



Instituto
Complutense
de Análisis
Económico

Simple Market Timing with Moving Averages

Jukka Ilomäki

Faculty of Management University of Tampere Finland

Hannu Laurila

Faculty of Management University of Tampere Finland

Michael McAleer

Department of Finance Asia University, Taiwan and
Discipline of Business Analytics
University of Sydney Business School, Australia and
Econometric Institute, Erasmus School of Economics
Erasmus University Rotterdam, The Netherlands and
Department of Economic Analysis and ICAE
Complutense University of Madrid, Spain and
Institute of Advanced Sciences Yokohama National University, Japan

Abstract

Consider using the simple moving average (MA) rule of Gartley (1935) to determine when to buy stocks, and when to sell them and switch to the risk-free rate. In comparison, how might the performance be affected if the frequency is changed to the use of MA calculations? The empirical results show that, on average, the lower is the frequency, the higher are average daily returns, even though the volatility is virtually unchanged when the frequency is lower. The volatility from the highest to the lowest frequency is about 30% lower as compared with the buy-and-hold strategy volatility, but the average returns approach the buy-and-hold returns when frequency is lower. The 30% reduction in volatility appears if we invest randomly half the time in stock markets and half in the risk-free rate.

Keywords Market timing, Moving averages, Risk-free rate, Returns and volatility

JEL Classification G32, C58, C22, C41, D23

Working Paper n° 1814
May, 2018



UNIVERSIDAD
COMPLUTENSE

MADRID

ISSN: 2341-2356

WEB DE LA COLECCIÓN: <http://www.ucm.es/fundamentos-analisis-economico2/documentos-de-trabajo-del-icae> Working papers are in draft form and are distributed for discussion. It may not be reproduced without permission of the author/s.

Simple Market Timing with Moving Averages*

Jukka Ilomäki

Faculty of Management
University of Tampere
Finland

Hannu Laurila

Faculty of Management
University of Tampere
Finland

Michael McAleer

Department of Finance
Asia University, Taiwan
and
Discipline of Business Analytics
University of Sydney Business School, Australia
and
Econometric Institute, Erasmus School of Economics
Erasmus University Rotterdam, The Netherlands
and
Department of Economic Analysis and ICAE
Complutense University of Madrid, Spain
and
Institute of Advanced Sciences
Yokohama National University, Japan

May 2018

* For financial support, the third author acknowledges the Australian Research Council and the Ministry of Science and Technology (MOST), Taiwan.

Corresponding author: hannu.laurila@uta.fi

Abstract

Consider using the simple moving average (MA) rule of Gartley (1935) to determine when to buy stocks, and when to sell them and switch to the risk-free rate. In comparison, how might the performance be affected if the frequency is changed to the use of MA calculations? The empirical results show that, on average, the lower is the frequency, the higher are average daily returns, even though the volatility is virtually unchanged when the frequency is lower. The volatility from the highest to the lowest frequency is about 30% lower as compared with the buy-and-hold strategy volatility, but the average returns approach the buy-and-hold returns when frequency is lower. The 30% reduction in volatility appears if we invest randomly half the time in stock markets and half in the risk-free rate.

Keywords: Market timing, Moving averages, Risk-free rate, Returns and volatility.

JEL: G32, C58, C22, C41, D23.

1. Introduction

According to the standard investing separation theorem of Tobin (1958), investors allocate investments between risk-free and risky assets. If the risk-free rate is low (high), the investors shift their wealth to (from) the risky assets. Fama (1972) divides forecasters into two categories, namely macro forecasters (or market timers) and micro forecasters (or security analysts), who try to forecast individual stock returns relative to the market returns.

Merton (1981) defines a market timer to forecast when stocks will outperform (underperform) the risk-free asset, indicating that, when $r_t^m > r_t^f$ ($r_t^m < r_t^f$), where r_t^m is average stock market returns, r_t^f is the risk-free asset, $r_t^i = r_t^f + \beta^i(r_t^m - r_t^f) + \varepsilon_t^i$, r_t^i is the return for individual stock i included in the market portfolio m , β^i is a positive parameter, and $E[\varepsilon_t^i | r_t^m] = E[\varepsilon_t^i]$. That is, a market timer only forecasts the statistical properties of $r_t^m - r_t^f$, indicating that their forecasts contain only the differential performance among individual stocks arising from systematic risk in the markets.

Merton (1981) shows theoretically that when investors have heterogeneous beliefs and imperfect information, the value of a random market timing forecast is zero, and if the forecast variable is distributed independently or the forecast is based on public information, its value is zero, too. In fact, Merton shows that the maximum value of skilled market timing is the value of the protective put against buy-and-hold strategy.

Henriksson and Merton (1981) present an empirical procedure whereby correct forecasts can be analyzed statistically. However, if it is assumed that ε_t^i follows an approximate normal distribution, this leads to the CAPM of Sharpe (1964) and Lintner (1965).

We use a simple MA rule for the timing aspect for individual Dow Jones Industrial Average (DJIA) stocks with different frequencies. Zhu and Zhou (2009) show analytically that MA trading rules, as a part of asset allocation rules, can outperform standard allocation rules when stock returns are partly forecastable. The standard rule

means investing a fixed proportion of wealth in risky assets and the rest in risk-free assets, with the ratio determined by the risk tolerance of an investor.

This is the well-known reward/risk (or mean-variance) principle in the spirit of Markowitz (1952), Tobin (1958) and Sharpe (1964). Zhu and Zhou (2009) argue that the fixed allocation rule is not optimal if returns are forecastable by using the MA rule. Therefore, assuming that risk tolerance and the forecast performance of stock market returns are constant, the linear combination rule means that, when the MA rule suggests an uptrend (downtrend), the rule suggests that the total weight should be allocated to stock markets (the risk-free rate).

The empirical findings suggest a low volatility anomaly that might be explained by investors' affection to high volatility, as suggested by Baker et al. (2011) and noted in Ang et al. (2009). On the other hand, the reported predictability of risk premia (see, for example, Cochrane 2008, and Fama 2014) can explain why, for instance, MA rules forecast better than using random highs and lows in the stock market (as noted in Jagannathan and Korajczyk 2017). The topic is important as Friesen and Sapp (2007), among others, report that mutual fund investors had negative outcomes, on average, in their timing to invest and withdraw cash from US mutual funds from 1991 to 2004. Munoz and Vicente (2018) report similar results with more recent data in US markets.

The plan of the remainder of the paper is as follows. Section 2 provides a literature review, and alternative model specifications are presented in Section 3. The empirical analysis is conducted in Section 4, while Section 5 gives some concluding comments.

2. Literature Review

In efficient markets, investors earn above average returns only by taking above average risks (Malkiel 2003). Samuelson (1998) conforms with Fama (1972) by noting that market efficiency can be divided into micro and macro efficiency. The former concerns the relative pricing of individual stocks, and the latter, for markets as a whole. The CAPM by Sharpe (1964), and Lintner (1965) argues that beta is a proper definition for systematic risk for stock i , if unexplained changes in risk adjusted returns for the stock follow approximately normal distribution with zero mean.

Black (1972) states that the slope of the security market line (SML) is flatter if there exists restrictions in borrowing, that is, leverage constraints in the model. Starting from Black et al. (1972), many studies have reported that the security market line is too flat in US stocks compared with the SML suggested by the CAPM version of Sharpe and Lintner.

Ang et al. (2009), Baker et al. (2014), and Frazzini and Pedersen (2014) find that low-beta stocks outperform high-beta stocks statistically significantly. In fact, Frazzini and Pedersen report that significant excess profits in US stocks can be achieved by shorting high-beta stocks and buying low-beta stocks with leverage, but that leverage constraints make them disappear. Using Black (1972), investors often have leverage constraints, thereby making them place too much weight on risky stocks, which results in lower required return for high-beta stocks than would be justified by the Sharpe-Lintner CAPM.

Markowitz (1952) defines portfolio risk simply as the volatility of portfolio returns. Clarke et al. (2010) find that the volatility of stock returns contains potentially an additional risk factor with respect to systematic risk that can be defined in the betas of CAPM by Sharpe and Lintner. Moreover, Ang et al. (2009) report that the total volatility of international stock market returns is highly correlated with US stock returns, thereby suggesting a common risk factor for US stocks.

Baker et al. (2011) suggest that the low-volatility anomaly is due to investor irrational behaviour, mainly because an average fund manager seeks to beat the buy-and hold strategy by overinvesting in high-beta stocks. The explanations include preference for lotteries (Barberis and Huang 2008; Kumar 2009; Bali et al. 2011), overconfidence (Ben-David et al. 2013), and representativeness (Daniel and Titman 2006)), which means that people assess the probability of a state of the world based on how typical of that state the evidence seems to be (Kahneman and Tversky 1974).

Baker and Wurgler (2015) argue that the anomaly is also related to the limits of arbitrage. In fact, the extra costs of shorting prevents to take advantage of overpricing (Hong and Sraer 2016). More importantly, Li et al. (2016) report that the excess returns of low-beta portfolios are due to mispricing in US stocks, indicating that the low-volatility anomaly does not exist because of systematic risk by some rational, stock

specific volatility risk factor. They tested the low-volatility anomaly with monthly data from January 1963 to December 2011 in NYSE, NASDAQ, and AMEX stocks.

Market timing is closely related to technical trading rules. Brown and Jennings (1987) show theoretically that using past prices (like the MA rule in Gartley (1935)) has value for investors, if equilibrium prices are not fully revealing, and signals from past prices have some forecasting qualities. More importantly, Zhu and Zhou (2009) indicate that the MA rules are particularly useful for asset allocation purposes among risk averse investors, when markets are forecastable (quality of signal).

Moskowitz et al. (2012) argue that there are significant time series momentum (TSM) effects in financial markets that are not related to the cross-sectional momentum effect (Jegadeesh and Titman 1993). However, TSM is closely related to MA rules, since it gives a buy (sell) signal according to some historical price reference points, whereas MA rules give a buy (sell) signal, when the current price moves above (below) the historical average of the chosen calculated rolling window measure.

Starting from LeRoy (1973) and Lucas (1978), the literature in financial economics states that financial markets returns in efficient markets are partly forecastable, when investors are risk averse. This leads to the time-varying risk premia of investors, as noted by Fama (2014). For example, Campbell and Cochrane (1999) present a consumption-based model, which indicates that when the markets are in recession (boom), risk averse investors require larger (smaller) risk premium for risky assets. More importantly, Cochrane (2008) notes that the forecastability of excess returns may lead to successful market timing rules.

