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The optimal sequential information acquisition structure: A rational utility-maximizing perspective

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ABSTRACT

We consider a rational utility maximizer decision maker (DM) who must gather two pieces of information from a set of multidimensional products before making a choice. We analyze the resulting sequential information acquisition process where the DM tries to find the best possible product subject to his information acquisition constraint. In addition, we introduce publicly observable signals that allow the DM to update his expected utility functions following a standard Bayesian learning rule. Even though it seems intuitively plausible to assume that the transmission of positive and credible information may lead DMs to accept any product signaled more eagerly, this paper illustrates how transmitting credible positive information is not sufficient to decrease the rejection probability faced by the information sender on its set of products. A significant difference in product rejection probabilities arises depending on the characteristic on which signals are issued, as will be illustrated numerically for both risk-neutral and risk-averse DMs.

1. Introduction and literature review

The current paper formalizes and studies the optimal information acquisition behavior of a rational decision maker (DM) when choosing among multidimensional products defined by vectors of characteristics. We analyze the case where the decision process is based on the possibility of collecting two pieces of information. The proposed model accounts for the assimilation of publicly observable *credible* signals through a standard Bayesian learning setting. The introduction of signals within a multi-dimensional information acquisition framework allows us to account explicitly for the effects that different risk attitudes, number and type of signals have on the optimal information acquisition behavior of DMs. Signals will be introduced both on the first and the second characteristic spaces defining a product in order to intuitively differentiate the role played by observations from that played by expectations in the formation of DMs' acceptance/rejection probabilities.

Economists have tried to explain why the introduction of newly developed technologies takes such a long time to be diffused [assimilated] among the population, see [1,2] for a review of the literature. This is the case, since it seems intuitively

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plausible to assume that once a DM recognizes a new product as an improvement over a previous version, he should immediately purchase it [if we abstract from considerations regarding excessive prices and other market frictions]. This is also the main idea behind the operational research literature, where the dynamic programming models employed to analyze the introduction of technological innovations rely on the *ad hoc* assumption that DMs recognizing these improvements will immediately purchase the corresponding product, see [3,4]. It should be highlighted that Smith and Ulu [3] constitutes a partial exception to this point, as they also obtain delays in the adoption of technologically superior products when repeated purchases are allowed among DMs. This research line remains however focused on the importance that search costs have in limiting the information processing capacity of, generally risk-neutral, DMs when deciding whether to continue or stop their search within settings defined by the adoption of a given technology, see, for example, [5].

The literature analyzing the speed of product updates concentrates traditionally on the cost effects for the firm, such as product cannibalization and product development costs, and the resulting firm's incentives regarding optimal new-product introduction timing and product-quality decisions, see [6,7] for a duopolistic case, and [8]. It is generally the supply side of the system the one considered when studying whether a technologically superior product should be introduced in the market. If the introduction of a new and improved version of a product is delayed, it is the supplier the one that may decide to do so due to the already mentioned time inconsistency problem that causes its old and new products to cannibalize each other, see [9] for a monopolistic case. This is assumed to be the case even if the underlying technologies required for the development of the new product are already available.

As Mallik and Chhajed [10] state, once a premium product is introduced, the valuations of the consumers change. Similarly to the previous papers, these authors discuss the conditions under which a firm should expand its product line and study the effects of cost savings and customer valuation changes. Once again, this type of approach imposes a firm's perspective on the product acquisition process, as seems to also be the case when considering dynamic advertising models under a wide range of market competition structures, see [11] for a literature review. That is, these models assume that as soon as DMs recognize the technological superiority of the new products being introduced they will readily purchase them or at least do so with a higher probability. As a result, it is generally assumed that delaying the introduction of a new [technologically superior] product translates into a failure to capitalize on customer willingness-to-pay for the improved technology. Moreover, there is empirical evidence illustrating how delays have a statistically significant negative effect on profitability [via returns on the assets] of the corresponding firms, see [12].

Information scientists have started to take an interest in the strategic consequences that the process of information transmission has for the information acquisition of and subsequent choices made by DMs. However, the corresponding literature seems to concentrate on the long run stability properties of the population distributions that follow from the survival rates defined in different agent-based models, see [13]. In this regard, the immediate effects that follows from the limited capacity of DMs to assimilate information when considering multiple characteristics are rarely analyzed. That is, DMs are assumed to purchase information continuously and assimilate it rationally. Consequently, we tend to believe that any product improvement constitutes an increase in the purchase probability if it is credible and rationally assimilated by DMs. The current paper illustrates how, even in the simplest, thought non-trivial, scenario, this is not necessarily the case and how the characteristic on which positive and credible information is issued constitutes an important determinant of the acceptance/rejection probability defined by DMs.

We will bias our notation towards the economic side of the literature and express the information acquisition process of DMs in utility terms. This notational choice is made to differentiate our model from the value functional forms required by the dynamic programming and operations research literatures. Due to formal reasons that will become evident below, the current model cannot be defined in the standard dynamic programming terms commonly considered within these branches of the literature. That is, the information acquisition process described through the paper should be redefined after each observation is gathered by the DM and recalculated in terms of *all previously observed variables, their sets of possible combinations and corresponding expected payoffs*. In this sense, adding dimensions in the form of additional observations [based on the general environment introduced in the next section] and calculating the resulting strategic possibilities should constitute immediate extensions of the current paper.

Intuitively, the small dimensionality of the model could be justified to some extent in terms of the satisficing capacity constraints defined by Simon [14] within a fully rational environment. Similarly, the value of information could be used to impose a limit on the number of observations acquired, see [15], and strategic considerations in terms of information costs could be addressed in future extensions of the paper. Moreover, the literature usually concentrates on a small number of attributes when describing the products available to DMs, i.e., quality and preference in the consumer choice environment of [16], performance and cheapness in the economic setting of [17], and variety and quality in the operational research one of [18].

We provide some additional intuition regarding the constraints imposed on the number of observations acquired in the following subsection.

1.1. Motivation

We illustrate how the willingness of DMs to purchase a newly introduced product depends on their degree of risk aversion and the unobserved characteristic on which signals are issued. In doing so, we reach the same type of conclusion as [19,20,4] regarding the effect that first order stochastic dominance improvements on expected revenue/utility have on the incentives of DMs to adopt a new technology. That is, signaling the development of a more advanced technology does not necessarily lead to faster adoption. We show how a positive *credible* signal on the development of a technologically superior product leads to higher acceptance or rejection probabilities depending on the characteristic being improved.

Our model is however in sharp contrast with those developed within the economic and operational research branches of the literature, which, as stated above, remain focused on the supply side when analyzing the introduction of new (technologically superior) products in the market. This is still the case even though it is known that the amount of information available on and novelty of the product under consideration have a direct effect on the demand that reflects immediately on the supply side of the market, see [21]. The formal approach followed in this paper allows us to calculate subjective probabilities of acceptance for a product based on any degree of risk aversion of DMs while also being able to adapt the behavior of DMs to any number and type of signals issued by the firms composing the market. As we will illustrate in Section 6, accounting explicitly for the complexity of the information acquisition process generates a considerable amount of potential scenarios and conclusions that are not reachable when following a standard supply approach.

It may initially seem that allowing DMs to acquire only two observations constitutes a substantial constraint on the set of information available. However, as will be emphasized later, these observations do not necessarily account for a unique property of the product, but a series of them whose combination defines a characteristic element endowed with a given subjective probability function. For example, when considering the purchase of a smartphone, the first characteristic could be defined as portability and consist of the average of weight, size, and battery life, while the second characteristic set may include screen resolution, applications, and connectivity and be defined as manageability. In any case, it can be argued that there is plenty of information on the characteristics of smartphones available online. Indeed, one may also say that there is too much information available. Thus, given the considerable amount of free information available, one may ask why DMs consider only two sets of characteristics from a product. In order to provide some intuition in this respect, we approach this constraint from two different perspectives.

First, we refer the reader to the literature on information management. Several papers illustrate how, when facing large amounts of information, DMs tend to cut short their information acquisition and processing, see [22], and to reduce the costs of cognitive effort rather than maximize accuracy, see [23]. In particular, as the time and effort required to complete a task increases, it has been shown that DMs tend to reduce information search at the expense of decision quality, see [24,25].

Second, we consider the consumer psychology side. For example, Chen et al. [26] elaborate on the theory of information overload, which predicts that, beyond a threshold, more information leads to worse quality of the buying decisions. These authors conclude that this problem persists despite the existence of information filtering tools and online shopping experience. In a more direct relation to our constraint, Sanbonmatsu et al. [27] illustrate through several experiments that when DMs make decisions based on incomplete or limited knowledge of the relevant attributes they tend to overvalue the information available. These authors also show that this effect is not necessarily sensitive to the amount of information acquired.

