

Far field of binary phase gratings with errors in the height of the strips

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ABSTRACT

Diffraction gratings are not always ideal but, due to the fabrication process, several errors can be produced. In this work we show that when the strips of a binary phase diffraction grating present certain randomness in their height, the intensity of the diffraction orders varies with respect to that obtained with a perfect grating. To show this, we perform an analysis of the mutual coherence function and then, the intensity distribution at the far field is obtained. In addition to the far field diffraction orders, a “halo” that surrounds the diffraction order is found, which is due to the randomness of the strips height.

1. INTRODUCTION

Diffraction gratings are essential components in many metrological applications. They periodically modulate the phase and amplitude of the impinging electromagnetic beam. In the far field, the beam is split into diffraction orders [1,2,3], following the grating equation

$$p \sin \theta_j = j \lambda \quad (1)$$

where λ is the wavelength, p is the grating period, j is an integer and θ_j is the angle of deviation of the diffracted order j in the grating.

For ideal gratings, the orders characteristics are obtained from the Fourier decomposition [4-10]. Nevertheless real gratings are not purely periodic, since they are affected by flaws due to different fabrication processes [10-13]. Localized defects are commonly found in phase and amplitude gratings, not to mention roughness in the topography of the grating. Also, changes in the period of the grating are observable and it is not rare to find other deviations from the expected, ideal structure of a grating [14-19]. Among them, phase gratings can suffer from variations in the height of their strips, thus giving rise to a performance that can be far from the simple interpretation in terms of a Fourier decomposition of the grating transmittance.

In this communication, and as a first approximation, we manage the problem of exploring the properties and changes due to random variations in the heights of the strips of a pure phase grating in the frame of the thin-element approximation [20,21]. As a consequence it is valid for gratings whose period is much larger than the wavelength $p \gg \lambda$.

This communication has been organized as follows: Section 2 accounts for the expression of the random perturbation of the transmittance just after the grating in terms of a correlation matrix. The field in a plane just after the grating will show dependence with the transmittance. Hence, the field will depend on this correlation matrix. Section 3 takes into consideration the propagation of the mutual coherence function of such a field. As it will be seen, it depends on the correlation matrix that explains the correlation in the topography of the grating. The average intensity in the far field, understood as an ensemble average, will be analytically computed from the knowledge of the correlation function of the field. It is remarkable that the expression thus obtained is easily reduced to the well-know equation for a finite grating when randomness disappears from the analysis. This confirms the approach followed here to compute it. Also, in this section, two limit situations will be studied, namely, high and low-randomness approximations. The behavior and performance of the grating is very different in each situation. The analysis in the previous sections will be reinforced with a direct calculation based on a numerical approach for the Rayleigh-Sommerfeld integral. A number of samples of the perturbed grating will be generated in a Montecarlo-like technique, and the corresponding averages will be obtained so as to compare them with those previously computed. The similarity between both methods to study the grating under this kind of random perturbation will confirm the analysis. Finally, in Section 4, a summary of the main procedures and results will be done.

2.- PERTURBED GRATING AND CORRELATIONS IN ITS STRUCTURE

The structure of the grating under analysis can be observed in Figure 1.

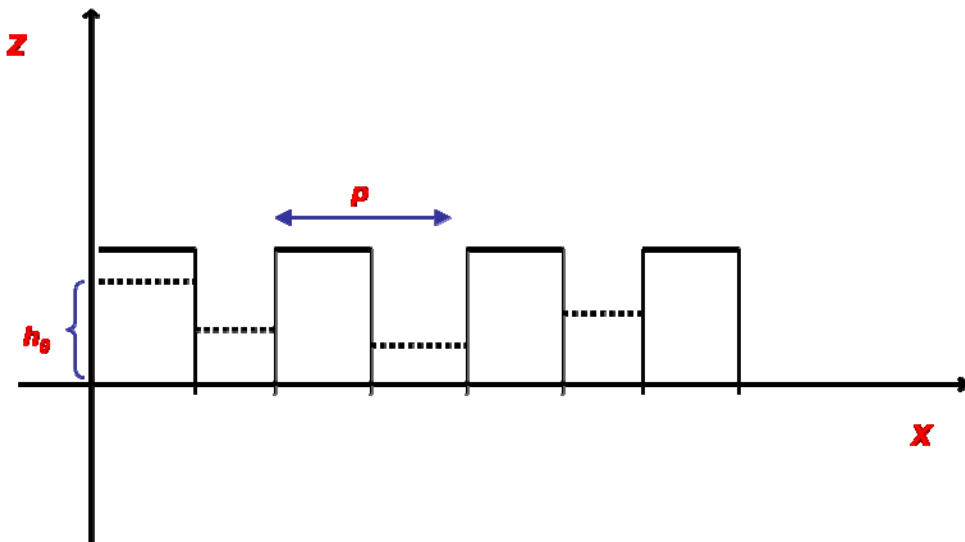


Figure 1: Finite grating with random perturbations on their strips. The continuous line represents the unperturbed grating, whereas the dashed line depicts the actual grating.

