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A General Theory of Money

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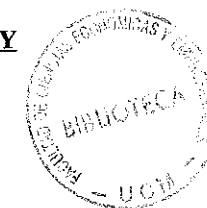
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**A GENERAL THEORY OF MONEY**

by

Arthur B. Treadway



**ABSTRACT** (keywords underlined)

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This General Theory of Money (GTM) is proposed as an appropriate Conceptual Model (CM) for any contemporary empirical time series analysis involving a measured variable corresponding to the conceptual variable of the aggregate nominal quantity of money. It is relevantly more general than existing CM's for the same purposes. Two illustrative empirical analyses relating to the demand and supply of money in the Spanish economy are summarized and an alternative CM, popular in the contemporary neutrality testing literature, is critically compared with this GTM. (JEL E40)

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**RESUMEN** (palabras clave subrayadas)

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Esta Teoría General del Dinero (TGD) se propone como un Modelo Conceptual (MC) apropiado para cualquier análisis empírico de series temporales contemporáneo que trata una variable observada correspondiente con la variable conceptual de la cantidad nominal de dinero agregada. Este MC es más general, en sentidos relevantes, que los MM. CC. existentes para los mismos fines. Se resumen dos análisis empíricos ilustrativos relacionados con la demanda y la oferta de dinero en España y un MC alternativo, popular en la literatura contemporánea de la contrastación de la neutralidad, se compara de forma crítica con esta TGD.

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## 1. Introduction

"My main conclusion is that equally plausible models yield fundamentally different results", wrote Jerome Stein in the introduction of his 1970 survey of monetary growth theory. Two decades later all we have is more reasons for reaching the same conclusion.

This statement by Orphanides and Solow (1990), p. 224, indicates a very sad state of affairs in monetary economics, reflecting the so far highly inconclusive character of its empirical findings. This may well arise from widespread incompetence in time series data analysis, but it may also arise from a lack of relevant generality in the conceptual models used to frame existing empirical research in monetary economics. The present paper is addressed to the latter issue.

This paper considers the Conceptual Model (CM) basic to any empirical time series analysis in which a measured variable is taken to correspond to the conceptual variable of the aggregate nominal quantity of money. This General Theory of Money (GTM) is general in the sense that it posits that this conceptual variable is the only aggregate variable and the only nominal variable essential to useful monetary theory: (1) a useful CM here must not depend at all on the concrete economic content taken for the other variables, except that (2) all of the other variables must be taken as real, i.e. their units of measurement do not include the monetary unit. This GTM is also general in the sense of being stated in terms of the general linear, nonexplosive, invertible, multivariate stochastic time series model, which is more general than most models employed in existing empirical monetary analyses but is nevertheless capable of comprehensive treatment using empirical methods available today. The GTM described is simple and the author has found it useful in many practical applications, some of which are described.

Monetary theory, like most other areas of economic theory, has traditionally been stated in a static and deterministic language that severely hampers or makes impossible the rigorous thinking about the dynamic and stochastic features of observable economic time series that is necessary for researchers to adequately model them in practice. Once sufficient generality is allowed, however, one may begin to clarify confusions inherent in the use of inadequate language and this, it is hoped, will establish a conceptual starting point for diverse empirical analyses in the monetary field that may, one day soon, become more conclusive.

The existing literature relevant to the subject of this paper is massive and by no means well integrated. It will be discussed at different points in the course of the paper rather than being reviewed systematically here. Section 5 summarizes two illustrative studies applying the GTM and touching on two major subject areas in the literature, the demand and supply of money. Section 6 presents a detailed critical comparison of the GTM presented here with one CM likely to be taken as alternative by readers conversant with the contemporary literature on neutrality testing.

Section 2 sets up the maintained hypotheses of the GTM, Section 3 states two relevant testable hypotheses within the GTM, Section 4 discusses certain special theories of money, Section 5 summarizes two applications of the GTM, Section 6 compares the GTM with a leading contemporary CM and Section 7 offers concluding remarks.

## 2. Maintained hypotheses

Before performing any empirical analysis of relationships between time series it is essential for the researcher to formulate an Initial Conceptual Model (ICM) as the

starting point for the iterative model building process, which itself will typically involve both theoretical and empirical features in specification, diagnosis and reformulation. This ICM should be as general as possible so that it may be a useful tool and not a censor prior to data analyses.

The ICM will always contain a set of maintained hypotheses, i.e., hypotheses not subject to empirical test within the confines of the specific research to which the ICM is being applied. These maintained hypotheses should be simple and plausible, the researcher should be thoroughly conscious of them and he should expose them openly in reporting his empirical results, indicating alternative maintained hypotheses and their implications for the interpretation of empirical results, if possible.

It is desirable for the ICM to be general in the sense of being just identified, if possible, so that it may deliver, unencumbered by untested but testable hypotheses, an economic interpretation of any empirical regularities discovered by the researcher. It is also desirable for the researcher to formulate hypotheses that are testable within the ICM, hypotheses which necessarily overidentify parameters so that, when not found inconsistent with the data in a specific case, they can be imposed on the Empirical Model (EM) to simplify it and gain efficiency in the estimation of remaining parameters and in the testing of further hypotheses.

The CM presented in this section appears to conform to these prescriptions. It also seems to merit the designation of a "general theory of money" (GTM), because, as compared with the CM's typical of existing econometric analyses of monetary phenomena, it is more general in many empirically relevant senses and appears to be less general in none.

Many concepts supposedly central to contemporary monetary theory are not, in

the author's opinion, at all essential for a GTM. This is the case of such conceptual variables as "the general level of prices", "the general level of production (or income)", etc., "the aggregate demand for money" and "the aggregate demand and supply of goods and services". The GTM proposed here does not exclude any of these concepts, but neither does it need them, which means that useful applications of this GTM may be feasible even when observations do not exist for the empirical counterparts of these conceptual variables.

The one aggregate conceptual variable that is essential to a GTM is the *nominal quantity of money*, a stock variable measured in monetary units (dollars in the U.S., e.g.). In the case of a specific empirical study, the researcher must choose one of several candidate time series as the empirical measure of this conceptual variable, relevant for his purposes, e.g. Monetary Base, M1, M2, etc. This choice is not relevant to the statement of the GTM, though it may well be relevant for the purposes of a specific study. In this paper  $M_t$  stands for this conceptual variable at the point in time  $t$ , taken here to be the end of the time interval  $t$  in discrete time. Extensive empirical analyses of candidate series for different national economies indicate that the logarithmic transformation is sensible for this variable within the family suggested by Box and Cox (1964) to induce approximate linearity, homoskedasticity and normality. With "ln" indicating the logarithm, write  $X_t \equiv \ln M_t$ .