Furthermore, DMs are limited in their capacity to process information and make complex decisions, which links the consumer psychology literature directly to the managerial evidence described above. Decisions consist of many different mental activities, including the weighting of attributes, the retrieval of information and the resulting comparisons between attributes, all of which deplete the neural and cognitive process of DMs, see [28]. This depletion, together with the peculiarities of the information acquisition processes taking place in online environments, limit the capacity of DMs to acquire and process large amounts of information, see [29].

The current market for smartphones constitutes an example of a market whose products are subject to a substantial amount of technological improvements, with Apple and Samsung acting as a potential duopoly, see [30]. Even though we will return to this specific example later in the paper, we should emphasize that, for example, in the iPhone case, the amount of netnographic information available to DMs is substantial, varied and highly subjective, see [31,32]. As a result, we will be assuming uniform densities on the distribution of characteristics through the paper, which allow for the highest information entropy (lower information content) among DMs. This reflects the subjective uncertainty generated by the information overflow to which DMs are subject. Moreover, technological improvements will be defined in terms of second order stochastic dominance over the original uniform distribution so as to provide a reference point with respect to the original uncertain situation.

The paper proceeds as follows. Section 2 deals with the standard notation and basic assumptions needed to develop the model. Section 3 defines the expected search utility functions, while Sections 4 and 5 introduce the Bayesian learning process and study the behavior and properties of the corresponding search utilities. Section 6 illustrates numerically the results obtained. Section 7 summarizes the main findings, highlights their managerial significance and suggests further extensions.

2. Basic notations and main assumptions

The notations and initial assumptions of our model are those of Di Caprio and Santos Arteaga [33]. However, for the sake of completeness, this section partially reproduces some formal definitions and related comments already included in Section 2 (Preliminaries and basic notations) and Section 3 (Main assumptions) of [33].

Definition 2.1 (*Mas-Colell et al.* [34]). Let X be a nonempty set. A binary relation $\succcurlyeq$ on X, satisfying reflexivity, completeness and transitivity, is said *preference relation*. A preference relation $\succcurlyeq$ on X is said to be *representable by a utility function* if there exists a function u: $X \to \mathcal{R}$ such that:

$$\forall x, y \in X, \quad x \geq y \Longleftrightarrow u(x) \geqslant u(y). \tag{1}$$

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Henceforth, G will denote the set of all products about which the DM can gather information. Also, fix $n \in \aleph$.

Definition 2.2. For every $i \leqslant n$, let X_i be a nonempty set. X_i will be called the *ith characteristic factor*. The Cartesian product $X = \prod_{i \leqslant n} X_i$ will be referred to as the *characteristic space*.

For every $i \leqslant n$, X_i represents the set of all possible variants for the ith characteristic of a generic product belonging to G. Consequently, every product in G can be described by an n-tuple $\langle x_1, ..., x_n \rangle$ in X.

As Di Caprio and Santos Arteaga [33], we follow the classical approach to information demand by economic agents proposed by Wilde [35]. Accordingly, we restrict our attention to the case where each X_i is a compact and connected non-degenerate real subinterval of $[0, +\infty)$. Consequently, following [33], we work under the following assumptions:

Assumption 1. For every $i \leqslant n$, $X_i = [x_i^m, x_i^M]$, whith $x_i^m, x_i^M > 0$ and $x_i^m \neq x_i^M$. The topology on X_i is the Euclidean topology, while the preference relation on X_i is the standard linear order > of $\mathcal{R}$.

Assumption 2. The characteristic space X is endowed with the product topology τ_p and a strict preference relation $\succ$.

Assumption 3. There exists a continuous additive utility function u representing $\succ$ on X such that each one of its components u_i: $X_i \to \mathcal{R}$, where $i \leqslant n$, is a continuous utility function on X_i.

The utility function u: $X \to \mathcal{R}$ being *additive* (Wakker [36]) means that, for every $i \leqslant n$, there exists u_i: $X_i \to \mathcal{R}$, such that $\forall \langle x_1, ..., x_n \rangle \in X$, $u(\langle x_1, ..., x_n \rangle) = u_1(x_1) + \cdots + u_n(x_n)$.

Assumption 4. For every $i \leqslant n$, μ_i: $X_i \to [0, 1]$ is a continuous probability density on X_i, whose support, the set $\{x_i \in X_i$: $\mu_i(x_i) \neq 0\}$, will be denoted by $Supp(\mu_i)$.

The probability densities $\mu_1, \ldots, \mu_n$ represent the subjective "beliefs" of the DM. For $i \leqslant n$, $\mu_i(Y_i)$ corresponds to the subjective probability assigned by the DM to the event that an element $x_i \in Y_i \subseteq X_i$ is the value of the ith characteristic of a randomly observed product from G. The probability densities $\mu_1, \ldots, \mu_n$ are assumed to be independent. However, the current model allows for subjective correlations to be defined among different characteristic within a given product. It should be noted that each observation acquired may not necessarily consist of a unique property of the product under consideration, but a series of them whose combination defines a characteristic element within X_i, and whose average is distributed according to a given μ_i probability function, with $i \leqslant n$. That is, the characteristic spaces could be assumed to represent average numerical valuations of, for example, design requirements and parts characteristics, complementing models such as the fuzzy linear programming one of [37].

Definition 2.3 (*Mas-Colell et al.* [34]). For every $i \leqslant n$, the *certainty equivalent of* μ_i *and* u_i, denoted by ce_i, is a characteristic in X_i that the DM is indifferent to accept in place of the expected one to be obtained through μ_i and u_i. That is, for every $i \leqslant n$, $ce_i = u_i^{-1}(E_i)$, where E_i denotes the expected value of u_i .

Since each u_i is assumed to be strictly increasing and continuous on X_i, ce_i is the unique element of X_i such that $ce_i = u_i^{-1}(E_i)$.

3. Expected search utilities

The set of all products, G, is identified with a compact and convex subset of the n-dimensional real space R^n. In the simplest non-trivial scenario, G consists of at least two products and the DM is allowed to collect two pieces of information, not necessarily from the same product. The DM uses the first observation to check the first characteristic from any of the products in G. After checking the value of the first characteristic from an initial product, the DM must decide whether to keep acquiring information on the same product and, hence, check the value of its second characteristic, or start collecting information on a new product.

Di Caprio and Santos Arteaga [33] show that the choice between continuing with the initial product and starting with a new one relies on the comparison between the values taken at the observed point x_1 by two real-valued functions defined on X_1. These functions, denoted by F and H, are defined by considering the expected utility values that the DM computes as the potential payoffs derived from his information acquisition process. The reference value for the DM's calculations is the sum $E_1 + E_2$, that is, the sum of the expected utility values defined by the pairs $\langle u_1, \mu_1 \rangle$ and $\langle u_2, \mu_2 \rangle$. The function F provides the DM's expected utility value associated with the option "continuing with the initial product". The function H allows the DM to evaluate the expected utility value associated with the option "starting with a new product". For the sake of completeness, we reproduces the definition of F and H.

The function F: $X_1 \to \mathcal{R}$ is defined by:

$$F(x_1) \stackrel{def}{=} \int_{P^+(x_1)} \mu_2(x_2)(u_1(x_1) + u_2(x_2))\,dx_2 + \int_{P^-(x_1)} \mu_2(x_2)(E_1 + E_2)\,dx_2. \quad (2)$$

The variable x_1 is given by the value of the first characteristic from the initial product already observed. The integration sets

$$P^+(x_1) = \{x_2 \in X_2 \cap Supp(\mu_2) : u_2(x_2) > E_1 + E_2 - u_1(x_1)\} \tag{3}$$

and

$$P^-(x_1) = \{x_2 \in X_2 \cap Supp(\mu_2) : u_2(x_2) \leqslant E_1 + E_2 - u_1(x_1)\}. \tag{4}$$

contain all the values x_2 that the second characteristic of the initial product should have in order to deliver either a higher utility than a random product from G, see Eq. (3), or a lower-equal utility than a random product from G, see Eq. (4).

Suppose that, after the value x_1 has been observed as the first characteristic for the initial product, the DM chooses to continue acquiring information on the initial product. Then, if $u_1(x_1) + u_2(x_2) \leqslant E_1 + E_2$, choosing a product from G randomly delivers a higher expected utility to the DM than choosing the initially observed product.

