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Introduction to the black hole physics

Introducción a la física de agujeros negros

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Superradiance in Black Holes

Resumen:

El fenómeno de la superradiancia en agujeros negros, es una consecuencia de la métrica de Kerr, debido a la aparición de la ergosfera. Este proceso es explicado en un contexto de la termodinámica y la mécanica cúantica, a partir de un modelo de interacción entre un campo cúantico y un baño térmico. Se explica la relación entre la población de modos de alta energía en un campo cúantico con la amplificación de una radiación tras interactuar con una agujero negro. Previamente se aborda el predecesor clásico de la superradiancia, el proceso de Penrose, para un mejor entendimiento. Se estudian dos de los principales mecanismos que podrían generar evidencias astrofísicas observables: la interacción de un agujero negro con bosones ultraligeros y la producción de energía en los jets de los AGNs.

Abstract:

The phenomenon of superradiance in black holes is a consequence of the Kerr metric, due to the appearance of the ergosphere. This process is explained in a context of thermodynamics and quantum mechanics, from a model of interaction between a quantum field and a thermal bath. It explains the relationship between the population of high energy modes in a quantum field and the amplification of a radiation after interacting with a black hole. Previously, the classic predecessor of superradiance, the Penrose process, is addressed for a better understanding. Two of the main mechanisms that could generate observable astrophysical evidence are studied: the interaction of a black hole with ultralight bosons and the production of energy in the jets of AGNs.

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1 Introduction

Black holes are one of the most captivating consequence of Einstein's theory of gravity. They arise as solutions to Einstein's equations for a vacuum spacetime. Black holes (BH) encloses regions of spacetime where curvature diverges, called the singularities, hidden behind a surface from which not even light can escape, known as the event horizon. Today these objects are still being one of the biggest challenges of the modern physics and a gate to help us understand better the universe.

The first exact solution of Einstein's equations was discovered by Schwarzschild in 1916 for the spacetime around a static, spherically symmetric object. Then half a century later, Kerr discovered the solution for the spacetime around a rotating BH and this is by far, in an astrophysical context, the most important one.

In this framework, I have decided that the purpose of this paper is to give an approximation to the theory of extracting energy from a rotating BH, through one of the most modern proposed process: the superradiance.

As we will see this phenomenon can take place in a rotating BH, due to the existence of an ergosphere. In this region an observer will always have a non-zero angular velocity due to an effect called frame dragging, so the inertial frames nearest to the event horizon are dragged by it at nearly the angular momentum of the BH. A discussion of the Kerr metric and the appearance of an ergoregion will be explore in the Section 2 of this work.

The first to suggest the idea of extracting energy from a BH, was Penrose [1], who proposed a process where a particle could emerge from a BH with more energy than with the one it had entered by spinning down the BH. The Penrose process will be explain in Section 3.

Zel'dovich picture this phenomenon applied to waves [2]. He proposed that an incident wave, which satisfies :

$$\omega < m\Omega_H, \tag{1}$$

will grow in amplitude by extracting energy and angular momentum from the BH. Here ω is the angular phase wave velocity, Ω_H the angular velocity of the BH and m an azimuthal quantum number. This is the condition for the superradiance to happen and it can occur for any dissipative system.

In Section 4, I will give a different approach from the classical one, which treats the superradiance process as a scattering problem. I am going to provide a treatment based on the linear coupling of a quantum field to a rotating heat bath [3]. The computations of this section will rely on the formalism of the Markovian master equation for an open quantum system [4].

However, BH superradiance is yet to be confirmed by observational evidence. In section 5, I will go over two main mechanisms that could lead to observational signs of this phenomenon. One of them is the interaction of the BH with a massive field, which could entail superradiant instabilities that can be a source of gravitational waves [5]. These will provide BHs with the remarkable ability of enabling detection of particles that have been proposed with different theoretical motivations. Of particular interest are axion-like particles that are considered strong candidates for dark matter. The other mechanism, is related with the jets of the AGNs, proposing superradiance as a way of explaining why do we observe some of them channelising such a huge amount of energy.

In this last section, my aim is to present a more didactic and conceptual approach rather than mathematical due to the complexity of the field and the context in which this paper has being made.

2 Properties of the Kerr metric

2.1 Kerr Metric

The first approximation to describe the spacetime geometry outside a spherically massive object is ,by and large, the Schwarzschild solution. The problems lies in the fact that this metric is symmetric because it does not take into account the rotation that the real cosmic objects have. This rotation entails an asymmetric solution of the metric, because the rotation axis of the object defines a special direction which destroys the isotropy of the solution. Therefore, not only will the mass, M,characterise the object, but also the angular momentum,J.

Here we are not going to develop thoroughly the mathematical produce to obtain the Kerr's metric, however, we are going to be mentioning noted considerations that have been made presented in the following book [6].

Firstly, the stationary and asymmetric character of the spacetime requires that the metric coefficients $g_{\mu\nu}$ have to be independent of the timelike coordinate, $x^0 = t$, and the azimuthal angle, about the axis of symmetry, $x^3 = \phi$, this entails that: $g_{\mu\nu} = g_{\mu\nu}(x^1, x^2)$, where x^1 and x^2 are two remaining spacelike coordinates. Moreover, it has also been required the invariability of the line element to simultaneous inversion of the coordinates t and ϕ, which implies that : $g_{01} = g_{02} = g_{13} = g_{23} = 0$. This condition, physically means that source of gravitational field has motions that are essentially rotational about the axis of symmetry. Furthermore, the metric can be display in the form $g_{ab} = \Omega^2(x)\eta_{ab}$, where $\Omega^2(x)$ is an arbitrary function of the coordinates and η_{ab} is a diagonal matrix with $diag(\pm 1, \pm 1)$ where the signs depend on the signature of the manifold.

All considered, we introduce the following expression of the line element:

$$ds^2 = Adt^2 - B(d\phi - \omega dt)^2 - Cdr^2 - Dd\theta^2. \tag{2}$$

The arbitrary functions A, B, C, D and ω depend only on r and θ, which are indeed the coordinates x^1 and x^2. This arbitrary functions are related to the metric coefficients $g_{\mu\nu}$ by :

$$g_{tt} = A - B\omega^2; g_{t\phi} = B\omega; g_{\phi\phi} = -B; g_{rr} = -C; g_{\theta\theta} = -D. \tag{3}$$

Note that $\omega = -\frac{g_{t\phi}}{g_{\phi\phi}}$ and if the body is not rotating then $\omega = 0$, thus, $g_{t\phi} = 0$.

We now want to very if this line element satisfy Einstein's gravitational field equations by obtaining explicit forms of the metric coefficients. To do so we impose that the line element (2) satisfies the empty-space Eintein's gravitational field equations,$R_{\mu\nu} = 0$, as we are interested in the space geometry outside the rotating matter distribution. Where $R_{\mu\nu}$ are the components of the Ricci tensor.

But Einstein equations alone are insufficient to determine all the unknown functions uniquely, because the requirement of axisymmetry is much less restrictive than the condition of spherical symmetry in the Schwarzschild geometry.
Hence, to find a unique solution we impose some additional conditions, which are flat geometry (Minkowski) when $r \to \infty$ and that somewhere there exist a smooth closed convex event horizon outside which the geometry is non-singular.

