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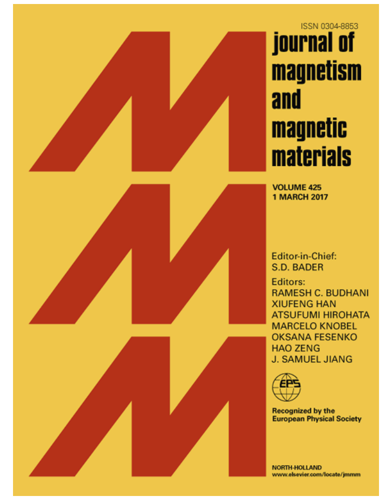

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Direct and inverse spin Hall effects: Zeeman energy

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Abstract

It is shown that magnetic forces as the force $\mp \mu\nabla\mathrm{B}$, exerted on electron spins at rest, account for both the transverse spin imbalance typical of spin Hall effect and the transverse charge imbalance associated with pure spin currents (inverse spin Hall effect). Considering that for stationary currents the laboratory reference frame, and those for which the spin up and spin down carriers are at rest, are inertial systems one can easily find the forces exerted by the lattice on both spin sub-bands, as well as the force between sub-bands.

The main idea underlying this article is based on the fact that the magnetic moments associated with the spin of the electric current carriers are subject to a force generated by the motion of the lattice in the opposite direction. This force is the same considered in the atomic spin-orbit coupling with the only difference that in this case the orbital current becomes a linear current and consequently it turns out to be a *macroscopic spin-orbit* coupling. Provided that the whole system is electrically neutral, the current seen at the laboratory frame of reference, *LF*, due to the carrier motion, is the same that the current seen at the frames, $\mu^{\uparrow}F$ and $\mu^{\downarrow}F$,in which the carriers are at rest. This equality is due to the relative lattice motion with opposite speed and opposite charge sign that the corresponding to the carriers at *LF*. The corresponding force exerted on the magnetic moments can be obtained from the Zeeman energy as $\mp \mu\nabla\mathrm{B}$, μ being the spin magnetic moment, the different sign corresponding to different spin polarization along the direction defined by the local magnetic field. Furthermore, it will be shown below, that the force obtained in this way can be formally expressed as two Hall effects induced by the magnetization of each spin sub-band on the current seen at $\mu^{\uparrow}F$ and $\mu^{\downarrow}F$ respectively. Under this framework it has been shown[1] that the force $\mp \mu\nabla\mathrm{B}$ induces a transverse spin imbalance similar to that known as spin Hall effect[2] and with strength of the order of magnitude of that reported for imbalance and Hal angle experiments with polarized conduction band[1,2,7]. In particular spin accumulation and Hall angle were shown[1] to be of the order of magnitude of that predicted by assuming intrinsic or extrinsic impurities and atomic spin-orbit scattering[3-15]. However, it seems at first sight that the lack of overall electrical current in a pure spin current makes it impossible to explain the inverse spin Hall effect as due to magnetic fields. In this article it is shown that magnetic forces, originated by the relative motion of the background charge respect to the carriers also account for the so-called inverse spin Hall[2], even though the net electric current in *LF* is zero.