A second basic concept for the GTM is the *state of the real economy*. All variables relevant to any economy, except  $M$ , may be taken as real, that is, they may be taken as measured in units that do not include the monetary unit. For the purposes of any specific study, the state of the real economy may thus be represented by a vector of real variables, each appropriately transformed to induce approximate properties of

linearity, homoskedasticity and normality. The  $(n-1) \times 1$  vector of such transformed real variables will here be called  $Z$ .

It is important to recognize the generality (ambiguity) of the definition of  $Z$  in the GTM, which is intended to yield a useful CM for any content for  $Z$  that a researcher may choose. Components of  $Z$  may, e.g., be physical quantities, deflated nominal quantities, flows or stocks of goods or services, index numbers of same, relative prices, real cash balances somehow defined, velocities of monetary circulation, interest rates, aggregates or disaggregates. The only essential property for a component of  $Z$  is that it be measured in units that do not include the monetary unit. Note that any nominal variable, i.e. a variable whose units of measurement include the monetary unit, can be transformed to a real variable by taking a ratio of it to  $M$  or to some other nominal variable or by taking the first temporal difference of its logarithm, the logarithmic rate of change. Note that all interest rates are real economic variables in the definition used here for drawing the nominal-real distinction.

Each researcher must, for the particular ends of his empirical study, select a specific content for  $Z$ . He may do this well or not, but that depends on his particular objectives and not on the GTM.

## 2.1 Definitions of some mathematical terms

An observed time series variable at time  $t$  will be written e.g.  $X_t$  and no distinction in notation will be made in this paper between this realized value and the random variable of the stochastic process that generates it. The backshift operator  $B$ , such that  $BX_t \equiv X_{t-1}$ , is extensively employed, as is the first difference operator  $\nabla \equiv 1 - B$ , such that  $\nabla X_t \equiv X_t - X_{t-1}$ .

A Linear Transfer Function (LTF) is written  $v(B) \equiv \sum_{k=0}^{\infty} v_k B^k$  where each  $v_k$  is a real constant. The concept of the long run gain of a LTF,  $g \equiv v(1)$ , is often useful. Such a LTF is said to be stable when  $v(B)$  converges for  $|B| \leq 1$ ; the long run gain  $g$  is thus finite under stability. In practice  $v(B)$  will be given an  $(s, b, r)$  approximation, where  $s, b, r$  are non-negative integers, so that  $v(B) \equiv \omega_s(B) B^b / \delta_r(B)$  where  $\omega_s(B) = \omega_0 - \omega_1 B - \dots - \omega_s B^s$  is of order  $s$ ,  $\delta_r(B) = 1 - \delta_1 B - \dots - \delta_r B^r$  is normalized ( $\delta_0 = 1$ ) and of order  $r$  and  $b$  is the dead time. The LTF is thus stable when  $\delta_r(B) = 0 \Rightarrow |B| > 1$ . Vectors or matrices with LTF components are here taken as stable when their components are.

An  $nx1$  vector  $Y_t$  of time series, taken as deviations from their deterministic components, will here be said to follow a general linear, nonexplosive, invertible, multivariate stochastic process (or MS for short) when:

$$\Pi(B) Y_t = a_t \quad (\Pi \text{ form}) \quad (1.1)$$

where  $a_t$  is an  $nx1$  random vector such that  $a_t$  is iid with  $E(a_t) = 0$  and  $E(a_t a_t^T) = \Sigma$  is symmetric positive definite ( $a_t$  is a white noise vector), (1.2) where the superindex "T" indicates matrix transposition,

$$\Pi(B) = \Theta^{-1}(B) \Phi(B) \quad (\text{ARMA form}) \quad (1.3)$$

for

$$\Theta(B) = I - \Theta_1 B - \dots - \Theta_q B^q \quad (1.4)$$

$$\Phi(B) = I - \Phi_1 B - \dots - \Phi_p B^p$$

where  $I$  is the  $nxn$  identity matrix,  $\Theta_k$  and  $\Phi_k$  are  $nxn$  matrices of real constants,

$$|\Phi(B)| = 0 \Rightarrow |B| \geq 1 \quad (\text{nonexplosiveness}) \quad (1.5)$$

where  $|\cdot|$  indicates the determinant for square matrices, such as  $\Phi(B)$ , and the absolute value for complex numbers, such as  $B$  in (1.5),

$$|\Theta(B)| = 0 \Rightarrow |B| > 1 \quad (\text{invertibility}) \quad (1.6)$$

and  $\Phi(B)$  and  $\Theta(B)$  have no common finite-order left factor except  $I$ . (1.7)

Note that a general MS so defined can have any kind of homogeneously nonstationary structure including common homogeneously nonstationary factors arising under cointegration. In principle, an empirical analysis of a sample of  $Y_t$  for  $t = 1, \dots, N$ , may reveal any specific structure for (1). Thus the first task of an ICM is to give interpretation to such general possibilities.

In most empirical practice, one would specify  $a_t$  to follow the multivariate normal density.

## 2.2 Behavioral distinction

The universe of economic agents is divided, for purposes of stating the GTM, into two sectors: the *sector H* of those who *hold the existing nominal stock of money*  $X_t$  at any *point*  $t$  in time and the *sector C* of those who *create the flow of increments*  $\nabla X_t$  to the stock over any *interval*  $t$  of time. The operational sectors for a given choice of empirical  $M$  should naturally be taken into account by the researcher, but the GTM merely draws such a behavioral distinction.

In words, the GTM postulates that: (1) sector H sets  $Z_t$ , given  $X_t$  and given the history of all variables  $\{Z_{t-\tau}, X_{t-\tau}; \tau > 0\}$ , (2) sector C sets  $\nabla X_t$ , given  $Z_t$  (until and unless the hypothesis of Absence of Contemporary Feedback, ACF, is assumed) and given the history of all variables and (3) the influences of all other (i.e. omitted) variables determining behavior of the two sectors are independently distributed between them. The rest of this subsection specifies this behavioral distinction in mathematical terms.

The behavior of sector H is specified with:

$$Z_t = v_z(B)X_t + N_{zt} \quad (2.1)$$

$$\Pi_z(B)N_{zt} = a_{zt}^* \text{ iid with } E(a_{zt}^*) = 0, E(a_{zt}^* a_{zt}^{*\tau}) = \Sigma_z^* \quad (2.2)$$

where  $v_z(B)$  is an  $(n-1) \times 1$  vector of stable LTF's and  $N_{zt}$  is the  $(n-1) \times 1$  vector stochastic error process for this behavior, specified to follow an  $(n-1)$  dimensional MS indicated as in (2.2) but specified to have the general MS properties of (1). The vector of LTF's  $v_z(B)$  thus describes the effects of the nominal quantity of money  $X_t$  on the real economy as described by  $Z_t$  and (2.2) describes the behavior of the real economy in the absence of these effects.

