Eur. Phys. J. C (2008) 58: 471–481
DOI 10.1140/epjc/s10052-008-0776-2

THE EUROPEAN
PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

Towards the effective potential of the littlest Higgs model

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Received: 1 February 2008 / Revised: 24 September 2008 / Published online: 29 October 2008

Abstract We compute the relevant parameters of the combined Higgs and ϕ scalar effective potential in the littlest Higgs (LH) model. These parameters are obtained as the sum of two kinds of contributions. The first one is the one-loop radiative corrections coming from fermions and gauge bosons. The second one is obtained at tree level from the higher-order effective operators needed for the ultraviolet completion of the model. Finally, we analyze the restrictions that the requirement of reproducing the standard electroweak symmetry breaking of the SM set on the LH model parameters.

1 Introduction

The discovery of a Higgs boson and the elucidation of the mechanism responsible for electroweak symmetry breaking are some of the major goals of present and future searches in particle physics. The quadratically divergent contributions to the Higgs mass and the electroweak precision observables imply different scales for physics beyond the standard model (SM) (1 and 10 TeV respectively). This is the so-called little hierarchy problem. As is well known, the mass of the Higgs boson receives loop corrections that are quadratic in the loop momenta. The largest contributions come from the top-quark loop, with smaller corrections coming from loops of the electroweak gauge bosons and of the Higgs boson itself. Cancellations between the top sector and other sectors must occur in order to have a Higgs mass smaller than 200 GeV as expected from the electroweak precision test of the SM, which requires a fine-tuning of one part in 100. As this situation is quite unnatural, various theories and models have been designed to solve this problem. For example, in supersymmetric models the problem of quadratic Higgs mass divergences is resolved by the introduction of an opposite-statistics partner for each particle of the SM. The more recent idea of the littlest Higgs model (LH) [1], inspired by an old suggestion by Georgi and Pais [2, 3], tries to solve the little hierarchy problem by adding new particles with masses O(TeV) and symmetries which protect the Higgs mass from those dangerous quadratically divergent contributions (see [4] and [5] for reviews). These particles include the Goldstone bosons (GB) corresponding to a global spontaneous symmetry breaking (SSB) from the SU(5) to the SO(5) group, a new third-generation vector quark called T and the gauge boson corresponding to an additional gauge group which contains at least a $\mathrm{SU}(2)_R$ and eventually a new hypercharge U(1). In this case, cancellation occurs between same-statistics particles. However, LH models typically leave uncanceled logarithmic mass contributions, which requires additional new contributions at some high scale to preserve a small Higgs boson mass. All of these new states could give rise to a very rich phenomenology, which could be probed at the CERN Large Hadron Collider (LHC) [6, 7].

Nevertheless, it is clear that any viable model has to fulfill the basic requirement of reproducing the SM model at low energies. In particular, from the LH model it is, in principle, possible to compute the Higgs low-energy effective potential and then, by comparing with the SM potential, to obtain their phenomenological consequences including new restrictions on the parameter space of the LH model itself. For example, once obtained, with the one-loop corrections to the parameters of the standard Higgs potential, $V = -\mu^2 HH^\dagger + \lambda(HH^\dagger)^2$, where μ^2 and λ denote the well-known Higgs mass and Higgs self-coupling parameters, restrictions over the LH parameters space can be obtained by imposing the condition $\mu^2 = \lambda v^2$. Here v is the SM vacuum expectation value ($H = (0, v)/\sqrt{2}$) with $v \simeq 245$ GeV. The μ^2 sign and value are well known [1, 7], and effectively they are the right ones to produce the electroweak symmetry breaking, giving a Higgs mass $m_{\mathrm{H}}^2 = 2\mu^2$. However, the full expression for the radiative corrections to λ has not been analyzed in detail.

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In principle, both μ^2 and λ receive contributions from fermion, gauge-boson and scalar loops, besides others that could come from the ultraviolet completion of the LH model. We have previously computed the contributions to the Higgs effective potential in the LH model coming from the fermion and gauge-boson sectors [8, 9]. On the other hand, several relations for the threshold corrections to the λ parameter in the presence of a 10 TeV cut-off, depending of the UV-completion of the theory, has been reported before (see, for example [10]).

In this work, we continue our program consisting in the computation of the relevant terms of the Higgs low-energy effective potential in the LH model and to analyze their phenomenological consequences. As has been mentioned before, we have started to developed this program in two previous articles [8, 9]. First, we have computed and analyzed the fermion contributions to the low-energy Higgs effective potential and we have illustrated the kind of constraints on the possible values of the LH parameters that can be set by requiring the complete LH Higgs effective potential to reproduce exactly the SM potential [8]. Second, the effects of virtual heavy and electroweak gauge bosons present in the LH model have been included in the analysis [9]. The radiative corrections to λ, at the one-loop level, have not previously been computed. First results are presented in the above-mentioned two articles. We want to note that the computation of λ is important for several reasons. First, it must be positive, for the low-energy effective action to make sense (otherwise the theory would not have any vacuum). In addition, from the effective potential above, one gets the simple formula $m_{\mathrm{H}}^2 = 2\lambda v^2$ or, equivalently, $\mu^2 = \lambda v^2$, v being set by experiment (for instance from the muon lifetime) at $v \simeq 245$ GeV. Our phenomenological discussions in [8, 9] have shown that the one-loop effective potential of the LH model cannot reproduce the SM potential with a low enough Higgs mass, $m_{\mathrm{H}}^2 = 2\lambda v^2 = 2\mu^2$, to agree with the standard expectations. However, there are some indications suggesting that the effects of including interaction terms between Goldstone bosons (GB) and the other particles, i.e. fermions and gauge bosons, and/or higher-order GB loops, could reduce the Higgs boson mass so that complete compatibility with the experimental constraints could be obtained.

The main objective of this work is to compute the effective potential for the doublet Higgs and the triplet ϕ, both being scalar fields of the LH model coming from some of the GB corresponding to the global SU(5) to SO(5) global symmetry breaking of the LH model. Its relevant terms read [6]

$$V_{\mathrm{eff}}(H,\phi) = -\mu^2 H H^\dagger + \lambda\big(HH^\dagger\big)^2 + \lambda_{\phi^2} f^2 \mathrm{tr}\big(\phi\phi^\dagger\big) + \mathrm{i}\lambda_{H^2\phi} f \big(H\phi^\dagger H^{\mathrm{T}} - H^*\phi H^\dagger\big). \tag{1.1}$$

This potential gets contributions from radiative corrections and from effective operators coming from the ultraviolet completion of the LH model. With this potential we shall study the regions of the LH parameter space giving rise to SM electroweak symmetry breaking. Although radiative corrections from fermion and gauge-boson loops are discussed in [8, 9], the radiative contributions to λ_{ϕ^2} and $\lambda_{H^2\phi}$ have not been computed so far. A new constraints over the LH parameter space emerge once we impose the new relation between coefficients of the effective Higgs potential, imposed by the diagonalization of the Higgs mass matrix.

This work is organized as follows: In Sect. 2 we briefly explain the LH model and set the notation. Section 3 is devoted to the computation of the radiative-correction contributions to the effective potential at one-loop level. The next section is dedicated to the effective operator contribution. In Sect. 5 we analyze the constraints that our computation establishes on the LH parameters and, finally, in Sect. 6 we present our conclusions. The Goldstone boson couplings to fermions and gauge bosons, needed for our computations, are listed in the appendix.