The function H: $X_1 \to \mathcal{R}$ is defined as follows:

$$H(x_1) \overset{def}{=} \int_{Q^+(x_1)} \mu_1(y_1)(u_1(y_1) + E_2)\,dy_1 + \int_{Q^-(x_1)} \mu_1(y_1)(\max\{u_1(x_1), E_1\} + E_2)\,dy_1. \tag{5}$$

The variable x_1 is still given by the value of the first characteristic from the initial product already observed. The integration sets

$$Q^+(x_1) = \{y_1 \in X_1 \cap Supp(\mu_1) : u_1(y_1) > \max\{u_1(x_1), E_1\}\} \tag{6}$$

and

$$Q^-(x_1) = \{y_1 \in X_1 \cap Supp(\mu_1) : u_1(y_1) \leqslant \max\{u_1(x_1), E_1\}\}. \tag{7}$$

contain all the values y_1 that the first characteristic of a new product should have in order to deliver either a higher utility than that of both a random product from G and the initial (partially observed) product, see Eq. (6), or a lower-equal utility than that of either a random product from G or the initial (partially observed) product, see Eq. (7).

Suppose that, after the value x_1 has been observed as the first characteristic for the initial product, the DM chooses to start acquiring information on a new product. Then, if, for instance, $u_1(y_1) \leqslant \max\{u_1(x_1), E_1\}$, then the DM would be better off choosing between the initial (partially observed) product and a randomly chosen one.

Note that the domain of both F and H is the support of μ_1.

The process of information acquisition is clearly determined by the expected utility functions F and H. Consider, for example, the case where the DM observes x_1 from an initial product. Then, the decision between continuing acquiring information on this product or shifting to a new one is based on the value taken by the functions F and H at x_1. It may also happen that both functions cross at a given point. At the crossing point, the DM should be indifferent between continuing acquiring information on a product and switching to a different one. These crossing points between both functions define optimal thresholds in the sequential information acquisition process of DMs. As a result, X_1 will be partitioned in subintervals that determine whether the DM continues acquiring information on the first product observed or switches to a different one. Di Caprio and Santos Arteaga [33] illustrate how the existence of optimal threshold values in the DM's information acquisition process can be guaranteed under common non-pathological assumptions. For example, it can be easily shown that $H(x_1^M) \leqslant F(x_1^M)$, with $H(x_1^M) = F(x_1^M)$ if and only if $u_1(x_1^M) + u_2(x_2^m) \geqslant E_1 + E_2$. Therefore, $u_1(x_1^M) + u_2(x_2^m) < E_1 + E_2$ suffices to guarantee the existence of at least one threshold value whenever $P^+(x_1^m) = \emptyset$.

4. Signals and learning

The analysis performed through this and the next section assumes that signals are issued on X_2. It should be noted that we will also be considering signals on X_1 through the numerical simulations' section. However, the formal analysis that must be performed to study the effect of credible X_1 signals on the information acquisition behavior of DMs is almost identical to that performed on X_2. Thus, we focus our attention on X_2 since it provides a more intuitive approach to the role played by expectations as opposed to immediate observations, which would constitute the intuitive focus of X_1. With this remark in mind, consider a standard uniform density function defined on the second characteristic space

$$\mu_2(x_2) = \begin{cases} \frac{1}{\beta - \alpha} & \text{if } x_2 \in [\alpha, \beta], \\ 0 & \text{otherwise.} \end{cases} \tag{8}$$

We will assume that receiving a credible positive signal, θ, regarding the distribution of characteristics on X_2 implies that the probability mass accumulated on the lower half of the distribution halves. At the same time, the probability mass eliminated from the lower half of the distribution is shifted to the upper one. Thus, given the distribution of X_2 characteristics defined by $\mu_2(x_2) = \frac{1}{\beta-\alpha}$ for $x_2 \in [\alpha, \beta]$, with $\alpha, \beta \geqslant 0$ and $\alpha < \beta$, the corresponding conditional density function is given by

$$\pi(\theta|x_2) = \begin{cases} \frac{3}{2(\beta-\alpha)} & \text{if } x_2 \in (\frac{\alpha+\beta}{2}, \beta], \\ \frac{1}{2(\beta-\alpha)} & \text{if } x_2 \in [\alpha, \frac{\alpha+\beta}{2}]. \end{cases} \tag{9}$$

After receiving a positive signal, rational DMs update their initial beliefs, given by $\mu_2(x_2)$, following Bayes' rule. Therefore, if a signal is received, i.e. $\theta = 1$, the updated beliefs of DMs will be given by

$$\mu_2(x_2|\theta = 1) = \frac{\pi(\theta|x_2)\mu_2(x_2)}{\int_{X_2} \pi(\theta|x_2)\mu_2(x_2)dx_2}. \tag{10}$$

Similarly, a second signal, providing DMs with the same qualitative information, i.e. $\theta = 2$, would lead to a second Bayesian updating process and a new distribution of beliefs on X_2

$$\mu_2(x_2|\theta = 2) = \frac{\pi(\theta|x_2)\mu_2(x_2|\theta = 1)}{\int_{X_2} \pi(\theta|x_2)\mu_2(x_2|\theta = 1)dx_2}. \tag{11}$$

This process can be assumed to continue as rational decision makers keep on updating their beliefs using Bayes' rule after receiving further signals. The resulting effect that these signals have on the F and H functions, determining the information acquisition behavior of DMs, is studied in the following section.

5. Learning and search processes

This section analyzes the effect that observing a positive signal has on the behavior of the expected search utilities of rational decision makers. Consider the $F(x_1)$ function in the first place. Intuitively, one may infer that the updated Bayesian density $\mu_2(x_2|\theta = 1)$ should lead to a higher expected utility value on the X_2 interval, i.e. $E_{(2|\theta=1)} \geqslant E_2$, which should, at the same time, result in the set $P^+(x_1|\theta = 1)$ shrinking relative to $P^+(x_1)$. In addition, the type of positive signal received implies that $\mu_2(x_2|\theta = 1) \geqslant \mu_2(x_2)$ over the newly defined interval $P^+(x_1|\theta = 1)$. Thus, receiving a positive signal leads to a stricter $P^+(x_1)$ interval displaying a larger probability mass than its presignal counterpart. The intuitive effect of the signal on the $F(x_1)$ function remains ambiguous.

Regarding the $H(x_1)$ function, note that the $Q^+(x_1)$ and $Q^-(x_1)$ intervals do not depend on E_2 and are therefore not affected by the signal. However, the direct dependence of $H(x_1)$ on E_2 should lead to an upward shift of the function if the value of E_2 increases after the signal is received.

In order to verify the previous intuitive description a formal treatment of the signal effects on both the $F(x_1)$ and $H(x_1)$ functions is provided. The following definition and proposition, taken from [34], are required for the subsequent analysis.

Definition 5.1 (*Definition 6.D.1 in Mas-Colell et al.* [34]). The distribution F(·) first-order stochastically dominates G(·) if, for every nondecreasing function u: R → R, we have

$$\int u(x)\,dF(x) \geqslant \int u(x)\,dG(x). \tag{12}$$

Proposition 5.2 (*Proposition 6.D.1 in Mas-Colell et al.* [34]). *The distribution of monetary payoffs F(·) first-order stochastically dominates the distribution G(·) if and only if* $F(x) \leqslant G(x)$ *for every x.* Therefore, given the updated definition of both the F and H functions

$$F(x_1|\theta = 1) \stackrel{def}{=} \int_{P^+(x_1|\theta=1)} \mu_2(x_2|\theta = 1)(u_1(x_1) + u_2(x_2))\,dx_2 + \int_{P^-(x_1|\theta=1)} \mu_2(x_2|\theta = 1)(E_1 + E_{(2|\theta=1)})\,dx_2, \tag{13}$$

$$H(x_1|\theta = 1) \stackrel{def}{=} \int_{Q^+(x_1)} \mu_1(y_1)(u_1(y_1) + E_{(2|\theta=1)})\,dy_1 + \int_{Q^-(x_1)} \mu_1(y_1)(\max\{u_1(x_1), E_1\} + E_{(2|\theta=1)})\,dy_1 \tag{14}$$

and the corresponding $E_{(2|\theta=1)} = \int_{X_2} \mu_2(x_2|\theta = 1)u_2(x_2)\,dx_2$ value, the following propositions, whose respective proofs are presented below, can be stated:

Proposition 5.3. $F(x_1|\theta = 1) \geqslant F(x_1)$.

Proposition 5.4. $H(x_1|\theta = 1) \geqslant H(x_1)$.