After a lengthy calculation [7] we can display Kerr metric in Boyer-Lindquist coordinates (in natural units, i.e., setting $c = 1, G = 1$, etc):

$$ds^2 = (1 - \frac{r_s r}{\Sigma})dt^2 + \frac{2r_s arsin^2\theta}{\Sigma}dtd\phi - \frac{\Sigma}{\Delta}dr^2 - \Sigma d\theta^2 - (r^2 + a^2 + \frac{r_s ra^2 sin^2\theta}{\Sigma})sin^2\theta d\phi^2. \quad (4)$$

Taking into account the following definitions :

$$\begin{aligned} r_s &= 2M \\ \Sigma &= r^2 + a^2 cos^2\theta \\ \Delta &= r^2 - r_s r + a^2 \end{aligned} \quad (5)$$

Therefore, we are dealing with a spacetime geometry outside of a body of mass M rotating with angular momentum $J = aM$, where a is the spin parameter. This parameter a is a measure of how fast the BH spins in relation with it's own mass.

2.2 Structure of a Kerr black hole

Before we start analysing the different parts of the Kerr black hole, we are going to introduce an intuitive approach to picture what may be happening in a geometrical sense. As before, this section is going to based in the Hobson book [6].

As we might observe the Kerr metric depends on two parameters M and a. If we take the following limits:

- $a \to 0$: Kerr metric tends to the Schwarzschild form (i.e. non rotating BH)

  $$\begin{aligned} \Delta &\to r^2(1 - \frac{r_s}{r}) \\ \Sigma &\to r^2 \end{aligned} \quad (6)$$

  Hence,

  $$ds^2 \to (1 - \frac{r_s}{r})dt^2 - (1 - \frac{r_s}{r})^{-1}dr^2 - r^2 d\theta^2 - r^2 sin^2\theta d\phi^2. \quad (7)$$

  The fact that the Kerr metric tends to the Schwarzschild metric as $a \to 0$ help us giving a geometrical meaning to the coordinates r and θ in the slow rotating limit, but in the general case, this coordinates are not the same as Schwarzschild polar coordinates, where r is the radial distance from the center of the BH, and θ the angle respect to the polar-axis.

- $M \to 0$: The spacetime tends to Minkowski (i.e. absence of a gravitating mass). Bearing in mind that $r_s = 2M$, we have:

  $$ds^2 \to dt^2 - (\frac{\Sigma}{r^2 + a^2})dr^2 - \Sigma d\theta^2 - (r^2 + a^2)sin^2\theta d\phi^2. \quad (8)$$

Analysing this last case, we can appreciate that it is the Minkowski metric $ds^2 = dt^2 - dx^2 - dy^2 - dz^2$, but written in terms of spatial coordinates (r,θ,ϕ) that are related to Cartesian coordinates by :

$$\begin{aligned} x &= \sqrt{r^2+a^2}\sin\theta\cos\phi \\ y &= \sqrt{r^2+a^2}\sin\theta\sin\phi \\ z &= r\cos\theta \end{aligned} \tag{9}$$

Where $r \geq 0$, $0 \leq \theta \leq \pi$ and $0 \leq \phi \leq 2\pi$.

In Fig. 1, where we are in the $\phi = 0$ plane, we can appreciate that the surfaces of $r = constant$ are oblate ellipsoids of rotation about the z-axis, with the special case of $r = 0$ which corresponds to a disc of radius a in the equatorial plane, centred on the origin of the Cartesian coordinates. Moreover, we can see that the surfaces of $\theta = constant$ correspond to an hyperbolae of revolution about the z-axis

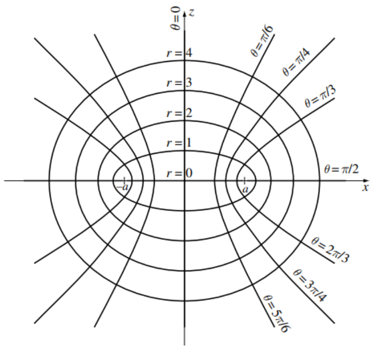

Figure 1: Geometrical approximation of the Boyer-Lindquist coordinates of Kerr's metric, when $M \to 0$, in the $\phi = 0$ plane [6].

2.2.1 Singularities and horizons

The event horizon of a black hole, is a surface that defines the region of the space-time from which nothing, not even light, could escape out of it. The event horizon, hides the central intrinsic singularity from the external viewers. In theory, all the matter should be concentrated in the singularity with zero volume, infinity density and space-time curvature. We are going to be analysing the singularities of the Kerr metric (2), distinguishing the intrinsic (or real) singularities from the coordinate singularities, which resulted simply from choosing coordinates with a restricted domain of validity, and that they can be remove by making appropriate transformations of coordinates.

Therefore, we see that the intrinsic singularity comes from:

$$\Sigma = r^2 + a^2\cos^2\theta = 0. \tag{10}$$

For this to happen we must impose $r = 0$ and $\theta = \frac{\pi}{2}$. As we have just seen $r = 0$ represents a disc of coordinate radius a in the equatorial plane, precisely, the collections of points with $r = 0$ and $\theta = \frac{\pi}{2}$ constitutes the outer edge of this disc, which means that the singularity has the form of a *ring* , rather than a point like the Schwarzschild solution.

In the Kerr metric the event horizon will occur where r= constant (because is a null 3-surface, see more information about this in the section 13.4 of the Hobson [6]) and this is given by the condition $g^{rr} = 0$ or, equivalently, $g_{rr} = -\frac{\Sigma}{\Delta} = \infty$. Hence, we dive into the study the roots of Δ, that actually are coordinate singularities:

$$r_{\pm} = M \pm \sqrt{M^2 - a^2}. \tag{11}$$

Thus, the Kerr metric has two event horizons, in contrast with the only one from Schwarzschild metric. We have the outer horizon r_+ and a Cauchy horizon r_-, both being a surface resembling an axisymmetric ellipsoid, flattened along the rotation axis.

As we can see in (11), for the horizons to exist : $a^2 < M^2$ (remember we have set $c = 1$). Hence, the magnitude of angular momentum, $J = Ma$, of a rotating black is limited by it's squared mass. An extreme Kerr BH is the one that adopts the limiting value $a^2 = M^2$, where the event horizons r_+ and r_- coincide at $r = M$.

2.2.2 Frame Dragging effect

Firstly,we consider an observer known as the ZAMO (Zero Angular Momentum Observer), which falls into the BH with with zero angular momentum. We are interested in the expression for it's angular velocity, as measured at infinity, which is given by:

$$\Omega \equiv \frac{\dot{\phi}}{\dot{t}} = -\frac{g_{t\phi}}{g_{\phi\phi}} = \frac{2Mar}{r^4 + r^2a^2 + 2a^2Mr}. \tag{12}$$

Here $\dot{\Phi}$ and $\dot{t}$ are obtained from the geodesic equations[7], imposing $L = 0$, being L the angular momentum per unit rest mass of the massless particles. The geodesics describe the motion of free pointlike particles in the equatorial plane of Kerr's geometry.

Being consistent with the fact that these are zero angular momentum observers at infinity: $\Omega = 0$. But at any finite distance $\Omega \neq 0$ and specifically at the outer horizon we find :

$$\Omega_H^{\text{ZAMO}} = \frac{a}{2Mr_+}. \tag{13}$$

Therefore, observer are force to co-rotate with the geometry. This phenomenon is called *frame-dragging effect*. Next, we are going to present the region where this phenomenon takes place: the *ergoregion*.

2.2.3 Ergoregion

In the Schwarzschild solution the surfaces of infinite redshift [1] and the event horizon coincide, but in the Kerr metric they don't. With the BH rotation there are going to appear two new surfaces, S_+ and S_-, define by the condition of $g_{tt} = 0$ which implies a stationary limit (no static observer is allowed inside this surfaces) and infinite redshift surface. Imposing the mentioned condition for the Kerr metric we have that these surfaces occur at: $r_{S\pm} = M \pm \sqrt{M^2 - a^2\cos^2\theta}$.