2 The littlest Higgs model Lagrangian

The LH model is based on the assumption that there is a physical system with a global SU(5) symmetry that is spontaneously broken to a SO(5) symmetry at a high scale Λ through a vacuum-expectation value of order f. Thus, 14 Goldstone bosons (GB) are obtained as a consequence of this breaking. In this work, we shall consider two different versions of the LH model. In the first one the global SU(5) symmetry is explicitly broken by the gauge group $[\mathrm{SU}(2)\times \mathrm{U}(1)]^2$. We refer to this version as *Model I* [8, 9]. In the second one the gauge group is $[\mathrm{SU}(2)^2 \times \mathrm{U}(1)]$ (*Model II*) [8, 9]. In both cases some of the GB become pseudo-GB and acquire their masses through radiative corrections coming from gauge bosons and t, b and T fermions loops.

The LH low-energy dynamics is then described by a nonlinear sigma model Lagrangian plus the appropriate Yukawa terms. The corresponding Lagrangian is given by [1, 6, 7],

$$\begin{aligned} L &= L_{\mathrm{kin}} + L_{\mathrm{YK}} \\ &= \frac{f^2}{8}\mathrm{tr}\big[(D_\mu\Sigma)\big(D^\mu\Sigma\big)^\dagger\big] - \frac{\lambda_1}{2} f \overline{u}_R \epsilon_{mn}\epsilon_{ijk}\Sigma_{im}\Sigma_{jn}\chi_{Lk} \\ &\quad - \lambda_2 f \overline{U}_{\mathrm{R}} U_{\mathrm{L}} + \mathrm{h.c.}, \end{aligned} \tag{2.1}$$

where

$$\Sigma = \mathrm{e}^{2\mathrm{i}\Pi/f}\Sigma_0 \tag{2.2}$$

is the GB matrix field. Σ_0 is

$$\Sigma_0 = \begin{pmatrix} 0 & 0 & \mathbf{1} \\ 0 & 1 & 0 \\ \mathbf{1} & 0 & 0 \end{pmatrix}, \tag{2.3}$$

with $\mathbf{1}$ being the 2×2 unit matrix. The Π matrix can be parametrized as

$$\Pi=\begin{pmatrix} 0 & \frac{-\mathrm{i}}{\sqrt{2}}H^\dagger & \phi^\dagger \\ \frac{\mathrm{i}}{\sqrt{2}}H & 0 & \frac{-\mathrm{i}}{\sqrt{2}}H^* \\ \phi & \frac{\mathrm{i}}{\sqrt{2}}H^{\mathrm{T}} & 0 \end{pmatrix}. \tag{2.4}$$

Here $H=(H^0,H^+)$ is the SM Higgs doublet and ϕ is the triplet given by

$$\phi=\begin{pmatrix} \phi^0 & \frac{1}{\sqrt{2}}\phi^+ \\ \frac{1}{\sqrt{2}}\phi^+ & \phi^{++} \end{pmatrix}. \tag{2.5}$$

The covariant derivative D_μ is defined by

Model I

$$D_\mu\Sigma=\partial_\mu\Sigma-\mathrm{i}\sum_{j=1}^{2}g_jW_j^a\big(Q_j^a\Sigma+\Sigma Q_j^{a\mathrm{T}}\big)$$
$$-\mathrm{i}\sum_{j=1}^{2}g'_jB_j\big(Y_j\Sigma+\Sigma Y_j^{\mathrm{T}}\big),$$

Model II

$$D_\mu\Sigma=\partial_\mu\Sigma-\mathrm{i}\sum_{j=1}^{2}g_jW_j^a\big(Q_j^a\Sigma+\Sigma Q_j^{a\mathrm{T}}\big)$$
$$-\mathrm{i}g'B\big(Y\Sigma+\Sigma Y^{\mathrm{T}}\big), \tag{2.6}$$

where g and g' are the gauge couplings, W_j^a $(a=1,2,3)$ and B_j, B are the SU(2) and U(1) gauge fields, respectively, $Q^a_{1ij}=\sigma^a_{ij}/2$ for $i,j=1,2$, $Q^a_{2ij}=\sigma^{a*}_{ij}/2$ for $i,j=4,5$ and zero otherwise. We have $Y_1=\mathrm{diag}(-3,-3,2,2,2)/10$, $Y_2=\mathrm{diag}(-2,-2,-2,3,3)/10$ and $Y=\mathrm{diag}(-1,-1,0,1,1)/2$.

The Yukawa Lagrangian in (2.1), L_{YK}, describes the interactions between GB and fermions, more exactly, the third generations of quarks plus the extra T quark appearing in the LH model. The indices in L_{YK} are defined such that $m,n=4,5$, $i,j=1,2,3$, and

$$\overline{u}_{\mathrm{R}}=c\,\overline{t}_{\mathrm{R}}+s\,\overline{T}_{\mathrm{R}},$$
$$\overline{U}_{\mathrm{R}}=-s\,\overline{t}_{\mathrm{R}}+c\,\overline{T}_{\mathrm{R}}, \tag{2.7}$$

with

$$c=\cos\theta=\frac{\lambda_2}{\sqrt{\lambda_1^2+\lambda_2^2}},$$
$$s=\sin\theta=\frac{\lambda_1}{\sqrt{\lambda_1^2+\lambda_2^2}}, \tag{2.8}$$

and

$$\chi_{\mathrm{L}}=\begin{pmatrix} u\\ b\\ U\end{pmatrix}_{\mathrm{L}}=\begin{pmatrix} t\\ b\\ T\end{pmatrix}_{\mathrm{L}}. \tag{2.9}$$

The SU(5) to SO(5) spontaneous breaking gives rise to four massless gauge bosons (the SM gauge bosons) and four or three massive gauge bosons corresponding to *Model I* or *Model II*, respectively. In the fermion sector, we obtain one massive T quark and two massless quarks, namely the top and the bottom quarks.

In order to compute the gauge-boson loops, the Lagrangian L must be supplemented by the standard terms depending only on the gauge fields. For the sake of simplicity we shall work in the Landau gauge. Then these terms can symbolically be written in the mass eigenstate basis

$$L_\Omega=\frac{1}{2}\Omega^\mu\big(\big(\Box+M_\Omega^2\big)g_{\mu\nu}-\partial_\mu\partial_\nu+2\tilde{I}\,g_{\mu\nu}\big)\Omega^\nu, \tag{2.10}$$

where Ω stands for any of the gauge bosons:

$$\textit{Model I}\quad \Omega^\mu=\big(W'^{\mu a},W^{\mu a},B'^{\mu},B^\mu\big),$$
$$\textit{Model II}\quad \Omega^\mu=\big(W'^{\mu a},W^{\mu a},B^\mu\big), \tag{2.11}$$

with the mass matrix eigenstates

$$\textit{Model I}\quad M_\Omega=(M_{W'}1_{3\times 3},0_{3\times 3},M_{B'},0),$$
$$\textit{Model II}\quad M_\Omega=(M_{W'}1_{3\times 3},0_{3\times 3},0), \tag{2.12}$$

with $M_{W'}=f\sqrt{g_1^2+g_2^2}/2$ and $M_{B'}=f\sqrt{g_1'^2+g_2'^2}/\sqrt{20}$. Finally, $\tilde{I}$ is the interaction matrix between the gauge bosons and the H and ϕ scalars as given in the appendix.

For the quarks, the complete Lagrangian is

$$L_\chi=\overline{\chi}_{\mathrm{R}}(\mathrm{i}\not{\partial}-M+\hat{I})\chi_{\mathrm{L}}+\mathrm{h.c.}, \tag{2.13}$$

where

$$\chi_{\mathrm{R}}=\begin{pmatrix} t\\ b\\ T\end{pmatrix}_{\mathrm{R}},$$

$M=\mathrm{diag}(0,0,m_{\mathrm{T}})$ with $m_{\mathrm{T}}=f\sqrt{\lambda_1^2+\lambda_2^2}$, and $\hat{I}$ being the scalar-quark interaction matrix, also given in the appendix. For more details of the model, including Feynman rules and also some phenomenological results, see, for example, [6].