Brock et al. (1992) test different MA lag rules for US stock markets, and find that they gain profits compared with holding cash. On the other hand, Sullivan et al. (1999) find that MA rules do not outperform the buy-and-hold strategy, if transaction costs are accounted for. Allen and Karjalainen (1999) use a genetic algorithm to develop the best ex-ante technical trading rule model using US data, and find some evidence of outperforming the buy-and-hold strategy. Lo et al. (2000) find that risk averse investors benefit from technical trading rules because they reduce volatility of the portfolio without giving up much returns when compared against the buy-and-hold strategy.

More recently, Neely et al. (2014) use monthly data from January 1951 to December 2011, and report that MA rules forecast the risk premia in US stock markets statistically significantly. Marshall et al. (2017) find that MA rules give an earlier signal than TSM, suggesting better returns for MA rules, but they both work best with large market value stocks.

Moskowitz et al. (2012) use monthly data from January 1965 to December 2009, and report that TSM provides significant positive excess returns in futures markets. However, Kim et al. (2016) report that these positive excess returns produced by TSM are due to the volatility scaling factor used by Moskowitz et. al. (2012).

3. Model Specifications

Consider an overlapping generation economy with a continuum of young and old investors $[0,1]$. A young risk-averse investor j invests their initial wealth, w_t^j , in infinitely lived risky assets $i = 1, 2, \dots, I$, and in risk-free assets that produce the risk-free rate of return, r^f . A risky asset i pays dividend D_t^i , and has x_t^i outstanding. Assuming exogenous processes throughout, the aggregate dividend is D_t .

A young investor j maximizes their utility from old time consumption through optimal allocation of initial resources, w_t^j , between risky and risk-free assets:

$$\max x_t^j \left(\frac{E_t(P_{t+1} + D_{t+1})}{P_t} - (1 + r^f) \right) - \frac{\nu^j}{2} x_t^{j^2} \sigma^2$$

s.t.

$$x_t^j P_t \leq w_t^j$$

where E_t is the expectations operator, P_t is the price of one share of aggregate stock, ν^j is a constant risk-aversion parameter for investor j , σ^2 is the variance of returns for the aggregate stock, and x_t^j is the demand of risky assets for an investor j . The first-order condition is:

$$\frac{E_t(P_{t+1} + D_{t+1})}{P_t} - (1 + r^f) - \nu^j x_t^j \sigma^2 = 0,$$

which results in optimal demand for the risky assets:

$$x_t^j = \frac{E_t((P_{t+1} + D_{t+1})/P_t) - (1 + r^f)}{\nu^j \sigma_i^2}. \quad (1)$$

Suppose that an investor j is a macro forecaster who allocates their initial wealth, w_t^j , between risky stocks and risk-free assets according to their forecast about the return of the risky alternative. Then, equation (1) says that the investor invests in the risky stocks only if the numerator on the right hand side is positive.

4. Empirical Analysis

This section presents the empirical results from seven frequencies for the (MA) trend-chasing rules. The data consist of 29 companies included in the Dow Jones Industrial Average (DJIA) index in January 2018. The trading data (daily closing prices) cover 30 years from 1 January 1988 to 31 December 2017. Choosing the current DJIA companies for the last 30 years creates a “survivor bias” in the buy-and-hold results. However, this should not be an issue as we intend to compare the performance of the alternative MA frequency rules.

The rolling window is 200 trading days. The first rule is to calculate MA in every trading day; the second frequency takes into account every 5th trading day (thereby providing a proxy for the weekly rule); the third frequency takes into account every 20th trading day (proxy for the monthly rule); the fourth rule is to calculate MA for every 40th trading day (proxy for every other month); the fifth rule takes into account every 60th trading day (proxy for every third month); the sixth rule takes into account every 80th trading day (proxy for every fourth month); and the seventh rule takes into account every 100th trading day (proxy for every fifth month).

For the 29 DJIA companies, 26 of them have daily stock data available from 27 March 1987, thereby giving 4 January 1988 as the first trading day. The data for Cisco are available from 12 February 1990, for Goldman Sachs from 4 May 1999, and for Visa from 19 March 2008. There are 217 569 observations of daily returns from DJIA stocks. Thus, there are $217569 \times 9 = 1\,958\,121$ daily returns for the first three frequencies (rules), $217\,569 \times 4 = 870\,276$ daily returns for the fourth rule, $217\,569 \times 3 = 652\,707$ daily returns for the fifth rule, $217569 \times 2 = 435\,138$ daily returns for the sixth rule, and 217 569 daily returns for the seventh rule.

The trading rule for all cases is to use a simple crossover rule. When the trend-chasing MA turns lower (higher) than the current daily closing price, we invest the stock (three-month US Treasury Bills) at the closing price of the next trading day. Thus, the trading rule provides a market timing strategy where we invest all wealth either in stocks (separately, every stock included in DJIA), or to the risk-free asset (three-month U.S. Treasury bill), where the moving average rule advises the timing.

At the first frequency (every trading day), we calculate daily returns for MA200, MA180, MA160, MA140, MA120, MA100, MA80, MA60, and MA40. For example, MA200 is calculated as:

$$\left(\frac{P_{t-1} + P_{t-2} + \dots + P_{t-200}}{200} \right) = X_{t-1}.$$

At the lowest frequency, where every 100th daily observation is counted, MAC2 is calculated as:

$$\left(\frac{P_{t-1} + P_{t-100}}{2} \right) = X_{t-1}.$$

If $X_{t-1} < P_{t-1}$, we buy the stock at the closing price, P_t , thereby giving daily returns as

$$R_{t+1} = \ln\left(\frac{P_{t+1}}{P_t}\right).$$

Tables 1-7 in Appendix 1 show that the annualized average log returns of MA200 - MA40 are **+0.053** after transaction costs (with 0.1% per change of position). Recall that there are 200 closing day prices in the rolling window MA200, whereas MA40 means that there are 40 closing day prices in the window. The respective log returns for MAW40-MAW8 (weekly) are **+0.063**; for MA10 - MA2 (monthly) **+0.071**; for MAD5 - MAD2 (every other month) **+0.078**; for MAT4 - MAT2 (every third month) **+0.084**, for MAQ3 - MAQ2 (every fourth month) **+0.094**; and for MAC2 (every fifth month) **+0.088** after transaction costs.

Tables 1-7 show that, as the frequency decreases until every fourth month frequency (MAQ3 - MAQ2), average returns tend to increase, and decrease thereafter. In comparison, the biased buy-and-hold strategy produces **+0.117** with equal weights among all DJIA stocks, and with **0.295** annual volatility. A random investment (half the time in the risk-free rate, and half in the equally weighted portfolio from 4 January 1988) produces $(0.117*0.5+0.022*0.5) = \mathbf{+0.070}$ annually, on average, with $(1-\sqrt{0.5} = 0.293) = 29.3\%$ reduction in volatility, indicating **0.209** annual volatility for that portfolio.

The data are dividend excluded, but the average annual dividend yield in DJIA stocks over the last thirty years has been +0.026, so that the biased buy and hold strategy produces +0.143 annually with equal weights among DJIA stocks before taxes. Thus, the random investment strategy produces +0.083 annually, with survivor bias.

Appendix 1 (that is, the second column of Tables 1-7) also reports the annualized average log returns calculated in the largest sample (full 200 observations) in every category: MA200 **+0.065**; MAW40 **+0.073**; MA10 **+0.079**; MAD5 **+0.083**; MAT4 **+0.089**; MAQ3 **+0.091**; and MAC2 **+0.088** after transaction costs and before dividends. Adding +0.013 produces after dividends and before taxes: MA200 **+0.078**; MAW40 **+0.086**; MA10 **+0.092**; MAD5 **+0.096**; MAT4 **+0.102**; MAQ3 **+0.104**; and MAC2

+0.101. These results imply that starting from every fifth trading day frequency, a macro forecaster beats the buy and hold strategy **in returns**.

Figure 1 below illustrates the effects of frequency on the returns to volatility ratio (the second column in Tables 1-7).

< Figure 1 goes here >

In Figure 1, the straight line illustrates the return to volatility ratio of portfolios, where wealth is randomly invested in combinations of the three-month Treasury Bill (risk-free rate), with stocks included in the DJIA between 4 January 1988 and 31 December 2017. The red crosses represent the average return/volatility points calculated in the 200-day rolling window with the following frequencies: daily, every five days, every 20 days, every 40 days, every 60 days, every 80 days, and every 100 days (with only the most observations in each frequency giving 200, 40, 10, 5, 4, 3, and 2 observations). The red crosses plot a convex curve that deviates increasingly from the straight return to volatility ratio line, thereby symbolizing superior portfolio efficiency.

Tables 8-14 in Appendix 2 show that the annualized volatility of daily returns read, on average: MA200-MA40 **0.2044**; MAW40-MAW8 **0.205**; MA10-MA2 **0.2091**; MAD5-MAD2 **0.213**; MAT4-MAT2 **0.219**; MAQ3-MAQ2 **0.221**; and MAC2 **0.218**. Thus, there is virtually no difference between the MA frequencies, while the biased buy-and-hold strategy produces **0.295**.

Figure 1 presents the volatilities calculated in the largest sample (full 200 day rolling window in every category, the second column in Tables 8-14). They read MA200 **0.207**; MAW40 **0.208**; MA10 **0.211**; MAD5 **0.213**; MAT4 **0.218**; MAQ3 **0.215**; and MAC2 **0.218** after transaction costs. Investing randomly half of the time in the risk-free rate and the other half in the equally weighted portfolio, produces **0.209**. Thus, the difference between the annual volatilities produced in profitable market timing MA rules (MA10 – MAC2) and random market timing (half and half) ranges from **0.009** to **0.002**.

< Figure 2 goes here >

In Figure 2, the straight line again presents the return to volatility ratio of portfolios with random investment in the risk-free rate and the stocks in DJIA between 4 January 1988 and 31 December 2017. The red crosses plot the average return to volatility ratios, calculated by using a 200 day rolling window, with the following frequencies: daily, every five days, every 20 days, every 40 days, every 60 days, every 80 days, and every 100 days. The averages of every lag are reported in Tables 1-14, Appendices 1 and 2. Thus, all daily returns from Tables 1-14 are included.

Comparing Figures 1 and 2, it is clear that using the whole 200 daily observation windows in the MA rules produces more efficient results in market timing. That is, comparing the products of shorter and longer MA rule rolling windows, say, the last two monthly observations compared with ten monthly observations, average realized returns drop from **+0.079** to **+0.059** before dividends, while volatility remains approximately unchanged (from 0.211 to 0.207). This suggests that, in both cases, about half and half is invested in the equally-weighted DJIA portfolios and in the risk-free rate, and the MA rules advice the timing. More importantly, Tables 8-14 in Appendix 2 show that the range in volatilities with all MA rules varies between 0.202 – 0.227 (with 0.02 difference), whereas Tables 1-7 in Appendix 1 show that realized returns vary between 0.096 – 0.033 before dividends (with 0.063 difference).

