NECESSARY CONDITIONS FOR SIGNAL PROCESSING BY RESONANT NEURONS

Villacorta JA, Sánchez-Jiménez A, Panetsos F

Departamento de Matemática Aplicada, Facultad de Biología, Universidad Complutense de Madrid, Ciudad Universitaria S/N, 28040, Madrid, Spain

fivos.panetsos@bio.ucm.es

Summary

We study a mathematical model for information processing and coding by means of groups of resonant neurons. We conclude that incoming signals can be expressed by means of their Fourier series which coefficients are represented by the value of the membrane potential of the resonant neurons.

Introduction

A great amount of data suggests that oscillating behavior of neural cells has an important role in information processing (Murthy and Fetz 1996, Salmelin and Hari 1994, Vaadia et al. 1995) and several advanced and very powerful models have been developed to describe such processes. They include concepts that range from information coding by spiking time-advances based on subthreshold oscillations (Wang and Wang 2000) to complex correlations between rhythmical fluctuations of the membrane potential and the spikes of distant neurons (Wehr and Laurent 1996) or information coding by spontaneous oscillations of neuronal groups ((Hopfield 1995). But, although rhythmical behavior is taken into account of such models and theories (Kurrer and Schulten 1995, Robinson et al. 1998, Wang et al. 1997), the role of oscillations of the membrane potential must be further investigated. Moreover, although, it has been studied in the last years, the role of resonance in information processing still remains to be cleared up (Liu et al. 1999, Puil et al. 1994).

In the present communication we study the information processing and coding capabilities of a group of non-coupled neurons that present subthreshold oscillations of their membrane potential. We propose that a continuous incoming signal $S(t)$ provoques a resonant behavior of the membrane potential of each neuron and that we can obtain the coefficients of its Fourier serie by means of the values that reach membrane potentials. In the next section the mathematical study is explained, and the performance of the model is computationally studied by ted signal in the section for results.

Theory

We consider a sys sonant neurons represented by N RLC circuits (Fig.1). Th on the oscillation frequency of each neuron and on th nce of the system. For the development of the mode uron, represented by the corresponding RLC circuit a mic behavior will be derived from the Kirchoff's laws.

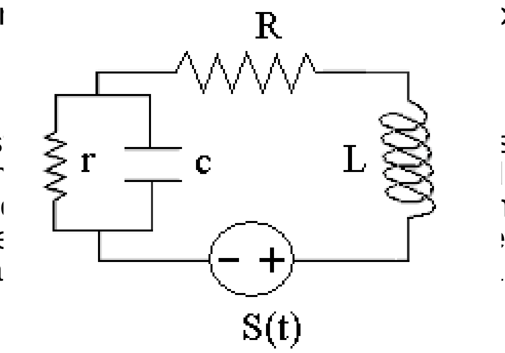

Figure 1. Model of the resonant neuron. The parameter r represents the behavior for passive channels, c represents the capacitative behavior of the cellular membrane, R and L represent the first order behavior of the active channels and $S(t)$ is the external signal in voltage.

First we define the currents present in the k-th RLC circuit. Then we apply the principle of conservation of charge in every node, getting the equation

$$I = I_1 + I_2, \qquad (1)$$

where I is the current in resistance R and inductance L, $I_1 (= dQ_1/dt)$ is the current in capacitor and I_2 is the current in resistance r. Finally we apply the conservation of voltage in the two loops. In the loop with r and c we obtain

$$I_2 r = \frac{Q_1}{c} \Rightarrow \frac{dI_2}{dt} = \frac{I_1}{cr} \tag{2}$$

and for the loops with c, R and L

$$L\frac{dI}{dt} + IR + \frac{Q_1}{c} = S(t). \tag{3}$$

Combining Eq. 1, Eq. 2 and Eq. 3 in terms of Q_1 we get

$$L\frac{d^2Q_1(t)}{dt^2} + \left(\frac{L}{cr}+R\right)\frac{dQ_1(t)}{dt} + \left(\frac{R}{cr}+\frac{1}{c}\right)Q_1(t) = S(t) \tag{4}$$

with initials conditions given by

$$Q_1(0) = \frac{dQ_1(0)}{dt} = 0. \tag{5}$$

The Eqs. 4 and 5 describes the response of the k-th resonant neuron in terms of its membrane potential $V_1 = Q_1/c$ (for simplicity we have not used any subindex k for the neuron parameters r, c, R and L).