3 One-loop effective potential

As is well known, electroweak symmetry breaking in the LH model is triggered, in principle, by the Higgs potential generated by one-loop radiative corrections, including both fermion and gauge-boson loops. Obviously this potential is invariant under the electroweak gauge group $SU(2) \times U(1)$. Its relevant terms are given in (1.1), μ^2 and λ being the Higgs mass and the Higgs self-coupling parameters, respectively. Quartic terms involving ϕ^4 and $H^2\phi^2$ are not included, since they are not relevant in our present computation. The coefficients λ, λ_{ϕ^2} and $\lambda_{H^2\phi}$ appearing in the potential (1.1) receive contributions from the tree-level higher-order operators coming from the ultraviolet completion of the LH model (see Sect. 4) and also from the gauge-boson and fermion radiative corrections as will be discussed in this section.

As described in detail in our previous articles [8, 9], we first focused on the effective potential for the H doublet, obtaining the first two terms of the potential,

$$V_{\text{eff}}(H) = -\mu_{\text{fg}}^2 H H^\dagger + \lambda_{\text{fg}} \big(H H^\dagger\big)^2, \tag{3.1}$$

where μ_{fg}^2 and λ_{fg} denote the sum of fermionic and the gauge-boson contributions to μ^2 and λ. By imposing that these parameters should reproduce the SM relation $m_{\text{H}}^2 = 2\lambda v^2 = 2\mu^2$, where m_{H} is the Higgs mass and v is the vacuum-expectation value, we found that with this potential it is not sufficient to find a light Higgs mass and to satisfy the relation $\mu_{\text{fg}}^2 = v^2 \lambda_{\text{fg}}$. Notice that v is set by experiment (for instance from the muon lifetime) to be $v \simeq 245$ GeV, and μ is forced by the data to be at most of order 200 GeV. However, the inclusion of the Goldstone boson (GB) sector could channel the situation towards a complete compatibility with the SM and the experimental constraints. In this way, the next objective is to obtain the effective potential for the H and ϕ fields, including the radiative contributions from fermion and gauge-boson loops and the ones coming from the effective higher-order operators (tree-level contribution) [1, 6, 11]. In this work we concentrate on the computation of the fermion and gauge-boson contributions to the remaining coefficients of the complete one-loop effective potential defined in (1.1), λ_{ϕ^2} and $\lambda_{H^2\phi}$. For this purpose, we consider constant GB fields, i.e. $\partial H = \partial \phi = 0$. This assumption makes the computation easier, since we have

$$S_{\text{eff}}[H, \phi] = -\int d^4x \, V_{\text{eff}}(H, \phi). \tag{3.2}$$

On the other hand, the action is quadratic in the fermionic fields. Therefore, this one-loop contribution is exactly computed.

We split the calculation in two parts: the first one is dedicated to the fermion sector and the second one to the gauge-boson sector. Details on how the effective action is computed, by using standard techniques (see for instance [12]), are given in [8, 9]. In the following we summarize just the main steps needed for the calculation.

3.1 Fermionic contribution

Following the idea in [8], the fermionic part of the effective action can be expanded:

$$\begin{aligned} S_{\text{eff}}^{\text{f}}[H, \phi] &\simeq -\mathrm{i}\,\mathrm{Tr} \log(1 + G \tilde{I}_{\text{f}}) \\ &= -\mathrm{i}\,\mathrm{Tr} \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} (G \tilde{I}_{\text{f}})^k, \end{aligned} \tag{3.3}$$

where we have neglected a constant irrelevant for the computation of the effective action. The fermion propagator, $G^{ab}(x, y)$, is given by

$$G^{ab}(x, y) \equiv \int d\tilde{k} e^{-\mathrm{i}k(x-y)} (\not{k} - m_{\text{f}})^{-1}_{ab}, \quad a, b \equiv t, b, T, \tag{3.4}$$

where $d\tilde{k} \equiv d^4k/(2\pi)^4$, and the interaction operators are

$$\hat{\tilde{I}}_{\text{f}}^{ab}(x, y) = (\tilde{I}_{\text{f1}} + \tilde{I}_{\text{f2}} + \tilde{I}_{\text{f3}} + \tilde{I}_{\text{f4}}) \delta(x - y) \delta^{ab}. \tag{3.5}$$

Here the subindex indicates the number of GB interacting with two fermions.

In order to obtain the fermionic contribution to λ_{ϕ^2} and $\lambda_{H^2\phi}$ we only need consider the terms $k = 1$ and $k = 2$ in the expansion (3.3), respectively. The generic one-loop diagrams are shown in Fig. 1. For $k = 1$ we get

$$S_{\text{f}}^{(1)}[H, \phi] = -\mathrm{i}\,\mathrm{Tr}\big[G^a \big(\tilde{I}_{\text{f2}}^{aa} + \tilde{I}_{\text{f4}}^{aa}\big)\big]. \tag{3.6}$$

For the case $k = 2$ one obtains

$$\begin{aligned} S_{\text{f}}^{(2)}[H, \phi] = \frac{\mathrm{i}}{2} \mathrm{Tr}\big[&2G^a \tilde{I}_{\text{f1}}^{ab} G^b \tilde{I}_{\text{f2}}^{ba} + G^a \tilde{I}_{\text{f2}}^{ab} G^b \tilde{I}_{\text{f2}}^{ba} \\ &+ 2G^a \tilde{I}_{\text{f1}}^{ab} G^b \tilde{I}_{\text{f3}}^{ba}\big]. \end{aligned} \tag{3.7}$$

By using well-known methods, and after some work in which the divergent integrals that emerge are regularized by using an ultraviolet cutoff Λ, we obtain the various contributions to the couplings. The fermionic contributions are

$$\lambda_{\phi^2 \text{f}} = \frac{8N_{\text{c}}}{(4\pi f)^2} (\lambda_{\text{t}}^2 + \lambda_{\text{T}}^2) \left(\Lambda^2 - m_{\text{T}}^2 \log\left(\frac{\Lambda^2}{m_{\text{T}}^2} + 1\right)\right), \tag{3.8}$$

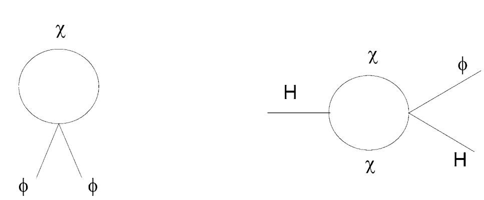

Fig. 1 Fermionic one-loop diagrams contributing to λ_{ϕ^2} and $\lambda_{H^2\phi}$, with $\chi = t, b$ and T. All possible combinations of these particles appear in the loops

and

$$\lambda_{H^2\phi\mathrm{f}} = -\frac{4N_\mathrm{c}}{(4\pi f)^2}\left[\left(\lambda_\mathrm{t}^2+\lambda_\mathrm{T}^2\right)\Lambda^2 - \lambda_\mathrm{T}^2 m_\mathrm{T}^2 \log\left(\frac{\Lambda^2}{m_\mathrm{T}^2}+1\right)\right], \tag{3.9}$$

where N_c is the number of colors, and λ_t and λ_T are, respectively, the SM top Yukawa coupling and the heavy top Yukawa coupling, given by

$$\lambda_\mathrm{t} = \frac{\lambda_1\lambda_2}{\sqrt{\lambda_1^2+\lambda_2^2}}, \qquad \lambda_\mathrm{T} = \frac{\lambda_1^2}{\sqrt{\lambda_1^2+\lambda_2^2}}. \tag{3.10}$$