These results indicate that a macro market timing with 200 days rolling window produces a reduction in volatility from **0.295** (the buy-and hold) to between 0.207-0.218, but the average annualized returns (dividends included) tend to rise as the MA frequency falls (+0.078 with all 200 observations to +0.104 with every fourth month observations). Thus, the results indicate that MA market timing finds long term stochastic trends more efficiently than short term stochastic trends.

The Sharpe ratio of random market timing (half and half) with dividends is **0.292**; for MA200 **0.271**; for MAW40 **0.308**; for MA10 **0.332**; for the MAD5 **0.347**; for MAT4 **0.370**; for MAQ3 **0.381**; and for MAC2 it is **0.362**.

< Figure 3 goes here >

Figure 3 shows that when the volatility changes 1% in the DJIA stocks, then the average returns change is 0.39%. Figures 1 and 2 suggest that the theoretical change should be such that when the volatility changes 1%, then the average returns change is 0.50%, suggesting a flatter SML line in the data. This suggests strongly that DJIA investors have overweight high-beta stocks in the last 30 years.

It is obvious that transaction costs are crucial in MA performance. In the above calculations, the transaction costs are 0.1% per transaction from current wealth. Tables 15 and 16 in Appendix 3 report the transaction costs for the MA200-MA40 and MA10-MA2 rules. In the MA200-MA40 rules, the average annualized transaction costs are **0.0133**, such that the rules have about 13 changes in positions per year. Meanwhile, for the MA10-MA2 rules, the average annualized transaction costs are **0.0032**, suggesting about 3 changes in positions per year.

Allen and Karjalainen (1999) give reasons for a cost of 0.2% per transaction in their sample, but since technological progress has reduced transaction costs since the mid-nineties, 0.1% per transaction should be fair, on average. Nevertheless, a trial with 0.2% transaction costs shows that, for example, the average annualized daily returns become 0.0403 for the MA200-MA40 rules, and 0.0674 for the MA10-MA2 rules. Note that the returns grow 67%, on average, for the MA10-MA2 rules (with about the same volatility) compared with costs of 0.1% per transaction.

Note that the model prohibits short selling since we have only long positions in stocks or investing in the risk-free rate. Then the limits of arbitrage argument of Baker et al. (2015) are consistent with our results.

5. Concluding Remarks

The analysis suggests that a macro forecaster can obtain higher returns with equal volatility (30 % below that of the buy-and-hold strategy) by reducing the frequency used in MA rules. The return to volatility ratio for risk-averse investors with MA market timing significantly outperforms the random benchmark strategy, when the frequency in

the MA rules is reduced. This indicates that the forecasts are more accurate the longer is the time frame.

The results suggest that a flatter SML in the CAPM can be followed by the irrational preference of investors in high-beta stocks, as suggested by Baker et al. (2011) and Li et al. (2016), since the empirically efficient frontier of portfolios becomes flatter than the theoretically efficient SML (random timing) (see Figure 1). In other words, the empirical results suggests that market timing with the few past observations (for example, every fourth month) in the past 200 rolling window daily prices, have produced significantly better returns to risk ratio for the portfolio of DJIA equally weighted stocks in the past 30 years than random timing. The finding points to the low-volatility anomaly.

One explanation for the results is that they are due to time-varying risk premiums. This is emphasized by Neely et al. (2014), who claim that MA rules, in effect, forecast changes in the risk premium. If the results are rational products of time-varying risk premiums, the results suggest that investor sensitivity to risk must be extremely high, and their risk premium is larger (smaller) in downs (ups), as suggested by Campbell and Cochrane (1999). As volatility rises (decreases), usually in downs (ups), the results suggest that when volatility is high, investors as a group tolerate significantly more risk (that is, volatility) than in calmer periods.

Consider the following numerical example: Assume that the risk premium is 0.08 in volatile downs, and 0.04 in calm ups, and the variance of returns is 0.03 in downs and 0.09 in ups. Then the risk aversion coefficient must be 0.89 in volatile down periods, and 1.33 in calm up periods. As market timing with MA rules works better in longer periods with few observations, it seems to be more accurate in longer stochastic (up or down) trends.

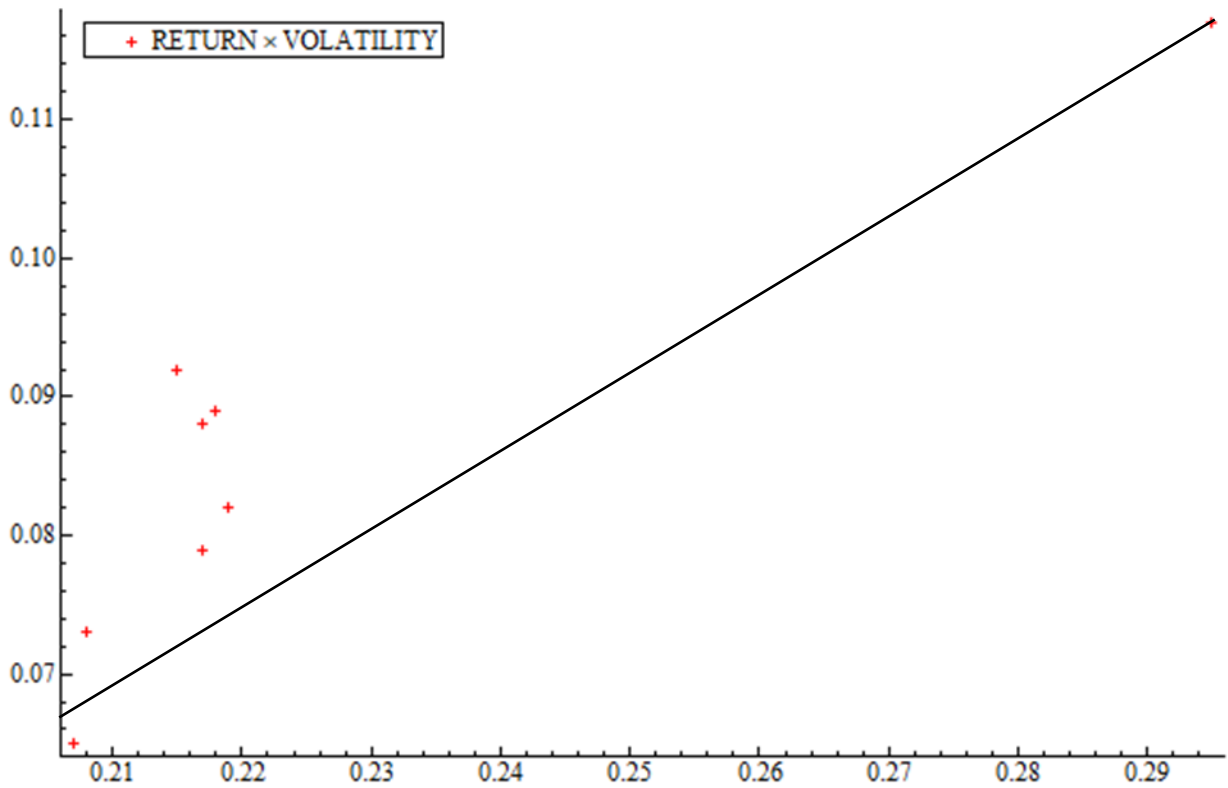


Figure 1: Returns to volatility ratio in MA200, MAW40, MA10, MAD5, MAT4, MAQ3, MAC2, and the theoretical random timing efficient SML

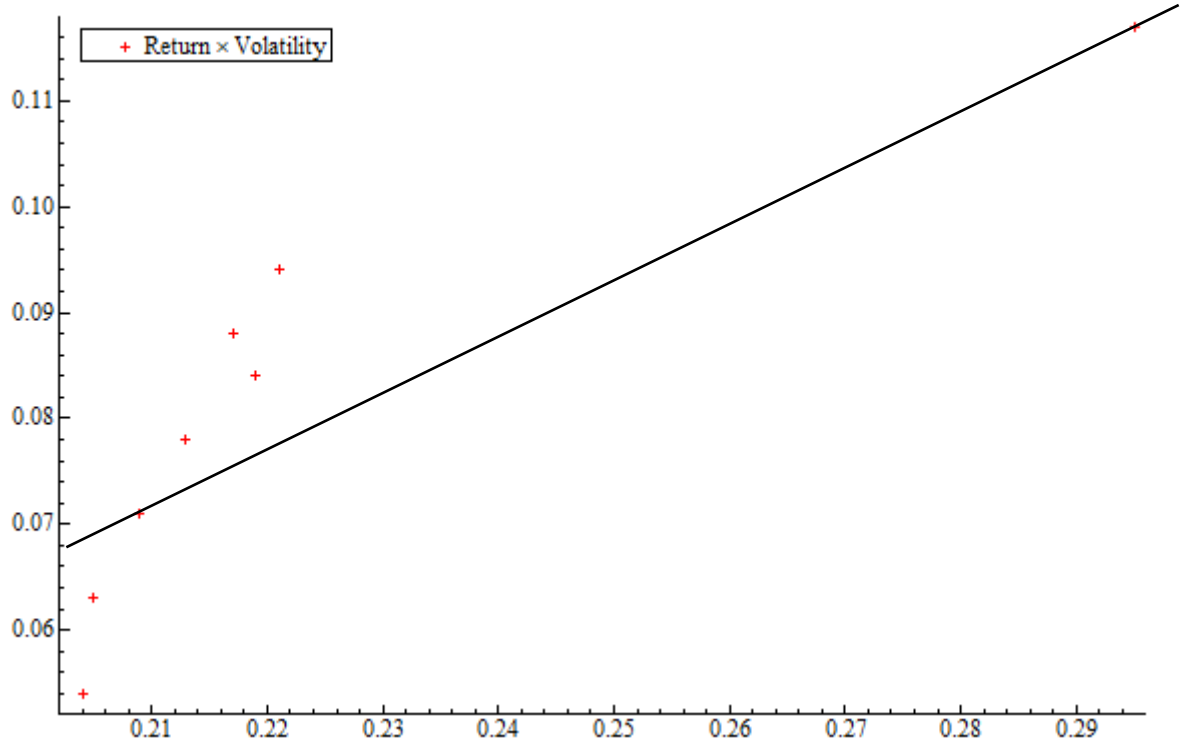


Figure 2: Returns to volatility ratio in MA200-MA40, MAW40-MAW8, MA10-MA2, MAD5-MAD2, MAT4-MAT2, MAQ3-MAQ2, MAC2, and the theoretical random timing efficient SML

