^2}\sum_{k=1}^{M}\sigma_k{}^4 \simeq \frac{2}{M^2}\sum_{k=1}^{M} s_k{}^4. \tag{A11}$$

In Eq. (A7) the second term corresponds with the averaging of y^2. For the calculation, we use $\langle y^2\rangle = \sigma_y{}^2 + \mu_y{}^2$. Then, after again applying the ergodicity principle, we find that the second term of expression (A8) is

$$-3\left\langle\left[\frac{1}{M}\sum_{k=1}^{M} (Y_i^k)^2\right]^2\right\rangle = -3\left[\frac{2}{M^2}\sum_{k=1}^{M} s_k{}^4 + \left(\frac{1}{M}\sum_{k=1}^{M} s_k{}^2\right)^2\right]. \tag{A12}$$

By grouping the two terms, the mean value of $k_4{}^i$ is given by

$$\langle c_4{}^i\rangle = c_4 = \left(3 - \frac{6}{M}\right)c_{2,2} - \frac{6}{M}\lambda^2. \tag{A13}$$

This estimator does not depend on the chosen principal component i. However, owing to the large value of M, those terms having M in the denominator are neglected. The result is

$$c_4 \simeq 3c_{2,2}. \tag{A14}$$

With the previous analysis, the variance–covariance matrix [Eq. (A1)] has the same element along the diagonal, $(2\lambda^2 + c_4)/M$, and then in the nondiagonal elements we find $c_{2,2}/M$. This structure means that the actual value follows a normal distribution[14] $N(\mu, \sigma^2)$, where

$$\mu = \frac{1}{r}\sum_{i=1}^{r} l_i = \lambda, \tag{A15}$$

$$\sigma^2 = \frac{1}{M}\left\{\frac{1}{r}\left[2\lambda^2 + c_4 + (r-1)c_{2,2}\right]\right\} = \frac{\lambda^2}{M}\left[\frac{4}{r} + \frac{6}{r(r-1)}\right]. \tag{A16}$$

Therefore, to check if a given set of eigenvalues can be considered having the same value, it is necessary to locate those values within the previous normal distribution and decide, for a given level of confidence, if the calculated values are within the previous normal distribution. In our case, we are interested in the definition of diplets, therefore $r = 2$. The level of confidence used in this paper for grouping the eigenvalues is 99.9%.

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