The surface S_- coincides with the ring-shaped singularity located in the equatorial plane. Besides, S_- resides completely within the inner horizon $r = r_-$, except at the poles where both surfaces touch.

On the other hand, the surface S_+ has a coordinate radius of $2M$ at the equator and for all values of θ completely encloses the outer horizon $r = r_+$, except at the poles where they also touch. It is indeed, between this S_+ surface and the outer horizon r_+ where the *ergosphere* is contained. This can be better visualise in Figure 2.

The key property of an ergosphere is that is a region for which $g_{tt} < 0$ [2] and from which particles **can escape**. In the Schwarzschild geometry this condition was only satisfied within the event

[1]This means that the frequency of any light signal emitted from this surface will be indefinitely decrease for a distance viewer.

[2]The condition $g_{tt} < 0$ is because we are taking the signature $(+, -, -, -)$ for the metric, otherwise with $(-, +, +, +)$ it will be $g_{tt} > 0$. Therefore the key property really is that in the ergoshere this metric coefficient changes it's sign.

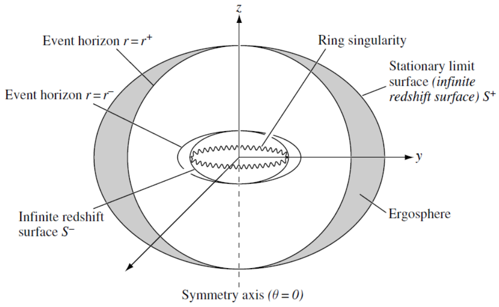

Figure 2: The structure of a Kerr Black Hole [6]

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horizon, so it doesn't have an ergoregion. Ergo, this region is a **clear distinction of the Kerr metric, i.e , of a rotating geometry** and as we shall see **it's existence makes possible the extraction of rotational energy from the black hole**, and as a consequence, the superradiance process.

Since $g_{tt} < 0$ at all points in the ergoregion, an observer cannot remain at a fixed (r, θ, ϕ) position, because it's 4-velocity will be $[u^{\mu}] = (u^t, 0, 0, 0)$ but the requirement:

$$u \cdot u = g_{tt}(u^t)^2 = c^2, \tag{14}$$

(in natural coordinates = 1) cannot be satisfied if $g_{tt} < 0$ [3].

However, it is possible to have an observer fixed at r and θ coordinates by rotating around the black hole (with respect to an observer at infinity) in the same sense as the hole's rotation. Therefore, the 4-velocity of such an observer is:

$$[u^{\mu}] = (u^t, 0, 0, \Omega), \tag{15}$$

where Ω is the angular velocity with respect to the observer at infinity.

We are interested in calculating the allowed values of this Ω. For that we again require (14) and by using (15), we end up with:

$$g_{tt}(u^t)^2 + 2g_{t\phi}u^t u^{\phi} + g_{\phi\phi}(u^{\phi})^2 = (u^t)^2(g_{tt} + 2g_{t\phi}\Omega + g_{\phi\phi}\Omega^2) = c^2. \tag{16}$$

Hence, for u^t to be real we require that :

$$g_{\phi\phi}\Omega^2 + 2g_{t\phi}\Omega + g_{tt} > 0. \tag{17}$$

Let consider the zeroes of the above, as a result we get:

$$\Omega_{\pm} = -\frac{g_{t\phi}}{g_{\phi\phi}} \pm \left[\left(\frac{g_{t\phi}}{g_{\phi\phi}}\right)^2 - \frac{g_{tt}}{g_{\phi\phi}}\right]^{1/2} = \omega_{\pm}\left(\left(\frac{\omega^2 - g_{tt}}{g_{\phi\phi}}\right)^{1/2}\right). \tag{18}$$

Thus the allowed values for Ω are between: $\Omega_- < \Omega < \Omega_+$.

Here we find two particular cases of interest:

- $g_{tt} = 0 \rightarrow \Omega_- = 0$ and $\Omega_+ = 2\omega$. As we said before this condition corresponds to the stationary limit surface $r = r_{S+}$ which is the outer defining surface of the ergoregion (see again Figure 2)

[3]The condition for the 4-velocity modulus $u \cdot u = g_{tt}(u^t)^2 = c^2$, comes from the fact that we are using the convention for the metric of the form : $(+, -, -, -)$.

- $\omega^2 = \frac{g_{tt}}{g_{\phi\phi}} \to \Omega_{\pm} = \omega$. This condition holds where $\Delta = 0$,i.e at the outer event horizon $r = r_+$, which is the inner defining surface of the ergoregion.

  In the event horizon we said that $\Omega = \Omega_H$ (13), which is the maximum allowed value of the angular velocity for any observer at fixed r and θ at the ergoregion.

3 Penrose process

In this section we are going to explain in a nutshell the Penrose process [1]. This step is necessary for understanding why the existence of an ergoregion allows the energy extraction of the BH. In Section 4.3 we will find an equivalence between the needed condition for the Penrose process and the superradiance phenomenon.

Suppose that a particle A located at infinity with enery $E^{(A)}$ is introduce in the ergosphere of a BH. The energy of this particle measure by the observer at infinity in the moment of the emission is:

$$E^{(A)} = p^{(A)}(\epsilon) \cdot u_{\text{obs}} = p_t^{(A)}(\epsilon), \tag{19}$$

where the $p_t^{(A)}(\epsilon)$ is the 4-momentum of the particle at this event and $u_{\text{obs}} = (1, 0, 0, 0)$ is the 4-velocity of the observer.

Inside the ergosphere the particle A decay into two particles, B and C. Because of the momentum conservation we will have in the moment of the decay :

$$p^{(A)} = p^{(B)} + p^{(C)}. \tag{20}$$

Moreover due to fact that the covariant time component of the 4-momentum of the particle, p_t, is conversed a long the geodesics in the Kerr metric (because the metric is stationary $\partial_t g_{\mu\nu} = 0$), we have conservation of the energy:

$$E^{(A)} = E^{(B)} + E^{(C)}. \tag{21}$$

Let's say now that particle C escapes into the infinity and B is absorbed by the BH. We want to see how their respective energies change. From (21) and expressing $E^{(B)}$ in the same way as we express $E^{(A)}$ in (19) we can write:

$$E^{(C)} = E^{(A)} - p_t^{(B)}, \tag{22}$$

The key step here is to analyze the sign of $E^{(B)} = p_t^{(B)}$ and it's relation with the g_{tt} coefficient. We have that:

$$p_t^{(B)} = e_t \cdot p^{(B)}, \tag{23}$$

where e_t (or ∂_t) is the basis vector of the t coordinate where it's norm is given by : $e_t \cdot e_t = g_{tt}$.

Inside the ergoregion $g_{tt} < 0$, so e_t is spacelike (with the signature that we are taking a vector with negative modulus is spacelike). In this case, $p_t^{(B)}$ would be a component of spatial momentum, which might be positive or negative. For decays where is negative we have that from (22) and (21) we see that:

$$E^{(C)} > E^{(A)}, \quad E^{(B)} < E^{(A)}. \tag{24}$$

Hence, the particle that escapes from the BH, has more energy than with the one it had entered. Whereas the one which states inside inside the BH reduces it's energy.

From these procedure, we reach to an important condition for this process to happen, that is the possibility of having ,for a co-mobile reference frame with the ergoregion, negative energies for particles:

$$E(B) = p_t^{(B)} < 0, \tag{25}$$

This is the condition that in Section 4.3 we will find it's analogue for the superradiance phenomenon.