Consider a thin metallic film in which along its longitudinal direction, *y*, flow, in the more general case, two *LF* steady density currents $j_{\uparrow y} = n_\uparrow q v_{\uparrow y}$ and $j_\downarrow = n_\downarrow q v_{\downarrow y}$ corresponding to carriers with spin magnetic moment up, $\mu_z^\uparrow$, and, down, $\mu_z^\downarrow$, respectively (see Figure 1). Notice that the third component of the magnetic moment is related to that of the spin carrier, S_z, as $\mu = \frac{q}{m} S_z$, where *q* is the carrier charge, *q*=-*e* for electrons or *q*=*e* for holes. This third component of μ has been defined along the film thickness direction, *z*, because it is determined by the local magnetic field produced by the currents[1]. The carrier concentration is given by $n = n_\uparrow + n_\downarrow$. ***B*** is the inhomogeneous magnetic field seen at $\mu^\uparrow F$ and $\mu^\downarrow F$, that is produced by the relative lattice current $J_{\uparrow y}$ and $J_{\downarrow y}$, which for films with thickness narrower than the width its gradient along the film width is found to be[1] $B_z(\mu^{\uparrow or \downarrow} F) = \alpha \mu_0 J_{\uparrow y\, or\, \downarrow y}\, x$. α is the geometrical factor[1], $\alpha = \frac{8 l_z}{l_x}$, l_z and l_x being the thickness and width of the film, respectively. Therefore, the force acting on the sub-band $\mu_z^\uparrow$ and $\mu_z^\downarrow$ along the x direction is given by $F_x = -\mu \alpha \mu_0 J_y$. It is important to remark that the *LF* velocity of both sub-bands can be the same when a battery is connected to the film ends or opposite to each other when is generated by a spin gradient. The relative current of the background charge respect to $\mu^\uparrow F$ is $J_{\uparrow y} = (n_\uparrow + n_\downarrow) q v_{\uparrow y}$ and respect to $\mu^\downarrow F$ becomes $J_{\downarrow y} = (n_\uparrow + n_\downarrow) q v_{\downarrow y}$. Thus, two cases should be treated: *spin Hall experiments* for which $J_{\uparrow y} = J_{\downarrow y}$ and *inverse spin Hall experiments* for which $J_{\uparrow y} = -J_{\downarrow y}$. All the forces act along the *x* direction since the magnetic moments or the magnetization lie along the *z*- axis and the currents are oriented along the *y*-axis. It is assumed a unique type of carriers which only can differ in the third component of its spin or intrinsic magnetic moment.

It is easy to observe that the forces $\mp \mu \nabla B$ exerted by the inhomogeneous field in the two sub-bands can be expressed in vector form as

$\boldsymbol{F}_{\uparrow \boldsymbol{B}} = -\boldsymbol{J}_\uparrow \times \mu_0 \boldsymbol{M}^\uparrow$ and $\boldsymbol{F}_{\downarrow \boldsymbol{B}} = -\boldsymbol{J}_\downarrow \times \mu_0 \boldsymbol{M}^\downarrow$ [1]

where, $\boldsymbol{M}^\uparrow = n_\uparrow \boldsymbol{\mu}_{\boldsymbol{B}}^\uparrow$, $\boldsymbol{M}^\downarrow = n_\downarrow \boldsymbol{\mu}_{\boldsymbol{B}}^\downarrow$

Therefore, it is concluded that the force exerted by the background current on the opposite spin sub-bands of the carriers can be considered the reaction force to the Hall effect exerted by the magnetization of the two sub-bands on the background current. Expressions [1] were found by assuming that the unbalanced charge acts as a pure electric current, but in case it has some spontaneous magnetization, ***M**** a new Lorentz force, ($\boldsymbol{j}_{\uparrow \boldsymbol{or} \downarrow} \times \alpha \mu_0 \boldsymbol{M}^*$) acting on the carriers has to be added so resulting in

$\boldsymbol{F}_{\uparrow \boldsymbol{B}} = -\boldsymbol{J}_\uparrow \times \mu_0 \boldsymbol{M}^\uparrow + \boldsymbol{j}_\uparrow \times \mu_0 \boldsymbol{M}^\uparrow$ and $\boldsymbol{F}_{\uparrow \boldsymbol{B}} = -\boldsymbol{J}_\downarrow \times \mu_0 \boldsymbol{M}^\downarrow + \boldsymbol{j}_\downarrow \times \mu_0 \boldsymbol{M}^\downarrow$ [2]

The *LF* electric currents, $j_\uparrow$ and $j_\downarrow$ generate magnetic forces, $F_{\uparrow\downarrow}$, between the assembly of carriers with $\mu^\uparrow$ and $\mu^\downarrow$, and forces exerted by the relative current of the positive background on both systems $\mu^\uparrow$ and $\mu^\downarrow$.