Proof of Proposition 5.3. We will make use of the following Lemmas to illustrate Proposition 5.3. □

Lemma 5.5. *$\mu_2(x_2|\theta = 1)$ first-order stochastically dominates $\mu_2(x_2)$.*

Proof. Consider the basic learning model defined in the previous section. The decision maker, who is assumed to have a [subjective] uniform probability density function defined on X_2, receives a signal halving the probability mass allocated to the lower half density values while reallocating the subtracted mass uniformly among the upper half ones. That is, the initial uniform density, the signal received and the corresponding Bayesian updating rule defining the learning process of the decision maker are respectively given by

$$\mu_2(x_2) = \frac{1}{\beta - \alpha} \quad \text{if } x_2 \in [\alpha, \beta], \tag{15}$$

$$\pi(\theta|x_2) = \begin{cases} \frac{3}{2(\beta-\alpha)} & \text{if } x_2 \in \left(\frac{\alpha+\beta}{2}, \beta\right], \\ \frac{1}{2(\beta-\alpha)} & \text{if } x_2 \in \left[\alpha, \frac{\alpha+\beta}{2}\right], \end{cases} \tag{16}$$

$$\mu_2(x_2|\theta = 1) = \frac{\pi(\theta|x_2)\mu_2(x_2)}{\int_{X_2} \pi(\theta|x_2)\mu_2(x_2)dx_2}. \tag{17}$$

Thus, the following updated density results from applying Bayes' rule after receiving a signal

$$\mu_2(x_2|\theta = 1) = \begin{cases} \frac{\frac{3}{2(\beta-\alpha)^2}}{\frac{1}{(\beta-\alpha)}} = \frac{3}{2(\beta-\alpha)} & \text{if } x_2 \in \left(\frac{\alpha+\beta}{2}, \beta\right], \\ \frac{\frac{1}{2(\beta-\alpha)^2}}{\frac{1}{(\beta-\alpha)}} = \frac{1}{2(\beta-\alpha)} & \text{if } x_2 \in \left[\alpha, \frac{\alpha+\beta}{2}\right]. \end{cases} \tag{18}$$

Clearly, x_2 variants situated on the upper half of the distribution are assigned $\int_{\frac{\alpha+\beta}{2}}^{\beta} \mu_2(x_2|\theta = 1)\,dx_2 = \frac{3}{4}$ of the total density while those on the lower half are endowed with the remaining $\int_{\alpha}^{\frac{\alpha+\beta}{2}} \mu_2(x_2|\theta = 1)\,dx_2 = \frac{1}{4}$.

In order to illustrate the first order stochastic dominance of $\mu_2(x_2|\theta = 1)$ over $\mu_2(x_2)$ it must be shown that $\int_{\alpha}^{\beta} \mu_2(x_2|\theta = 1)\,dx_2 < \int_{\alpha}^{\beta} \mu_2(x_2)\,dx_2$, for all $x_2 \in [\alpha, \beta]$. Therefore, the resulting distribution functions must be calculated and compared for both codomains within $[\alpha, \beta]$

(i) Consider first $x_2 \in \left[\alpha, \frac{\alpha+\beta}{2}\right]$ codomain. Clearly, $\mu_2(x_2|\theta = 1)$ first order stochastically dominates $\mu_2(x_2)$ if and only if (Mas-Colell et al. [34]) $\forall x \in [\alpha, \beta]$.

$$\int_{\alpha}^{\frac{\alpha+\beta}{2}} \mu_2(x_2)\,dx_2 = \frac{x - \alpha}{2(\beta - \alpha)} > \int_{\alpha}^{\frac{\alpha+x}{2}} \mu_2(x_2|\theta = 1)\,dx_2 = \frac{x - \alpha}{4(\beta - \alpha)}. \tag{19}$$

This inequality is satisfied whenever $x > \alpha$. Thus, $\mu_2(x_2|\theta = 1)$ stochastically dominates $\mu_2(x_2)$ for every $x > \alpha$, that is, $\forall x_2 \in \left[\alpha, \frac{\alpha+\beta}{2}\right]$.

(ii). Consider now $x_2 \in \left[\frac{\alpha+\beta}{2}, \beta\right]$ codomain. Once again, $\mu_2(x_2|\theta = 1)$ first order stochastically dominates $\mu_2(x_2)$ if and only if $\forall x \in \left[\frac{\alpha+\beta}{2}, \beta\right]$

$$\int_{\frac{\alpha+\beta}{2}}^{x} \mu_2(x_2)\,dx_2 + \int_{\alpha}^{\frac{\alpha+\beta}{2}} \mu_2(x_2)\,dx_2 > \int_{\frac{\alpha+\beta}{2}}^{x} \mu_2(x_2|\theta = 1)\,dx_2 + \int_{\alpha}^{\frac{\alpha+x}{2}} \mu_2(x_2|\theta = 1)\,dx_2. \tag{20}$$

In the case under consideration the above inequality corresponds to

$$\frac{2x - \alpha - \beta}{2(\beta - \alpha)} + \frac{\beta - \alpha}{2(\beta - \alpha)} > \frac{6x - 3\alpha - 3\beta}{4(\beta - \alpha)} + \frac{\beta - \alpha}{4(\beta - \alpha)}, \tag{21}$$

which simplifies to

$$\frac{x - \alpha}{\beta - \alpha} > \frac{3x - 2\alpha - \beta}{2(\beta - \alpha)}. \tag{22}$$

This inequality is satisfied for every $x < \beta$, that is, $\forall x_2 \in \left[\frac{\alpha+\beta}{2}, \beta\right]$. Therefore, $\mu_2(x_2|\theta = 1)$ first-order stochastically dominates $\mu_2(x_2)$ for every $x_2 \in [\alpha, \beta]$.

A similar proof can be used to illustrate the first-order stochastic dominance of $\mu_2(x_2|\theta = 2)$ over $\mu_2(x_2|\theta = 1)$. □

Even though we have only verified the first-order stochastic dominance resulting from the signal for the uniform density case defined in the paper, the analysis could be generalized to any other density function whose probability mass is redistributed to generate higher expected utilities, refer to Chapter 6 in [34].

Lemma 5.5 together with Definition 5.1 imply directly that

Corollary 5.6. $E_{(2|\theta=1)} \geqslant E_2$.

The signal received affects the $F(x_1)$ function both through the new induced value of E_2 and the updated $\mu_2(x_2|\theta = 1)$ density. While the previous corollary describes the effect that signal-induced changes in the $\mu_2(x_2)$ density have on expected utilities, the following lemma concentrates on the effect that changes in the E_2 value have on $F(x_1)$ for a given constant

$\mu_2(x_2)$. If the latter effect is positive, then coupling a signal-based increment in E_2 with a first-order stochastic dominance spread on $\mu_2(x_2)$ would lead to an increase of the $F(x_1)$ function.

Lemma 5.7. $\left.\frac{dF(x_1)}{dE_2}\right|_{\mu_2(x_2)} > 0 \ \forall x_1 \in X_1$, *if and only if* $P^-(x_1^M) \neq \emptyset$

Proof. We start by expressing $F(x_1)$ as a function of E_2.

$$F(x_1) \overset{def}{=} \int_{u_2^{-1}(E_1+E_2-u_1(x_1))}^{x_2^M} \mu_2(x_2)(u_1(x_1)+u_2(x_2))\,dx_2 + \int_{x_2^m}^{u_2^{-1}(E_1+E_2-u_1(x_1))} \mu_2(x_2)(E_1+E_2)\,dx_2. \tag{23}$$

Applying Leibnitz's rule to the above definition while keeping $\mu_2(x_2)$ fixed allows us to isolate the effect that changes in the E_2 value have on the $F(x_1)$ function.

$$\begin{aligned}
\left.\frac{dF(x_1)}{dE_2}\right|_{\mu_2(x_2)} &= \int_{u_2^{-1}(E_1+E_2-u_1(x_1))}^{x_2^M} \frac{\partial}{\partial E_2}\left[\mu_2(x_2)(u_1(x_1)+u_2(x_2))\right]dx_2 + \left[\mu_2(x_2^M)(u_1(x_1)+u_2(x_2^M))\right]\frac{dx_2^M}{dE_2} \\
&- \left[\mu_2(u_2^{-1}(E_1+E_2-u_1(x_1)))(u_1(x_1)+u_2(u_2^{-1}(E_1+E_2-u_1(x_1))))\right]\frac{d}{dE_2}\left[u_2^{-1}(E_1+E_2-u_1(x_1))\right] \\
&+ \int_{x_2^m}^{u_2^{-1}(E_1+E_2-u_1(x_1))} \mu_2(x_2)\,dx_2 + \left[\mu_2(u_2^{-1}(E_1+E_2-u_1(x_1)))(E_1+E_2)\right]\frac{d}{dE_2}\left[u_2^{-1}(E_1+E_2-u_1(x_1))\right] \\
&- \left[\mu_2(x_2^m)(E_1+E_2)\right]\frac{dx_2^m}{dE_2} = 0 + 0 - \left[\mu_2(u_2^{-1}(E_1+E_2-u_1(x_1)))(E_1+E_2)\right]\frac{d}{dE_2}\left[u_2^{-1}(E_1+E_2-u_1(x_1))\right] \\
&+ \int_{x_2^m}^{u_2^{-1}(E_1+E_2-u_1(x_1))} \mu_2(x_2)\,dx_2 + \left[\mu_2(u_2^{-1}(E_1+E_2-u_1(x_1)))(E_1+E_2)\right]\frac{d}{dE_2}\left[u_2^{-1}(E_1+E_2-u_1(x_1))\right] - 0 \\
&= \int_{x_2^m}^{u_2^{-1}(E_1+E_2-u_1(x_1))} \mu_2(x_2)\,dx_2.
\end{aligned} \tag{24}$$