This grating is a binary phase structure engraved in glass or another different dielectric with refraction index n . A monochromatic light beam with amplitude A_0 impinges normally on the grating, thus giving rise in the far field, in the ideal case, to a set of diffraction orders. Nonetheless, those steps affected by perturbations can be described using the following equation

$$h(x) = h_0(x) + \delta h(x), \quad (2)$$

where $h(x)$ represents the actual transmittance of the grating and $\delta h(x)$ is the random perturbation on $h_0(x)$, the original phase transmittance of the grating. The phase of any strip is related to its height throughout the following relation

$$\Delta = k(n-1)h_0, \quad (3)$$

where k is the wavenumber. If a step is affected by a random perturbation, the corresponding phase for the random contribution for this step is

$$\Delta T_{step} = e^{i\phi}, \quad (4)$$

where $\phi(z) = k(n-1)z$ and z is a random process, linked to a statistical distribution, called $w(z)$. The key point to describe the transmittance for the whole structure is splitting the random and fixed contributions into two different transmittances, thus making easier further computations and analysis, $T(x) = T_0(x)\Delta T(x)$, where $T_0(x)$ is the unperturbed transmittance and can be Fourier-analyzed [22]:

$$T_0(x) = \sum_j a_j e^{iqjx}, \quad (5)$$

where $q = 2\pi/p$ is the spatial period and a_j are the Fourier coefficients of the unperturbed grating. The perturbation is understood as a finite grating with stochastic heights,

$$\Delta T(x) = \sum_{m=0}^N s_m \Pi\left(\frac{x-x_m}{p/2}\right), \quad (6)$$

where $\Pi(x)$ is the rectangle function. We assume that the fill factor of the grating is 1/2. The term $x_m = (2m+1)p/4$ denotes the position of the different strips of the gratings. $N+1$ are the number of strips in the perturbation and

$$s_m = e^{i\phi}. \quad (7)$$

Therefore, s_m is a random process in each step too. Assuming a Gaussian distribution for z

$$w(z) = e^{-z^2/2\sigma^2} / \sqrt{2\pi}\sigma. \quad (8)$$

The mutual coherence function of the field just after the grating plane can be computed from its definition [23]

$$J(x, x') = |A_0|^2 \langle T(x)T^*(x') \rangle, \quad (9)$$

The average is made on a representative ensemble of the fluctuations over the unperturbed grating. Hence, developing the expression above in terms of the quantities defined in previous paragraphs,

$$J(x, x') = |A_0|^2 T_0(x)T_0^*(x') \sum_{m=0}^N \sum_{n=0}^N \Gamma_{nm} \Pi\left(\frac{x-x_m}{p/2}\right) \Pi\left(\frac{x'-x'_n}{p/2}\right), \quad (10)$$

and the correlations between different points in the plane of the grating are explained by the matrix $\Gamma_{nm} = \langle s_n s_m^* \rangle$ whose index run over the slits of the grating. The entries of Γ will depend on the average $\langle \Delta T(x) \rangle$. Making use of the expression for the probability distribution for z , [24]

$$\langle \Delta T_{step} \rangle = \langle e^{i\phi} \rangle = \int_{-\infty}^{\infty} w(z) e^{i\phi(z)} dz, \quad (11)$$

then

$$\langle \Delta T_{step} \rangle = e^{-g/2} = \alpha, \quad (12)$$

where $g = [k(n-1)\sigma]^2$ and Γ is expressed as

$$\Gamma = (1 - \alpha^2) \mathbf{I} + \alpha^2 \mathbf{C}(\mathbf{1}), \quad (13)$$

\mathbf{I} accounts for the identity matrix, and $\mathbf{C}(\mathbf{1})$ is a constant matrix whose entries are equal to one.