The behavior of sector C is specified with:

$$\nabla X_t = v_x(B)Z_t + N_{xt} \quad (3.1)$$

$$\Pi_x(B)N_{xt} = a_{xt} \text{ iid with } E(a_{xt}) = 0, E(a_{xt}^2) = \sigma_x^2 \quad (3.2)$$

where  $v_x(B)$  is a  $1 \times (n-1)$  vector of stable LTF's and  $N_{xt}$  is the scalar stochastic error process for this behavior, specified to follow a scalar MS indicated as in (3.2) but

specified to have the general MS properties of (1) with  $n = 1$ . The vector of LTF's  $v_x(B)$  describes the feedback effects of the real economy as described by  $Z_t$  on the rate of money creation  $\nabla X_t$  and (3.2) describes the behavior of sector C aside from such feedback effects.

To effectively draw the behavioral distinction between sectors H and C it is essential to specify the independence of the innovation processes  $a_{zt}^*$  and  $a_{xt}$ :

$$E(a_{zt}^* a_{xt}) = 0 \quad \forall t, \tau. \quad (4)$$

The specifications (2) - (3) set up the terms for the distinction, but the real content of the distinction is found in (4) and this is a key maintained hypothesis.

How might (4) be invalid in a specific empirical study? One way it could be invalid arises when the researcher omits a component of  $Z$  that feedbacks to  $\nabla X$  (has a nonzero component in  $v_x(B)$ ) and is also related, through  $\Pi_z(B)$  and/or  $\Sigma_z^*$ , with the included components of  $Z$ . Another potential source of trouble is measurement error in  $X$  correlated with one or more components of  $Z$ , e.g., if a  $Z$  component contains the variable  $X$  as in a real-cash-balances or velocity variable.

There are at least three apparently reasonable lines of defense against criticism of the full CM on the basis of the possible failure of (4). One is that this or some similar hypothesis is so essential that it has been maintained by virtually every alternative CM in the existing literature, in many cases unconsciously. A second defense is that such criticisms (omitted variables, measurement errors), when made in the absence of concrete empirical evidence in support of their relevance for a specific case, are necessarily sterile; they constitute theory without practice. In any case, the third point is that the error on the part of the researcher in a specific case does not

undermine the GTM. The critical lesson is that the empirical researcher must mind the possibility of failure in his maintained hypotheses, here clearly set out.

Note, of course, that only the contemporary independence of  $a_{zt}^*$  and  $a_{xt}$  is questionable at any level, because strictly lagged dependence is already covered by  $v_z(B)$  and  $v_x(B)$ .

### 2.3 The Full System

The full system can now be summarized in:

$$\begin{bmatrix} \Pi_z(B) & | & -\Pi_z(B)v_z(B) \\ \hline -\Pi_x(B)v_x(B) & | & \Pi_x(B)\nabla \end{bmatrix} \begin{bmatrix} Z_t \\ X_t \end{bmatrix} = \begin{bmatrix} a_{zt}^* \\ a_{xt} \end{bmatrix} \quad (5.1)$$

where the full  $nxn$  variance-covariance matrix is:

$$\Sigma^* \equiv E \begin{bmatrix} a_{zt}^* \\ \hline a_{xt} \end{bmatrix} \begin{bmatrix} a_{zt}^{*T} & , & a_{xt} \end{bmatrix} = \begin{bmatrix} \Sigma_z^* & | & \mathbf{0}_{n-1} \\ \hline \mathbf{0}_{n-1}^T & | & \sigma_x^2 \end{bmatrix} \quad (5.2)$$

where  $\mathbf{0}_{n-1}$  denotes a zero column vector of size  $n-1$ .

The equations (5) summarize all that has been specified so far but do not yet constitute an MS for the  $nx1$  vector  $[Z_t^T, X_t^T]^T$  in the sense of (1). Note that (5) is not normalized. That is, the matrix on the left in (5.1) when evaluated at  $B = 0$  is not an identity matrix, but takes the form:

$$A = \begin{bmatrix} \mathbf{I} & | & -v_{z0} \\ \hline -v_{x0} & | & \mathbf{I} \end{bmatrix} \quad (5.3)$$

where  $\mathbf{I}$  is an  $(n-1) \times (n-1)$  identity matrix, where  $v_{z0} \equiv v_z(0)$  and  $v_{x0} \equiv v_x(0)$ .

The further basic assumption of the GTM is that the vector  $[Z_t^T, X_t^T]^T$  follows an MS as specified in (1). System nonexplosiveness requires that:

$$\nabla - v_x(B)v_z(B) = 0 \Rightarrow |B| \geq 1 \quad (6)$$

which implies that  $1 - v_{x0}v_{z0} \neq 0$  and that  $g_x g_z \leq 0$  where  $g_x \equiv v_x(1)$  and  $g_z \equiv v_z(1)$ . System invertibility is assured by specifications already given.

Taking the partitioned inverse of  $A$ :

$$A^{-1} = \frac{1}{(1 - v_{x0}v_{z0})} \begin{bmatrix} (1 - v_{x0}v_{z0})\mathbf{I} + v_{z0}v_{x0} & | & v_{z0} \\ \hline v_{x0} & | & 1 \end{bmatrix} \quad (7)$$

and premultiplying (5.1) by it, the normalized MS form is found. The same notation as that used for (1) can be employed here. The reader must distinguish between the general content for the notation when used in (1) and the concrete content when used here. Take the  $nx1$  vector  $Y_t = [Z_t^T, X_t^T]^T$  and partition (1) in the same sense of  $n-1$  and  $1$  terms. That is  $a_t = [a_{zt}^T, a_{xt}]^T$  and

$$\Pi(B) = \left[ \begin{array}{c|c} \Pi_{zz}(B) & \Pi_{zx}(B) \\ \hline \Pi_{xz}(B) & \Pi_{xx}(B) \end{array} \right]$$

where  $\Pi_{zz}(B)$  is  $(n-1) \times (n-1)$ ,  $\Pi_{zx}(B)$  is  $(n-1) \times 1$ ,  $\Pi_{xz}(B)$  is  $1 \times (n-1)$  and  $\Pi_{xx}(B)$  is scalar. The following expressions for these elements of  $\Pi(B)$  are obtained:

$$\Pi_{zz}(B) = \frac{[(1 - v_{x0} v_{z0}) \mathbf{I} + v_{z0} v_{x0}] \Pi_z(B) - v_{z0} \Pi_x(B) v_x(B)}{1 - v_{x0} v_{z0}} \quad (8.1)$$

$$\Pi_{xz}(B) = \frac{v_{x0} \Pi_z(B) - \Pi_x(B) v_x(B)}{1 - v_{x0} v_{z0}} \quad (8.2)$$

$$\Pi_{zx}(B) = \frac{v_{z0} \Pi_x(B) \nabla - [(1 - v_{x0} v_{z0}) \mathbf{I} + v_{z0} v_{x0}] \Pi_z(B) v_z(B)}{1 - v_{x0} v_{z0}} \quad (8.3)$$

$$\Pi_{xx}(B) = \frac{\Pi_x(B) \nabla - v_{x0} \Pi_z(B) v_z(B)}{1 - v_{x0} v_{z0}} \quad (8.4)$$