The force $n_\uparrow f_\uparrow$, acting on the assembly of $\mu^\uparrow$ carriers can be decomposed into two components: the force $n_\uparrow f_{\uparrow\downarrow}$ due to the assembly of $\mu^\downarrow$ carriers plus the force $F_{\uparrow B} = n_\uparrow f_{\uparrow B}$ exerted by the positive background formed by the lattice sites. The same decomposition holds for the $\mu^\downarrow$ carriers

After considering $n_\uparrow f_{\uparrow\downarrow}$ =-$n_{\uparrow\downarrow} f_{\downarrow\uparrow}$, the net force F^H acting on the total carrier system becomes

$$F^H = n_\uparrow f_\uparrow + n_\downarrow f_\downarrow = n_\uparrow(f_{\uparrow\downarrow} + f_{\uparrow B}) + n_\downarrow(f_{\downarrow\uparrow} + f_{\downarrow B})\ F^H = n_\uparrow f_{\uparrow B} + n_\downarrow f_{\downarrow B} \qquad [2]$$

F^H generates the charge imbalance along the film width or Hall effect.

The difference between the forces acting on both sub-bands F^S is the force inducing spin imbalance or transverse spin separation along the same *x* direction, i.e. spin Hall effect. Since as shown in the supplementary information appendix, $f_{\uparrow\downarrow}$ vanishes, the difference can be written as

$$F^S = (n_\uparrow f_{\uparrow B} - n_\downarrow f_{\downarrow B}) + 2n_\uparrow f_{\uparrow\downarrow} = (n_\uparrow f_{\uparrow B} - n_\downarrow f_{\downarrow B}) \qquad [3]$$

According to [1] expression for F^H and F^S derived from Ref. 1 and valid for a demagnetized background, have been obtained with vector expressions as

$$\boldsymbol{F^H} = -\boldsymbol{J}_\uparrow \times \alpha\mu_0\boldsymbol{M}^\uparrow - \boldsymbol{J}_\downarrow \times \alpha\mu_0\boldsymbol{M}^\downarrow + (\boldsymbol{j}_\uparrow + \boldsymbol{j}_\downarrow) \times \alpha\mu_0\boldsymbol{M}^* \qquad [4]$$

$$\boldsymbol{F^S} = -\boldsymbol{J}_\uparrow \times \alpha\mu_0\boldsymbol{M}^\uparrow + \boldsymbol{J}_\downarrow \times \alpha\mu_0\boldsymbol{M}^\downarrow + (\boldsymbol{j}_\uparrow - \boldsymbol{j}_\downarrow) \times \alpha\mu_0\boldsymbol{M}^* \qquad [5]$$

The generality of these results, that enclose all the phenomenology reported on spin Hall physics, can be illustrated after considering the following different scenarios. We consider below three cases in which as concerns the sub-band density currents verify: $\boldsymbol{j}_\uparrow = \boldsymbol{j}_\downarrow$; $\boldsymbol{j}_\uparrow = -\boldsymbol{j}_\downarrow$ and $\boldsymbol{j}_\downarrow = 0$.

Spin Hall effect.

Let us consider a demagnetized background, ***M*=0***. The film is connected to a battery that promotes an electric current ***j=j***$_\uparrow + \boldsymbol{j}_\downarrow$, as it is shown in Figure 2. In this case: $v_\uparrow = v_\downarrow$ and $\boldsymbol{J}_\uparrow = \boldsymbol{J}_\downarrow$. $\boldsymbol{M} = \boldsymbol{M}^\uparrow + \boldsymbol{M}^\downarrow$; $\boldsymbol{M_s} = \boldsymbol{M}^\uparrow - \boldsymbol{M}^\downarrow$ There are two possible situation a) demagnetized conduction band for which [4] and [5] yield

$$\boldsymbol{F^H} = \boldsymbol{0}$$

$$\boldsymbol{F^S} = -\boldsymbol{J}_\uparrow \times \alpha\mu_0\boldsymbol{M}^\uparrow = -\boldsymbol{j} \times \alpha\mu_0\boldsymbol{M_s} \qquad [6]$$

And b) polarized conduction band,

$$\boldsymbol{F^H} = -\boldsymbol{J}_\uparrow \times \alpha\mu_0\boldsymbol{M} = -\boldsymbol{j} \times \alpha\mu_0\boldsymbol{M} \qquad [7]$$

$$\boldsymbol{F^S} = -\boldsymbol{J}_\uparrow \times \alpha\mu_0\boldsymbol{M_s} = -\boldsymbol{j} \times \alpha\mu_0\boldsymbol{M_s}$$