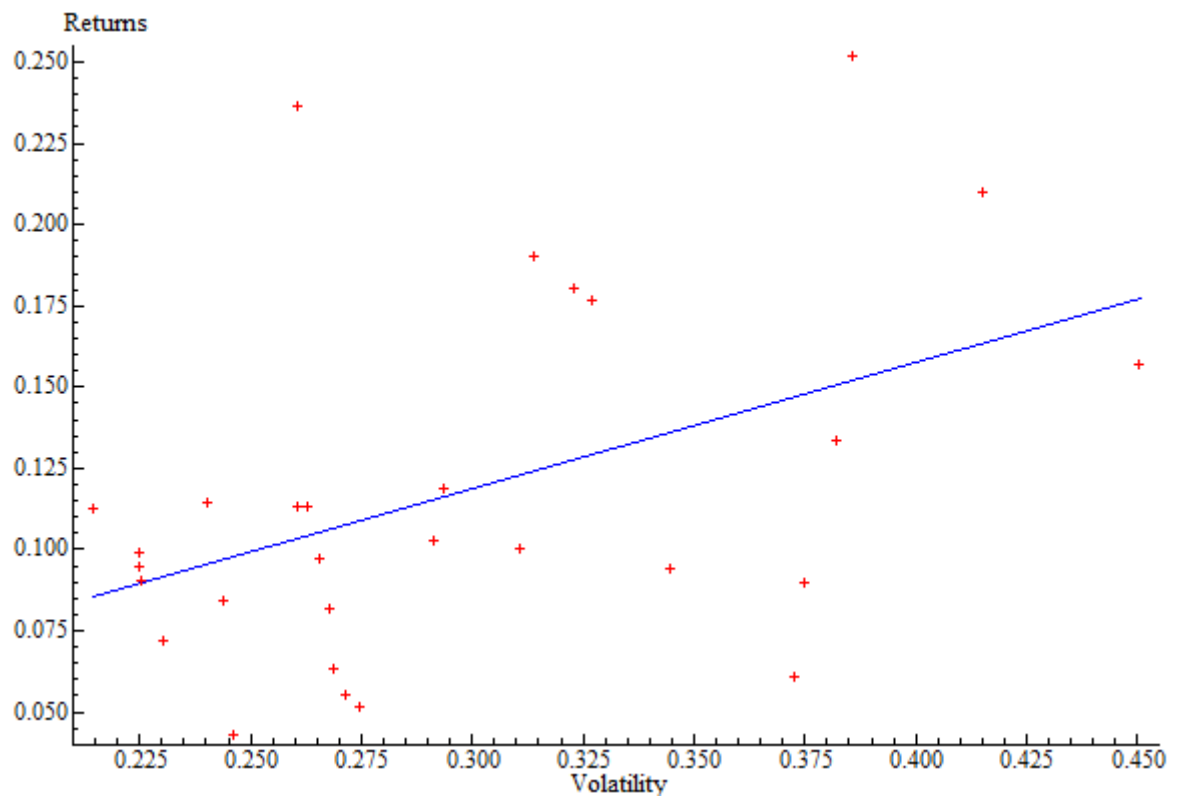


Figure 3: Returns to the volatility ratio in current DJIA stocks in annual averages from 4 January 1988 to 31 December 2017

References

- Allen, F., Karjalainen, R. (1999), Using genetic algorithms to find technical trading rules, *Journal of Financial Economics* 51: 245-271.
- Ang, A., Hodrick, R., Xing, Y., Zhang, X. (2009), High idiosyncratic volatility and low returns: International and further U.S. evidence, *Journal of Financial Economics* 61: 1-23.
- Baker, M., Bradley, B., Wurgler, J. (2011a), Benchmark as limits to arbitrage: Understanding the low-volatility anomaly, *Financial Analysts Journal* 67: 1-15.
- Baker, M., Bradley, B., Taliaferro, R. (2014), The Low-risk anomaly: a decomposition into micro and macro effects, *Financial Analysts Journal* 70: 45-58.
- Baker, M., Wurgler, J. (2015), Do strict requirements raise the cost of capital? Bank regulation, capital structure, and the low-risk anomaly, *American Economic Review: Papers & Proceedings* 105: 315-320.
- Bali, T., Cakici, N., Whitelaw, R. (2011), Maxing out: stock as lotteries and the cross-section of expected returns, *Journal of Financial Economics* 99: 427-446.
- Barberis, N., Huang, M. (2008), Stocks as lotteries: The implications of probability weighting for security prices, *American Economic Review* 98: 2066-2100.
- Ben-David, I., Graham, J., Harvey, C. (2013), Managerial miscalibration, *Quarterly Journal of Economics* 128: 1547-1584.
- Black, F. (1972), Capital market equilibrium with restricted borrowing, *Journal of Business* 45: 444-455.

Black, F., Jensen, M., Scholes, M. (1972), The capital asset market model: some empirical tests. In: Jensen, *Studies in the theory of capital markets*, Praeger, New York, 79-121.

Brock, W., Lakonishok, J., LeBaron, B. (1992), Simple technical trading rules and the stochastic properties of stock returns, *Journal of Finance* 47: 1731-1764.

Brown, D., Jennings, R. (1989), On technical analysis, *Review of Financial Studies* 2: 527-551.

Campbell, J., Cochrane, J. (1999), By force of habit: Consumption-based explanation of aggregate stock market behavior, *Journal of Political Economy* 107: 205-251.

Cochrane, J. (2008), The dog that did not bark: A defense of return predictability, *Review of Financial Studies* 21: 1533-1573.

Daniel, K., Titman, S. (2006), Market reactions to tangible and intangible information, *Journal of Finance* 61: 1605-1643.

Fama, E. (1972), Components on investment performance, *Journal of Finance* 27: 551-568.

Fama, E. (2014), Two pillars of asset pricing, *American Economic Review* 104: 1467-1485.

Frazzini, A., Pedersen, L. (2014), Betting against betas, *Journal of Financial Economics* 111: 1-25.

Friesen, G., Sapp, T. (2007), Mutual fund flows and investor returns: An empirical examination of fund investor timing ability, *Journal of Banking and Finance* 31: 2796-2816.

Gartley, H. (1935), *Profits in the Stock Markets*, Washington: Lambert-Gann Publishing.

Henriksson, R., Merton R., (1981), On market timing and investment performance II: Statistical procedures for evaluating forecasting skills, *Journal of Business* 54: 513-533.

Hong, H., Sraer, D., (2016), Speculative betas, *Journal of Finance* 71: 2095-2144.

Jagannathan, R., Korajczyk, R. (2017), Market timing. In: Guerard, *Portfolio Construction, Measurement and Efficiency*, Springer International Publishing Switzerland, 49-71.

Jegadeesh, N., Titman, S. (1993), Returns to buying winners selling losers: Implications for stock market efficiency, *Journal of Finance* 48: 65-91.

Kahneman, D., Tversky, A. (1974), Judgement under uncertainty: Heuristics and biases, *Science* 185: 1124-1131.

Kim, A., Tse, Y., Wald, J. (2016), Time series momentum and volatility scaling, *Journal of Financial Markets* 30, 103-124.

Kumar, A. (2009), Who gambles in the stock markets? *Journal of Finance* 64: 1889-1933.

LeRoy, S. (1973), Risk aversion and the martingale property of stock prices, *International Economic Review* 14: 436-446.

Li, X., Sullivan, R., Garcia-Feijoo, L. (2016), The low-volatility anomaly: Market evidence on systematic risk vs. mispricing, *Financial Analysts Journal* 72: 36-47.

Lintner, J. (1965), The valuation of risky assets and the selection of risky investments in stock portfolios and capital budgets, *Review of Economics and Statistics* 47, 13-37.

Lo, A., Mamaysky, H., Wang, J. (2000), Foundations of technical analysis: Computational algorithms, statistical inference, and empirical implementation, *Journal of Finance* 54: 1705-1770.

- Lucas, R. (1978), Asset prices in an exchange economy, *Econometrica* 46: 1429-1445.
- Markowitz, H. (1952), Portfolio selection, *Journal of Finance* 7: 77-91.
- Malkiel, B. (2003), The efficient market hypothesis and its critics, *Journal of Economic Perspectives* 17: 59-82.
- Marshall, B., Nguyen, N., Visaltanachoti, N. (2017), Time series momentum and moving average trading rules, *Quantitative Finance* 17: 405-421.
- Merton, R. (1981), On market timing and investment performance. I. An equilibrium theory of value for market forecast, *Journal of Business* 54: 363-406.
- Moskowitz, T., Ooi, Y., Pedersen, L. (2012), Time series momentum, *Journal of Financial Economics* 104: 228-250.
- Munoz, F., Vicente, R. (2018), Hindsight effect: What are actual cash flow timing skills of mutual fund investors?, *Journal of Empirical Finance* 45: 181-193.
- Neely, C., Rapach, D., Tu, J., Zhou, G. (2014), Forecasting equity risk premium: The role of technical indicators, *Management Science* 66: 1772-1791.
- Samuelson, P. (1998), Summing up on business cycles: Opening address, in *Beyond Shocks; What Causes Business Cycles?* Edited by J. Fuhrer and S. Schuh, Boston: Federal Reserve Bank of Boston.
- Sharpe, W. (1964), Capital asset prices: A theory of market equilibrium under conditions of risk, *Journal of Finance* 19: 425-442.
- Sullivan, R., Timmermann, A., White, H. (1999), Data-snooping, technical trading rule performance and the bootstrap, *Journal of Finance* 53: 1647-1691.
- Tobin, J. (1958), Liquidity preference as behavior towards risk, *Review of Economic Studies* 67: 65-86.

Zhu, Y., Zhou, G. (2009), Technical analysis: An asset allocation perspective on the use of moving averages, *Journal of Financial Economics* 91: 519-544.