Since  $\mathbf{a}_t = [\mathbf{a}_{zt}^T, a_{xt}]^T = \mathbf{A}^{-1} [\mathbf{a}_{zt}^{*T}, a_{xt}]^T$ :

$$\Sigma \equiv E(\mathbf{a}_t \mathbf{a}_t^T) = \mathbf{A}^{-1} \Sigma^* (\mathbf{A}^{-1})^T \quad (8.5)$$

It is evident that the system so far specified is under identified. One can expect empirical analysis to set values for the parameters of  $\Pi(B)$  and  $\Sigma$ , but  $\Sigma$  contains only  $n(n+1)/2$  parameters, which is  $n-1$  less than the total of  $(n^2 + 3n - 2)/2$  parameters found by summing the  $2(n-1)$  from  $\mathbf{A}$  with the  $(n-1)n/2$  from  $\Sigma_z^*$  and the one from  $\sigma_x^2$ .

## 2.4 Identifying hypotheses

At the level of the GTM there is no justification for further restricting the elements of  $\Sigma^*$  in any way. Thus the reasonable place to add restrictions is in  $\mathbf{A}$ , that is, in the two vectors  $v_{z0}$  and  $v_{x0}$  characterizing the contemporaneous relationships between the two sectors.

One hypothesis that seems plausible for many cases is that of Absence of Contemporaneous Feedback (ACF):

$$v_{x0} = \mathbf{0}. \quad (9)$$

Under this hypothesis, the general system simplifies to:

$$\Pi_{zz}(B) = \Pi_z(B) - v_{z0} \Pi_x(B) v_x(B) \quad (10.1)$$

$$\Pi_{xz}(B) = -\Pi_x(B) v_x(B) \quad (10.2)$$

$$\Pi_{zx}(B) = v_{z0} \Pi_x(B) \nabla - \Pi_z(B) v_z(B) \quad (10.3)$$

$$\Pi_{xx}(B) = \Pi_x(B) \nabla \quad (10.4)$$

$$\Sigma = \left[ \begin{array}{c|c} \Sigma_z^* + \sigma_x^2 v_{z0} v_{z0}^T & \sigma_x^2 v_{z0} \\ \hline \sigma_x^2 v_{z0}^T & \sigma_x^2 \end{array} \right] \quad (10.5)$$

and the parameters of this CM are clearly just identified.

The ACF hypothesis is by no means the only possible or potentially useful identifying hypothesis, but it may be expected to be widely useful, because there are many  $\mathbf{Z}$  variables in contemporary economies that probably receive lagged effects of

the nominal quantity of money (under some operational definition) but that simply are not observed quickly enough to be taken into account by sector C in setting  $\nabla X_t$ , at least not on short enough sampling intervals.

One example of a case in which the researcher might well not want to employ the ACF hypothesis is when the economy is small and subject to a truly fixed exchange rate; in such a case one might want to use the alternative hypothesis of  $v_{z0} = 0$  and let  $v_{x0}$  be free.

It should be evident that the length of the sampling interval may be critical in identifying the system. This is true here as it is in most systems. For example, a system modeled in annual data may present identification problems that are easily and plausibly resolved in monthly data on the same variables.

In practice, the researcher must select the identifying restrictions on A that seem appropriate for the specific case. This set of restrictions should be utterly irrelevant, however, for a GTM, that is, the GTM does not include the identifying restrictions, though these will constitute maintained hypotheses in concrete applications.

### 2.5 Comment

In practice, when the researcher has chosen empirical time series for Z and X, he will attempt to elaborate an EM of these series and the GTM is helpful but not complete in specifying an ICM. The researcher must face the specification of that part of the ICM that treats the MS submodel for  $N_{z1}$ . Further economic theory must be brought into play to help think about  $\Pi_z(B)$  and  $\Sigma_z^*$ .

But this further economic theory can, by virtue of the GTM, be based on a thoroughly real kind of thought, because  $N_{z1}$  describes the real variables, indicating

the state of the economy, purged of all dependence on the history of the nominal quantity of money.

One of the principal weaknesses of currently dominant forms of monetary theory is that they are usually purely real theories in the first place, that is, they specify restrictions on  $\Pi_z(B)$  and/or  $\Sigma_z^*$  while taking  $v_z(B) = 0$  and  $v_x(B) = 0$ , which means that they cannot deliver a degree of simplicity comparable to that offered by the GTM without overburdening the data with untested *ad hoc* hypotheses, usually unconsciously employed.

The empirical study of  $v_z(B)$  will describe the direct effects of X on Z, but the structure of  $\Pi_z(B)$  and  $\Sigma_z^*$  allows for indirect effects. Some modern monetary theories posit that the nominal quantity of money influences the real economy only through its effects on interest rates. Such an hypothesis is testable in the GTM; it is, in fact, rejected in some cases, e.g., in economies with administered interest rates and poor capital markets such as Spain in the 1960's.

### 3. Testable hypotheses

The GTM is based on the maintained hypotheses of the previous section, none of which appears to have convincing alternatives. But the GTM also contains testable hypotheses that lead to simplifications of the MS. Recall that the GTM is general in essence because it consists of hypotheses that do not require a particular choice of Z variables. The testable hypotheses of the GTM are then hypotheses on the two vectors of LTF's,  $v_z(B)$  describing the effects of X on Z and  $v_x(B)$  describing the effects of Z on  $\nabla X$ . Two useful examples are presented; others may arise from future research.