Note, that for a fully polarized band, $\boldsymbol{M} = \boldsymbol{M_s}$ both forces are identical and the spin Hall effect coincides with the Hall effect. The force is that obtained as the opposite to the Hall effect produced by the conduction band magnetization on the background current in $\mu^\uparrow F$

Inverse spin Hall effect

When a spin gradient is kept constant between two opposite points placed along the y-axis a spin current flows along this axis (See Figure 3). In this situation $v_\uparrow = -v_\downarrow$ and $\boldsymbol{J}_\uparrow = -\boldsymbol{J}_\downarrow$**.** Therefore, the Hall and spin Hall components becomes from [4] and [5]

$$\boldsymbol{F^H} = -\boldsymbol{J}_\uparrow \times \alpha\mu_0\boldsymbol{M_s} \qquad [8]$$

$$\boldsymbol{F^S} = -\boldsymbol{J}_\uparrow \times \alpha\mu_0\boldsymbol{M}$$

It must be emphasized that even though when the current ***j*** is zero in the *L F*, which is the case for a depolarized conduction band ***M=0***, there exists a Hall voltage. The reason is that the net current is zero because we add two opposite real currents, each of them transported by carriers of opposite spin. In this case the background current, as well as its corresponding magnetic fields, are opposite to each other at $\mu^{\uparrow}$ and $\mu^{\downarrow}$. Then, the corresponding generated forces, with the spins being also opposite, act on the same direction in both sub-bands, $\mu^{\uparrow}$ and $\mu^{\downarrow}$, thereby contributing to the charge imbalance given by [8], as illustrated by Figure 4

It can be concluded that the Lorentz forces contribute to both spin Hall and inverse spin Hall.

Spin Hall effect in a conduction band when the background holds spontaneous magnetization M*.

According to [4] and [5] and after considering [7] one finds

$$\boldsymbol{F^H} = -\boldsymbol{J}_{\uparrow} \times \alpha\mu_0\boldsymbol{M} = -\boldsymbol{j} \times \alpha\mu_0(\boldsymbol{M} - \boldsymbol{M}^*) \qquad [9]$$

$$\boldsymbol{F^S} = -\boldsymbol{J}_{\uparrow} \times \alpha\mu_0\boldsymbol{M_s} = -\boldsymbol{j} \times \alpha\mu_0\boldsymbol{M_s} + (\boldsymbol{j}_{\uparrow} - \boldsymbol{j}_{\downarrow}) \times \alpha\mu_0\boldsymbol{M}^*$$

For a depolarized band [9] becomes

$$\boldsymbol{F^H} = \boldsymbol{j} \times \alpha\mu_0\boldsymbol{M}^* \qquad [10]$$

$$\boldsymbol{F^S} = -\boldsymbol{j} \times \alpha\mu_0\boldsymbol{M_s}$$

The Hall effect is the expected for a current $\boldsymbol{j}$ flowing under the action of a transverse magnetic field of strength $\alpha\mu_0\boldsymbol{M}^*$. Equations [10] are coincident with equations [7] save in sign, they describe a similar physics of either a magnetized background acting on a demagnetized conduction band or a magnetized conduction band on a demagnetized background. Action and reaction principle accounts for the sign shift. Forces given by [9] and [10] are expected to contribute to the anomalous Hall effect in ferromagnetic materials.

The calculations were restricted to the spin and charge imbalance induced in the conduction band by the force exerted from the background or lattice current. However, a similar but opposite force is exerted on the spin and charge of the lattice. Therefore, the macroscopic or measured imbalances should be the sum of both. The macroscopic imbalance should be that calculated here when the charge and spin polarizability of the lattice, mainly governed by the nature of its atoms, is vanishingly small. This lattice influence can be linked to the experimental results that indicate such type of dependence on the film material.