Appendix 1

Table 1: Annualized daily returns of MA40-MA200, average annualized returns

	Buy & Hold	MA200	MA180	MA160	MA140	MA120	MA100	MA80	MA60	MA40
3M	0.090	0.042	0.034	0.017	0.015	0.019	0.014	0.006	-0.009	6E-04
American Express	0.094	0.035	0.037	0.039	0.055	0.039	0.042	0.043	0.041	0.008
Apple	0.157	0.147	0.145	0.147	0.142	0.156	0.149	0.150	0.146	0.164
Boeing	0.119	0.088	0.089	0.060	0.055	0.061	0.061	0.058	0.046	0.048
Caterpillar	0.100	0.075	0.079	0.058	0.058	0.049	0.034	0.028	0.039	0.025
Chevron	0.084	0.005	0.013	0.002	-0.000	-0.000	0.003	-0.01	-0.025	-0.05
Coca-Cola	0.099	0.058	0.055	0.030	0.035	0.039	0.027	0.023	0.009	0.003
Walt Disney	0.103	0.072	0.078	0.079	0.074	0.077	0.074	0.076	0.056	0.048
Exxon	0.072	-0.011	-0.010	-0.020	-0.030	-0.020	-0.025	-0.01	-0.044	-0.05
GE	0.052	0.072	0.071	0.058	0.039	0.039	0.033	0.018	0.013	9E-04
Home Depot	0.190	0.125	0.116	0.102	0.092	0.087	0.076	0.067	0.068	0.058
IBM	0.055	0.016	0.029	0.033	0.028	0.016	0.021	0.031	0.029	0.048
Intel	0.134	0.083	0.082	0.083	0.073	0.091	0.082	0.080	0.077	0.078
Johnson & Johnson	0.113	0.062	0.058	0.053	0.042	0.032	0.044	0.028	0.008	-0.00
JP Morgan	0.090	0.013	0.014	0.003	0.010	0.017	0.013	0.031	0.038	0.025
McDonalds	0.114	0.047	0.048	0.040	0.044	0.040	0.035	0.043	0.030	0.018
Merck	0.063	0.050	0.048	0.044	0.032	0.033	0.029	0.022	0.016	-0.02
Microsoft	0.180	0.117	0.128	0.105	0.102	0.104	0.095	0.090	0.070	0.062
Nike	0.177	0.087	0.093	0.085	0.102	0.108	0.107	0.119	0.133	0.112
Pfizer	0.097	0.059	0.056	0.043	0.042	0.052	0.044	0.040	0.024	0.009
Procter & Gamble	0.095	0.037	0.045	0.037	0.036	0.037	0.029	0.023	0.004	0.017
Travellers	0.082	0.036	0.035	0.038	0.029	0.008	-0.004	-9E-04	-0.001	0.006
United Technologies	0.113	0.051	0.057	0.046	0.059	0.057	0.049	0.049	0.041	0.017
United Health Group	0.252	0.181	0.182	0.157	0.147	0.136	0.130	0.118	0.125	0.076
Verizon	0.043	-0.017	-0.020	-0.010	-0.000	-0.020	-0.020	-0.02	-0.029	-0.02
Wal-Mart	0.113	0.019	0.016	0.010	0.012	0.012	0.016	0.012	0.020	0.024
Cisco	0.210	0.198	0.194	0.210	0.208	0.198	0.205	0.152	0.096	0.085
Goldman Sachs	0.061	0.038	0.029	0.033	0.038	0.050	0.057	0.078	0.076	0.063

Visa	0.236	0.112	0.118	0.129	0.141	0.128	0.132	0.120	0.094	0.085	
Average	0.117	0.065	0.066	0.059	0.058	0.057	0.053	0.05	0.041	0.033	0.054

**Table 2: Annualized daily (every fifth trading day) returns of MAW8-MAW40
(W = number of weeks), average annualized returns**

	Buy&Hold	MAW40	MAW36	MAW32	MAW28	MAW24	MAW20	MAW16	MAW12	MAW8	
3M	0.090	0.035	0.033	0.020	0.021	0.019	0.012	0.019	0.032	0.026	
American Express	0.094	0.058	0.053	0.062	0.063	0.047	0.046	0.035	0.034	0.015	
Apple	0.157	0.130	0.137	0.143	0.131	0.134	0.131	0.188	0.174	0.144	
Boeing	0.119	0.089	0.079	0.075	0.074	0.080	0.082	0.066	0.074	0.076	
Caterpillar	0.100	0.057	0.062	0.058	0.058	0.061	0.054	0.049	0.043	0.023	
Chevron	0.084	0.005	0.015	3E-04	0.004	0.008	0.009	0.004	0.004	-0.03	
Coca-Cola	0.099	0.055	0.054	0.054	0.041	0.054	0.047	0.047	0.029	0.011	
Walt Disney	0.103	0.071	0.073	0.062	0.080	0.076	0.080	0.078	0.065	0.051	
Exxon	0.072	0.018	0.016	0.007	0.008	0.010	0.013	0.020	0.011	0.005	
GE	0.052	0.061	0.046	0.047	0.047	0.045	0.023	0.018	0.031	0.023	
Home Depot	0.190	0.135	0.133	0.124	0.112	0.110	0.088	0.076	0.096	0.077	
IBM	0.055	0.020	0.037	0.044	0.040	0.051	0.027	0.028	0.008	0.016	
Intel	0.134	0.088	0.091	0.075	0.061	0.075	0.073	0.070	0.076	0.085	
Johnson & Johnson	0.113	0.074	0.079	0.071	0.059	0.050	0.050	0.048	0.042	0.027	
JP Morgan	0.090	0.040	0.036	0.027	0.033	0.033	0.048	0.051	0.042	0.020	
McDonalds	0.114	0.086	0.068	0.059	0.058	0.052	0.052	0.059	0.058	0.044	
Merck	0.063	0.051	0.039	0.029	0.034	0.034	0.030	0.033	0.024	0.029	
Microsoft	0.180	0.128	0.125	0.116	0.116	0.116	0.105	0.099	0.062	0.078	
Nike	0.177	0.087	0.091	0.098	0.093	0.087	0.094	0.102	0.119	0.091	
Pfizer	0.097	0.070	0.061	0.057	0.053	0.063	0.049	0.050	0.044	0.050	
Procter & Gamble	0.095	0.050	0.044	0.050	0.051	0.040	0.043	0.042	0.031	0.033	
Travellers	0.082	0.020	0.006	0.010	0.014	0.006	0.005	0.008	0.017	0.015	
United Technologies	0.113	0.071	0.077	0.062	0.072	0.071	0.056	0.061	0.051	0.053	
United Health Group	0.252	0.171	0.133	0.130	0.151	0.124	0.134	0.123	0.113	0.087	
Verizon	0.043	-0.00	-0.01	0.002	0.006	-0.01	-0.01	-0.01	-0.009	-0.00	
Wal-Mart	0.113	0.050	0.049	0.045	0.038	0.028	0.033	0.026	0.038	0.029	
Cisco	0.210	0.209	0.211	0.219	0.222	0.219	0.204	0.164	0.120	0.094	
Goldman Sachs	0.061	0.050	0.030	0.031	0.040	0.036	0.071	0.089	0.078	0.077	
Visa	0.236	0.143	0.142	0.131	0.171	0.167	0.159	0.113	0.119	0.080	
Average	0.117	0.073	0.069	0.066	0.067	0.065	0.062	0.061	0.056	0.046	0.063

Table 3: Annualized daily (every 20s trading day) returns of MA2-MA10, average annualized returns

	Buy and Hold	MA10	MA9	MA8	MA7	MA6	MA5	MA4	MA3	MA2	
3M	0.090	0.033	0.035	0.023	0.023	0.024	0.023	0.038	0.021	0.012	
American Express	0.094	0.086	0.087	0.091	0.107	0.088	0.062	0.062	0.036	0.038	
Apple	0.157	0.057	0.069	0.056	0.076	0.076	0.094	0.069	0.099	0.071	
Boeing	0.119	0.122	0.122	0.102	0.099	0.115	0.110	0.100	0.091	0.077	
Caterpillar	0.100	0.065	0.062	0.071	0.083	0.081	0.063	0.057	0.009	0.051	
Chevron	0.084	0.022	0.021	0.025	0.026	0.019	0.032	0.032	0.013	0.005	
Coca-Cola	0.099	0.083	0.072	0.087	0.071	0.073	0.072	0.069	0.046	0.026	
Walt Disney	0.103	0.061	0.066	0.073	0.077	0.071	0.079	0.081	0.073	0.057	
Exxon	0.072	0.040	0.038	0.028	0.028	0.034	0.020	0.027	0.025	0.026	
GE	0.052	0.079	0.078	0.080	0.072	0.070	0.063	0.018	0.038	0.037	
Home Depot	0.190	0.126	0.133	0.134	0.136	0.120	0.14	0.119	0.118	0.110	
IBM	0.055	0.029	0.033	0.032	0.038	0.036	0.026	0.033	0.026	0.03	
Intel	0.134	0.079	0.080	0.096	0.095	0.085	0.063	0.082	0.110	0.116	
Johnson & Johnson	0.113	0.078	0.076	0.071	0.059	0.057	0.058	0.050	0.052	0.031	
JP Morgan	0.090	0.057	0.051	0.051	0.063	0.046	0.070	0.079	0.067	0.067	
McDonalds	0.114	0.077	0.077	0.057	0.055	0.045	0.056	0.042	0.045	0.033	
Merck	0.063	0.069	0.069	0.054	0.059	0.05	0.045	0.027	0.011	3E-04	
Microsoft	0.180	0.122	0.127	0.123	0.099	0.112	0.093	0.095	0.090	0.108	
Nike	0.177	0.128	0.136	0.130	0.127	0.115	0.111	0.109	0.082	0.089	
Pfizer	0.097	0.070	0.069	0.067	0.068	0.066	0.068	0.056	0.040	0.034	
Procter & Gamble	0.095	0.057	0.060	0.055	0.042	0.043	0.021	0.024	0.038	0.039	
Travellers	0.082	0.045	0.049	0.047	0.041	0.034	0.016	0.009	0.002	0.017	
United Technologies	0.113	0.064	0.062	0.074	0.078	0.063	0.046	0.037	0.050	0.050	
United Health Group	0.252	0.158	0.162	0.167	0.154	0.168	0.176	0.174	0.180	0.158	
Verizon	0.043	0.002	9E-04	0.011	0.017	0.025	-0.00	0.01	-0.00	-0.02	
Wal-Mart	0.113	0.046	0.046	0.040	0.044	0.032	0.041	0.037	0.023	0.038	
Cisco	0.210	0.228	0.227	0.222	0.221	0.191	0.186	0.184	0.160	0.134	
Goldman Sachs	0.061	0.029	0.030	0.020	0.052	0.067	0.065	0.070	0.041	0.068	
Visa	0.236	0.171	0.161	0.162	0.149	0.122	0.113	0.115	0.142	0.097	
Average	0.117	0.079	0.079	0.078	0.078	0.073	0.069	0.066	0.059	0.055	0.071