The consequences of the Penrose process for the BH are the following:

$$M \to M + \frac{p_t^{(B)}}{c^2}, \tag{26}$$

$$J \to J - p_\phi^{(B)}, \tag{27}$$

being M the mass and J angular momentum of the BH. In the last equation we should recall that for the particle orbits in general, p_ϕ is minus the angular momentum component of the particle along the rotation axis of the BH. From (26), we see that he negative value of $p_t^{(B)}$ reduces the total mass of the BH.

Moreover, Penrose process also reduces the angular momentum of the BH and this is what is meant when we speak of extracting rotational energy from the BH. To prove this, we will look for the sign of $p_\phi^{(B)}$. For such a purpose we will be needing the 4-velocity of particle B, with respect to an observer at infinity , given by (15). This observer would measure the energy of particle B to be:

$$E^{(B)} = p^{(B)} \cdot u = u^t \left(p_t^{(B)} + p_\phi^{(B)} \Omega \right). \tag{28}$$

Since this energy must be positive (for this observer), we require:

$$L < \frac{p_t^{(B)}}{\Omega}, \tag{29}$$

where $L = -p_\phi^{(B)}$ is the component of the angular momentum of the particle along the rotational axis of the BH. Because $p_t^{(B)}$ is negative in the Penrose process and Ω is positive as we discuss in Section 2.2.3 , we see that $L < 0$. Therefore, a particle that falls into the BH must have negative angular momentum, which leads to a reduction of the net angular momentum of the BH.

4 Interaction of a quantum field with a rotating heat bath

4.1 Model and approximations

In this section we are going to be explaining the superradiance phenomenon taking an approximation via the thermodynamics and the quantum mechanics, based on the following article [3] from Alicki and Jenkins. The idea is going to be to relate the population of the quantum field modes with the amplification of an incident radiation in to the BH.

Our model is going to be a system form by a rotating heat bath (that is the analogue to the Kerr´s BH) and a quantum field. Therefore, we consider a large system that acts as an equilibrium heat bath with a fix temperature. This system rotates along the its symmetry axis, which we take to be the **z**-axis, with a angular velocity Ω .

A free quantum field, either bosonic or fermionic, is interacting with this source and is described by a set of annihilation and creations operators $a_{m\alpha}(\omega)$, $a^\dagger_{m\alpha}(\omega)$ corresponding to the field modes $|\omega, m, \alpha\rangle$ and that satisfy the the (anti-)commutation relations:

$$\begin{aligned} [a_{m\alpha}(\omega), a^\dagger_{m'\alpha'}(\omega')]_\pm &= \delta_{\omega\omega'}\delta_{mm'}\delta_{\alpha\alpha'} \\ [a_{m\alpha}(\omega), a_{m'\alpha'}(\omega')]_\pm = [a^\dagger_{m\alpha}(\omega), a^\dagger_{m'\alpha'}(\omega')]_\pm &= 0. \end{aligned} \tag{30}$$

Where the $+$ denotes the anticommutator and the $-$ commutator and we have set $\hbar = 1$.

In the expression of the field modes we have, the azimuthal quantum number $m = 0, \pm 1, \pm 2, \ldots$ related with the angular momentum along the z-axis , $\omega \geq 0$ the frequency, and α the spin together with any other quantum numbers needed to specify the field's state.
Our main aim is to describe the interaction between these two elements. On account of this, we need the quantum field hamiltonian and angular momentum (z-component):

$$H_f = \sum_{\omega,m,\alpha} \omega a^\dagger_{m\alpha}(\omega) a_{m\alpha}(\omega) \qquad L^z_f = \sum_{\omega,m,\alpha} m a^\dagger_{m\alpha}(\omega) a_{m\alpha}(\omega). \tag{31}$$

The bath has its own Hamiltonian H_b and its own z-component of the angular momentum L^z_b. Moreover, the most general field–bath interaction that is linear in the field operators is given by an additional term in the Hamiltonian of the form

$$H_{\text{int}} = \sum_{\omega,m,\alpha} \left(a_{m\alpha}(\omega) \otimes B^\dagger_{m\alpha}(\omega) + a^\dagger_{m\alpha}(\omega) \otimes B_{m\alpha}(\omega) \right). \tag{32}$$

where $B_{m\alpha}(\omega)$ is a proper bath operator. This linearity of the equations with respect to the quantum field is a valid approximation as long as the field is weak enough that it's self-coupling can be neglected, which it will be the case for our discussion. To take into account the bath's rotation we use an effective Hamiltonian of the form

$$H^{\text{eff}}_b = H_b - \Omega L^b_z. \tag{33}$$

The bare hamiltonian H_b is independent of the rotation and the arbitrary sign in front of the Ω is taken negative for later notation convenience. This equation is a well-grounded approximation as long as the bath's coherent rotation does not entail energies that large to excite internal degrees of freedom.

We could see this separation of the pieces of the system each with it's corresponding hamiltonian, linked by an interaction hamiltonian, H_{int}, as an analogous to the Born-Oppenheimer approximation in the molecular physics.

4.2 Coupling spectrum

In this discussion as we said before, we are going to be dealing with a regime were the bath and the quantum field are that weakly couple, that we can assume the interaction as a perturbation. As a consequence we can express the physical properties of the bath that will affect the field using correlations of second order in the bath operators $B_{m\alpha}(\omega), B^\dagger_{m\alpha}(\omega)$, evaluated for the bath's stationary state. Using a short-hand notation B_k, $B^\dagger_l$ with multi-index $k \equiv \{\omega, m, \alpha\}$, these correlations are expressed as matrix elements of the coupling spectrum.

$$\gamma^0_{kl}(x) = \int_{-\infty}^{\infty} e^{ixt} \langle B^\dagger_k(t) B_l \rangle_b \, dt, \tag{34}$$

where $\langle \cdot \rangle_b$ denotes the expectation value with respect to the given stationary state of the bath, while $B_k(t)$ is the bath observable B_k as it evolves in the Heisenberg picture for a bath governed by its internal Hamiltonian H_b. The function $\gamma^0_{kl}(x)$ represents the matrix elements of the coupling spectrum. It describes how different modes of the bath are coupled between them at different frequencies x.

The diagonal elements of the coupling spectrum matrix are directly related with the rates of creation $\gamma \uparrow$ and annihilation $\gamma \downarrow$ of particles in the modes of the quantum field, which express the probability per unit time of having a process of annihilation or creation in the field due to it's interaction with the thermal bath . We obtain this diagonal terms by replacing in Eq.(34) the internal Hamiltonian by the effective one on Eq. (33) and applying rotational invariability:

$$\gamma^{\Omega}_{kk}(x) = \gamma^{0}_{kk}(x + m\Omega). \tag{35}$$

This expression evaluated in a certain frequency $x = \omega$ (we use the symbol $\Gamma^{\Omega}_{m\alpha}(\omega)$ to denote this) is equal to the decay rate:

$$\gamma_{\downarrow}(k) \equiv \Gamma^{\Omega}_{m\alpha}(\omega) = \gamma^{0}_{kk}(\omega + m\Omega). \tag{36}$$

Moreover, to relate this with the creation rate, we introduce the Kubo-Martin-Schwinger (KMS) condition for an equilibrium state of a rotating bath :

$$\gamma^{\Omega}_{kk}(-x) = e^{-\beta(x-m\Omega)}\gamma^{\Omega}_{-k-k}(x), \tag{37}$$

where $-k \equiv \omega, -m, T_{\alpha}$, with T_{α} the time reversal of α. The operator T is an operator of temporal inversion, that changes the sign of angular momentum and other relevant index. This KMS condition is a temporal symmetry that establish a relation between the decay rates and its time-reverse (i.e, pumping) for a field coupled to the stationary bath.
Therefore,

$$\gamma_{\uparrow}(k) \equiv e^{-\beta(\omega-m\Omega)}\Gamma^{\Omega}_{m\alpha}(\omega). \tag{38}$$

4.3 Inversion of population

If the body does not rotate then we expect that after some time the field will reach thermal equilibrium at the same temperature. However to explain what would happen if the body does rotates, we will be needing the tools of the quantum theory of open system to explain it .