In summary, it has been shown that the Zeeman force $\mp \mu\nabla\mathrm{B}$ acting on the spin carriers, at the reference frames at which they are at rest, account for the properties and characteristics of the spin and inverse spin Hall effect. By carefully looking of the relations [4] and [5] it is inferred that the sign of spin Hall and inverse spin Hall depend on different conditions. The total spin and charge imbalance depend on geometry, through the α parameter, and on the particular film metal or semiconductor, through its conductivity, concentration of carrier, and lattice polarizability. In fact, the results shown in this article indicate that both spin and charge imbalance originated by electric currents can be naturally understood in the framework of classical electrodynamics plus the concept of the spin magnetic moment. It is the force exerted on the spin moments of the carriers, in the system in which they are at rest, that contributes to both transverse imbalances. It can be concluded that, despite the validity of other approaches, the forces found in this article must be taken into account when trying to understand the effects produced by electric and spin currents.

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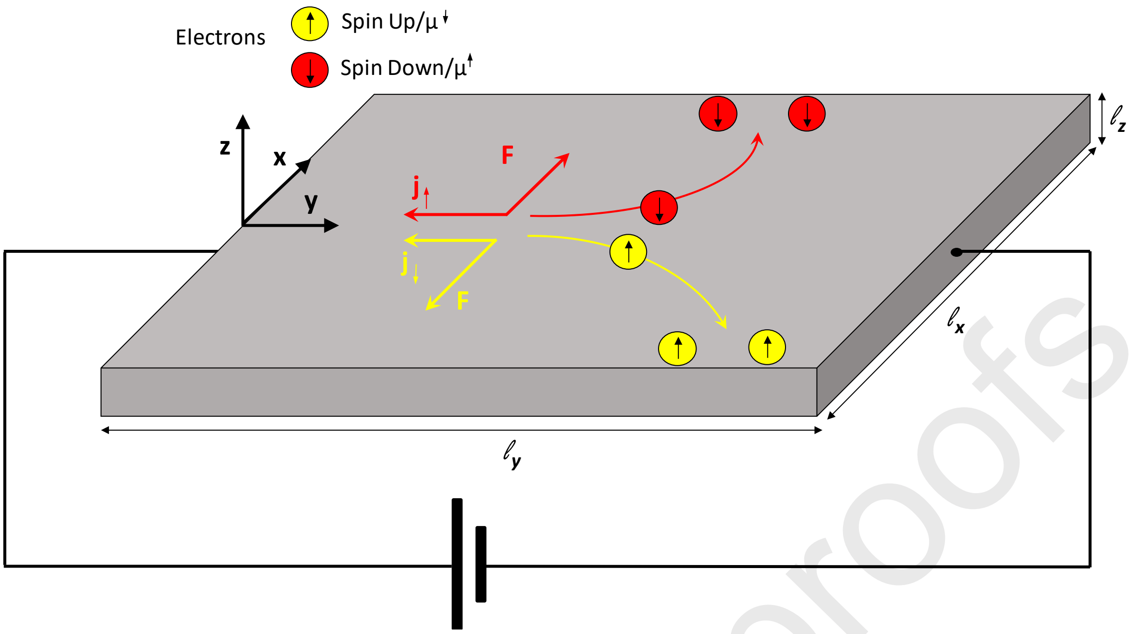

Figure 1. Spin Hall effect in a thin metallic film which contains a current (generated by a battery) running along its *y* direction as seen from the laboratory reference frame *LF*.