Therefore, increments in E_2 have a strictly positive effect on $F(x_1)$, $\forall x_1 \in X_1$, if and only if $P^-(x_1^M) \neq \emptyset$. □

Corollary 5.6 and Lemma 5.7 imply that if $P^-(x_1^M) \neq \emptyset$ and the signal received leads to $E_{(2|\theta=1)} > E_2$, then $F(x_1|E_{(2|\theta=1)}) > F(x_1)$. This result together with Lemma 5.5 provide the required conclusion, i.e. $F(x_1|\theta = 1) \geqslant F(x_1)$ for all x_1 values in X_1. In particular, if either $E_{(2|\theta=1)} \geqslant E_2$ or $P^-(x_1^M) = \emptyset$, or both, then

$$\begin{aligned}
&F(x_1|\theta=1) \overset{def}{=} \int_{P^+(x_1|\theta=1)} \mu_2(x_2|\theta=1)(u_1(x_1)+u_2(x_2))\,dx_2 + \int_{P^-(x_1|\theta=1)} \mu_2(x_2|\theta=1)(E_1+E_{(2|\theta=1)})\,dx_2 \geqslant \\
&F(x_1|E_{(2|\theta=1)}) \overset{def}{=} \int_{P^+(x_1|\theta=1)} \mu_2(x_2)(u_1(x_1)+u_2(x_2))\,dx_2 + \int_{P^-(x_1|\theta=1)} \mu_2(x_2)(E_1+E_{(2|\theta=1)})\,dx_2 \geqslant \\
&F(x_1) \overset{def}{=} \int_{P^+(x_1)} \mu_2(x_2)(u_1(x_1)+u_2(x_2))\,dx_2 + \int_{P^-(x_1)} \mu_2(x_2)(E_1+E_2)\,dx_2.
\end{aligned} \tag{25}$$

Proof of Proposition 5.4. The result follows directly from Corollary 5.6 and the fact that

$$\frac{dH(x_1)}{dE_2} = \int_{Q^+(x_1)} \mu_1(y_1)\,dy_1 + \int_{Q^-(x_1)} \mu_1(y_1)\,dy_1 = 1 > 0. \tag{26}$$

□

6. Numerical simulations

6.1. The evolution of the threshold values

This section presents several numerical simulations that illustrate the behavior of the threshold values as the number of signals received by DMs on the characteristics defining a given set of products increases. Decision theoretical models, mainly in their economic and operational research variants, tend to assume risk-neutral decision makers, a trend recently modified by information theorists, see [38]. Even though the analytical simplifications derived from a risk-neutral environment are substantial, the consequences are far from innocuous. Therefore, numerical simulations will be provided for both risk-neutral and risk-averse DMs.

Figs. 1 and 2 illustrates the zero, one, two, three and four signals cases, denoted by *ns*, 1*s*, 2*s*, 3*s* and 4*s*, respectively, and the evolution of the corresponding threshold values within a basic risk-neutral scenario.

Figs. 3 and 4 do the same within a risk-averse setting, where the utilities with which the DM is endowed has been shifted from a linear shape to a concave one. In all these figures the horizontal axis represents the set of x_1 realizations that may be observed by the DM, with the corresponding subjective expected utility values defined on the vertical axis and the certainty equivalents explicitly identified through a vertical line whenever possible without complicating the presentation.

Throughout this section, the reference risk-neutral case will be described by the following parameter values

- characteristic spaces: $X_1 = [5, 10]$, $X_2 = [0, 10]$;
- utility functions: $u_1(x_1) = x_1$, $u_2(x_2) = x_2$;
- probability densities, both continuous and uniform: $\forall x_1 \in X_1$, $\mu_1(x_1) = \frac{1}{5}$; $\forall x_2 \in X_2$, $\mu_2(x_2) = \frac{1}{10}$;

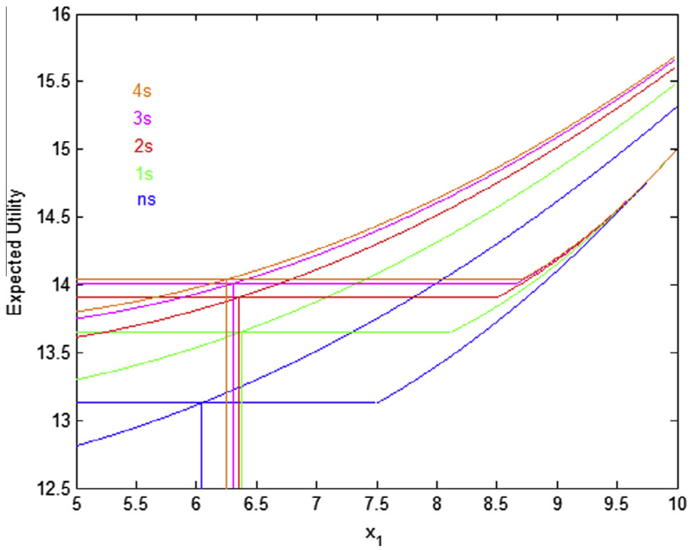

Fig. 1. Evolution of optimal threshold values given risk-neutral utility functions and uniform risk distributions. Threshold values: *ns* = 6.0355; 1*s* = 6.3725; 2*s* = 6.3460; 3*s* = 6.2909; 4*s* = 6.2647.

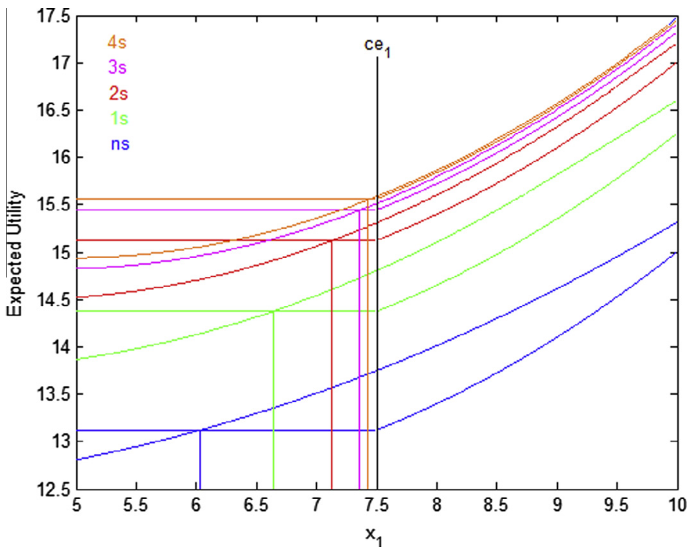

Fig. 2. Evolution of optimal threshold values given risk-neutral utility functions and uniform risk distributions. Threshold values: *ns* = 6.0355; 1*s* = 6.6368; 2*s* = 7.1352; 3*s* = 7.3673; 4*s* = 7.4544.

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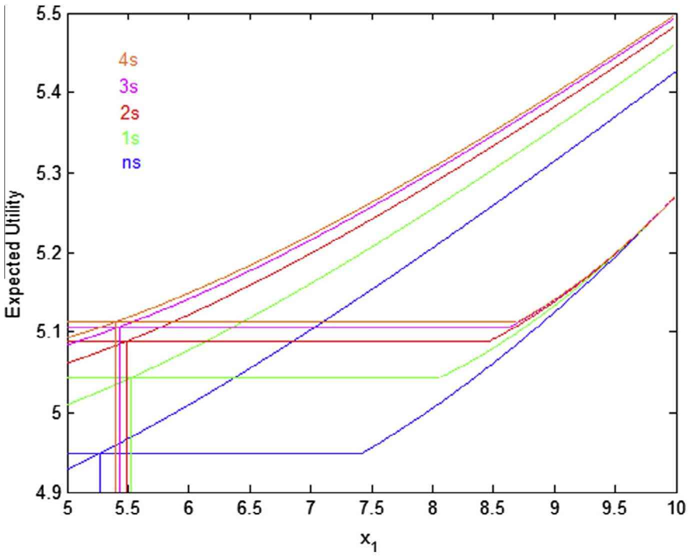

Fig. 3. Evolution of optimal threshold values given risk-averse utility functions and uniform risk distributions. Threshold values: *ns* = 5.2695; 1*s* = 5.5282; 2*s* = 5.4810; 3*s* = 5.4225; 4*s* = 5.3960.