Thus, we consider the density matrix of the open quantum system (in this case this system will be our quantum field), $\rho(t)$, and we look for it´s evolution, that is given by the reduce density matrix of the system :

$$\rho(t) = U(t)\rho(0) \otimes \sigma U(t)^{\dagger}, \tag{39}$$

being $\rho(0)$ the initially density matrix, σ the density matrix of the bath, *U(t)* the unitary evolution operator given by the hamiltonian of the system that includes weak interaction between the field and the bath, and then we take the trace to average over the degrees of freedom of the bath.

This expression is very complex to solve, so we need some approximation, the standar one is the Markovian approximation, that using Davie´s weak coupling limit technique [4] we can obtain the Markovian master equation, that for the model in the previous subsection we have:

$$\begin{aligned} \dot{\rho}(t) = -i[H_f, \rho(t)] + L\rho(t) = -i[H_f, \rho(t)] \\ +\frac{1}{2}\sum_k \gamma_{\downarrow}(k)([a_k, \rho(t)a_k^{\dagger}] + [a_k\rho(t), a_k^{\dagger}]) \\ +\sum_k \gamma_{\uparrow}(k)([a_k^{\dagger}, \rho(t)a_k] + [a_k^{\dagger}\rho(t), a_k]). \end{aligned} \tag{40}$$

Here L refers to the dissipate part and H_f is the renormalized Hamiltonian, which takes into account the field-bath interaction in the bare Hamiltonian. The term $-i[H_f, \rho(t)]$, represents the unitary evolution of the system, the reversible and coherent dynamics due to the own field Hamiltonian ,H_f.

As we said in the previous section, $\gamma_{\downarrow}$ and $\gamma_{\uparrow}$ are the decay and pumping rate, respectively, for the mode k. The term that follows $\gamma_{\downarrow}$ describes the energy dissipation of the system into the environment (thermal bath) and the one that goes with $\gamma_{\uparrow}$ captures the injection of energy from the environment into the system.

As we have seen in the previous section the creation rate can be related with the annihilation rate by the following expression:

$$\gamma_{\uparrow}(k) = e^{-\beta(\omega - m\Omega)}\gamma_{\downarrow}(k). \tag{41}$$

Furthermore, we can express the relation between these rates in terms of a Boltzmann factor with the "local " (depends on ω) inverse temperature $\beta_{loc}(\omega)$:

$$e^{-\beta_{\text{loc}}[\omega]\omega} \equiv \frac{\gamma_{\uparrow}(k)}{\gamma_{\downarrow}(k)} = e^{-\beta(\omega - m\Omega)} \quad . \tag{42}$$

So we can see that this local inverse temperature is define as:

$$\beta_{\text{loc}}[\omega] = \beta \left(1 - \frac{m\Omega}{\omega}\right). \tag{43}$$

To relate (43), with how the particles distribute in a system in contact with a thermal bath, we are going to dive briefly into the canonical ensemble in the framework of the Stadistical Physics [8].

The probability $P(E_i)$ that a system in thermal equilibrium is in a state with energy E_i is given by the Boltzmann distribution:

$$P(E_i) = \frac{e^{-\beta E_i}}{Z}, \tag{44}$$

where $\beta = \frac{1}{k_B T}$ is the inverse temperature (with k_B being the Boltzmann constant) and Z is the canonical partition function, defined as $Z = \sum_i e^{-\beta E_i}$.

In thermal equilibrium, the ratio of the populations N_i and N_j of states i and j is:

$$\frac{N_i}{N_j} = \frac{P(E_i)}{P(E_j)} = e^{-\beta(E_i - E_j)}. \tag{45}$$

Being N_i a state of lower energy than N_j, so that $E_i < E_j$.
Therefore, if the temperature is positive, the whole exponent is positive and $N_i > N_j$, what means that the states of lower energy have a larger population than the ones with higher energy.
But if the system have a negative temperature, the opposite scenario will take place. In our analysis a negative temperature $\beta_{\text{loc}}[\omega] < 0$, will take place when $\omega < m\Omega$, this implies that :

$$\beta_{\text{loc}}[\omega] < 0 \Leftrightarrow \omega < m\Omega. \tag{46}$$

This scenario of negative temperature implies that the modes of lower energy ($\omega < m\Omega$) are less occupy than the ones with high energy, this phenomenon is called *population inversion.* As we will see in the next point, this event makes easier the stimulated emission and an amplification of the radiation, consequently, (46) is a condition for superradiance to occur.

If we look carrefully at (43), we can appreciate that the possibility of β_{loc} of being negative, comes from the introduction of a rotation factor.

Furthermore, the modes with $\beta_{loc}(\omega) < 0$ are associated with negative energy in the comoving bath's frame of reference. Here is where we see that the condition from the Penrose process of displaying particles with negative energy in order to make possible the extraction of energy from the BH, translates in to $\omega < m\Omega$ when we are talking about superradiance.

4.4 Feedback and stimulated emission

The probability $P_n(k,t)$, per unit time, of creating a new particle if n particles are already present in a given mode $|k\rangle \equiv |\omega, m, \alpha\rangle$, is given by the diagonal matrix elements of $\rho(t)$, computed in the corresponding population number representation basis, and have the following form:

$$P_n(k,t) = \gamma_\uparrow(k)(1 \pm n). \tag{47}$$

Here the 1 corresponds to spontaneous emission and the n-dependence can be interpreted as resulting from a feedback between the field and the bath. This feedback is positive for bosons, due to stimulated emission. For fermions the feedback is negative, due to the Pauli exclusion principle.

The stimulated emission is a process, that was proposed by Albert Einstein in 1917, and is the basis of the laser functioning. It occurs when a photon interacts with an exited atom (that is unstable) and causes the atom to emit a second photon that has the same phase, frequency and direction that the incident photon (due to a decay of the electron to a lower level). This coherence is vital for the amplification, because it allows that the radiation waves could generate a constructive sum. In the lasers, it exist what is call an "active medium" that contains atoms that could be exited to a higher level of energy.

In our context this process can take place, in the case where the roll of the atom would be play by the quantum field, the exited photons would be bosons in unstable modes of high energy and the gravitational field of the BH acts like the "active medium" .

For the stimulated emission to be significant, there has to be more particles in the exited modes (modes with high energy) than in the low energy modes, i.e, it has to be an inversion in the population.

As we said the n-dependence of the Eq.(47) is related with the stimulated emission. So as many n particles are in a state the higher the probability of creating a new particle in that state. Therefore, stimulated emission makes easier the amplification of the specific modes of the quantum field due to the positive feedback with the thermal bath (rotating black hole) which supplies the needed energy to maintain and increment the population of the field modes, which, involves a constant amplification of the radiation, ending in an superradiance instability that could be observed as intense and coherent emission.

The stimulated emission is the process that help us understand how we connected the concept of amplification of an incident radiation and the over-population of high energy modes of the quantum field due to to states of negative energy.