3.- PROPAGATION OF THE CORRELATION FUNCTION AND AVERAGE INTENSITY IN THE FAR FIELD

The propagation of the mutual coherence function is carried out on the grounds of the Fraunhofer approximation for the integration kernel in the diffraction integral. Thus,

$$J(x_2, x_2') = e^{\frac{ik}{2z}(x_2^2 - x_2'^2)} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} J(x_1, x_1') e^{\frac{ik}{z}(x_2' x_1' - x_2 x_1)} dx_1 dx_1'. \quad (14)$$

Taking into account the expression for J in the plane of the grating, the above equation is

$$\overline{J(\theta_2, \theta_2')} = \sum_{m=0}^N \sum_{n=0}^N \Gamma_{mn} \left\{ \int_{-\infty}^{\infty} T_0^*(x_1') \Pi_m(x_1') e^{ik\theta_2 x_1'} dx_1' \int_{-\infty}^{\infty} T_0(x_1) \Pi_n(x_1) e^{-ik\theta_2 x_1} dx_1 \right\}, \quad (15)$$

where $\theta_2 = x_2 / z$, and $\theta_2' = x_2' / z$. Then the average intensity in the far field zone results in

$$\langle \overline{I(\theta_2)} \rangle = \overline{J(\theta_2, \theta_2)} = \sum_{j,w} a_j a_w^* M_{jw}(\theta_2) \text{sinc} \left[\frac{\pi}{2} \left(\frac{p\theta_2}{\lambda} - j \right) \right] \text{sinc} \left[\frac{\pi}{2} \left(\frac{p\theta_2}{\lambda} - w \right) \right]. \quad (16)$$

In this relation, it has been introduced the parameters $M_{jw}(\theta_2)$, which surmise all the influence of random variations in the far field intensity. They can be expressed as

$$M_{jw}(\theta_2) = \sum_{n,m} \Gamma_{nm} e^{i[\phi_{mj}(\theta_2) - \phi_{mw}(\theta_2)]}. \quad (17)$$

When $p \gg \lambda$ the intensity does not depend of the crossed products in the sum of different orders, and the average intensity is approximately

$$\langle \overline{I(\theta_2)} \rangle \approx \sum_j |a_j|^2 M_{jj}(\theta_2) \text{sinc}^2 \left[\frac{\pi}{2} \left(\frac{p\theta_2}{\lambda} - j \right) \right]. \quad (18)$$

The role played by the quantities $M_{jj}(\theta_2)$ is to modify both the peak of the diffraction orders and their shape. When $\alpha = 1$ (no perturbation), the value for $M_{jj}(\theta_2)$ is

$$M_{jj}(\theta_2) = \sum_{n,m} e^{i(m-n)(qj - k\theta_2)p/2} = \frac{\sin^2 \left[\frac{\pi N}{2} \left(\frac{p\theta_2}{\lambda} - j \right) \right]}{\sin^2 \left[\frac{\pi}{2} \left(\frac{p\theta_2}{\lambda} - j \right) \right]}, \quad (19)$$

and the relation between the intensity and the orders of the grating recovers its usual form,

$$\langle \overline{I(\theta_2)} \rangle = \sum_j |a_j|^2 \text{sinc}^2 \left[\frac{\pi N}{2} \left(\frac{p\theta_2}{\lambda} - j \right) \right]. \quad (20)$$

This confirms the general equation for the intensity in the far-field.

Although the relations between Γ and the field have been put forward above, it is worthwhile to explore two limits for the intensity, i.e, low and high randomness. In the first case, the correlation matrix can be computed in the limit of $\alpha \approx 1 - g/2$. Consequently,

$$\Gamma \approx g\mathbf{I} + (1 - g)\mathbf{C}(\mathbf{1}). \quad (21)$$

Figure 2 shows a comparison between the direct, numerical computation of the diffraction pattern with the calculation made using the expressions above for a grating with phase shift π .