### 3.1 Neutrality of Money (NM)

The most important testable hypothesis of the GTM is certainly that called the Neutrality of Money (NM), defined by:

$$\mathbf{g}_z \equiv \mathbf{v}_z(1) = \mathbf{0} \quad \text{for any selection of } \mathbf{Z}. \quad (11)$$

This hypothesis captures, in the author's opinion, the essence of classical thought in monetary theory. It is also an hypothesis that simplifies the general system in a very powerful way. Under NM,  $\mathbf{v}_z(\mathbf{B}) = \mathbf{v}_z^*(\mathbf{B})\nabla$  for an  $(n-1) \times 1$  vector of stable LTF's  $\mathbf{v}_z^*(\mathbf{B})$  and, consulting (8), one finds that:

$$\Pi_{zx}(\mathbf{B}) = \left\{ \frac{\mathbf{v}_{z0} \Pi_x(\mathbf{B}) - [(1 - \mathbf{v}_{x0} \mathbf{v}_{z0}) \mathbf{I} + \mathbf{v}_{z0} \mathbf{v}_{x0}] \Pi_z(\mathbf{B}) \mathbf{v}_z^*(\mathbf{B})}{1 - \mathbf{v}_{x0} \mathbf{v}_{z0}} \right\} \nabla \quad (12.1)$$

$$\Pi_{xx}(\mathbf{B}) = \left\{ \frac{\Pi_x(\mathbf{B}) - \mathbf{v}_{x0} \Pi_z(\mathbf{B}) \mathbf{v}_z^*(\mathbf{B})}{1 - \mathbf{v}_{x0} \mathbf{v}_{z0}} \right\} \nabla \quad (12.2)$$

which means that the  $\mathbf{Y}_t$  vector can be redefined to  $\mathbf{Y}_t = [\mathbf{Z}_t^T, \nabla X_t]^T$  instead of  $\mathbf{Y}_t = [\mathbf{Z}_t^T, X_t]^T$  and the equations (8.1), (8.2) and (8.5) are unchanged, but (8.3) and (8.4) are redefined to (12.1) and (12.2) without the  $\nabla$  on the right hand side. Note that system nonexplosiveness will imply  $\mathbf{g}_x \mathbf{g}_z^* \leq 1$  under NM where  $\mathbf{g}_z^* \equiv \mathbf{v}_z^*(1)$ .

It is thus clear that NM can always be tested without imposing any identification hypothesis. If an empirically adequate MS model in  $[\mathbf{Z}_t^T, X_t]^T$  can be validly

written in  $[\mathbf{Z}_t^T, \nabla X_t]^T$ , i.e. if the last column of  $\Pi(\mathbf{B})$  has  $\nabla$  as a common factor, then NM necessarily holds. If not, then NM is rejected.

The NM hypothesis has never been convincingly rejected, as far as the author is aware. The researcher is justified in imposing it to help build an empirical MS in the vector  $[\mathbf{Z}_t^T, \nabla X_t]^T$  and then, using the same parametrization for the vector  $[\mathbf{Z}_t^T, X_t]^T$  with each of the elements of the last column of the AR matrix having its own extra factor  $(1 - \alpha_i \mathbf{B})$ , he can consider testing NM efficiently by testing the joint hypothesis of  $\alpha_i = 1 \quad \forall i = 1, \dots, n$ . In the author's experience, it can be very difficult to parameterize the MS in practice without using the NM hypothesis, but parameterization is much easier with it.

### 3.2 Absence of Feedback (AF)

A useful testable hypothesis of the GTM is the Absence of Feedback (AF):

$$\mathbf{v}_x(\mathbf{B}) \equiv \mathbf{0} \quad \text{for any selection of } \mathbf{Z}, \quad (13)$$

which restricts the form (10) under ACF to change (10.1) and (10.2) to

$$\Pi_{zz}(\mathbf{B}) = \Pi_z(\mathbf{B}) \quad (14.1)$$

$$\Pi_{xz}(\mathbf{B}) = \mathbf{0}, \quad (14.2)$$

also very easy to test because no lagged effects of real variables may appear in  $X$ .

### 4. Special theories of money

Special theories of money are all those theories that are framed within the GTM

but that specify conditions for particular (i.e. special) real variables.

One might be tempted, in the light of the existing literature, to specify the Superneutrality of Money (SNM) as a testable hypothesis of the GTM, conditional on the acceptance of NM. The specification might be taken as:

$$\mathbf{g}_z^* \equiv \mathbf{v}_z^*(1) = 0 \text{ for any selection of } \mathbf{Z}. \quad (15)$$

However, SNM can be clearly seen to be at most an hypothesis of special monetary theory. Consider the real variable  $\nabla \ln P$  formed from some nominal variable  $P$ ; the fact that  $\nabla \ln P$  is a real variable is evident and the acceptance of NM is highly plausible ( $g_{\nabla \ln P} = 0$ ), but NM itself, for the real-cash-balance variable  $\ln M_t/P_t$ , must imply  $g_{\nabla \ln P}^* = 1$ , which violates SNM. Hence all real variables formed as rates of change of nominal variables must be excluded from SNM, making SNM a special theory of money.

The SNM hypothesis is also violated by several well known monetary theories. The Fisher proposition that interest rates receive unit long run effects of the rate of inflation (= the rate of monetary expansion under NM in the long run) requires interest rates to be excluded from the list of real variables subject to SNM. The proposition in the theory of the demand for money, especially useful in studies of hyperinflation, by which real cash balances receive negative long run effects of the rate of inflation (= the rate of monetary expansion under NM in the long run) requires real-cash-balance variables to be excluded from the list of real variables satisfying SNM; see, e.g., Patinkin(1987). Thus the SNM hypothesis cannot be sustained as an hypothesis of a GTM even if one takes a classical quantity theory approach to monetary theory. If one allows for more keynesian ideas, the SNM hypothesis becomes even less sustainable as

part of a GTM. See Orphanides and Solow(1990) for a variety of monetary growth theories that violate SNM.

Aside from the fact that SNM must be excluded from a GTM, it is also true that many other elements of existing monetary theory are seen to be special monetary theories. The theory of the demand for money employs a very specific list of real variables; see, e.g., Friedman(1957). Theories of the supply of money also employ specific lists of real variables; see, e.g., Brunner(1961, 1973, 1976 ).

That a monetary theory is not part of the GTM does not, of course, imply that it is invalid or useless, but does imply that attempts to make it an all encompassing framework for monetary analysis are doomed to failure, because it lacks relevant generality.

## 5. Two illustrations

The following illustrations of the GTM are summaries of two time series studies of monetary phenomena in the Spanish economy. Space does not allow for sufficiently detailed reports of the data analyses to adequately defend the empirical results described and no pretense of such justification is intended. Detailed reports in Spanish are available to the interested reader along with the numerical data; an English version of Gonzalo(1996) is also available.

### 5.1 Money in the Spanish macroeconomic table

Treadway, Cabo y Gonzalo(1994), TCG in the sequel, report on time series analyses of the annual variables making up the so-called Macroeconomic Table for the Spanish economy. A part of that report is summarized here.