Table 4: Annualized daily (every other month) returns of MAD2-MAD2 (D = every other month, and 5,4,3,2 are the numbers of observations in the rolling window), average annualized returns

	Buy&Hold	MAD5	MAD4	MAD3	MAD2	
3M	0.090	0.062	0.063	0.042	0.049	
American Express	0.094	0.089	0.098	0.052	0.041	
Apple	0.157	0.040	0.042	0.030	0.085	
Boeing	0.119	0.112	0.110	0.102	0.110	
Caterpillar	0.100	0.079	0.09	0.089	0.084	
Chevron	0.084	0.033	0.036	0.026	0.028	
Coca-Cola	0.099	0.093	0.102	0.080	0.078	
Walt Disney	0.103	0.068	0.074	0.080	0.084	
Exxon	0.072	0.022	0.018	0.010	0.009	
GE	0.052	0.067	0.066	0.041	0.033	
Home Depot	0.190	0.174	0.175	0.156	0.160	
IBM	0.055	0.016	0.023	0.017	0.021	
Intel	0.134	0.093	0.098	0.089	0.112	
Johnson & Johnson	0.113	0.083	0.086	0.048	0.071	
JP Morgan	0.090	0.053	0.052	0.048	0.054	
McDonalds	0.114	0.094	0.098	0.071	0.070	
Merck	0.063	0.084	0.067	0.036	0.031	
Microsoft	0.180	0.138	0.136	0.106	0.088	
Nike	0.177	0.140	0.144	0.133	0.122	
Pfizer	0.097	0.062	0.051	0.061	0.059	
Procter & Gamble	0.095	0.048	0.054	0.048	0.034	
Travellers	0.082	0.018	0.015	0.018	2E-04	
United Technologies	0.113	0.066	0.073	0.096	0.060	
United Health Group	0.252	0.181	0.179	0.191	0.207	
Verizon	0.043	-0.018	-0.01	-0.02	-0.02	
Wal-Mart	0.113	0.067	0.065	0.050	0.061	
Cisco	0.210	0.217	0.226	0.207	0.196	
Goldman Sachs	0.061	0.041	0.059	0.060	0.039	
Visa	0.236	0.174	0.173	0.151	0.120	
Average	0.117	0.083	0.085	0.073	0.072	0.078

Table 5: Annualized daily (every third month) returns of MAT2-MAT4 (T = every third month, and 4,3,2 are the numbers of observations in the rolling window), average annualized returns

	Buy&Hold	MAT4	MAT3	MAT2	
3M	0.090	0.061	0.055	0.039	
American Express	0.094	0.113	0.091	0.066	
Apple	0.157	0.089	0.073	0.096	
Boeing	0.119	0.127	0.131	0.114	
Caterpillar	0.100	0.070	0.069	0.078	
Chevron	0.084	0.047	0.053	0.037	
Coca-Cola	0.099	0.077	0.078	0.072	
Walt Disney	0.103	0.043	0.042	0.068	
Exxon	0.072	0.055	0.049	0.037	
GE	0.052	0.084	0.080	0.047	
Home Depot	0.190	0.161	0.163	0.128	
IBM	0.055	0.054	0.048	0.028	
Intel	0.134	0.107	0.115	0.072	
Johnson & Johnson	0.113	0.094	0.094	0.074	
JP Morgan	0.090	0.058	0.076	0.007	
McDonalds	0.114	0.080	0.082	0.069	
Merck	0.063	0.062	0.062	0.049	
Microsoft	0.180	0.127	0.128	0.080	
Nike	0.177	0.146	0.151	0.099	
Pfizer	0.097	0.078	0.070	0.056	
Procter & Gamble	0.095	0.068	0.072	0.076	
Travellers	0.082	0.041	0.043	0.025	
United Technologies	0.113	0.077	0.089	0.079	
United Health Group	0.252	0.147	0.161	0.178	
Verizon	0.043	-0.00	-0.00	-0.02	
Wal-Mart	0.113	0.081	0.081	0.083	
Cisco	0.210	0.211	0.217	0.213	
Goldman Sachs	0.061	0.044	0.026	0.030	
Visa	0.236	0.183	0.199	0.177	
Average	0.117	0.089	0.089	0.075	0.084

**Table 6: Annualized daily (every fourth month) returns of MAQ2-MAQ3
(Q = every fourth month, and 3 and 2 are the numbers of observations
in the rolling window), average annualized returns**

	Buy&Hold	MAQ3	MAQ2	
3M	0.090	0.056	0.058	
American Express	0.094	0.089	0.094	
Apple	0.157	0.094	0.094	
Boeing	0.119	0.122	0.128	
Caterpillar	0.100	0.064	0.084	
Chevron	0.084	0.060	0.054	
Coca-Cola	0.099	0.083	0.093	
Walt Disney	0.103	0.061	0.062	
Exxon	0.072	0.056	0.064	
GE	0.052	0.069	0.081	
Home Depot	0.190	0.152	0.157	
IBM	0.055	0.048	0.031	
Intel	0.134	0.064	0.070	
Johnson & Johnson	0.113	0.080	0.079	
JP Morgan	0.090	0.085	0.091	
McDonalds	0.114	0.096	0.112	
Merck	0.063	0.056	0.061	
Microsoft	0.180	0.143	0.145	
Nike	0.177	0.181	0.199	
Pfizer	0.097	0.059	0.045	
Procter & Gamble	0.095	0.073	0.077	
Travellers	0.082	0.051	0.051	
United Technologies	0.113	0.080	0.077	
United Health Group	0.252	0.185	0.218	
Verizon	0.043	0.027	0.023	
Wal-Mart	0.113	0.087	0.076	
Cisco	0.210	0.195	0.180	
Goldman Sachs	0.061	0.042	0.056	
Visa	0.236	0.195	0.228	
Average	0.117	0.091	0.096	0.094

Table 7: Annualized daily (every fifth month) returns of MAC2 (C = every fifth month, and 2 = observations accounting in the rolling window), average annualized returns

	Buy & Hold	MAC2
3M	0.090	0.076
American Express	0.094	0.088
Apple	0.157	0.132
Boeing	0.119	0.080
Caterpillar	0.100	0.094
Chevron	0.084	0.047
Coca-Cola	0.099	0.094
Walt Disney	0.103	0.044
Exxon	0.072	0.049
GE	0.052	0.048
Home Depot	0.190	0.143
IBM	0.055	0.032
Intel	0.133	0.057
Johnson & Johnson	0.113	0.081
JP Morgan	0.090	0.045
McDonalds	0.114	0.079
Merck	0.063	0.080
Microsoft	0.180	0.094
Nike	0.177	0.141
Pfizer	0.097	0.099
Procter & Gamble	0.095	0.039
Travellers	0.082	0.068
United Technologies	0.113	0.056
United Health Group	0.252	0.152
Verizon	0.043	0.048
Wal-Mart	0.113	0.093
Cisco	0.210	0.225
Goldman Sachs	0.061	0.053
Visa	0.236	0.217
Average	0.117	0.088

Appendix 2

Table 8: Annualized daily volatility of MA40-MA200, average annualized volatility

	Buy & Hold	MA200	MA180	MA160	MA140	MA120	MA100	MA80	MA60	MA40	
3M	0.225	0.164	0.165	0.161	0.161	0.159	0.159	0.158	0.158	0.157	
American Express	0.345	0.227	0.228	0.221	0.225	0.224	0.225	0.224	0.228	0.229	
Apple	0.451	0.317	0.321	0.315	0.315	0.313	0.315	0.315	0.310	0.305	
Boeing	0.294	0.201	0.203	0.199	0.201	0.199	0.198	0.198	0.201	0.204	
Caterpillar	0.311	0.216	0.218	0.216	0.216	0.214	0.215	0.214	0.213	0.215	
Chevron	0.244	0.167	0.168	0.166	0.166	0.166	0.165	0.164	0.167	0.168	
Coca-Cola	0.225	0.164	0.166	0.161	0.160	0.159	0.158	0.158	0.156	0.155	
Walt Disney	0.291	0.196	0.201	0.199	0.200	0.199	0.198	0.203	0.204	0.203	
Exxon	0.230	0.162	0.163	0.159	0.159	0.159	0.157	0.156	0.155	0.157	
GE	0.275	0.174	0.175	0.172	0.173	0.173	0.171	0.168	0.168	0.168	
Home Depot	0.314	0.226	0.228	0.223	0.221	0.221	0.219	0.217	0.217	0.214	
IBM	0.271	0.187	0.189	0.185	0.184	0.181	0.179	0.177	0.176	0.174	
Intel	0.382	0.273	0.275	0.267	0.265	0.263	0.260	0.257	0.256	0.254	
Johnson & Johnson	0.215	0.163	0.164	0.161	0.159	0.157	0.155	0.153	0.152	0.149	
JP Morgan	0.375	0.223	0.226	0.223	0.224	0.227	0.237	0.242	0.245	0.248	
McDonalds	0.240	0.183	0.184	0.18	0.178	0.177	0.176	0.176	0.175	0.174	
Merck	0.269	0.177	0.179	0.173	0.173	0.174	0.172	0.174	0.174	0.177	
Microsoft	0.323	0.248	0.249	0.243	0.241	0.237	0.236	0.233	0.232	0.231	
Nike	0.327	0.243	0.245	0.238	0.236	0.235	0.235	0.232	0.232	0.233	
Pfizer	0.266	0.188	0.19	0.187	0.186	0.187	0.186	0.187	0.187	0.187	
Procter & Gamble	0.225	0.169	0.169	0.164	0.163	0.161	0.158	0.157	0.156	0.156	
Travellers	0.268	0.174	0.175	0.174	0.175	0.178	0.180	0.184	0.182	0.185	
United Technologies	0.261	0.179	0.181	0.179	0.178	0.177	0.177	0.177	0.176	0.173	
United Health Group	0.386	0.290	0.293	0.290	0.290	0.283	0.282	0.282	0.280	0.273	
Verizon	0.246	0.163	0.165	0.164	0.163	0.163	0.163	0.161	0.161	0.163	
Wal-Mart	0.263	0.203	0.204	0.200	0.198	0.195	0.191	0.19	0.189	0.191	
Cisco	0.415	0.300	0.302	0.297	0.295	0.291	0.290	0.285	0.282	0.275	
Goldman Sachs	0.373	0.222	0.226	0.22	0.222	0.223	0.228	0.230	0.227	0.229	
Visa	0.260	0.209	0.212	0.209	0.208	0.212	0.208	0.206	0.205	0.197	
Average	0.295	0.207	0.209	0.205	0.205	0.204	0.203	0.203	0.202	0.202	0.204