4.5 Stable and unstable modes

In this section, we are going to analyse specifically for bosons, under which conditions their modes are stable or unstable . The general expression for the average occupation number of a single mode is :

$$\bar{n}_{m\alpha}(\omega,t) \equiv \mathrm{Tr}\left[\rho(t) a^\dagger_{m\alpha}(\omega) a_{m\alpha}(\omega)\right], \tag{48}$$

which obeys :

$$\dot{n}_{m\alpha}(\omega,t) = -\left(\Gamma^{\Omega}_{m\alpha}(\omega)\left[1 - (\pm)e^{-\beta(\omega - m\Omega)}\right]\right) n_{m\alpha}(\omega,t) + \Gamma^{\Omega}_{m\alpha}(\omega) e^{-\beta(\omega - m\Omega)}. \tag{49}$$

Where the sign $(+)$ corresponds to bosons and $(-)$ to fermions.

We are going to be considering the case of zero temperature ($\beta \to \infty$) for superradiant modes in (49) for bosons. The form of this equation in this limit depends on if we are in a superradiance regime or not :

$$\lim_{\beta\to\infty} e^{-\beta(\omega - m\Omega)} = \begin{cases} 0 & \text{if } \omega > m\Omega \\ 1 & \text{if } \omega < m\Omega \end{cases} \tag{50}$$

For $\omega > m\Omega$:

$$\dot{n}_{m\alpha}(\omega,t) = -\Gamma^{\Omega}_{m\alpha}(\omega) n_{m\alpha}(\omega,t); \qquad n_{m\alpha}(\omega,t) = n_{m\alpha}(\omega,0) e^{-\Gamma^{\Omega}_{m\alpha}(\omega)t}. \tag{51}$$

This describes a exponential decay of the occupational number. So modes which fulfil this conditions are stable modes.

Whereas for $\omega < m\Omega$:

$$\dot{n}_{m\alpha}(\omega,t) = \Gamma^{\Omega}_{m\alpha}(\omega)\left[1 + n_{m\alpha}(\omega,t)\right]. \tag{52}$$

So even though, the creation rate tends to zero, the decay rate (36) is still being finite and positive, allowing an exponential growth in the occupation number of the superradiance modes:

$$n_{m\alpha}(\omega,t) = n_{m\alpha}(\omega,0) e^{\Gamma^{\Omega}_{m\alpha}(\omega)t}. \tag{53}$$

Therefore, modes which fulfil this superradiant condition, are unstable modes. This equation implies that a rotating body at zero temperature will produce a continuous spectrum of radiation, with a non-trivial spatial distribution determined by the superradiant condition $\omega < m\Omega$.

4.6 Black hole radiation

In this last part, we are going to be proving that a BH can be shape as a thermal heat bath at a fix temperature.

The modes of any quantum field around a black hole, can be separated into two types: the outer modes, those localised far outside the black hole and the inner modes, which are the ones close or below the event horizon.

The particles occupying the inner modes form a bath for the outer ones. The interaction between this two types of modes, is described by a Hamiltonian of the form of the Eq.(32), where the operator $a^{\dagger}_{m\alpha}(\omega)$ creates a particle in an outer mode and $B_{m\alpha}(\omega)$ annihilates particles in a certain superposition of the inner modes.

We can introduce a bath operator in the Eq.(32) of the following form:

$$B_k = \sum_{k'=\{\omega',m,\alpha'\}} \left[f_k(\omega') b_{k'} + g_k(\omega') b^{\dagger}_{-k'} \right]. \tag{54}$$

There is no summation over $m' \neq m$ because we assumed rotational symmetry. This expression is based on the theory shown by Hawking in [9], where he explains that due to the strong gravity of a black hole an uncertainty between the annihilation operator b_k and the creation operator $b^{\dagger}_K$ of an inner mode is created. But this indeterminacy is exponentially small and approximately given by:

$$\frac{|g_k(\omega)|^2}{|f_k(\omega)|^2} = e^{-\beta_H \hbar\omega}. \tag{55}$$

where $f_k(\omega)$, $g_k(\omega)$ are some factors and β_H is the inverse Hawking temperature of the black hole.

If we insert the Eq.(54) into Eq.(34) for the vacuum state of the inner modes we obtain :

$$\gamma^0_{kl}(x) = \int_{-\infty}^{\infty} e^{ixt} \left\langle \left(\sum_{k'=\{\omega',m,\alpha'\}} \left[f_k(\omega') b_{k'} + g_k(\omega') b^{\dagger}_{-k'} \right] \right)^{\dagger} (t) \left(\sum_{l'=\{\omega'',m,\alpha''\}} \left[f_l(\omega'') b_{l'} + g_l(\omega'') b^{\dagger}_{-l'} \right] \right) \right\rangle_b dt$$

Applying the dagger and gathering the summatories:

$$\gamma^0_{kl}(x) = \int_{-\infty}^{\infty} e^{ixt} \left\langle \sum_{k',l'} \left(f^*_k(\omega')b^\dagger_{k'} + g^*_k(\omega')b_{-k'} \right)(t) \left(f_l(\omega'')b_{l'} + g_l(\omega'')b^\dagger_{-l'} \right) \right\rangle_b dt$$

Know we evaluate the expected values. Due to the case that we are in the vacuum state $|0\rangle$ so that $b_k|0\rangle = 0$:

$$\left\langle b^\dagger_{k'}(t)b_{l'} \right\rangle_b = \delta_{k'l'}e^{-i\omega' t}\langle b^\dagger_{k'}b_{k'}\rangle_b = \delta_{k'l'}e^{-i\omega' t}$$

$$\left\langle b_{-k'}(t)b^\dagger_{-l'} \right\rangle_b = \delta_{k'l'}e^{i\omega' t}\langle b_{-k'}b^\dagger_{-k'}\rangle_b = \delta_{k'l'}e^{i\omega' t}$$

The other terms, $\left\langle b^\dagger_{k'}(t)b^\dagger_{-l'} \right\rangle_b$ and $\langle b_{-k'}(t)b_{l'}\rangle_b$, are zero in the vacuum state.

Know if we insert this and simplify:

$$\gamma^0_{kl}(x) = \int_{-\infty}^{\infty} e^{ixt} \sum_{k'} \left(f^*_k(\omega')f_l(\omega')e^{-i\omega' t} + g^*_k(\omega')g_l(\omega')e^{i\omega' t} \right) dt$$

$$= \sum_{k'} \left(f^*_k(\omega')f_l(\omega') \int_{-\infty}^{\infty} e^{ix(t-\omega')}dt + g^*_k(\omega')g_l(\omega') \int_{-\infty}^{\infty} e^{ix(t+\omega')}dt \right)$$

To solve the integral we use the Fourier relation: $\int_{-\infty}^{\infty} e^{i(x-\omega')t}dt = 2\pi\delta(x-\omega')$ (where the 2π is a convention of the normalization factor):

$$\gamma^0_{kl}(x) = 2\pi \left(f^*_k(x)f_l(x) + g^*_k(x)g_l(x) \right)$$

Introducing $\frac{|g_k(x)|^2}{|f_k(x)|^2} = e^{-\beta_H \hbar x}$, we obtain:

$$\gamma^0_{kl}(x) = 2\pi f^*_k(x)f_l(x) \left(1 + e^{-\beta_H \hbar x} \right). \tag{56}$$

Know we want to see if this equation satisfies the KMS condition: $\gamma^0_{kl}(-x) = e^{-\beta_H x}\gamma^0_{-l,-k}(x)$ Therefore changing the sign of x in (56) we have:

$$\gamma^0_{kl}(-x) = 2\pi f^*_k(-x)f_l(-x) \left(1 + e^{\beta_H \hbar x} \right) = e^{-\beta_H \hbar x} \left(2\pi f^*_{-l}(x)f_{-k}(x) \left(1 + e^{-\beta_H \hbar x} \right) \right)$$

We see that this satisfies the KMS condition with $\beta = \beta_H$, showing that the black hole behaves with respect to the external fields as a heat bath at the Hawking temperature.