Let  $Y$  indicate nominal (current peseta) Gross Domestic Product,  $Q$  real (constant peseta) GDP, and  $P \equiv Y/Q$  the implicit GDP deflator (divided by 100). Variables  $Y$  and  $Q$  are annual flows. Let  $M$  stand for the M4-type nominal money stock, the end-of-year value in the present case. This definition of  $M$  is used by most researchers for the Spanish case, the Bank of Spain used it as the definition for its monetary expansion targets for many years and, though definitely debatable, it is taken in this study as the initial choice, to be questioned in later research. All four variables are modeled in logarithms, a decision based on very clear evidence for analogous monthly and quarterly time series. In TCG, 29 annual observations for 1964-1992 are analyzed, though more recent updates show the results to be robust to data revisions and data for 1993-1995 so far available. Forecast and monitor results for the Spanish Macroeconomic Table were published several times a year in the bulletin *Seguimiento y Previsión de la Economía Española* (Monitoring and Forecasting the Spanish Economy) in 1992-1996 and were based on these econometric results.

Univariate analyses indicate that each of the four series is integrated of order two, that is,  $I(2)$ . Bivariate analyses indicate that the variables  $Y, P, Q$  do not have lagged effects on  $M$ . The ACF hypothesis is assumed and seems plausible in this case. Bivariate analyses further indicate that  $M$  has lagged effects on the variables  $Y, P, Q$  as well as a contemporary effect on  $Q$  but none on  $P$ . A univariate analysis of velocity ( $V_t \equiv Y_t/M_t$ ) indicates that this variable is  $I(1)$ , revealing a useful sense in which the old idea that "velocity is more stable than either of its components, nominal income and nominal money stock" is apparently true for this Spanish data; this cointegration result is not, however, necessary either for using the GTM or for the interpretation given below. The bivariate analysis of  $P$  and  $M$

indicates that  $M$  leads  $P$  with dead time of one year.

The following two-output, one-input transfer-function-noise model (Jenkins (1979), 23-25, 119-122) is constructed, assuming NM so that the input is  $\nabla \ln M$ :

$$\ln \frac{Y_t}{M_t} = - \left( \begin{array}{c} .61 \\ (.08) \end{array} + \begin{array}{c} .08B \\ (.12) \end{array} \right) \nabla \ln M_t + N_{yt} \quad (16.1)$$

$$\hat{\sigma}_y = -.69(.17)$$

$$\ln \frac{P_t}{M_{t-1}} = - \left( \begin{array}{c} .75 \\ (.11) \end{array} + \begin{array}{c} .27B \\ (.08) \end{array} \right) \nabla \ln M_{t-1} + N_{pt} \quad (16.2)$$

$$\hat{\sigma}_p = -1.02(.17)$$

$$(1 - .70B)(\nabla N_{yt} + .022) = a_{yt}, \hat{\sigma}_y = 1.9\% \quad (16.3)$$

(.09)                      (.007)

$$\nabla^2 N_{pt} = a_{pt}, \hat{\sigma}_p = 2.0\% \quad (16.4)$$

$$\nabla^2 \ln M_t = (1 - .45B)a_{mt}, \hat{\sigma}_m = 2.5\% \quad (16.5)$$

(.16)

where  $a_{yt}, a_{pt}, a_{mt}$  are uncorrelated in all senses except for a contemporaneous correlation between  $a_{yt}$  and  $a_{pt}$  of .78(.19).

This model is estimated with the Exact Maximum Likelihood criterion and implementation developed by Mauricio(1995, 1996, 1997) and the values in ( ) are the estimated large sample standard errors. Though details are not reported here, the estimation situation is well defined and diverse diagnostic checks reveal no misspecification. It should be noted that a common step effect in 1977 estimated at 4.5%, is also, in fact, included in both (16.1) and (16.2), to account for an extreme value in  $\nabla \ln P$  that is known to be due to a massive price-control liberalization that

was instituted immediately following the first democratic elections; the results on structure and relationship parameters do not depend on this at all.

The NM hypothesis is imposed in (16.1)-(16.2). To check it, the model is estimated with all specifications maintained, except with the  $(\omega_0 - \omega_1 B) \nabla$  forms of the relations to  $\ln M$  replaced by the more general  $\omega_0' - \omega_1' B - \omega_2' B^2$  and the hypothesis NM ( $\omega_0' - \omega_1' - \omega_2' = 0$ ) is evaluated. This hypothesis is found to be clearly consistent with this data.

Note, however, that the SNM hypothesis is found to be roundly inconsistent with the data, because the hypotheses  $g_y = 0$  and  $g_p = 0$  are both easily rejected. In this connection, it is interesting to solve for the implicit transfer-function-noise model of  $\ln Q$ :

$$\ln Q_t = \left( \begin{array}{ccc} .39 & + & .67B & + & .27B^2 \end{array} \right) \nabla \ln M_t + N_{yt} - N_{pt} \quad (17)$$

$\begin{array}{ccc} (.08) & (.11) & (.08) \end{array}$

$$\hat{g}_q = 1.33(.21)$$

The SNM hypothesis is rejected, though NM is acceptable and imposed. A step increase in  $\ln M$  will lead to higher output in the short runs of 0, 1, or 2 years, but the effect on output is zero for longer periods. Thus there is a reasonable reconciliation of keynesian and classical ideas.

Note that the output variable in (16.2) is the inverse of a variable interpretable as aggregate real cash balances (in terms of GDP). This variable, even after purging it of the effects of past monetary expansion, is I(2), not I(1) as occurs with the first output variable, velocity. The rate of inflation  $\nabla \ln P$ , even after purging it of the effects of past monetary expansion, is not stationary! This is a very important result, because it

says that, though monetary expansion matters in inflation, it is not the only nonstationary determinant of inflation in this case.

Model (16) has been used successfully in the forecasting operations for which it was designed. However, monitor operations and interpretation can be enriched by considering the interpretation of the contemporaneous correlation between  $a_{yt}$  and  $a_{pt}$ . Different identifying assumptions are possible, but it seems plausible to assume that  $P$  can influence  $Q$  within the year without contemporaneous feedback. This is, in the author's opinion, the most plausible hypothesis for identifying aggregate demand and supply relations, when and if one wants to use such concepts, because it reflects in an operational way the idea, basic to keynesian thinking, that the price level is fixed in the (shortest) short run.