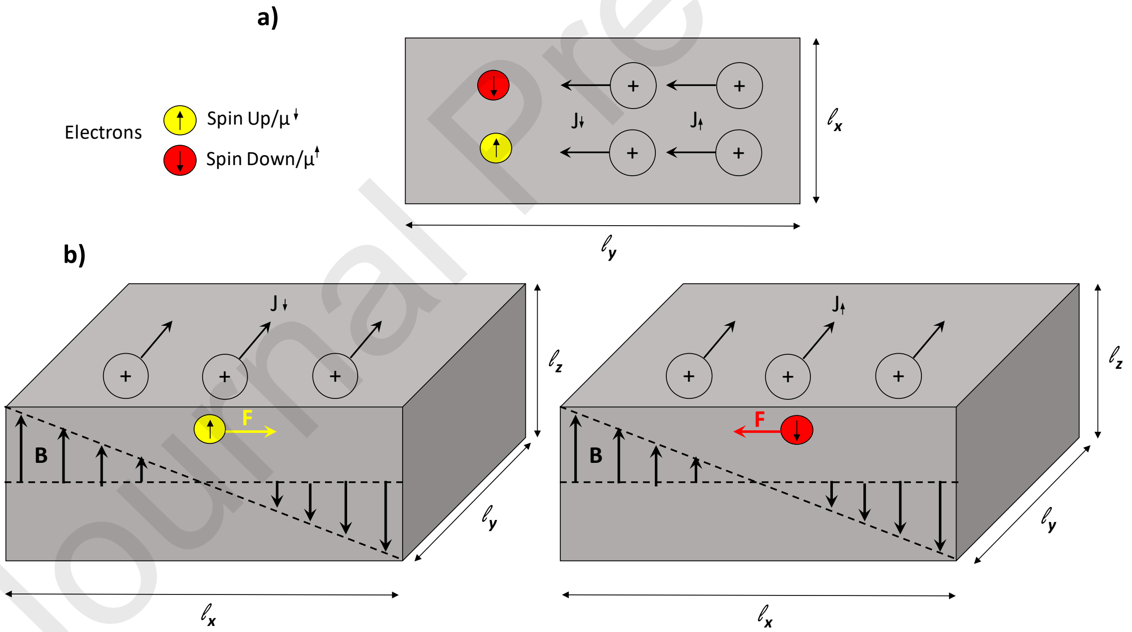

Figure 2. a) Currents due to the relative lattice motion as seen from the reference frame $F\mu^{\uparrow\, or \downarrow}$ for the scenario presented in Figure 1. b) Cross section of the thin film showing the magnetic field distribution created by the relative lattice current and the resulting forces acting on the carriers.

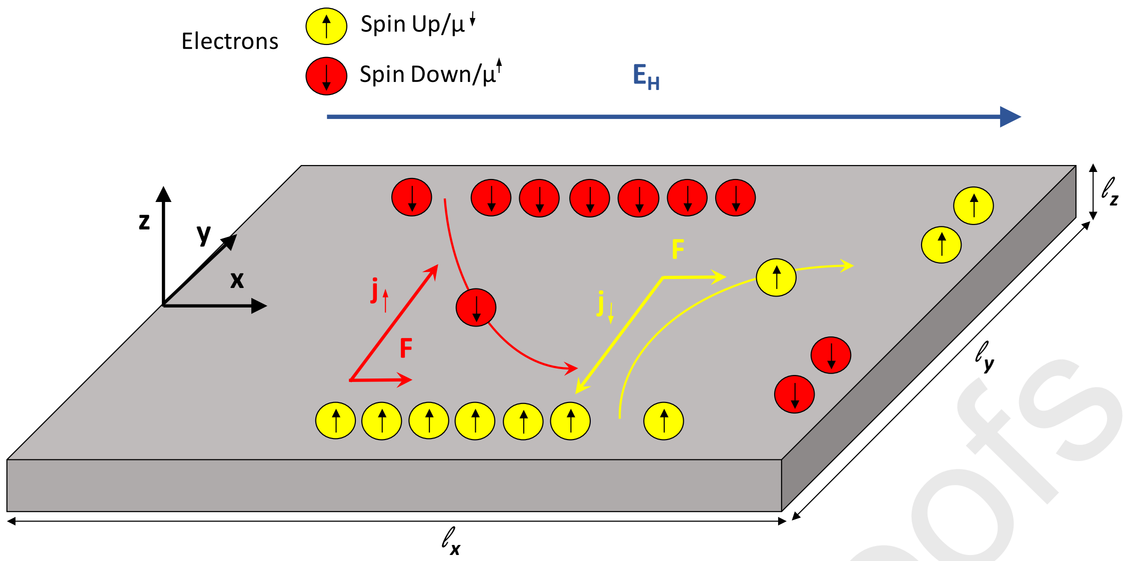

Figure 3. Inverse Spin hall effect in a thin metallic film with a spin gradient between opposite *y* edges which generates spin currents $j_↓$ and $j_↑$ as seen from the laboratory reference frame *LF*.