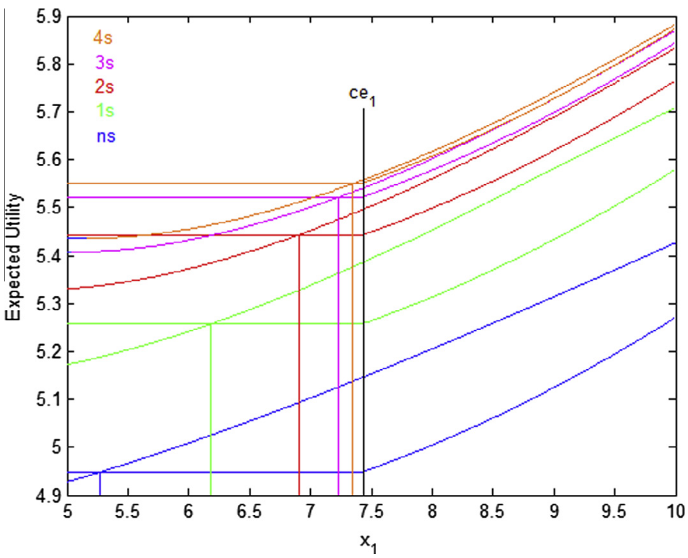

Fig. 4. Evolution of optimal threshold values given risk-averse utility functions and uniform risk distributions. Threshold values: *ns* = 5.2695; 1*s* = 6.1823; 2*s* = 6.8980; 3*s* = 7.2348; 4*s* = 7.3623.

while the risk-averse reference case will share the characteristic spaces and probability densities with the risk-neutral one, but its utility functions will be given by $u_1(x_1) = \sqrt{x_1}$ and $u_2(x_2) = \sqrt{x_2}$. There is not a particular formal reason to assume these functional forms or probability densities, besides the maximum information entropy provided by the latter ones. Modifying the functional values or the support of the densities does not affect the main results obtained. Refer to [33] for additional reference numerical examples providing further intuition on the unsignaled information acquisition behavior of DMs.

Clearly, positive signals generating first order stochastic dominant beliefs lead to higher expected utility levels for all possible x_1 values. This is the case independently of whether signals are issued on X_1 (Figs. 1 and 3) or X_2 (Figs. 2 and 4). However, as it is immediately clear when comparing the X_1 and X_2 settings, the effect that signals have on the resulting threshold values, which will be denoted by x_1^*, differs significantly. These simulations have been introduced to provide an intuitive description of the substantial behavioral differences generated by both signaling strategies. The main focus of the current analysis is however the rejection probabilities arising from both these signaling environments, which are described in the following subsection.

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6.2. Rejection probabilities

Given the behavior of the optimal threshold values identified in the simulations, a basic strategic structure can be defined to model the introduction of technologically superior products. In this case, the technological superiority of a set of products is reflected by the stochastically dominant distribution of either its X_1 or X_2 characteristics. The information acquisition incentives of DMs are determined by their expected utilities, which, at the same time, define the ability of firms to introduce the new products in a given market.

DMs will be reluctant to purchase a product whose expected utility value is lower than the one delivered by the certainty equivalent product defined within the corresponding signaled market. Given the value of x_1^* and the corresponding certainty equivalent values defined within each market, the following possibilities arise when a DM acquires information from a set of products: he either observes a product whose first characteristic is above x_1^* and continues acquiring information about and eventually purchases it or the characteristic observed is lower than x_1^* and he rejects the product. The process of information acquisition and the resulting purchase behavior of DMs are described in Fig. 5.

Thus, the rejection probabilities faced by a firm signaling the existence of a technologically superior product follow from the set of expected search outcomes attained by DMs within each potential market

$$\mu_1(x_1 < x_1^*|\theta = s) + \mu_1(x_1 > x_1^*|\theta = s)\,\mu_2(x_2 < x_2^*|x_1), \tag{27}$$

with x_2^* such that $u_1(x_1) + u_2(x_2^*) = E_{(1|\theta=s)} + E_{(2|\theta=s)}$, for each $s = 1, \ldots, 4$, which leads to

$$\mu_1(x_1 > x_1^*)\,\mu_2(x_2 < x_2^*|x_1) = \int_{x_1^*}^{x_1^M} \mu_1(x_1)\left[\int_0^{ce_{(1|\theta=s)}+ce_{(2|\theta=s)}-x_1} \mu_2(x_2|\theta = s)\,dx_2\right]dx_1, \tag{28}$$

for each $s = 1, \ldots, 4$. The resulting *initial rejection* [and acceptance] probabilities defined within each signaling scenario, i.e. $\mu_1(x_1 < x_1^*|\theta = s)$, with $s = 1, \ldots, 4$, are presented in Tables 1 and 2 for the risk-neutral and risk-averse cases, respectively.

Note that the initial rejection probabilities are much lower when signals are issued on X_1 than when they are issued on X_2. This is intuitively straightforward after observing the corresponding threshold values illustrated in Figs. 1–4. At the same time, Tables 3 and 4 introduce the *combined rejection* probabilities defined after two observations are acquired on a given product, i.e. $\mu_1(x_1 > x_1^*|\theta = s)\,\mu_2(x_2 < x_2^*|x_1)$, with $s = 1, \ldots, 4$, within a risk-neutral and a risk-averse setting, respectively.

Note the reverse in values with respect to the previous initial rejection probabilities in terms of the characteristic being signaled. The more lenient initial product acceptance behavior followed when signals are issued on X_1, relative to X_2, leads to a higher *combined* rejection probability when the product is fully observed.

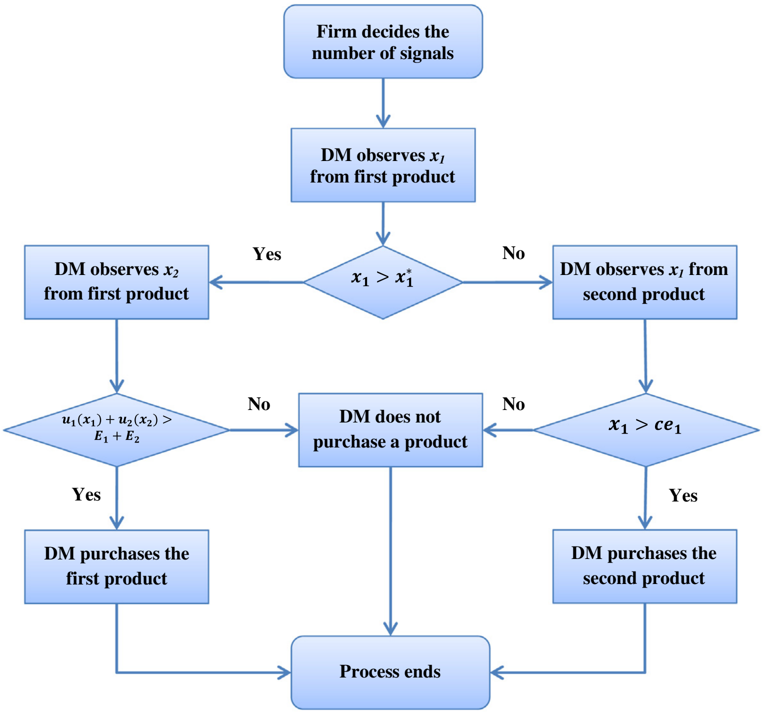

Fig. 5. Process of information acquisition and purchase behavior of DMs.

Table 1
Rejection || acceptance cumulative probabilities with risk-neutral-based threshold values: $\mu_1(x_1 < x_1^* | \theta = s) || \mu_1(x_1 > x_1^* | \theta = s)$, with $s = 1, \ldots, 4$.

Signals	X_1	X_2
ns	0.2071 || 0.7929	0.2071 || 0.7929
1*s*	0.1373 || 0.8627	0.3274 || 0.6726
2*s*	0.0538 || 0.9462	0.4270 || 0.5730
3*s*	0.0184 || 0.9816	0.4735 || 0.5265
4*s*	0.0062 || 0.9938	0.4909 || 0.5091

Table 2
Rejection || Acceptance cumulative probabilities with risk-averse-based threshold values: $\mu_1(x_1 < x_1^* | \theta = s) || \mu_1(x_1 > x_1^* | \theta = s)$, with $s = 1, \ldots, 4$.

Signals	X_1	X_2
ns	0.0539 || 0.9461	0.0539 || 0.9461
1*s*	0.0528 || 0.9472	0.2365 || 0.7635
2*s*	0.0192 || 0.9808	0.3796 || 0.6204
3*s*	0.0060 || 0.9940	0.4470 || 0.5530
4*s*	0.0019 || 0.9981	0.4725 || 0.5275

Table 3
Rejection probabilities after observing $x_1 > x_1^*$ such that the combination of x_1 and x_2 provides a lower utility than the CE product, risk-neutral case: $\mu_1(x_1 > x_1^* | \theta = s)\, \mu_2(x_2 < x_2^* | x_1)$.