For the purpose of illustration and the final discussion of this paper, we also summarize here the fermionic contribution to the other two parameters of the Higgs potential, μ^2 and λ, as have been obtained in [8]:

$$\mu_\mathrm{f}^2 = N_\mathrm{c}\frac{m_\mathrm{T}^2\lambda_\mathrm{t}^2}{4\pi^2}\log\left(1+\frac{\Lambda^2}{m_\mathrm{T}^2}\right), \tag{3.11}$$

and

$$\begin{aligned}\lambda_\mathrm{f} = \frac{N_\mathrm{c}}{(4\pi)^2}\Bigg[&2\left(\lambda_\mathrm{t}^2+\lambda_\mathrm{T}^2\right)\frac{\Lambda^2}{f^2} \\ &-\log\left(1+\frac{\Lambda^2}{m_\mathrm{T}^2}\right)\left(-\frac{2m_\mathrm{T}^2}{f^2}\left(\frac{5}{3}\lambda_\mathrm{t}^2+\lambda_\mathrm{T}^2\right)\right. \\ &\left.+4\lambda_\mathrm{t}^4+4\left(\lambda_\mathrm{T}^2+\lambda_\mathrm{t}^2\right)^2\right) \\ &-4\lambda_\mathrm{T}^2\frac{1}{1+\frac{m_\mathrm{T}^2}{\Lambda^2}}\left(\frac{m_\mathrm{T}^2}{f^2}-2\lambda_\mathrm{t}^2-\lambda_\mathrm{T}^2\right)-4\lambda_\mathrm{t}^4\log\left(\frac{\Lambda^2}{m^2}\right)\Bigg].\end{aligned} \tag{3.12}$$

Observe that the λ parameters, $\lambda_\mathrm{f}, \lambda_{\phi^2\mathrm{f}}$ and $\lambda_{H^2\phi\mathrm{f}}$, are quadratically divergent. This is due to the lack of any symmetry protecting them, unlike the μ parameter, which is protected by a SU(3) global symmetry. This will be the case for the gauge sector too, as will be seen in the following.

3.2 Bosonic contribution

Here we concentrate on the gauge-boson contributions at one-loop level. We use the Landau gauge, so that we do not have to consider any ghost field at this level. In this case the effective action can be expanded as

$$S_\mathrm{eff}^\mathrm{g}[H,\phi] = \frac{\mathrm{i}}{2}\mathrm{Tr}\log(1+G\tilde{I}_\mathrm{g}) = \frac{\mathrm{i}}{2}\mathrm{Tr}\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k}(2G\tilde{I}_\mathrm{g})^k, \tag{3.13}$$

where G is the gauge-boson propagator given by (Landau gauge)

$$\begin{aligned}G_{\mu\nu}^{ab}(x,y) &\equiv \int \mathrm{d}\tilde{k}\frac{\mathrm{e}^{-\mathrm{i}k(x-y)}}{k^2-M_\mathrm{g}^2}\left(-g_{\mu\nu}+\frac{1}{k^2}k_\mu k_\nu\right)_{ab}, \\ a,b &= W'^a, W^a, B', B,\end{aligned} \tag{3.14}$$

and the interaction operators are

$$\hat{\tilde{I}}_\mathrm{g}^{ab}(x,y) = (\tilde{I}_{\mathrm{g}2}+\tilde{I}_{\mathrm{g}3}+\tilde{I}_{\mathrm{g}4})\delta(x-y)\delta^{ab}. \tag{3.15}$$

The generic diagrams for this computation are given in Fig. 2. In this case, we only need to consider the term $k=1$ to obtain the two parameters λ_{ϕ^2} and $\lambda_{H^2\phi}$. We get

$$S_\mathrm{g}^{(1)}[H,\phi] = \mathrm{i}\mathrm{Tr}\left[G(\tilde{I}_{\mathrm{g}2}+\tilde{I}_{\mathrm{g}3}+\tilde{I}_{\mathrm{g}4})\right]. \tag{3.16}$$

As mentioned above, we consider in our analysis two different models: the original LH with two U(1) groups (*Model I*), and the other one with just one U(1) group (*Model II*). As there is no mixing between the SU(2) and U(1) groups, the only difference among these two models occurs in the U(1) sector.

3.2.1 Model I

The contributions from the gauge-boson sector to the λ_{ϕ^2} and $\lambda_{H^2\phi}$ parameters are given by

$$\begin{aligned}\lambda_{\phi^2\mathrm{g}}^\mathrm{I} = \frac{3}{4(4\pi f)^2}\Bigg[&\frac{g^2}{c_\psi^2 s_\psi^2}\Lambda^2 - g^2M_{W'}^2\log\left(\frac{\Lambda^2}{M_{W'}^2}+1\right) \\ &\times\left(\frac{(s_\psi^2-c_\psi^2)^2}{c_\psi^2 s_\psi^2}-4\right) \\ &+\frac{g'^2}{c_{\psi'}^2 s_{\psi'}^2}\Lambda^2 - g'^2M_{B'}^2\log\left(\frac{\Lambda^2}{M_{B'}^2}+1\right)\frac{(s_{\psi'}^2-c_{\psi'}^2)^2}{c_{\psi'}^2 s_{\psi'}^2}\Bigg],\end{aligned} \tag{3.17}$$

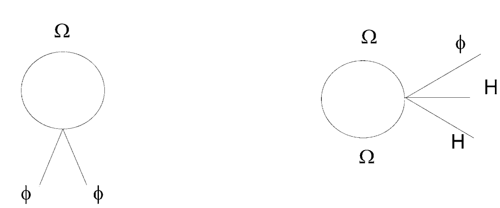

Fig. 2 One-loop gauge-boson diagrams contributing to λ_{ϕ^2} and $\lambda_{H^2\phi}$, where Ω represents the gauge-boson particles, $W'^{1,2,3}$, $W^{1,2,3}$, B' and B. All possible combinations of these particles appear in the loops

$$
\begin{aligned}
&\lambda^{\mathrm{I}}_{H^2\phi\mathrm{g}}\\
&\quad=\frac{3}{8(4\pi f)^2}\Bigg[g^2\frac{s_\psi^2-c_\psi^2}{c_\psi^2 s_\psi^2}\left(\Lambda^2-M_{W'}^2\log\left(\frac{\Lambda^2}{M_{W'}^2}+1\right)\right)\\
&\qquad+g'^2\frac{s_{\psi'}^2-c_{\psi'}^2}{c_{\psi'}^2 s_{\psi'}^2}\left(\Lambda^2-M_{B'}^2\log\left(\frac{\Lambda^2}{M_{B'}^2}+1\right)\right)\Bigg]. \qquad (3.18)
\end{aligned}
$$

Note that this last parameter only receives contributions from the heavy gauge-boson sector.