The results of the Section 4.5 imply that a Kerr black hole will superradiate bosons obeying the condition:

$$\omega < m\Omega_H, \tag{57}$$

where Ω_H is the angular velocity of the BH horizon show in (13).

5 Observational Evidence of Superradiance

The superradiance phenomenon, even though it is well establish theoretically, it presents meaningful challenges in terms of its direct observation in astrophysics context and it has not been detected yet.

We will be exploring two principal phenomenon than lead to observational evidence that may occur due to the superradiance process: the interaction of the black hole with ultralight bosons and the jets of the Active Galaxies Nuclei (AGNs).

5.1 Ultralight massive bosons

The detection of gravitational waves by LIGO and Virgo have opened a new window in the seek of superrandiance signs. In particular the existence of ultralight bosons (like axions) that could lead to superradiance instabilities, which may occur under certain conditions during the extraction of energy from the superradiance process.

One example is the "black hole bomb" by Press and Teukolsky [10], that consist in a rotating black hole surrounded by a spherical mirror. A single photon introduced in the system, or created by quantum fluctuations, with quantum numbers satisfying the superradiance condition Eq.46 gives rise to an exponentially growing number of photons inside the mirror, through the stimulated emission explained in Section 4.4 but know exaggerated with the presence of the mirror that makes the field interact over and over with the BH.

Nature provides such a mirror in the presence of a massive boson [11], if the Compton wavelength of the boson is of the order of the black hole gravitational radius. We will see that this instability turns rotating black holes into sensitive detectors of bosons with masses in the range $\mu \sim 10^{-9} \div 10^{-21}$ eV. The best particle candidate for this ultralight bosons is a QCD axion.

5.1.1 Brief context about Axions

The QCD axion is a hypothetical particle proposed to solve a particle physics problem, known as the "Strong CP problem", which we are not going to enter to explain.

Is a pseudoscalar particle, so it means that it has properties that they invert under a parity transformation. A symmetry break (PQ symmetry) gives mass to this particle, calculate as :

$$\mu_a \approx 6 \cdot 10^{-10} \text{ eV} \left(\frac{10^{16} \text{ GeV}}{f_a} \right). \tag{58}$$

Where f_a is a parameter that describes the energy in which the (PQ) symmetry is broken.

When f_a is really high ($\geq 10^{16} GeV$), the Compton wavelength of the axion (inversely related with it's mass) is comparable to the size of stellar black holes and, consequently, can affect their dynamics (due to superradiance inestabilities), suggensting that this part of the parameter space of the QCD axion can be explored through black hole observations.

Finding the QCD axion with f_a of the previous mention order of magnitude, would indicate that the baryon-to-dark matter ratio varies on length scales longer than the observed part of the Universe. This variation could be determinate by conditions that allow life existence.

5.1.2 The Axionic Cloud

Massive bosons, in our cause axions ,that have a wavelength comparable to the size of the BH, can occupy energy levels in the gravitational field of the black hole, similarly as how electrons occupy levels in an atom.

This energy levels, known as Keplerian levels, are orbits were axions could remain bonded to the black hole due to the gravitational interaction.

Through the superradiance process, the black hole release it's spin by populating specific levels that satisfy the superradiance condition. Eventually, this accumulation of axions in certain quantum states form a Bose-Einstein condensate (BEC)[4] cloud that rotates around the black hole. This is what is called an Axionic Black Hole Atom, see Figure 3 to get an idea of what could be happening.

[4]In stadistical physics a BEC is form when a boson system at a low temperature condenses in a state of low energy,i.e, loads of bosons occupy the same quantum state. This phenomenon is possible due to the Bose-Einstein stadistics, which allow multiple particles to be in the same quantum state. In this context when we talk about BEC rather than referring to minimum energy state where the bosons concentrate, we are speaking in a more general sense, where the BEC refers to a large occupation of specific quantum states.

Therefore, the energy spectrum of superradiant levels is quantized and really similar to the energy levels of the hydrogen atom.

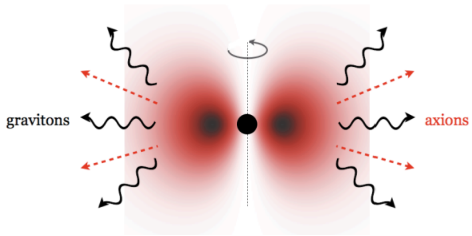

Figure 3: Axionic Black Hole Atom: The spinning black hole "feeds" superradiant states forming an axion Bose-Einstein condensate.[5]

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However, the superradiance effect is significant for a range of specific values of the BH and axion mass, as is shown in Figure 4.

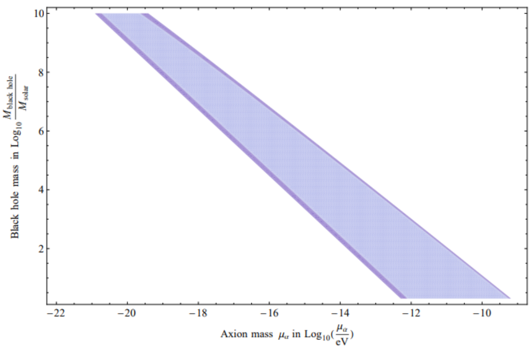

Figure 4: Part of the black hole and axion parameter space potentially affected by superradiance.[5]

.

Here the x-axis is the axion mass, μ_a , in $\log_{10}\left(\frac{\mu_a}{\text{eV}}\right)$ and the y-axis is the black hole mass divided by the mass of the Sun, $\frac{M_{\text{BH}}}{M_{\text{Sun}}}$ in $\log_{10}$.

The dark area mark the region where a superradiant cloud has enough time to be built up during the lifetime of the Universe for a maximally spinning BH. Whereas, the light region indicates the part of the parameter space where the spindown rate due to the superradiance is faster than the spinup rate because of the Eddington accretion [5].

So Eddington accreting black hole in this mass range will lose their spin as the cloud develops (having consequences on the own cloud).

Furthermore this axion cloud losses energy and angular momentum through several mechanisms:

1. Gravity wave emission

    - The transitions between axionic levels (analog to atomic levels) can emit gravitational waves, similar to how electrons in an atom can emit photons when they change of level.
    - Axion Annihilation into Gravitons:The axions in the BEC can annihilate between them and produce gravitons[6].

[5] Is a maximum theoretical limit of mass accretion rate. Beyond it, the radiation pressure would expel the additional material.

[6] Gravitons are hypothetical particles that would mediate the gravitational force in Quantum-Gravity theories

2. Axion non-linearities of its own potential

   - Bosenova: As the cloud grows the attractive interaction between the axions turn significant. If they get that strong that they exceed the energy of the gravitational bonding, which keeps the cloud tight, suddenly the cloud collapse.This collapse release a huge amount of energy in form of gravitational waves an other particles.
   - Shutdown of superradiance due to level mixing: In a lineal system the energy levels are well defined and they don't interact between them, however, in a non lineal system, because of the meaningful interactions in the cloud, the energy levels could mix up. This leads to an inefficient capacity of the axions to occupy certain energy levels, what could stop or "shut down" the superradiance process. The main consequence is that the axion cloud cannot continue extracting energy and angular momentum from the BH in an effective way.