Table 9: Annualized daily (every fifth trading day) volatility of MAW8-MAW40**(W = number of weeks), average annualized volatility**

	Buy&Hold	MAW40	MAW36	MAW32	MAW28	MAW24	MAW20	MAW16	MAW12	MAW8	
3M	0.225	0.165	0.165	0.163	0.163	0.16	0.159	0.157	0.157	0.159	
American Express	0.345	0.227	0.224	0.224	0.227	0.225	0.223	0.228	0.232	0.234	
Apple	0.451	0.316	0.316	0.313	0.318	0.316	0.343	0.317	0.312	0.309	
Boeing	0.294	0.204	0.203	0.204	0.204	0.203	0.203	0.201	0.201	0.206	
Caterpillar	0.311	0.216	0.215	0.215	0.217	0.214	0.215	0.215	0.213	0.214	
Chevron	0.244	0.169	0.168	0.169	0.168	0.168	0.167	0.166	0.168	0.172	
Coca-Cola	0.225	0.165	0.165	0.164	0.162	0.160	0.159	0.159	0.157	0.155	
Walt Disney	0.291	0.195	0.198	0.197	0.197	0.199	0.200	0.202	0.203	0.204	
Exxon	0.230	0.163	0.161	0.160	0.161	0.160	0.157	0.156	0.153	0.158	
GE	0.275	0.174	0.174	0.174	0.175	0.174	0.170	0.169	0.171	0.166	
Home Depot	0.314	0.228	0.228	0.226	0.225	0.222	0.224	0.219	0.219	0.214	
IBM	0.271	0.190	0.188	0.185	0.184	0.183	0.178	0.177	0.178	0.177	
Intel	0.382	0.267	0.267	0.268	0.264	0.263	0.259	0.256	0.259	0.259	
Johnson & Johnson	0.215	0.164	0.163	0.162	0.160	0.158	0.156	0.156	0.152	0.15	
JP Morgan	0.375	0.222	0.225	0.224	0.230	0.236	0.239	0.243	0.241	0.252	
McDonalds	0.240	0.185	0.182	0.181	0.179	0.177	0.177	0.176	0.174	0.171	
Merck	0.269	0.179	0.175	0.174	0.173	0.173	0.172	0.175	0.176	0.175	
Microsoft	0.323	0.250	0.247	0.245	0.244	0.24	0.236	0.236	0.230	0.232	
Nike	0.327	0.244	0.241	0.239	0.240	0.241	0.238	0.235	0.232	0.232	
Pfizer	0.266	0.189	0.187	0.186	0.187	0.188	0.190	0.189	0.189	0.184	
Procter & Gamble	0.225	0.170	0.168	0.167	0.165	0.164	0.161	0.158	0.160	0.156	
Travellers	0.268	0.175	0.175	0.175	0.178	0.177	0.177	0.184	0.184	0.185	
United Technologies	0.261	0.181	0.179	0.178	0.177	0.177	0.177	0.177	0.176	0.172	
United Health Group	0.386	0.292	0.291	0.292	0.291	0.290	0.289	0.287	0.282	0.278	
Verizon	0.246	0.163	0.162	0.162	0.162	0.164	0.162	0.161	0.160	0.159	
Wal-Mart	0.263	0.205	0.202	0.201	0.198	0.194	0.191	0.191	0.190	0.192	
Cisco	0.415	0.307	0.305	0.300	0.296	0.292	0.293	0.288	0.285	0.281	
Goldman Sachs	0.373	0.225	0.223	0.221	0.221	0.220	0.230	0.233	0.241	0.241	
Visa	0.260	0.203	0.210	0.209	0.208	0.210	0.208	0.206	0.203	0.195	
Average	0.295	0.208	0.207	0.206	0.206	0.205	0.205	0.204	0.203	0.203	0.205

Table 10: Annualized daily (rule in every 20s trading day) volatility of MA2-MA10, average annualized volatility

	Buy and Hold	MA10	MA9	MA8	MA7	MA6	MA5	MA4	MA3	MA2	
3M	0.225	0.167	0.169	0.162	0.163	0.161	0.161	0.157	0.156	0.156	
American Express	0.345	0.232	0.235	0.222	0.218	0.22	0.219	0.22	0.243	0.235	
Apple	0.451	0.343	0.347	0.342	0.339	0.339	0.338	0.342	0.335	0.331	
Boeing	0.294	0.207	0.210	0.202	0.202	0.199	0.200	0.197	0.207	0.205	
Caterpillar	0.311	0.216	0.220	0.217	0.215	0.214	0.217	0.218	0.221	0.224	
Chevron	0.244	0.169	0.171	0.172	0.17	0.169	0.169	0.167	0.181	0.171	
Coca-Cola	0.225	0.168	0.171	0.169	0.168	0.166	0.161	0.161	0.161	0.156	
Walt Disney	0.291	0.203	0.207	0.202	0.203	0.203	0.210	0.212	0.215	0.211	
Exxon	0.230	0.166	0.167	0.165	0.164	0.163	0.162	0.157	0.161	0.160	
GE	0.275	0.177	0.177	0.175	0.175	0.175	0.172	0.169	0.172	0.180	
Home Depot	0.314	0.234	0.235	0.228	0.221	0.230	0.228	0.233	0.225	0.219	
IBM	0.271	0.194	0.196	0.196	0.196	0.196	0.19	0.194	0.195	0.190	
Intel	0.382	0.273	0.277	0.272	0.272	0.268	0.266	0.266	0.264	0.259	
Johnson & Johnson	0.215	0.168	0.169	0.167	0.167	0.162	0.158	0.158	0.154	0.150	
JP Morgan	0.375	0.222	0.223	0.217	0.220	0.230	0.233	0.234	0.244	0.234	
McDonalds	0.240	0.189	0.189	0.186	0.185	0.185	0.179	0.170	0.171	0.180	
Merck	0.269	0.177	0.178	0.173	0.173	0.174	0.173	0.181	0.182	0.192	
Microsoft	0.323	0.250	0.251	0.247	0.239	0.233	0.235	0.237	0.233	0.234	
Nike	0.327	0.247	0.248	0.244	0.241	0.240	0.235	0.236	0.238	0.248	
Pfizer	0.266	0.188	0.190	0.186	0.186	0.186	0.187	0.187	0.191	0.189	
Procter & Gamble	0.225	0.173	0.174	0.171	0.167	0.165	0.163	0.164	0.158	0.155	
Travellers	0.268	0.171	0.172	0.17	0.169	0.171	0.191	0.186	0.192	0.198	
United Technologies	0.261	0.178	0.179	0.178	0.177	0.177	0.175	0.178	0.176	0.173	
United Health Group	0.386	0.300	0.302	0.299	0.298	0.294	0.289	0.280	0.283	0.275	
Verizon	0.246	0.167	0.167	0.164	0.162	0.160	0.164	0.157	0.160	0.163	
Wal-Mart	0.263	0.208	0.210	0.205	0.199	0.196	0.197	0.198	0.198	0.189	
Cisco	0.415	0.304	0.307	0.301	0.298	0.300	0.292	0.290	0.281	0.278	
Goldman Sachs	0.373	0.230	0.232	0.225	0.232	0.245	0.239	0.253	0.268	0.256	
Visa	0.260	0.204	0.203	0.212	0.225	0.221	0.219	0.217	0.217	0.196	
Average	0.295	0.211	0.213	0.209	0.208	0.208	0.208	0.208	0.210	0.207	0.209

Table 11: Annualized daily (every other month) volatility of MAD2-MAD2 (D = every other month, and 5,4,3,2 are the numbers of observations in the rolling window), average annualized volatility

	Buy&Hold	MAD5	MAD4	MAD3	MAD2	
3M	0.225	0.168	0.169	0.162	0.159	
American Express	0.344	0.222	0.226	0.216	0.211	
Apple	0.450	0.351	0.363	0.357	0.338	
Boeing	0.294	0.210	0.216	0.211	0.208	
Caterpillar	0.311	0.218	0.229	0.215	0.211	
Chevron	0.244	0.168	0.175	0.166	0.165	
Coca-Cola	0.225	0.168	0.173	0.165	0.158	
Walt Disney	0.291	0.197	0.200	0.198	0.203	
Exxon	0.230	0.172	0.174	0.159	0.156	
GE	0.274	0.175	0.181	0.176	0.182	
Home Depot	0.314	0.229	0.230	0.221	0.237	
IBM	0.271	0.196	0.199	0.200	0.200	
Intel	0.382	0.274	0.286	0.267	0.265	
Johnson & Johnson	0.215	0.173	0.175	0.165	0.154	
JP Morgan	0.375	0.236	0.241	0.246	0.237	
McDonalds	0.240	0.182	0.186	0.178	0.169	
Merck	0.269	0.185	0.196	0.188	0.199	
Microsoft	0.323	0.245	0.249	0.238	0.250	
Nike	0.327	0.252	0.258	0.253	0.253	
Pfizer	0.266	0.199	0.203	0.191	0.189	
Procter & Gamble	0.225	0.173	0.177	0.169	0.166	
Travellers	0.268	0.176	0.178	0.183	0.191	
United Technologies	0.261	0.182	0.187	0.178	0.177	
United Health Group	0.386	0.313	0.313	0.299	0.305	
Verizon	0.246	0.163	0.171	0.165	0.153	
Wal-Mart	0.263	0.197	0.199	0.194	0.193	
Cisco	0.415	0.312	0.317	0.315	0.285	
Goldman Sachs	0.373	0.229	0.245	0.239	0.265	
Visa	0.260	0.215	0.215	0.225	0.222	
Average	0.295	0.213	0.218	0.212	0.210	0.213

Table 12: Annualized daily (every third month) volatility of MAT2-MAT4 (T = every third month, and 4,3,2 are the numbers of observations in the rolling window), average annualized volatility