This assumption leads to a structural model of two equations. The first is merely (16.2), which one might regard as an aggregate supply relationship. The second, solving for  $\ln Q$  under the identifying assumption,  $a_{yt} = .74 a_{pt} + a_t$ , can be written:

$$\ln Q_t = \left[ \frac{.26 + .04B}{1 - .7B} \right] \ln \frac{M_{t-1}}{P_t} + \left[ \frac{.39 + .48B - .36B^2 - .20B^3}{1 - .7B} \right] \nabla \ln M_t + N_t \quad (18.1)$$

for

$$(1 - .71B)(\nabla N_t + .022) = a_t, \quad \hat{\sigma}_a = 1.2\% \quad (18.2)$$

Equation (18.1) can be interpreted as an aggregate demand for money relation with a unit long run elasticity relative to real output and a negative unit long run semielasticity relative to the rate of monetary expansion. The latter is plausibly due to

the opportunity costs of money holding, which should be higher at a higher expected inflation rate, in the long run equal to the observed inflation rate, which receives unit long run effects of the monetary expansion rate as a consequence of (16.2).

It is seen that a hitherto central concept in monetary theory, the aggregate demand for money, though not necessary for the GTM, is by no means ruled out by it. In the empirical analysis itself, however, in this illustration, the GTM is more useful as a CM and the partial interpretation of results in terms of the aggregate demand for money requires an untestable hypothesis, interpreting a contemporaneous correlation, that is additional to the GTM.

In connection with the TCG study, it is worth observing that empirically relevant effects of the rate of monetary expansion are found in several other real variables, relative implicit deflators and shares in nominal GDP. Such effects are understood by the GTM, but are largely ignored by researchers whose most comprehensive CM is the long run aggregate demand for money and/or an unjustified belief in SNM.

## 5.2 Money supply mechanisms

Gonzalo(1996) analyzes monthly end-of-month data for 1964-1990 for the Spanish economy on monetary base ( $B$ ), bank liquid assets ( $A$ ), currency in the hands of the nonbank public ( $E$ ) and the four key kinds of bank deposits ( $D1$ ,  $D2$ ,  $D3$ ,  $D4$ ). The identity  $B \equiv A + E$  is satisfied in this data and the different measures of money holdings by the nonbank public are defined by  $M1 \equiv E + D1$ ,  $M2 \equiv M1 + D2$ ,  $M3 \equiv M2 + D3$  and  $M4 \equiv M3 + D4$ . The objective of the analysis is to find the relationships between these series in order to characterize monetary policy itself and its effects on this set of variables, this to improve forecasting both of these variables and

of others on which they may be expected to have effects.

Earlier analyses revealed that, in the years since 1973 when the Bank of Spain had claimed to be setting  $B$  to achieve previously announced target rates of growth for  $M3$  (1974-1983) or  $M4$  (1984-1990), the data do not reveal such a policy to have been active: errors at target do not follow a white noise process, nor do they drive the monetary base. If the official policy had been found to be active, then the relevant variable to identify with  $\sqrt{\ln M}$  in the GTM would have been the target  $M3$  (or  $M4$ ) expansion rate. Since the policy was not found to have been active,  $B$  is identified with the  $M$  concept in the GTM for Gonzalo's analysis and is taken to be set by monetary policy. Interest rates were, for the most part, set by decree and almost unvarying throughout much of this period; hence, their very infrequent changes and their effects are treated by intervention analysis, Box and Tiao(1975), but are largely irrelevant.

Monetary policy also sets a legal minimum reserves ratio for banks and the accounting conventions for bank reporting. These variables are changed very infrequently and hence are treated as deterministic. The data reveal that the Bank of Spain alters  $B$  to compensate or reinforce such changes, in reserve requirements and accounting rules, in ways that differ from one case to another. These movements in  $B$  are modeled and the remaining part of  $B$ ,  $B^*$ , is then the variable actually identified with  $M$  in the GTM. Monetary policy is thus characterized by the deterministic reserves requirement and accounting rules series and by the corrected base series,  $B^*$ , which is uncorrelated with them by construction.

The real  $Z$  variables of the GTM are, in this case, taken to be  $E/D1$ ,  $D2/D1$ ,  $D3/D1$ ,  $D4/D1$  and  $A/D1$ . All but the last describe the composition of the monetary portfolio of the nonbank public. The last is taken to describe the behaviour

of banks.

All five real series were checked for evidence of the effects of deterministic monetary policy variables. Such effects were found in  $A/D1$  only, and  $A^*D1$  stands for  $A/D1$  purged of these effects. All five real series were also analyzed to evaluate the effects of the statistical monetary policy variable,  $\nabla \ln B^*$ , and to check for feedback. No feedback was detected. No effects of  $\nabla \ln B^*$  were found in any of the real variables, with the important exception of  $A^*D1$ , which reacts positively and slowly to  $\nabla \ln B^*$ . Contemporaneous correlations between the real variables is the only relevant correlation found between them.

Gonzalo evaluates all the models described in three nonoverlapping subsamples designed to coincide with three potentially different policy and institutional regimes. No outstanding differences are found, except in the bank behavior relation, and the qualitative description given above is applicable in all three regimes.

The GTM is useful in this case, because it leads the researcher to check for effects of  $\nabla \ln B^*$ , and feedback to it, in the real variables that together compose the so-called money multipliers ( $M1/B$ ,  $M2/B$ ,  $M3/B$ ,  $M4/B$ ). No relationship is found with the real variables describing the behavior of the nonbank public and this seems plausible at the same time that it does not threaten the SNM hypothesis. However, very substantial effects of  $\nabla \ln B^*$  are found in the variable describing banking behavior,  $A^*D1$ , these are useful in practical forecasting and monitoring, and these violate the SNM hypothesis, implicit and probably unconscious in most existing studies of money supply mechanisms. The CM that largely dominates existing econometric practice on this subject imposes the very restrictive  $v_x(B) = 0$  hypothesis, which is found to be clearly inconsistent with the data of the illustration.

## 6. Comments on an alternative formulation in the recent literature

The literature of econometric studies of monetary phenomena is voluminous and far from well integrated. The GTM should prove useful in virtually any of the existing individual areas of research and also in the task of integration. The aggregate demand for money emerged in the first illustration of the last section and the potential relevance of the GTM for studies of sets of variables thought to be related by such a relation should be clear. The second illustration, based on Gonzalo(1996), offers a study of money supply mechanisms that employs the GTM to considerable effect in this second line of work.

Another large part of the contemporary literature of econometric analyses of monetary issues is that on neutrality testing. See Weber(1994), pp. 68-69, for a summary of over 25 papers over the last 25 years. One of these, Fisher and Seater(1993), FS in the following, appears to offer the most highly developed CM to be found in this literature to date; King and Watson(1992) offer a CM very similar to that by FS. FS appear to reach conclusions that make sense of many apparently contradictory results found in the preceding literature and FS are cited approvingly by other authors publishing recently in this literature; see, e.g., King and Watson(1992), Weber(1994) and Bullard and Keating(1995). A comparison of the FS CM with the GTM is thus considered in the following.