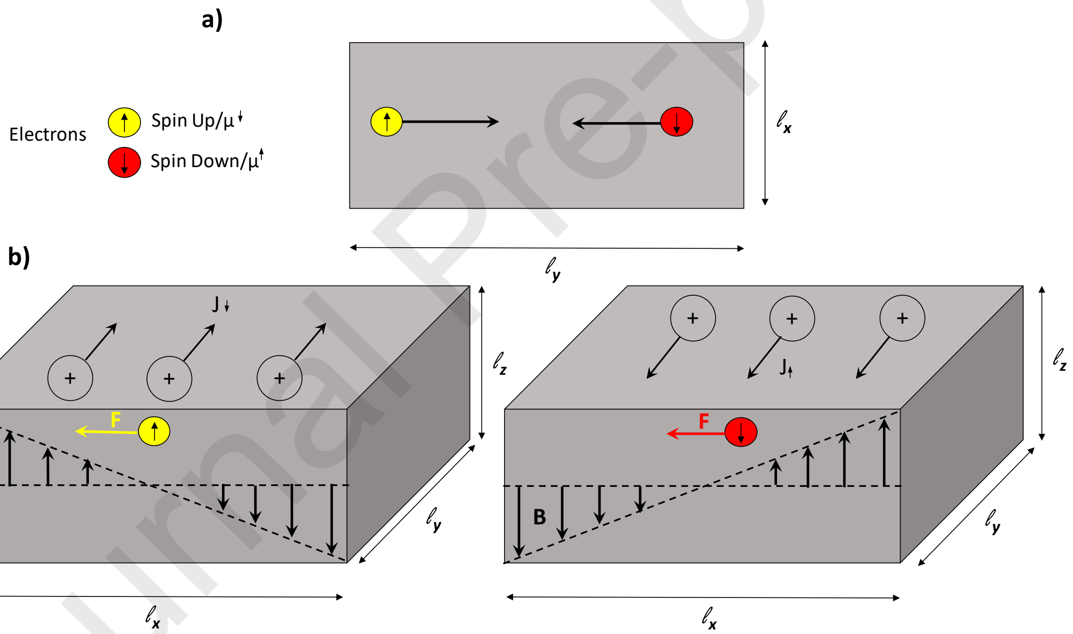

Figure 4. a) Currents due to the relative lattice motion as seen from the reference frame $F\mu^{\uparrow\, or\, \downarrow}$ for the scenario presented in Figure 3. b) Cross section of the thin film showing the magnetic field distribution created by the relative lattice current and the resulting forces acting on the carriers.

Supplementary information

Force between the two opposite spin carriers sub-bands.

The expression of the force exerted by the lattice electric current on each μ sub-bands can be directly obtained from Ref. 1 as follows. For the $\mu^{\uparrow}$subband, with $n^{\uparrow}$ carriers, the force is given by

$F_{\uparrow B=} - \mu^{\uparrow}\nabla_x B = -\alpha\mu_0\mu_B n_\uparrow n e v_{y\uparrow} = -\alpha\mu_0 M^{\uparrow} J_{y\uparrow} = -\alpha\mu_0 M_s j_{y\uparrow}$ [1´]

Consider now that the spin up and spin down sub-bands move with relative speed $v_{\uparrow\downarrow} = -v_{\downarrow\uparrow}$. The force exerted by the spin down on the spin up system can be written, according to [4] and [5], as $F_{\uparrow\downarrow} + F_{\uparrow\downarrow}{}^*$. The force $F_{\uparrow\downarrow}$due to the magnetic field produced by the relative current of the $\mu^{\downarrow}$ carriers respect to the $\mu^{\uparrow}$ is according to [1´]

$F_{\uparrow\downarrow} = -\alpha\mu_0\mu_B n_\uparrow n_\downarrow e v_{rel}$ [2´]

And, according to [5´] the Lorentz force exerted on $\mu^{\uparrow}$, $F_{\uparrow\downarrow}{}^*$, due to the magnetization of $\mu^{\downarrow}$ carriers exactly counterbalances $F_{\uparrow\downarrow}$

$F_{\uparrow\downarrow}{}^* = \alpha\mu_0\mu_B n_\uparrow n_\downarrow e v_{rel}$ [3´]

-Direct and inverse spin Hall effects can be explained with classical electrodynamics
-From their reference framework, charge carriers observe a background lattice current
-Background current coupling and force between spin sub-bands account for SHE and ISHE