For the sake of completeness and our phenomenological discussion we also list here the results for the gauge-boson contributions to μ^2 and λ, as obtained in [9]:

$$
\begin{aligned}
\mu_{\mathrm{g}}^{2\mathrm{I}}=&-\frac{3}{64\pi^2}\Bigg[3g^2M_{W'}^2\log\left(1+\frac{\Lambda^2}{M_{W'}^2}\right)\\
&+g'^2M_{B'}^2\log\left(1+\frac{\Lambda^2}{M_{B'}^2}\right)\Bigg], \qquad (3.19)
\end{aligned}
$$

$$
\begin{aligned}
\lambda_{\mathrm{g}}^{\mathrm{I}}=&-\frac{3}{(16\pi f)^2}\Bigg[-\left(\frac{g^2}{c_\psi^2 s_\psi^2}+\frac{g'^2}{c'^2_\psi s'^2_\psi}\right)\Lambda^2\\
&+g^2M_{W'}^2\log\left(1+\frac{\Lambda^2}{M_{W'}^2}\right)\Bigg(4+\frac{1}{c_\psi^2 s_\psi^2}\\
&+2g'^2\frac{(c_\psi^2 s'^2_\psi+s_\psi^2 c'^2_\psi)^2}{c_\psi^2 s_\psi^2 c'^2_\psi s'^2_\psi}\frac{f^2}{M_{W'}^2-M_{B'}^2}\Bigg)\\
&+g'^2M_{B'}^2\log\left(1+\frac{\Lambda^2}{M_{B'}^2}\right)\Bigg(\frac{4}{3}+\frac{1}{c'^2_\psi s'^2_\psi}\\
&+2g^2\frac{(c_\psi^2 s'^2_\psi+s_\psi^2 c'^2_\psi)^2}{c_\psi^2 s_\psi^2 c'^2_\psi s'^2_\psi}\frac{f^2}{M_{B'}^2-M_{W'}^2}\Bigg)\\
&+f^2\log\left(1+\frac{\Lambda^2}{M_{W'}^2}\right)\Bigg(3g^4+2\big(3g^2+g'^2\big)\\
&\times g^2\frac{(s_\psi^2-c_\psi^2)^2}{c_\psi^2 s_\psi^2}\Bigg)\\
&+f^2\log\left(1+\frac{\Lambda^2}{M_{B'}^2}\right)\Bigg(g'^4+2\big(g^2+g'^2\big)\\
&\times g'^2\frac{(s'^2_\psi-c'^2_\psi)^2}{c'^2_\psi s'^2_\psi}\Bigg)\\
&+f^2\log\left(\frac{\Lambda^2}{m^2}\right)\Big(3g^4+g'^4+8g^2g'^2\Big)\\
&-3f^2\frac{g^4}{1-\frac{M_{W'}^2}{\Lambda^2}}-f^2\frac{g'^4}{1-\frac{M_{B'}^2}{\Lambda^2}}\Bigg], \qquad (3.20)
\end{aligned}
$$

where

$$
g\equiv\frac{g_1g_2}{\sqrt{g_1^2+g_2^2}},\qquad s_\psi=\sin\psi=\frac{g_1}{\sqrt{g_1^2+g_2^2}},
$$

$$
c_\psi=\cos\psi=\frac{g_2}{\sqrt{g_1^2+g_2^2}} \qquad (3.21)
$$

and

$$
\begin{aligned}
&g'\equiv\frac{g'_1g'_2}{\sqrt{g'^2_1+g'^2_2}},\qquad s'_\psi=\sin\psi'=\frac{g'_1}{\sqrt{g'^2_1+g'^2_2}},\\
&c'_\psi=\cos\psi'=\frac{g'_2}{\sqrt{g'^2_1+g'^2_2}}.
\end{aligned} \qquad (3.22)
$$

3.2.2 Model II

The corresponding results for this model are

$$
\begin{aligned}
\lambda^{\mathrm{II}}_{\phi^2\mathrm{g}}=&\frac{3}{64\pi^2f^2}\Bigg[\frac{g^2}{c_\psi^2 s_\psi^2}\Lambda^2-g^2M_{W'}^2\log\left(\frac{\Lambda^2}{M_{W'}^2}+1\right)\\
&\times\left(\frac{(s_\psi^2-c_\phi^2)^2}{c_\psi^2 s_\psi^2}-4\right)\Bigg]+\frac{3g'^2}{(4\pi f)^2}\Lambda^2, \qquad (3.23)
\end{aligned}
$$

$$
\lambda^{\mathrm{II}}_{H^2\phi\mathrm{g}}=\frac{3g^2}{8(4f\pi)^2}\frac{s_\psi^2-c_\psi^2}{c_\psi^2 s_\psi^2}\left(\Lambda^2-M_{W'}^2\log\left(\frac{\Lambda^2}{M_{W'}^2}+1\right)\right). \qquad (3.24)
$$

As expected, the U(1) sector does not have any influence on $\lambda^{\mathrm{II}}_{H^2\phi\mathrm{g}}$.

In addition, the $\mu_{\mathrm{g}}^{2\mathrm{II}}$ and $\lambda_{\mathrm{g}}^{\mathrm{II}}$ parameters are given by [9]

$$
\mu_{\mathrm{g}}^{2\mathrm{II}}=-\frac{3}{64\pi^2}\left(3g^2M_{W'}^2\log\left(1+\frac{\Lambda^2}{M_{W'}^2}\right)+g'^2\Lambda^2\right), \qquad (3.25)
$$

and

$$
\begin{aligned}
\lambda_{\mathrm{g}}^{\mathrm{II}}=&-\frac{3}{(16\pi f)^2}\Bigg[-\frac{g^2}{c_\psi^2 s_\psi^2}\Lambda^2+\frac{4}{3}g'^2\Lambda^2\\
&+g^2M_{W'}^2\log\left(\frac{\Lambda^2}{M_{W'}^2}+1\right)\left(4+\frac{1}{c_\psi^2 s_\psi^2}\right)\\
&+f^2\log\left(1+\frac{\Lambda^2}{M_{W'}^2}\right)\Bigg(3g^4+2\big(3g^2+g'^2\big)\\
&\times g^2\frac{(s_\psi^2-c_\psi^2)^2}{s_\psi^2 c_\psi^2}\Bigg)\\
&+f^2\log\left(\frac{\Lambda^2}{m^2}\right)(3g^4+g'^4+8g^2g'^2)\\
&-3f^2\frac{g^4}{1-\frac{M_{W'}^2}{\Lambda^2}}\Bigg]. \qquad (3.26)
\end{aligned}
$$

With these results, the radiative contributions at one-loop level to the Higgs potential parameters are completed.

4 Effective operators

As discussed above, the Higgs potential gets gauge-boson and fermion one-loop contributions in the LH model. In addition, the potential coefficients also receive contributions from additional operators coming from the ultraviolet completion of the LH model. Thus, these operators must be consistent with the symmetry of the theory [1, 6, 11]. At the lowest order they can be parameterized by two unknown coefficients, a and $a' \sim \mathrm{O}(1)$. The form of these effective operators are, for the fermion sector [6],

$$O_{\mathrm{f}} = -a'\frac{1}{4}\lambda_1^2 f^4 \epsilon^{wx}\epsilon_{yz}\epsilon^{ijk}\epsilon_{kmn}\Sigma_{iw}\Sigma_{jx}\Sigma^{*my}\Sigma^{*nz}, \quad (4.1)$$

where i, j, k, m, n run over 1, 2, 3 and w, x, y, z run over 4, 5, and for the gauge sector (*Model I*)

$$O_{\mathrm{gb}} = \frac{1}{2}af^4\left\{g_j^2\sum_{a=1}^{3}\mathrm{Tr}\left[\left(Q_j^a\Sigma\right)\left(Q_j^a\Sigma\right)^*\right] + g_j'^2\mathrm{Tr}\left[(Y_j\Sigma)(Y_j\Sigma)^*\right]\right\}, \quad (4.2)$$

with $j = 1, 2$ and Q_j^a and Y_j being the generators of the $\mathrm{SU}(2)_j$ and $\mathrm{U}(1)_j$ groups, respectively.

In the case of *Model II*

$$O_{\mathrm{gb}} = \frac{1}{2}cf^4\left\{g_j^2\sum_{a=1}^{3}\mathrm{Tr}\left[\left(Q_j^a\Sigma\right)\left(Q_j^a\Sigma\right)^*\right] + g'^2\mathrm{Tr}\left[(Y\Sigma)(Y\Sigma)^*\right]\right\}, \quad (4.3)$$

where $j = 1, 2$, and Y is the generator of the unique U(1) group.