	Buy&Hold	MAT4	MAT3	MAT2	
3M	0.225	0.172	0.174	0.171	
American Express	0.344	0.230	0.237	0.206	
Apple	0.450	0.345	0.357	0.349	
Boeing	0.294	0.206	0.219	0.200	
Caterpillar	0.311	0.219	0.223	0.214	
Chevron	0.244	0.176	0.182	0.170	
Coca-Cola	0.225	0.177	0.179	0.181	
Walt Disney	0.291	0.220	0.228	0.205	
Exxon	0.230	0.168	0.176	0.158	
GE	0.274	0.178	0.185	0.177	
Home Depot	0.314	0.236	0.251	0.241	
IBM	0.271	0.205	0.209	0.193	
Intel	0.382	0.285	0.296	0.274	
Johnson & Johnson	0.215	0.185	0.188	0.165	
JP Morgan	0.375	0.242	0.248	0.240	
McDonalds	0.240	0.198	0.204	0.192	
Merck	0.269	0.191	0.191	0.180	
Microsoft	0.323	0.257	0.267	0.258	
Nike	0.327	0.264	0.265	0.258	
Pfizer	0.266	0.195	0.206	0.208	
Procter & Gamble	0.225	0.177	0.181	0.168	
Travellers	0.268	0.187	0.188	0.198	
United Technologies	0.261	0.192	0.199	0.187	
United Health Group	0.386	0.300	0.308	0.315	
Verizon	0.246	0.176	0.176	0.160	
Wal-Mart	0.263	0.202	0.208	0.208	
Cisco	0.415	0.310	0.311	0.303	
Goldman Sachs	0.373	0.226	0.232	0.235	
Visa	0.260	0.204	0.215	0.208	
Average	0.295	0.218	0.224	0.214	0.219

Table 13: Annualized daily (every fourth month) volatility of MAQ2-MAQ3 (Q = every fourth month, 3 and 2 are the number of observations in the rolling window), average annualized volatility

	Buy&Hold	MAQ3	MAQ3	
3M	0.225	0.168	0.176	
American Express	0.344	0.220	0.226	
Apple	0.450	0.360	0.373	
Boeing	0.294	0.213	0.224	
Caterpillar	0.311	0.222	0.239	
Chevron	0.244	0.167	0.177	
Coca-Cola	0.225	0.173	0.182	
Walt Disney	0.291	0.206	0.218	
Exxon	0.230	0.160	0.176	
GE	0.274	0.180	0.195	
Home Depot	0.314	0.237	0.242	
IBM	0.271	0.194	0.218	
Intel	0.382	0.274	0.293	
Johnson & Johnson	0.215	0.181	0.186	
JP Morgan	0.375	0.218	0.227	
McDonalds	0.240	0.177	0.193	
Merck	0.269	0.204	0.212	
Microsoft	0.323	0.248	0.260	
Nike	0.327	0.258	0.265	
Pfizer	0.266	0.198	0.207	
Procter & Gamble	0.225	0.173	0.174	
Travellers	0.268	0.182	0.192	
United Technologies	0.261	0.181	0.188	
United Health Group	0.386	0.299	0.314	
Verizon	0.246	0.167	0.177	
Wal-Mart	0.263	0.194	0.207	
Cisco	0.415	0.341	0.349	
Goldman Sachs	0.373	0.240	0.260	
Visa	0.260	0.212	0.225	
Average	0.295	0.215	0.227	0.221

Table 14: Annualized daily (every fifth month) volatility of MAC2 (C = every fifth month, 2 = observations in rolling window), average annualized volatility

	Buy & Hold	MAC2
3M	0.225	0.176
American Express	0.344	0.226
Apple	0.450	0.323
Boeing	0.294	0.218
Caterpillar	0.311	0.227
Chevron	0.244	0.165
Coca-Cola	0.225	0.168
Walt Disney	0.291	0.206
Exxon	0.230	0.166
GE	0.274	0.187
Home Depot	0.314	0.242
IBM	0.271	0.202
Intel	0.382	0.296
Johnson & Johnson	0.215	0.187
JP Morgan	0.375	0.244
McDonalds	0.240	0.182
Merck	0.269	0.194
Microsoft	0.323	0.250
Nike	0.327	0.249
Pfizer	0.266	0.191
Procter & Gamble	0.225	0.187
Travellers	0.268	0.183
United Technologies	0.261	0.204
United Health Group	0.386	0.298
Verizon	0.246	0.170
Wal-Mart	0.263	0.223
Cisco	0.415	0.333
Goldman Sachs	0.373	0.218
Visa	0.260	0.220
Average	0.295	0.218

Appendix 3

Table 15: Transaction costs per year of MA40-MA200, with one transaction costing 0.1% of total wealth, average annualized transaction costs

	MA200	MA180	MA160	MA140	MA120	MA100	MA80	MA60	MA40	
3M	0.010	0.011	0.010	0.011	0.012	0.013	0.016	0.019	0.022	
American Express	0.011	0.011	0.011	0.012	0.012	0.013	0.016	0.017	0.023	
Apple	0.007	0.008	0.008	0.009	0.010	0.012	0.014	0.015	0.020	
Boeing	0.008	0.009	0.010	0.011	0.011	0.012	0.014	0.015	0.020	
Caterpillar	0.008	0.009	0.010	0.011	0.012	0.013	0.015	0.015	0.019	
Chevron	0.011	0.012	0.012	0.013	0.014	0.016	0.018	0.020	0.024	
Coca-Cola	0.009	0.010	0.011	0.011	0.011	0.012	0.015	0.018	0.022	
Walt Disney	0.007	0.008	0.009	0.011	0.012	0.012	0.013	0.017	0.021	
Exxon	0.011	0.013	0.016	0.017	0.017	0.018	0.019	0.023	0.028	
GE	0.007	0.008	0.009	0.010	0.011	0.012	0.014	0.017	0.023	
Home Depot	0.008	0.009	0.010	0.011	0.013	0.014	0.016	0.018	0.021	
IBM	0.009	0.010	0.010	0.010	0.012	0.012	0.013	0.014	0.019	
Intel	0.007	0.009	0.010	0.010	0.012	0.014	0.014	0.016	0.019	
Johnson & Johnson	0.009	0.008	0.009	0.010	0.012	0.014	0.016	0.020	0.024	
JP Morgan	0.010	0.010	0.011	0.012	0.012	0.014	0.015	0.016	0.020	
McDonalds	0.010	0.011	0.011	0.013	0.012	0.014	0.016	0.018	0.023	
Merck	0.008	0.009	0.009	0.011	0.011	0.013	0.015	0.017	0.022	
Microsoft	0.008	0.009	0.010	0.010	0.011	0.013	0.015	0.015	0.020	
Nike	0.009	0.009	0.010	0.010	0.011	0.012	0.013	0.014	0.019	
Pfizer	0.008	0.010	0.010	0.011	0.011	0.012	0.014	0.017	0.021	
Procter & Gamble	0.010	0.010	0.010	0.011	0.012	0.014	0.016	0.019	0.022	
Travellers	0.010	0.011	0.012	0.012	0.013	0.015	0.016	0.018	0.024	
United Technologies	0.009	0.010	0.011	0.011	0.012	0.014	0.015	0.018	0.021	
United Health Group	0.008	0.008	0.010	0.010	0.011	0.012	0.014	0.017	0.021	
Verizon	0.011	0.011	0.011	0.011	0.013	0.014	0.017	0.018	0.023	
Wal-Mart	0.010	0.010	0.012	0.013	0.013	0.014	0.015	0.019	0.022	
Cisco	0.006	0.006	0.008	0.010	0.009	0.010	0.014	0.017	0.023	
Goldman Sachs	0.008	0.010	0.012	0.012	0.014	0.015	0.022	0.026	0.035	
Visa	0.008	0.008	0.009	0.009	0.008	0.010	0.011	0.014	0.022	
Average	0.009	0.0010	0.010	0.011	0.012	0.013	0.015	0.018	0.022	0.013

Table 16: Transaction costs per year of MA2-MA10, average annualized transaction costs

	MA10	MA9	MA8	MA7	MA6	MA5	MA4	MA3	MA2	
3M	0.003	0.003	0.003	0.003	0.003	0.004	0.004	0.005	0.006	
American Express	0.002	0.002	0.002	0.002	0.002	0.003	0.003	0.004	0.006	
Apple	0.002	0.002	0.002	0.002	0.003	0.003	0.004	0.005	0.006	
Boeing	0.002	0.002	0.002	0.002	0.002	0.003	0.004	0.004	0.006	
Caterpillar	0.002	0.002	0.002	0.002	0.003	0.003	0.004	0.005	0.006	
Chevron	0.002	0.003	0.003	0.003	0.003	0.003	0.004	0.005	0.007	
Coca-Cola	0.002	0.002	0.002	0.002	0.002	0.003	0.003	0.004	0.006	
Walt Disney	0.002	0.002	0.002	0.002	0.003	0.003	0.003	0.004	0.006	
Exxon	0.002	0.002	0.003	0.003	0.003	0.004	0.004	0.005	0.006	
GE	0.002	0.002	0.002	0.002	0.003	0.003	0.004	0.004	0.006	
Home Depot	0.002	0.002	0.002	0.002	0.003	0.003	0.003	0.004	0.006	
IBM	0.003	0.002	0.003	0.002	0.003	0.003	0.004	0.004	0.006	
Intel	0.002	0.003	0.003	0.003	0.003	0.003	0.004	0.004	0.006	
Johnson & Johnson	0.002	0.002	0.002	0.002	0.003	0.003	0.004	0.005	0.006	
JP Morgan	0.002	0.003	0.003	0.003	0.003	0.003	0.003	0.004	0.006	
McDonalds	0.002	0.002	0.003	0.003	0.003	0.003	0.004	0.005	0.006	
Merck	0.002	0.002	0.002	0.003	0.003	0.003	0.004	0.005	0.006	
Microsoft	0.002	0.002	0.002	0.003	0.003	0.003	0.004	0.004	0.006	
Nike	0.002	0.002	0.002	0.002	0.003	0.003	0.004	0.004	0.006	
Pfizer	0.002	0.002	0.002	0.003	0.003	0.003	0.004	0.004	0.006	
Procter & Gamble	0.002	0.002	0.003	0.003	0.003	0.004	0.004	0.005	0.006	
Travellers	0.003	0.002	0.003	0.003	0.003	0.004	0.004	0.005	0.007	
United Technologies	0.002	0.002	0.002	0.002	0.003	0.003	0.004	0.004	0.006	
United Health Group	0.002	0.002	0.002	0.003	0.003	0.003	0.003	0.004	0.006	
Verizon	0.003	0.003	0.003	0.003	0.003	0.004	0.004	0.005	0.006	
Wal-Mart	0.003	0.003	0.003	0.003	0.003	0.004	0.004	0.005	0.006	
Cisco	0.002	0.002	0.002	0.002	0.003	0.003	0.003	0.005	0.006	
Goldman Sachs	0.002	0.002	0.002	0.003	0.003	0.003	0.003	0.004	0.005	
Visa	0.002	0.001	0.002	0.002	0.002	0.003	0.003	0.003	0.005	
Average	0.002	0.002	0.002	0.003	0.003	0.003	0.004	0.004	0.006	0.003