It should be recognized at the outset that the *motivation* for FS is narrower than that for the GTM; FS are concerned only with neutrality testing while the GTM attempts to offer a CM of use in any econometric analysis including an  $M$  variable. Most econometric work in the monetary field is dominated by static, deterministic theories extended by *ad hoc* dynamic and stochastic assumptions, these being *ad hoc*

because they are neither justified by any kind of economic theory nor by their generality of representation in mathematical or statistical terms. The GTM presents a general linear dynamic, stochastic formulation that is well aligned with current possibilities for empirical analysis *and* is free of all dependence on previous static, deterministic theories, though these are not excluded.

Neutrality is a useful testable hypothesis for the GTM, but not the central issue, which is to facilitate thinking in interaction with data analysis in empirical model building when an *M* variable is present. In practice, testing NM is of very little interest, because NM cannot apparently be convincingly rejected. One may often use (impose) NM to help obtain an adequate parameterization, check NM by relaxing its restrictions within that parameterization, find NM to be entirely acceptable and proceed. The author takes rejection of NM to suggest incompetent data analysis procedures and/or an inadequate CM. That is, in fact, his interpretation of the FS evidence against NM for U.S. real income in 1869-1975.

A second point of difference between the GTM and FS is *dimension*. The FS CM is bivariate, including only the *M* variable and one other variable; King and Watson(1992), p.18, recognize and discuss this limitation of their CM. By contrast, the GTM is general multivariate and this constitutes relevant generality, because the econometric model builder needs to integrate results. There is not just one nominal quantity of money, one interest rate, one nominal price, one real output, etc. in a real world economy and, even if there were, the model builder needs to deal with them together. The existing neutrality testing literature seems to use only bivariate models and this is both conceptually incomplete and empirically inefficient. Left out variable biases are likely to abound. For example, FS report results favoring NM for U.S.

nominal income and prices in 1869-1975 but rejecting NM for real income in the same period, and their bivariate CM applied three times, with the three different "second" variables, does not force them to face this glaring contradiction, much less resolve it.

This contradiction is not even mentioned by Boschen and Otrok(1994), who present analyses of the relationship between real income and money to show that NM holds except for the anomalous 1930's, but ignore the data on nominal income and prices, as if nominal income, prices and real income were not related.

A third point for comparison is the *forms of nonstationarity allowed* by the GTM and the FS CM. FS exclude the forms of nonstationarity that arise in seasonality and all forms of regular and/or seasonal cointegration. The GTM excludes neither.

The *clarity (or lack of it) with which the behavioral distinction is drawn* constitutes a fourth point. The GTM consciously employs the independence assumption to draw a clear behavioral distinction between the agents who hold money and those who create it. FS do not assume contemporary independence ( $\sigma_{uw} = 0$  in their notation) until they discuss identification. But the drawing of clear behavioral distinctions is a matter of theory *per se* while identification is a matter of connecting theory with empirical analysis. FS multiply this lack of theoretical clarity by including the contemporary independence condition ( $\sigma_{uw} = 0$ ) as part of each of the four alternative identification hypotheses they consider. This is a point on which FS also offer useless generality.

A fifth issue concerns the *use of the distinction between real and nominal variables*. The GTM, with no loss of useful generality, assumes that all variables aside from the nominal quantity of money are real and defines explicitly what is meant when the terms "real" or "nominal" are used. FS allow nominal variables other than *M*,

which constitutes useless generality; they do not define the terms "real" and "nominal" at all.

FS do not recognize that rates of change of nominal variables are real and hence fail to recognize that SNM is a special monetary hypothesis in a sense that NM is not. FS exclude interest rates from the list of real variables when they define SNM. This is legitimate, but violates their pretense to "... derive implications that apply to *any* real variable." (Note 2, p.402, emphasis in original) as stated with respect to their nonexclusion of real-cash-balance variables. None of this leads them to recognize that NM is a highly general hypothesis while SNM is a highly special one.

A sixth point involves the *comparison of content*; see the FS Table 2 (p. 407). The GTM is so formulated that the logarithm of nominal money stock,  $X$ , is not stationary, but  $I(d)$ , integrated of order  $d$ , for  $d \geq 1$ . It is taken as axiomatic that the nominal quantity of money would not be of interest to economic research if it were stationary. That the FS CM does not exclude a stationary nominal quantity of money, and that FS along with other authors in the recent neutrality-testing literature discuss this case very earnestly, appears to be an example of research in the vast world of "true but irrelevant" propositions. Once the stationary  $X \equiv \ln M$  case is excluded, FS and the GTM (for  $n = 2$ ) coincide exactly with respect to the NM hypothesis.

FS fail to recognize the contradiction between SNM and NM in the case of real variables  $\nabla \ln P$  formed from a nominal variable  $P$ . This flaws the SNM part of their Table 2. When such variables are excluded, FS and the GTM (for  $n = 2$ ) give the same results when the order of integration of  $X$  is one or two, except that the GTM produces a testable specification when this order is one but the FS long run derivative (LRD) is undefined in this case.

The GTM excludes a stationary nominal quantity of money at the outset and denies generality to the SNM hypothesis. In the area remaining, FS and the GTM (for  $n = 2$ ) do not differ.

Most of the critical issues raised above with respect to the FS paper are directly applicable to many other papers in the neutrality testing literature or even to wider literatures. The FS paper, however, requires one specific comment, this regarding the LRD concept the authors claim to introduce. The FS LRD, whenever it is defined, is nothing more than the long run gain for a given LTF, a very old and by now standard concept in both time series analysis and econometrics, though it has appeared with different names in different literatures. The FS LRD is, furthermore, not defined in a number of contexts in which the long run gain is defined. That someone might misuse the long run gain concept or that many econometricians may have so misused it in the past, in these latter contexts, is no justification for the FS pretense of having invented something new. In any case, the long run gain is a property of a relationship and a relationship is conceptually independent of the behaviour of its input variable. To claim otherwise, as do FS and some others in the neutrality testing literature, is simply wrong.

## 7. Concluding remarks

Readers familiar with the literature of time series analysis will undoubtedly be aware that there are many directions in which the GTM proposed in this paper is less general than a theorist could conceive. Forms of nonlinear relationships or nonlinear stochastic error processes, forms of error heteroskedasticity, periodic forms of relationship and periodic forms of stochastic error are only a few of the long list of possible theoretical extensions. The relevant issue, however, is whether any of these

ideas is likely to be empirically fruitful. The author doubts that most of the conceivable extensions of the GTM will be found to be empirically relevant for a long time.

There is no point in this paper at which the term "money" is defined. This is left to the empirical researcher who uses the GTM. In fact, one might well consider this GTM to be a General Theory of Nominal Scale, because it is conceivable for the theory to be useful even when the variable taking the role of  $M$  is not, in fact, a plausible measure of money.

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