By expanding the GB field matrix Σ in these effective operators, we obtain their different contributions to the coefficients of the effective potential (1.1):

parameters	*Model I*	*Model II*
λ_{EO}	$\frac{a}{8}\left(\frac{g^2}{s_\psi^2 c_\psi^2} + \frac{g'^2}{s_\psi'^2 c_\psi'^2}\right) + 2a'\left(\lambda_{\mathrm{t}}^2 + \lambda_{\mathrm{T}}^2\right)$	$\frac{a}{8}\left(\frac{g^2}{s_\psi^2 c_\psi^2}\right) - \frac{a}{3}g'^2 + 2a'\left(\lambda_{\mathrm{t}}^2 + \lambda_{\mathrm{T}}^2\right)$
$\lambda_{\phi^2\mathrm{EO}}$	$\frac{a}{2}\left(\frac{g^2}{s_\psi^2 c_\psi^2} + \frac{g'^2}{s_\psi'^2 c_\psi'^2}\right) + 8a'\left(\lambda_{\mathrm{t}}^2 + \lambda_{\mathrm{T}}^2\right)$	$\frac{a}{2}\left(\frac{g^2}{s_\psi^2 c_\psi^2}\right) + 4ag'^2 + 8a'\left(\lambda_{\mathrm{t}}^2 + \lambda_{\mathrm{T}}^2\right)$
$\lambda_{H^2\phi\mathrm{EO}}$	$\frac{a}{4}\left(g^2\frac{c_\psi^2 - s_\psi^2}{s_\psi^2 c_\psi^2} + g'^2\frac{c_\psi'^2 - s_\psi'^2}{s_\psi'^2 c_\psi'^2}\right) + 4a'\left(\lambda_{\mathrm{t}}^2 + \lambda_{\mathrm{T}}^2\right)$	$\frac{a}{4}g^2\frac{c_\psi^2 - s_\psi^2}{s_\psi^2 c_\psi^2} + 4a'\left(\lambda_{\mathrm{t}}^2 + \lambda_{\mathrm{T}}^2\right)$
μ_{EO}^2	0	$af^2g'^2$

To summarize, the complete result for these parameters is the sum of the contributions coming from the effective operators, as given above, and the radiative contributions coming from the fermion and gauge-boson sector, which were given in Sect. 3.

5 Numerical results and phenomenological discussion

In this section we discuss the constraints on the possible values of the LH parameters. In our previous works we focused on the analysis of the constraints on the LH parameters by considering the effective potential only for the LH doublet (3.1) [8, 9]. Our computation included the effect of virtual heavy quarks, t, b and T, together with the heavy and electroweak gauge bosons, W', W, B' and B, present in the LH model. By imposing the potential (3.1) to have a minimum whenever $\mu^2 = \lambda v^2$ ($v = 246$ GeV), we found that the obtained values for the μ parameter were too high to be compatible with the expected Higgs mass, which should not be larger than about 200 GeV according to the electroweak precision data.

It is clear that a similar analysis should be done if we consider the complete effective Higgs potential as given in (1.1). In this case, by diagonalizing the Higgs mass matrix, the Higgs mass is given to leading order by $m_{\mathrm{H}}^2 \simeq 2(\lambda - \lambda_{H^2\phi}^2/\lambda_{\phi^2})v^2 = 2\mu^2$ [6]. Therefore, the LH parameters must satisfy the condition

$$v^2 = \frac{\mu^2}{(\lambda - \lambda_{H^2\phi}^2/\lambda_{\phi^2})}. \quad (5.1)$$

In the following we shall discuss the constraints that the condition (5.1) imposes on the LH parameters space. In this way, we should also take into account other constraints imposed by requiring the consistency of the LH models with the electroweak precision data. There exist several studies of the corrections to electroweak precision observables in the little Higgs models, exploring whether there are regions of the parameter space in which the model is consistent with data [4–7, 13–27]. In *Model I* with gauge group $SU(2) \times SU(2) \times U(1) \times U(1)$, one has a multiplet of heavy SU(2) gauge bosons and a heavy U(1) gauge boson. The latter leads to large electroweak corrections and some problems with the direct observational bounds on the Z' boson from Tevatron [13, 14]. Then, a very strong bound on the symmetry-breaking scale f, $f > 4$ TeV at 95% C.L., is found [13]. However, it is known that this bound is lowered to 1–2 TeV for some region of the parameter space [14] by gauging only $SU(2) \times SU(2) \times U(1)$ (*Model II*). Thus, in the following we focus on the LH version called *Model II*.

In order to avoid small values of the mass of the W' and a very strong coupling constant we shall set $M_{W'} > 0.6$ TeV and $g_R \leq 3$ in our numerical discussion. We have found that for very small or very large values of the gauge-group mixing angles, μ^2_{fg} is not positive and SSB does not occur. However, due to the dependence of the heavy gauge-coupling constants on the mixing angles,

$$g_R^2 \equiv \frac{1}{2}\left(g_1^2 s_\psi^2 + g_2^2 c_\psi^2\right), \tag{5.2}$$

it is found that $c_\psi < 0.1$ or $c_\psi \sim 1$ imply a very strong gauge coupling. Accordingly we shall work with $0.1 < c_\psi < 0.9$, and then we shall ensure that μ^2 has the right sign to generate SSB. Besides, taking into account the restrictions on the parameters given in [9], we also set the following ranges: $0.5 < \lambda_T < 2$, $0.8\ \mathrm{TeV} < f < 1\ \mathrm{TeV}$ and $10\ \mathrm{TeV} < \Lambda < 12\ \mathrm{TeV}$. The condition $\lambda_T \gtrsim 0.5$ is established setting the bounds on the couplings $\lambda_1, \lambda_2 \geq m_t/v$ or $\lambda_1\lambda_2 \geq 2(m_t/v)^2$ from the top mass [6]. The condition $m_T \lesssim 2.5$ TeV is required in order to avoid a large amount of fine-tuning in the Higgs potential [1, 7]. Since m_T grows linearly with f, f should be lesser than about 1 TeV [8]. Finally, Λ is restricted by the standard condition, $\Lambda \sim 4\pi f$ [28–30]. On the other hand the a and a' parameters are expected to be O(1) [1, 6, 11]. Values of the symmetry-breaking scale f around 4 TeV are also allowed by the electroweak precision observables [13, 14]. However, this value of f implies that m_T is always greater than 5.7 TeV, when $\lambda_T > 0.5$. A fine-tuning of 0.8% is estimated for a Higgs mass of 200 GeV [13]. Besides, one gets $M_{W'} > 2.6$ TeV. In addition, we have found that for $f = 4$ TeV, the allowed region for the LH parameter space, satisfying the condition imposed by the minimum of the Higgs potential, is smaller [8, 9]. In fact, values around 1–3 TeV are the preferred ones for our selected choices of the LH parameters.

By considering the LH parameters bounded as described above and imposing the condition (5.1), we analyze the constraints on the different LH parameters with the inclusion of the effects of both contributions, the radiative ones (for the *Model II*) and the effective operators. We found that the minimum μ value is $\mu = 0.39$ TeV; for $f = 0.8$ TeV, $\Lambda = 10$ TeV, $\lambda_T = 0.72$, $c_\psi = 0.34$ and $a = a' = 0$, and the maximum is $\mu = 0.761$ TeV; for $f = 0.95$ TeV, $\Lambda = 11.5$ TeV, $\lambda_T = 1.8$, $c_\psi = 0.71$, $a = 1$ and $a' = 0.3$. Clearly, the minimum value corresponds to the case of considering the radiative corrections alone. In this case, small values of $\cos\psi$ are preferred. This is due to the fact that the fermionic contribution to the μ parameter is very large for higher energies, while λ does not change strongly with f and Λ. In this way, to satisfy the condition (5.1), small values of $\cos\psi$ are needed in order to reduce μ. As expected, a large mass of ϕ, $M_\phi = 4.1$ TeV, is found for values of the parameters corresponding to the minimum value of μ.