Now we impose one of the conditions to guarantee the performing of the system:

$$\frac{R}{r} << \left(\frac{L}{cr}+R\right)\sqrt{\frac{c}{L}} << 1. \tag{6}$$

In order to simplify the exposition we will replace the effective capacitance $(R/cr + 1/c)^{-1}$ by C, apply the relation $L = \frac{1}{c\omega_k^2} = \frac{1}{C\omega_k^2} + O(R/r)$, with ω_k the frequency of the k-th neuron, define a new variable $x(t) = \frac{Q_1(t)}{C\omega_k}$ and replace the term $\left(\frac{L}{cr}+R\right)\sqrt{\frac{c}{L}}$ by $\hat{R}$ we get

$$\left.\begin{array}{l} \frac{1}{\omega_k}\ddot{x}(t) + \hat{R}\dot{x}(t) + \omega_k x(t) = S(t) \\ x(0) = \dot{x}(0) = 0 \end{array}\right|. \tag{7}$$

In fact the equal sign in the first equation in Eq. 7 is just “equal with order R/r” but that sign will be used because the final results will work with order $(L/cr + R)\sqrt{c/L}$, greater than R/r (Eq. 6).

The standard solution $x(t)$ of Eq. 7 is given by the sum of the solution of the homogenous equation (or temporal charge - TQ) denoted by $x_T(t)$, and a particular solution $x_P(t)$ (or stationary periodic charge - SPQ). The expression of the first depends on the $\hat{R}$ value: if $\hat{R} > 2$, $\hat{R} = 2$ or $\hat{R} < 2$ this solution decays exponentially or decays oscillating. With the above restriction given by Eq. 6 we are in the last case (sub-damped regime) and the TQ is written by

$$x_T(t) = e^{-pt}\left(c_1\cos(\omega t) + c_2\sin(\omega t)\right) \tag{8}$$

with $p = \hat{R}\omega_k/2$, $\omega = \sqrt{\omega_k^2 - p^2} = \omega_k\sqrt{1-\hat{R}^2/4}$ and c_1 and c_2 constants to be obtained from the initial conditions. In order to develop the SPQ, the Fourier series theory will be used. First of all we define a temporal window $[0,T]$, denoted by T, where the signal $S(t)$ is exhibited. $S(t)$ is simultaneously injected on the N neurons as an e.m.f., and is mathematically supposed continuous with piecewise continuous derivative and with $S(0) = S(T) = 0$ allowing to extent the signal out of T as a even or an odd function without lost of continuity. In this circumstances the external signal can be represented by its sine Fourier serie: $S(t) = \sum_{i=1}^{\infty} A_i\sin(\omega_i t)$ with $\omega_i = \frac{i\pi}{T}$ $\forall i \in \aleph$ (odd extension). On the other side exists an unique solution of C^2 class for the problem represented by Eq. 7 and it allows the particular solution $x_P(t)$ to be represented by its Fourier serie as

$$x_P(t) = C_0 + \sum_{i=1}^{\infty}\left[B_i\sin(\omega_i t) + C_i\cos(\omega_i t)\right]. \tag{9}$$

The Fourier coefficients can be established with no difficulties by integration by parts or by the superposition principle, and results

$$x_P(t) = \sum_{i=1}^{\infty}\frac{A_i}{\omega_k}\left[\frac{\left(1-\frac{\omega_i^2}{\omega_k^2}\right)\sin\left(\frac{i\pi}{T}t\right) - \hat{R}\frac{\omega_i}{\omega_k}\cos\left(\frac{i\pi}{T}t\right)}{\left(1-\frac{\omega_i^2}{\omega_k^2}\right)^2 + \left(\hat{R}\frac{\omega_i}{\omega_k}\right)^2}\right]. \tag{10}$$

The second condition over our system is in the frequencies of the resonant neurons: their values will be

$$\omega_k = \frac{k\pi}{T} \quad \forall k \in \{1,\ldots,N\}. \tag{11}$$

Now the particular solution $x_P(t)$ will give in the extremes of the temporal window

$$x_P(T) - x_P(0) = \sum_{\substack{i=1 \\ i\,odd}}^{\infty}\frac{2T}{i\pi}\left[\frac{A_i\hat{R}}{\left(\frac{k}{i}-\frac{i}{k}\right)^2 + \hat{R}^2}\right]. \tag{12}$$

To repeat the process for $x_T(t)$ first we need the constants c_1 and c_2 from the initial conditions, the Eq. 8 and the Eq. 10. This way we get

$$c_1 = \sum_{i=1}^{\infty}\frac{T}{i\pi}\left[\frac{A_i\hat{R}}{\left(\frac{k}{i}-\frac{i}{k}\right)^2 + \hat{R}^2}\right] \qquad c_2 = \frac{1}{k}\sum_{i=1}^{\infty}\text{[illegible]} \tag{13}$$

and expanding in Taylor serie around $\hat{R} = 0$ we obtain

$$x_T(T) - x_T(0) = \begin{cases} \text{[illegible]} - \frac{c_1\hat{R}k\pi}{2} + O\left(\hat{R}^2\right) & for\ k\ even \\ -c_1\left(2 - \frac{\hat{R}k\pi}{2}\right) + O\left(\hat{R}^2\right) & for\ k\ odd \end{cases}. \tag{14}$$