Signals	X_1	X_2
ns	0.3554	0.3554
1*s*	0.3979	0.2273
2*s*	0.4579	0.1575
3*s*	0.4852	0.1349
4*s*	0.4950	0.1282

Table 4
Rejection probabilities after observing $x_1 > x_1^*$ such that the combination of x_1 and x_2 provides a lower utility than the CE product, risk-averse case: $\mu_1(x_1 > x_1^* | \theta = s)\, \mu_2(x_2 < x_2^* | x_1)$.

Signals	X_1	X_2
ns	0.4160	0.4160
1*s*	0.4128	0.2483
2*s*	0.4336	0.1654
3*s*	0.4421	0.1417
4*s*	0.4447	0.1347

The signaling rejection probabilities described in Eq. (27) are represented in Figs. 6 and 7 for the risk-neutral and risk-averse cases, respectively.

Note that the rejection probabilities faced by firms within a risk-averse environment are consistently lower than those faced within a risk-neutral one. Note also the substantial differences existing between both signaling strategies, with improvements on X_1 decreasing the rejection probability while those on X_2 increase it. The intuition behind this result is simple: the increase in the initial rejection probability due to the higher x_1^* values and the shifts in $\mu_1(x_1|\theta = s)$ is not compensated by the corresponding improvements in $\mu_2(x_2|\theta = s)$, with $s = 1, \ldots, 4$. Thus, even within the current heuristic decision setting and despite the credibility of the signals issued, a substantial difference in rejection probabilities is observed depending on the characteristic on which the signal is issued.

6.3. On technological races

Technological races dealing with the strategic introduction of technologically superior products within a given market structure, generally a duopoly, remain one of the main study areas in economics and, in particular, industrial organization, see [39]. However, the perspective generally considered is that of the supply side, while leaving the demand aside. As stated in the introduction, a similar remark applies to the operational research and management literatures. The timing and introduction of technologically improved products is entirely determined by the payoffs of the interacting firms. Assuming that

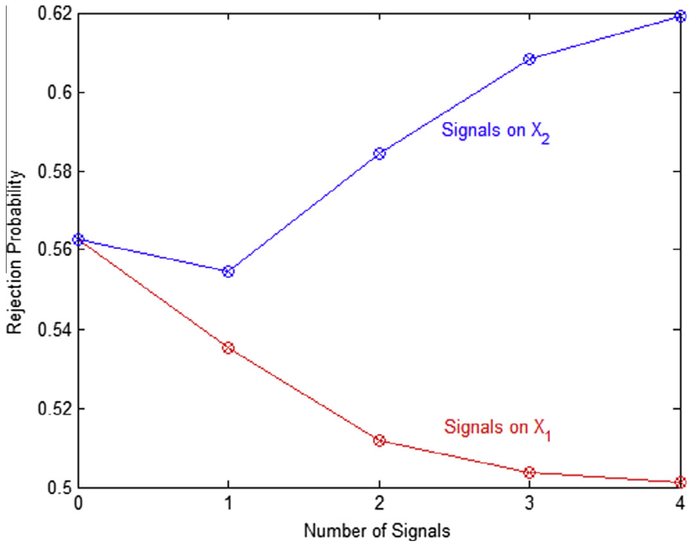

Fig. 6. Rejection probabilities with risk-neutrality when two observations may be acquired.

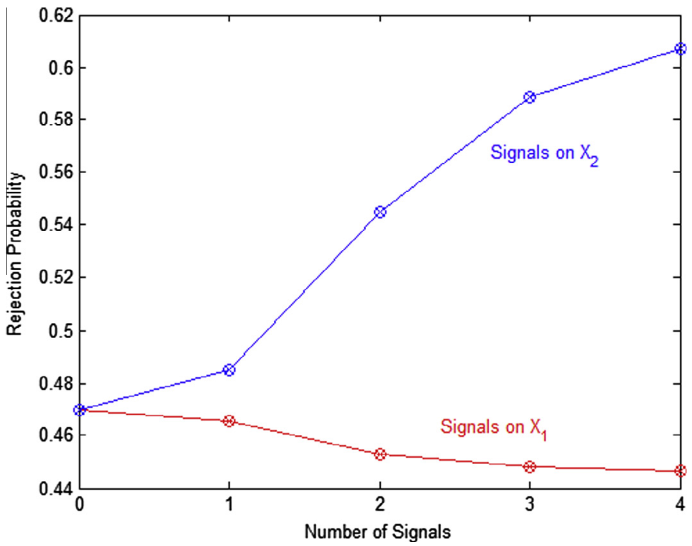

Fig. 7. Rejection probabilities with risk aversion when two observations may be acquired.

the improvement is acknowledged by DMs, these will accept the new product openly, leaving the laggard firm without a market to sell its obsolete products.

The current model has direct applications to both the economic and managerial branches of the literature from a strategic perspective. The numerical simulations provided in the previous subsection illustrate the incentives of a firm to introduce further improvements while enjoying a technological monopoly. This type of normative behavior justifies the continuous improvements observed in the main (most preferred) characteristics of the products defining monopolistic markets by the monopolist itself, see [40] for the iPod Nano case. That is, monopolists may improve a given subset of characteristics even if they have a considerable technological advantage over the laggard firms. It is therefore natural to consider the technological race faced by two firms when shifting from a duopoly to a technological monopoly. The smartphone market constitutes an example on which the current numerical simulations could be based, but others may be easily found such as the Windows versus Mac system upgrades, or the Boeing versus Airbus strategic developments of their airplanes.

The rejection probability faced by a monopolist when DMs acquire information on its product is given by Eq. (27). In this case, DMs check the product from the monopolist and purchase it if the realizations provide a utility higher than that of the certainty equivalent product. When dealing with a duopoly, DMs may check two different products before making a purchase. That is, after acquiring information on an initial product, DMs will obtain additional information about it if $x_1 > x_1^*$. If this is not the case, then the DMs will acquire information on a product from the rival firm and will acquire it if, and only if, it delivers a utility higher than that of the certainty equivalent product, i.e. iff $x_1 > ce_1$. The rejection probability faced by a firm competing within a duopoly is therefore given by

$$\frac{1}{2}\mu_1(x_1 < x_1^*|\theta = s) + \frac{1}{2}\mu_1(x_1 > x_1^*|\theta = s)\mu_2(x_2 < x_2^*|x_1) + \frac{1}{2}\mu_1'(x_1' > x_1^*|\theta = s) + \frac{1}{2}\mu_1'(x_1' < x_1^*|\theta = s)\mu_1(x_1 < x_1^*|\theta = s). \tag{29}$$

The tilde superscript refers to the rival firm. For simplicity, we will assume that both firms are identical and have an equal probability of being observed in the first place, i.e. $\frac{1}{2}$. Since both firms are assumed identical, we will also have that $\mu_1(x_1 < x_1^*|\theta = s) = \mu_1'(x_1' < x_1^*|\theta = s)$. These assumptions should be relaxed when studying potential extension arising from this game theoretical setting.

These rejection probabilities are introduced in Tables 5 and 6 for the risk neutral and risk averse cases, respectively. This setting suffices to generate a strategic environment accounting for the main characteristics of a non-trivial signaling market. Performing a formal analysis of the resulting signaling game and finding its set of Bayesian equilibria lies outside the scope of this paper. However, we provide some intuition on the complexities arising when considering explicitly the information acquisition behavior of DMs within the current environment.

The corresponding payoff matrices describing the transition incentives from a duopolistic environment to a monopolistic one are given by Figs. 8 and 9. These matrices describe the incentives of firms to signal the existence of a technologically superior product at different levels of improvement. The entrances of the matrices represent the rejection probabilities faced by the firms within the corresponding market environment. We concentrate on the risk averse setting, with signals issued on either X_1 orX_2.

Table 5
Rejection probabilities with two competing firms, risk-neutral case: Eq. (29).

Signals	X_1	X_2
ns	0.7295	0.7295
1*s*	0.7333	0.6955
2*s*	0.7424	0.6855
3*s*	0.7472	0.6858
4*s*	0.7491	0.6868

Table 6
Rejection probabilities with two competing firms, risk-averse case: Eq. (29).

Signals	X_1	X_2
ns	0.7211	0.7211
1*s*	0.7192	0.6816
2*s*	0.7215	0.6749
3*s*	0.7225	0.6794
4*s*	0.7228	0.6821

Player 2

Player 1	Signal	No Signal		
Signal	(0.7225, 0.7225)	(0.4481, 1)		
No Signal	(1, 0.4481)	(0.7215, 0.7215)	(0.4528, 1)	
		(1, 0.4528)	(0.7192, 0.7192)	(0.4656, 1)
			(1, 0.4656)	(0.7211, 0.7211)

Fig. 8. Market transition incentives with risk-aversion and signals on X_1.