In Fig. 3 we show how the viable region changes with different values of f, Λ, a and a'. Deviations from the condition (5.1) are allowed up to 20%. In the top row we have set the f and Λ values to 0.8 TeV and 10 TeV, and in the bottom row to 1 TeV and 12 TeV, respectively. Thus, the columns represent different a and a' values. Starting from left to right the values are as follows: $a = 0.3$ and $a' = 0.2$, $a = 0.3$ and $a' = 0.7$, $a = 0.9$ and $a' = 0.2$, and finally, $a = 0.9$ and $a' = 0.7$. One can see that small a' values are preferred in order to satisfy the SSB condition. Since in this case we are taking into account both radiative corrections and effective operators, the fermion-sector contributions become even more important than in the cases of considering some of these contributions alone. Therefore, the strong dependence on a' caused by the top's Yukawa couplings gives large values of the λ's parameters for high a' values, making it difficult to satisfy (5.1). However, the case of a is quite different. From Fig. 3 it is clear that the results do not depend strongly on this parameter since its contribution is suppressed by the gauge couplings g and g' and by the not so small values of c_ψ.

It is interesting to note that, if one would consider only the contributions to the potential coming from the effective operators (except for the μ parameter, which still will be dominated by the radiative corrections), the SSB condition is not easily satisfied. The reason is very simple; the λ parameters at tree level do not depend on f, nor on Λ, while μ does. For example, the dependence of μ on f appears through the heavy-particle masses and then μ is directly proportional to f. In this way, μ rises very quickly with f meanwhile the λ's parameters do not. Therefore high values of the parameters a and a' are needed in order to satisfy the condition (5.1). We find that only for $f = 0.8$ TeV, $\Lambda = 10$ TeV, and $a \geq 0.75$, $a' \geq 0.95$ the equality (5.1) is satisfied. A more detailed analysis of the allowed values of the constants a and a', in agreement with the electroweak precision fits, is given in [13]. The lowest value we found for μ is $\mu = 0.49$ TeV, corresponding to $f = 0.8$ TeV, $\Lambda = 10$ TeV, $c_\psi = 0.2$ $\lambda = 2.59$, $a = 0.75$ and $a' = 0.95$, the mass of the ϕ scalar being 2.86 TeV.

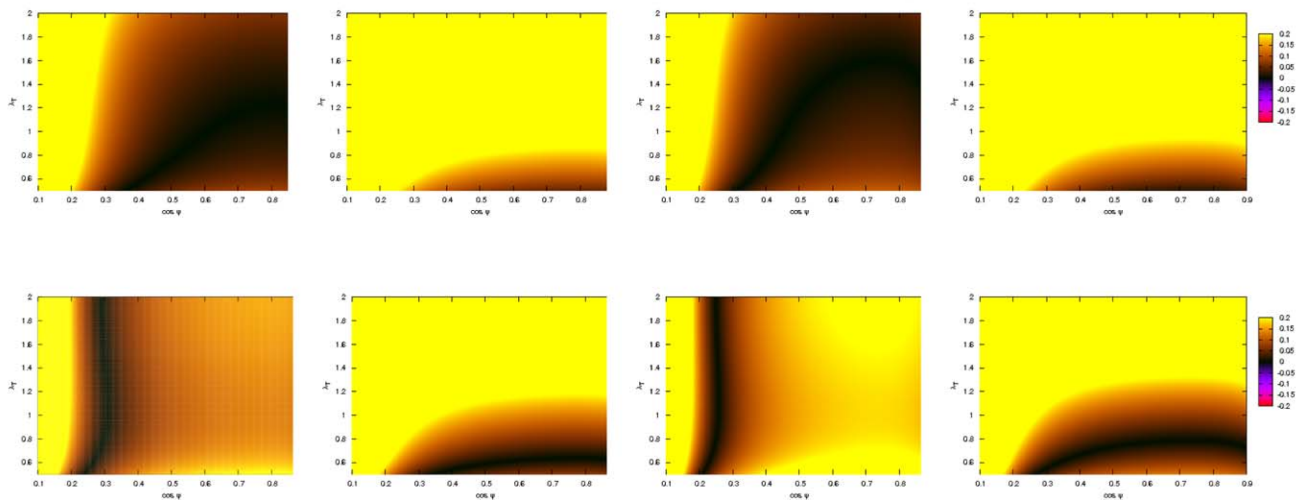

Fig. 3 Contours of the viable regions in the $\lambda_T - c\psi$ plane with the condition (5.1) for different values of a and a'. In the *top* and *bottom rows*, f and Λ are fixed at $f = 0.8$ TeV and $\Lambda = 10$ TeV, and $f = 1$ TeV and $\Lambda = 12$ TeV, respectively

From the discussion above we see that in all cases the μ values are higher than 350 GeV. This is far away from the expected bound of the order of 200 GeV predicted by the SM precision tests. Therefore, it is clear that the inclusion of the interactions terms between GB and the other particles is not enough to reduce the Higgs boson mass so that complete compatibility with the experimental constraints can be obtained. There are some indications suggesting that contributions coming from the scalar sector must reduce the absolute value of μ^2 and thus the Higgs mass. Although the scalar-loop contributions have not been analyzed before (except for the case of the radiatively generated scalar operators that have been discussed in [31]), the expression for the leading correction to the Higgs mass parameter, μ_ϕ^2, is presented in several articles. In particular, this correction is given by [1],

$$\mu_\phi^2 = -\frac{\lambda}{16\pi^2} M_\phi \log\left(1 + \frac{\Lambda^2}{M_\phi^2}\right). \tag{5.3}$$

Let us now estimate the size of the above contribution for the case in which we have obtained the minimum value for μ, $\mu = 0.39$ TeV; corresponding to $f = 0.8$ TeV, $\Lambda = 10$ TeV, $\lambda_T = 0.72$, $c_\psi = 0.34$ and $a = a' = 0$, with $M_\phi = 4.1$ TeV. Thus, by taking $\Lambda = 10$ TeV, $M_\phi = 4.1$ TeV and assuming that $\lambda \simeq \frac{1}{3}$ (at tree level) for having a Higgs mass of order 200 GeV [13] we get $\mu_\phi = -0.14$ TeV. This implies that the μ value is reduced to $\mu = 250$ GeV. Note, however, that the quartic coupling λ is fixed to a particular value in the above analysis. Since small changes in the input parameters of the model produce large changes in the value of λ (and therefore the value of μ could vary), the radiative corrections to this coupling coming from the scalar sector must also be taken into account in a full analysis [32].

6 Conclusions

In this work we have computed the Higgs and ϕ bosons effective potential of the LH model (1.1). We have considered two kinds of contributions to the parameters of this effective potential. Firstly, we have concentrated on the fermion and gauge-boson one-loop radiative corrections. In this case, we observe that the λ parameters are $O(\Lambda^2)$, since they are not protected as the Higgs mass is in the LH model. Secondly, we compute the contributions to the parameters from the effective operators coming from the ultraviolet completion of the LH model. Here, the parameters obtained depend exclusively on the two new unknown coefficients a and a', as well as the mixing angle c_ψ and the T Yukawa coupling. This is an important difference between the results obtained at one loop, which also depend on the cutoff Λ.