5.1.3 Observational signatures of the axionic cloud

Despite of the fact that there are several classes of observational signs associated with the axion cloud, we are going to be focusing in the gravitational waves.

Arvanitaki (the one who wrote the paper in which we are basing this section [5]) was the first to noticed that superradiant instabilities of a rotating BH can be a source of gravitational wavess.

As we have seen the gravitational waves, are the main emission of this cloud, due to several mechanisms of loss energy.

In Figure 5, it is presented an estimated gravitational wave signal strength as a function of the axion and black hole masses for a source at a 20 Mpc distance from the Earth.

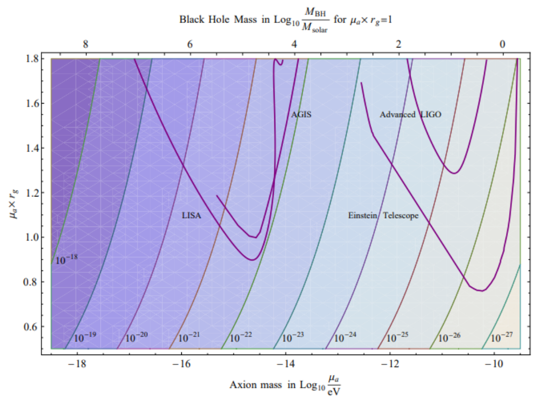

Figure 5: Contour plot of constant gravitational wave signal from axion transitions between the 6g and the 5g levels for a BH located at 20 Mpc away from the Earth .[5]

.

The shaded areas and label curves show the sensibility of the different detectors for a coherent time of integration of $10^4 s$.

We see that the experiments could be sensitive to the transition signal from the superradiant cloud over a large range of axion and black hole masses.

The fact to highlight here is that, the gravitational wave signal from the QCD axion cloud (heavy mass regime for axions) around stellar mass black holes, falls into the Advanced LIGO and Einstein telescope frequency band and turn them into a particle discovery machine.

In this paper of 2021 [12],that is one of the most recent last evidence I found of this field, it says that this observational signal from the axion cloud has not been detected yet but it remarks the importance of the investigation line because it speaks of the possibility that this QCD axions could be potentially constituents of dark matter.

5.2 Jets of the AGN

The superrelativist jets observed in AGNs[7] are collimated fluxes of charged particles and radiation that are expel throughout the rotational axis of the black hole at relativist speeds.

Even though the first AGNs (such as quasars and radio galaxies) were discovered four decades ago, the engine powering of these events is still largely unknown. The energy needed for the acceleration of these relativistic outflows of matter is widely believed to came from two possible sources.

One from the accretion of matter into the BH, which leads to a transfer of gravitational binding energy to the particles that conform the jet.

The other probable source comes from the BH rotational energy through processes similar to the superradiance or the Penrose process. This is the case of the Blandford-Znajek (BZ) mechanism which occurs for BHs inmmersed in magnetic fields.

In the following sections we will give a brief idea of this process and argue if the superradiance is also needed to explain the energy of the jets.

5.2.1 Blandford-Znajek Mechanism

The Blanford-Znajek mechanism (BZ) is a process proposed by Rofer Blandford and Roman Znajek in 1977 to explain the energy extraction from a rotating BH and it's conversion into electromagnetic radiation, which is observed as relativist jets [13]. This process is based in the interaction between a magnetic field and a Kerr BH.

In the neighborhood of the rotating BH, a magnetic field could be tie down by the surrounding plasma. The rotation of that black hole drags the magnetic field through the ergoregion bringing on electric currents, which generate an electric potential that can accelerate particles throughout the magnetic lines.

This creates a dynamo configuration[8] where the rotational energy of the BH is transferred into the electromagnetic field. The inducted currents generate a torsion of the magnetic field lines, which transport the energy far away from the black hole. This electromagnetic energy can be channelised in the shape of the relativist jets.

The efficiency of the process is quite difficult to estimate. In this paper [14] they concluded that the efficiency with which a spininng BH can generate jet power depends on the BH spin a via the angular momentum Ω_H and on the magnetic flux $\phi_B H$.

It is approximated that the efficient of this process is of the order of the 30% or even more depending in the factors previously mentioned. But observations said ,that there are indications of some AGNs in the universe may have extremely efficient jets of the order of more than the 100% of efficiency [15]. In this cases other process apart from the BZ mechanism must be operating, here is where the superradiance process appear.

[7]The AGNs (Active Galaxy Nuclei) are an extreme luminous region of the center of a galaxy, that emits enormous amounts of radiation in multiple wavelengths, from radio even to gamma rays. There are form by a supermassive black hole, an accretion disk, regions of fast and slow rotating gas clouds, a dust toroid and some of them with jets . There are several types, one example of them are the Quasars.

[8]A dynamo is a device which transforms mechanical energy into electrical energy through the use of an magnetic field. It functions with the Electromagnetic Induction Principle discovered by Faraday, where a conductor that moves through a magnetic field inducts an electric current.

5.2.2 Contribution of the Superradiance Process to the Jets

The superradiance process could increase the efficiency of the BZ mechanism through several ways:

1. Amplification of the electromagnetic waves: As we have discuss throughout this work, the superradiance can amplify incident waves into the BH. In this case, it will turn into the amplification of electromagnetic waves. On account of this, the intensity of the magnetic field could increase near the event horizon, which at the same time could raise the extraction energy efficiency through the BZ mechanism.

2. Interaction with the axion cloud: If there is an axion cloud around the BH due to the superradiance process, the transitions and the annihilations of the axions can liberate additional energy that could be capture by the magnetic field, thus increasing the luminosity and the efficiency of the jets.

3. Acceleration of the particles: The extracted energy from the BH by the superradiance process could be transmitted to the charged particles within the magnetic field, raising their kinetic energy and making the jets more powerful and bright.

6 Conclusion

After the development of this work, I hope I have convinced the reader that black hole superradiance is an extremely rich phenomenon that could entail really interesting consequences which are very likeable to be observed in the near future.

We have explore the most important repercussion of the metric of a rotating object: the appearance of an ergoregion. As we have seen, one of it's main characteristics is the change in sign of the coefficient g_{tt}, what brings the key outcome of allowing the energy extraction of the black hole.

For the study of this phenomenon we have started with a classical approach, explaining the Penrose process which introduces the concept of negative energies for particles in the ergoregion.

As we have seen, treating the problem via the thermodynamics and quantum mechanics, the requirement of negative energies leads to the superradiance condition: $\omega < m\Omega$.

Furthermore, we have discover than when this requisite is fulfill is because modes with higher energy are more populated than the one's with lower energy, i.e, an inversion of the population has been accomplished, leading to a significant stimulated emission which amplifies the incident radiation.

Therefore, one of the most remarkable conclusion of this work is that by the idea of overpopulating high energy modes of a quantum field due to a positive feedback with a thermal heat bath, we can get a model to explain the superradiance phenomenon.

Moreover, it should be noted that, as we have mention in the last part of this work, superradiance could turn black holes into particle detectors by phenomenons like the Axionic Black Hole Atom. Detecting gravitational waves from this events will lead to the discovery of potential constituents of dark matter.

Furthermore, superradiance could help explaining the mystery of one of the most powerful sources of the universe, the jets of the AGNs.

Clearly, this field has a huge potential that in the future could be the key to solve important questions about the universe that nowadays are still not figure out . As we have seen we are not far away of developing the needed technology for detecting this amazing phenomenon called superradiance.

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