Player 2

Player 1	Signal	No Signal		
Signal	(0.6794, 0.6794)	(0.5887, 1)		
No Signal	(1, 0.5887)	(0.6749, 0.6749)	(0.545, 1)	
		(1, 0.545)	(0.6816, 0.6816)	(0.4848, 1)
			(1, 0.4848)	(0.7211, 0.7211)

Fig. 9. Market transition incentives with risk-aversion signals on X_2.

The lower right (sub)matrix highlighted in blue corresponds to the transition from the zero-to-one signals scenario. In this case, if both firms are unable to signal they face a rejection probability of 0.7211 within their duopoly. There are two reasons why the rejection probability increases with competition. First, the characteristic required to be observed if the firm is checked in the second place shifts from x_1^* to ce_1. Second, firms must deal with the possibility of not having their product checked at all, which is a direct effect of the increment in competition.

The set of (sub)games ranges from the zero-to-one signals setting to the two-to-three signals top left (sub)matrix highlighted in red. When both firms send one signal, they find themselves in the non-signaling equilibrium of the next technological stage. This escalation continues until the number of improvements accounted for exhausts. Intuitively, the equilibrium strategies of a firm range from strictly dominant signaling incentives when shifting from the zero to the one signal setting to a prisoner dilemma-type structure in the remaining (sub)games with two and three potential signals. That is, there exist considerable incentives to signal in all subcases. Thus, as illustrated in the previous subsection, monopolies will try to stay as such and, more importantly, may keep on issuing signals on X_1 since this strategy decreases the rejection probability of their products.

Figs. 6, 7, 10 and 11 illustrate how the incentives faced by firms when signaling as monopolists or as duopolists are completely different. When signaling as monopolists, X_1 is the characteristic leading to a lower rejection probability. However, when signaling within a potential duopoly, X_2 is the characteristic that should be emphasized. The intuition for this result is

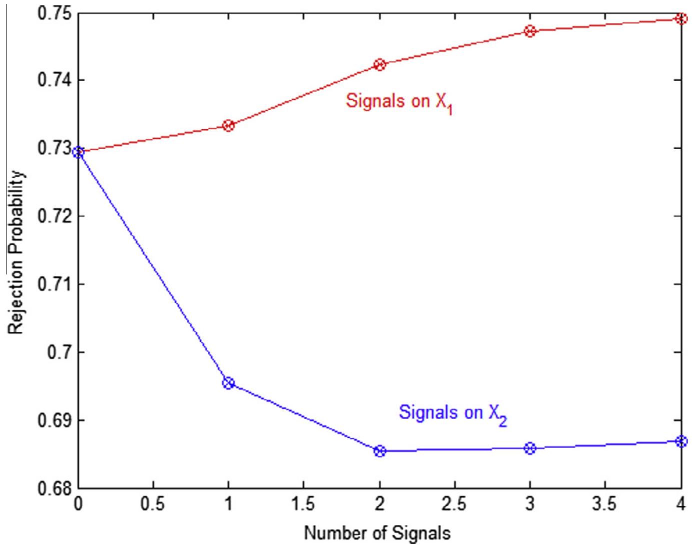

Fig. 10. Rejection probabilities with risk-neutrality and two competing firms when two observations may be acquired.

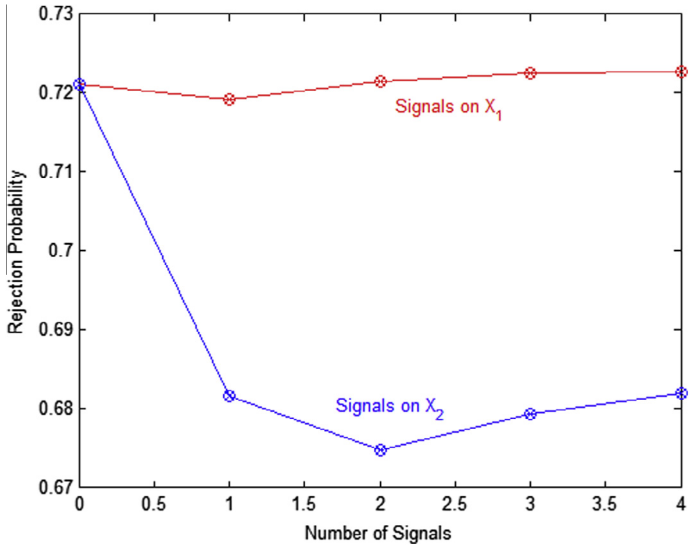

Fig. 11. Rejection probabilities with risk-aversion and two competing firms when two observations may be acquired.

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simple: if signals represent truthful reports, then being observed in the second place constitutes a serious disadvantage due to the higher updated probability defined on X_1. Thus, duopolies would bias towards improving X_2 while monopolies will try to preserve their position through X_1 improvements. Clearly, other variables such as brand dependence and credibility should also be considered or assumed within the constrains defining the behavior of DMs. For example, trust in the brand and the reports of other DMs are important factors, particularly in online shopping environments, see [41].

Finally, note that the rejection probabilities defining the signaling matrices may be assumed to depend on the number of users accepting the product, with a minimum number of DMs being required to adopt a technology before it generates its own niche market, see [1]. In this regard, habits and consumption inertia ([42]) would modify the entries and the resulting equilibria, which will be directly determined by the value of these variables and the proportion of DMs affected by them together with the number of signals.

7. Conclusions, managerial implications and extensions

The current paper constitutes a warning sign against the intuitive plausibility leading operational researchers and economists to assume that introducing an improvement on any of the characteristics defining a product and signaling it credibly should immediately lead to a lower rejection probability when DMs are presented with the product.

The results obtained provide a strategic dimension to the information acquisition process of DMs and the corresponding signaling behavior of firms. Therefore, the strategic nature of the information transmission process should be explicitly analyzed in fields that do not currently account for it, such as knowledge management, see [43]. This is particularly important at the organizational level, where relatively small sets of decision variables are generally considered, see [44]. In this regard, the model could be used to extend in an heterogeneous direction papers such as that of [45], where a group of homogeneous experts must be hired, either simultaneously or sequentially, to forecast the stochastic market demand for a new product that is about to be introduced. Similar remarks apply to the design of decision support systems and information dashboards that guide the process of managerial decision making, see [46].

Note how the function $F(\cdot)$ approaches $H(\cdot)$ as the number of signals increases, leading to both functions almost overlapping for $x_1 \geqslant ce_1$ after four signals are observed, both in the risk-neutral and risk-averse cases but particularly so in the former. This result should be further developed so as to establish a link between the current model and the fuzzy decision theoretical literature. As already stated, the standard dynamic programming techniques employed by the operational research and management literatures are not suitable in the design of the current information acquisition process. A similar comment applies to the use of multicriteria decision making techniques. However, the decision fuzziness that results from signaling within the current environment opens the way for a direct relation to be established with the literature on fuzzy multicriteria decision making. Several comparisons with recent developments achieved by this branch of the literature should be considered as potential extensions of the current model.

An immediate extension of the current model would imply increasing the number of characteristics defining the products as well as the number of observations that may be acquired by DMs. In this regard, it should be noted that the dimension of the information acquisition process increases in the number of characteristics defining the products and the number of potential observations. This would complicate considerably the computation and basic applicability of the model. A potential solution would consist of defining values determining a satisficing ([14]) level for each characteristic based on the realizations observed, which may be used as reference values. These satisficing values will be the ones to improve upon if their combination leads to a higher expected utility than the certainty equivalent product.

At the same time, this increase in dimension would also bring the model closer to the literature on multi-armed bandit problems, see [47,48]. In this case, DMs will have to decide when it is optimal to stop acquiring information on a given product and start acquiring information on a new one based on their subjective beliefs and the history of observations retrieved from the current and any other previous product. Ideally, this extension could allow us to generate a version of the Gittins allocation index applicable to a finite sequential information acquisition environment with multidimensional products.

Finally, one could consider allowing for optimism and pessimism in the sequential information acquisition process, which would complement the recursive fuzzy dynamic programming approach of [49], introducing information uncertainty [together with qualitative considerations] and fuzzy preferences, which would relate to the analysis of Ekel et al. [50], and endowing DMs with fuzzy linguistic preference relations when defining the search utility functions and performing pairwise comparisons, see [51,52]. Besides, the current paper has illustrated normatively how the fact that DMs assign a higher expected utility value to the new [technologically superior] product introduced does not necessarily imply that their probability of purchase increases. This result should be accounted for when analyzing the influence of behavioral intentions in predicting the market acceptance of new products, see [53], or the approach to product design and selection beyond fuzzy multi-criteria decision making, see [54].

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