The resulting potential has the right form to produce spontaneous symmetry breaking, since $\mu^2 > 0$ for some regions of the LH model parameter space. Thus, if the condition (5.1) is satisfied, the electroweak symmetry is broken. By using the obtained effective potential we have analyzed the constraints imposed on the LH parameters in order to reproduce the SM electroweak symmetry breaking. We observe that if one only considers the effective operator contribution, the SSB condition is not easily satisfied. However, if the radiative corrections or both contributions are taken into account, the allowed ranges of the parameters are much wider. The explanation comes from the way in which the coefficients of the potential, μ and the λ, depend on f and Λ, as discussed above.

Finally, we numerically analyze the LH parameter space that can be set by requiring the LH Higgs effective potential to reproduce exactly the SM potential, and requiring its compatibility with the present phenomenological constraints on the Higgs boson mass. The lowest value found for the μ parameter is 390 GeV, which implies a Higgs boson mass of $m_{\mathrm{H}} \simeq 550$ GeV, which is far from the current bound of about 200 GeV. As a consequence, we conclude that radiative corrections, coming from the Higgs itself and ϕ fields could also provide relevant contributions to the effective potential if the LH model is able to reproduce the SM at low energies. An estimation of the scalar contribution to the μ parameter leads to a value of $\mu = 250$ GeV, and thus $m_{\mathrm{H}} \simeq 350$ GeV. Nevertheless, the full contribution from the triplet, and thus the triplet mass M_ϕ, is required to correct the Higgs mass in improved computations. The value of the Higgs quartic coupling, λ, receives several contributions which have a nontrivial dependence on the various parameters of the model and have not been computed so far. Work is in progress in order to compute these contributions and to check if the value of the Higgs mass will get closer to the current bound [32].

Acknowledgements This work is supported by DGICYT (Spain) under project number BPA2005-02327 and by the Universidad Complutense/CAM project: number 910309. The work of S.P. has been supported by a *Ramón y Cajal* contract from MEC (Spain) and partially by CICYT (grant FPA2006-2315) and DGIID-DGA (grant 2008-E24/2). The work of L.T.-C. is supported by a FPU grant from the Spanish M.E.C. We would like to thank J.R.Espinosa and J.No Redondo for useful discussions.

Appendix

The Goldstone bosons couplings to fermions and gauge bosons, needed for our computations, are listened here.

A.1 Couplings between fermions and Goldstone bosons

a-Three particles

$$-\sqrt{2}\lambda_{\mathrm{T}} H_0 \bar{t}(1+\gamma_5)T,$$
$$-\sqrt{2}\lambda_{\mathrm{t}} H_0 \bar{t}(1+\gamma_5)t,$$
$$-\sqrt{2}\lambda_{\mathrm{T}} H^+ \bar{b}(1+\gamma_5)T,$$
$$-\sqrt{2}\lambda_{\mathrm{t}} H^+ \bar{b}(1+\gamma_5)t.$$

b-Four particles

$$-\frac{\mathrm{i}\sqrt{2}}{f}\lambda_{\mathrm{T}} H_0^* \phi_0 \bar{t}(1+\gamma_5)T,$$
$$-\frac{\mathrm{i}}{f}\lambda_{\mathrm{T}} H_0^* \phi^+ \bar{b}(1+\gamma_5)T,$$
$$-\frac{\mathrm{i}}{f}\lambda_{\mathrm{T}} H^{+*} \phi^+ \bar{t}(1+\gamma_5)T,$$
$$-\frac{\mathrm{i}}{f}\lambda_{\mathrm{T}} H^{+*} \phi^{++} \bar{b}(1+\gamma_5)T,$$
$$-\frac{\lambda_{\mathrm{t}}}{f}\mathrm{tr}(\phi\phi^\dagger)\overline{T}(1+\gamma_5)t,$$
$$-\frac{\mathrm{i}\sqrt{2}}{f}\lambda_{\mathrm{t}} H_0^* \phi_0 \bar{t}(1+\gamma_5)T,$$
$$-\frac{\mathrm{i}}{f}\lambda_{\mathrm{t}} H_0^* \phi^+ \bar{b}(1+\gamma_5)t,$$
$$-\frac{\mathrm{i}}{f}\lambda_{\mathrm{t}} H^{+*} \phi^+ \bar{t}(1+\gamma_5)t,$$
$$-\frac{\mathrm{i}}{f}\lambda_{\mathrm{t}} H^{+*} \phi^{++} \bar{b}(1+\gamma_5)t,$$
$$-\frac{\lambda_{\mathrm{T}}}{f}\mathrm{tr}(\phi\phi^\dagger)\overline{T}T.$$

A.2 Couplings between gauge bosons and Goldstone bosons

a-Four particles

$$\frac{g^2}{2}(\phi_0\phi_0^* + 2\phi^+\phi^- + \phi^{++}\phi^{--})W_{1\mu}W_1^\mu,$$
$$\frac{g^2}{2}(\phi_0\phi_0^* + 2\phi^+\phi^- + \phi^{++}\phi^{--})W_{2\mu}W_2^\mu,$$
$$2g^2(\phi_0\phi_0^* + \phi^{++}\phi^{--})W_{3\mu}W_3^\mu,$$
$$\left(-\frac{1}{2}\frac{g_1^2 g_2^2}{g_1^2+g_2^2}\mathrm{tr}(\phi\phi^\dagger) + \frac{1}{4}\frac{g_1^4+g_2^4}{g_1^2+g_2^2}\phi^+\phi^-\right)W'_{1\mu}W_1'^\mu,$$
$$\left(-\frac{1}{2}\frac{g_1^2 g_2^2}{g_1^2+g_2^2}\mathrm{tr}(\phi\phi^\dagger) + \frac{1}{4}\frac{g_1^4+g_2^4}{g_1^2+g_2^2}\phi^+\phi^-\right)W'_{2\mu}W_2'^\mu,$$
$$\left(-\frac{1}{2}\frac{g_1^2 g_2^2}{g_1^2+g_2^2}\mathrm{tr}(\phi\phi^\dagger) + \frac{1}{4}\frac{g_1^4+g_2^4}{g_1^2+g_2^2}(\phi_0\phi_0^* - \phi^+\phi^- + \phi^{++}\phi^{--})\right)W'_{3\mu}W_3'^\mu,$$
$$g'^2\mathrm{tr}(\phi\phi^\dagger)b_\mu B^\mu,$$
$$\frac{1}{4}\frac{(g_1'^2-g_2'^2)^2}{g_1'^2+g_2'^2}\mathrm{tr}(\phi\phi^\dagger)B'_\mu B'^\mu,$$

a-Five particles

$$\frac{\mathrm{i}(g_1^2-g_2^2)}{8f}(H_0\phi^{--}H_0 + H^+\phi_0^*H^+ + \sqrt{2}H_0\phi^-H^+)W'_{1\mu}W_1'^\mu + \mathrm{h.c.},$$
$$-\frac{\mathrm{i}(g_1^2-g_2^2)}{8f}(H_0\phi^{--}H_0 + H^+\phi_0^*H^+ - \sqrt{2}H_0\phi^-H^+)W'_{2\mu}W_2'^\mu + \mathrm{h.c.},$$
$$\frac{\mathrm{i}(g_1^2-g_2^2)}{8f}(H_0\phi^{--}H_0 + H^+\phi_0^*H^+ - \sqrt{2}H_0\phi^-H^+)W'_{3\mu}W_3'^\mu + \mathrm{h.c.},$$
$$\frac{\mathrm{i}(g_1'^2-g_2'^2)}{8f}(H_0\phi^{--}H_0 + H^+\phi_0^*H^+ + \sqrt{2}H_0\phi^-H^+)B'_\mu B'^\mu + \mathrm{h.c.},$$

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