The complete solution $x(t)$ at $t = T$ can be written as

$$x(T) = x(T) - x(0) = x_P(T) - x_P(0) + x_T(T) - x_T(0),$$

where substituting the above expressions

$$x(T) = \begin{cases} \sum_{\substack{i=1 \\ i\,odd}}^{\infty}\frac{2T}{i\pi}\left[\frac{A_i\hat{R}}{\left(\frac{k}{i}-\frac{i}{k}\right)^2 + \hat{R}^2}\right] - \frac{c_1\hat{R}k\pi}{2} + O\left(\hat{R}^2\right) & for\ k\ even \\ \sum_{\substack{i=1 \\ i\,odd}}^{\infty}\frac{2T}{i\pi}\left[\frac{A_i\hat{R}}{\left(\frac{k}{i}-\frac{i}{k}\right)^2 + \hat{R}^2}\right] - c_1\left(2 - \frac{\hat{R}k\pi}{2}\right) + O\left(\hat{R}^2\right) & for\ k\ odd \end{cases} \tag{15}$$

Applying now the relation

$$\sum_{\substack{i=1\\ i\neq k}}^{\infty}\frac{A_i\hat{R}\left(\frac{k}{i}\right)}{\left(\frac{k}{i}-\frac{i}{k}\right)^2}=O(\hat{R}) \qquad (16)$$

in Eq. 13 and Eq. 15, and from the definition of $x(t)$ we get

$$Q_1(T)=\begin{cases}\frac{C\omega_k T}{2}\hat{R}^2 A_k+O\left(\hat{R}\right) & for\ k\ even\\ \frac{C\omega_k T}{2}A_k+O\left(\hat{R}\right) & for\ k\ odd\end{cases}. \qquad (17)$$

In realistic situations we can not know the charge $Q_1(t)$ but we are able to measure the potential $V(t)$ in the capacitor. The relation between charge and potential is

$$V(t)=\frac{Q_1(t)}{c}\Rightarrow\Delta Q_1=Q_1(T)-Q_1(0)=Q_1(T)=V(T)c, \qquad (18)$$

and taking into account the Eq. 6 and commentaries about the orders of R/r and $\left(L/cr+R\right)\sqrt{c/L}=\hat{R}$, we definitively obtain the following result:

$$A_k=\begin{cases}\frac{c2}{\omega_k T\hat{R}}V(T)+O\left(\left(\frac{L}{cr}+R\right)\sqrt{\frac{c}{L}}\right) & for\ k\ even\\ \frac{2}{\omega_k T}V(T)+O\left(\left(\frac{L}{cr}+R\right)\sqrt{\frac{c}{L}}\right) & for\ k\ odd\end{cases} \quad \forall k\in\{1,\dots,N\} \qquad (19)$$

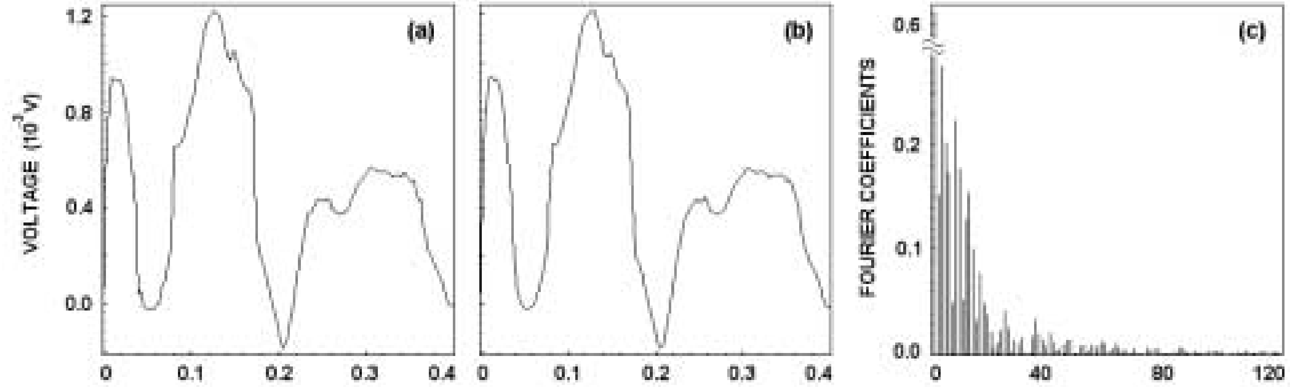

Fig 2. (a) Original signal simultaneously injected to 100 resonant neurons oscillating between 1.25 and 125 Hz. (b) The signal reconstructed by means of the membrane potentials of the 100 resonant neurons. (c) Frequency spectrum of the incoming signal presented in (a).

Conclusions

The membrane potential of oscillating neurons can be used for signal processing and coding. The ranges of the system parameters of the RLC circuits we used to prove that correspond to those of real neurons commonly found in the Central Nervous System.

Acknowledgments

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