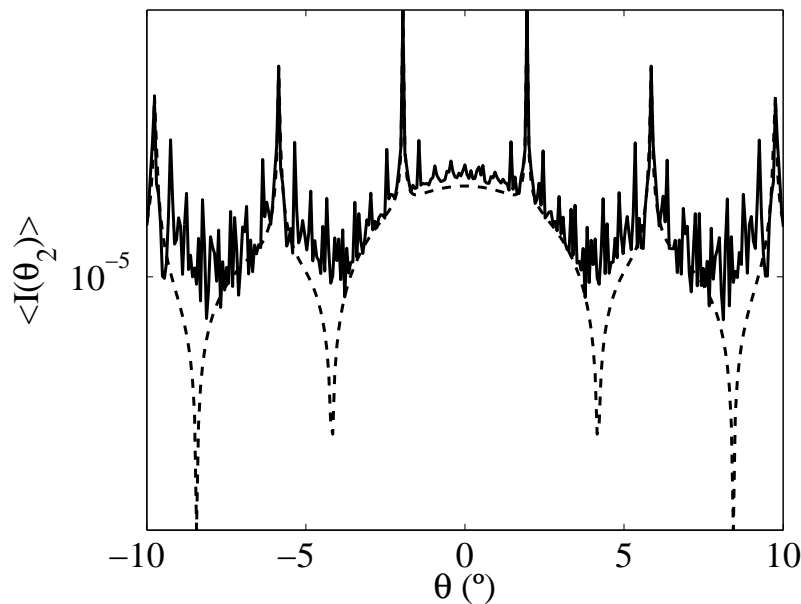


Figure 2: Intensity vs diffraction angle in log scale. Dashed line – Theoretical result obtained using eq. (18). Dashed line - numerical average has been computed using 50 samples. The randomness of the grating is $\sigma = 0.1\lambda$. The period of the grating is $p = 25 \mu m$, the wavelength is $680 nm$, and the index of refraction of the grating is $n = 1.5$.

As the randomness grows, the lobes take more importance. Figure 3 depicts this situation where it can be noticed out the increasingly importance of the lobes in comparison to the diffraction peaks.

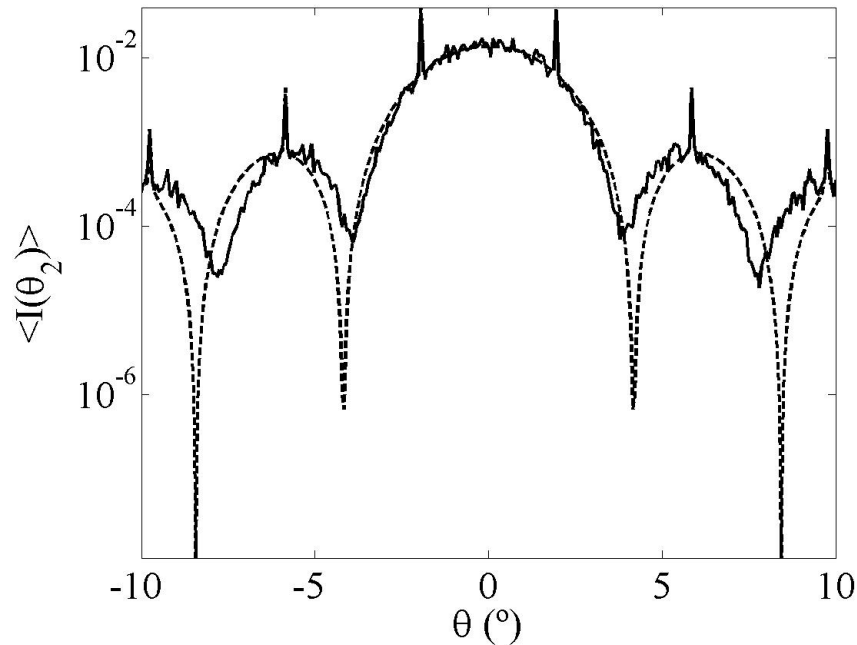


Figure 3: Intensity pattern in log scale for high randomness ($\sigma = 0.5\lambda$). The values for the rest of the parameters are the same than in the previous figure.

From this analysis it can be inferred that the larger the randomness, the less important the peaks in favor of the lobes, which are related to the diagonal of the Γ matrix, always present whatever would be the approximation under study.

4. CONCLUSIONS

In this work, we examine the behavior of phase gratings affected by random perturbations in the height of their strips. We have developed a theoretical method, based on the Fraunhofer approximation of the mutual correlation function for the field. This method has been compared with a direct numerical propagation for members of an ensemble representing the perturbed grating. The far field pattern is characterized by diffractions peaks in the original directions of the unperturbed grating and lobes. As the randomness increases, the energy of the peaks diminishes in favor of the energy in the lobes, as expected according to the form of the low and high randomness limit for the correlation matrix of the structure. This approach can serve to analyze further extensions of the grating geometry, such as multilevel gratings or Diffractive Optical Elements.

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