Matching the 2HDM to the HEFT and the SMEFT: Decoupling and perturbativity

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We consider the 2 Higgs doublet model (2HDM) and compare two effective field theory (EFT) approaches to it, according to whether the heavy degrees of freedom are integrated out before [standard model EFT (SMEFT)] or after [Higgs EFT (HEFT)] spontaneous symmetry breaking. By requiring decoupling and perturbativity in the 2HDM, we define a consistent EFT expansion in inverse powers of the heavy masses which is applied to both the SMEFT and the HEFT tree-level matchings to the 2HDM. We organize this expansion with a dimensionless parameter ξ, and investigate the tree-level scatterings $hh \to hh$ and $WW \to hh$ up to $\mathcal{O}(\xi^2)$. We find no differences between the HEFT and the SMEFT approaches at this order. We show scenarios where even including dimension-8 operators of the SMEFT is insufficient to obtain an accurate matching to the 2HDM.

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I. INTRODUCTION

Since the discovery of the Higgs boson in 2012, the experimental focus in the electroweak sector at the LHC has turned to precision measurements of Higgs observables and the search for heavy Higgs-like particles. To date, no significant deviation from the Standard Model (SM) predictions has been observed, suggesting that beyond the SM (BSM) physics, if it exists, must be at a much higher energy scale than that probed at the LHC. In this scenario, effective field theories (EFTs) are the tool of choice in the search for deviations from the SM. In principle, the EFTs represent a model-independent formalism which can then be matched to the predictions of specific ultraviolet (UV) complete models.

Two types of EFTs can be used to model the unknown BSM physics that potentially affects the Higgs sector; they are the SM effective field theory (SMEFT) [1–3] and the Higgs effective field theory (HEFT) [4–6] (cf. Ref. [7] for a review). Both use exclusively SM degrees of freedom, and both are invariant under the SM gauge groups $\mathrm{SU}(3) \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}_\mathrm{Y}(1)$. However, while the SMEFT considers the Higgs field h and the electroweak (EW) would-be Goldstone bosons, ω^a, to be embedded in the $\mathrm{SU}(2)_\mathrm{L}$ Higgs doublet, the HEFT treats h as a gauge singlet and classifies the ω^a as an $\mathrm{SU}(2)_\mathrm{L}$ triplet. As a consequence, the SMEFT starts from the SM as it is before spontaneous symmetry breaking (SSB) of $\mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}_\mathrm{Y}(1) \to \mathrm{U}_\mathrm{EM}(1)$, and adds to it a tower of higher dimensional operators, O_i^n, built out of the (before-SSB) SM fields:

$$\mathcal{L}_\mathrm{SMEFT} = \mathcal{L}_\mathrm{SM} + \sum_{n,i} \frac{C_i^n O_i^n}{\Lambda^{n-4}}, \tag{1}$$

where $n > 4$ is the dimension of the operator, C_i^n are coefficients (usually known as Wilson coefficients, WCs) and Λ the UV scale. By contrast, the HEFT starts by treating h and the ω^a separately, in such a way that the latter are embedded into a unitary matrix U. Moreover, the HEFT is an expansion in the number of covariant derivatives; at the lowest order, the part of the HEFT Lagrangian relevant for the scattering processes discussed in this article is[1]

$$\mathcal{L}_\mathrm{HEFT} \supset \frac{v^2}{4}\mathcal{F}(h)\mathrm{Tr}\{D_\mu U^\dagger D_\mu U\} + \frac{1}{2}(\partial_\mu h)^2 - V(h), \tag{2}$$

where $v = 246$ GeV represents the vacuum expectation value (vev) of the Higgs field in the SM, D_μ is the covariant

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[1]Only terms relevant for our current purposes are shown. In particular, fermions will not be relevant, and will be omitted in what follows.

derivative, and $\mathcal{F}(h)$ and $V(h)$ are generic functions of h. In general, one has $D_\mu U = \partial_\mu U + igW_\mu^a \frac{\sigma^a}{2} U - ig' U \frac{\sigma^3}{2} B_\mu$, with $U = 1$ in the unitary gauge. The fact that h is a gauge singlet means that symmetry invariance allows $\mathcal{F}(h)$ and $V(h)$ to be arbitrary power series in h. Considering again the lowest order HEFT Lagrangian, we find

$$\mathcal{F}(h) = 1 + 2a\frac{h}{v} + b\frac{h^2}{v^2} + \ldots,$$
$$V(h) = \frac{1}{2}m_h^2 h^2\left(1 + d_3\frac{h}{v} + \frac{d_4}{4}\frac{h^2}{v^2} + \ldots\right), \tag{3}$$

where m_h is the h mass, the dots stand for terms with higher powers of h, and a, b, d_3 and d_4 are arbitrary couplings. These are normalized so that the SM is recovered when both $a = b = d_3 = d_4 = 1$ and the remaining terms with higher powers of h are set to zero.

A significant effort has been made in recent years to derive techniques to distinguish the SMEFT and the HEFT from one another from a pure bottom-up approach, i.e. without assuming knowledge about any possible BSM model [8–14]. Yet, since the EFTs are ultimately effective descriptions of a particular UV model, it is also relevant to discuss a top-down approach. In this case, the BSM model is assumed to be known, and a matching between the EFTs and the UV model is obtained by integrating out the heavy degrees of freedom. This exercise has been done in the recent literature especially for the SMEFT, considering several different UV models [15–29].

In this paper, we follow the top-down approach taking the 2 Higgs doublet model (2HDM) [30] as the BSM model, and discuss the matching to both the SMEFT and the HEFT. Reference [19] performed an exercise along these lines, choosing as the BSM model a singlet extension of the SM with a $\mathbb{Z}_2$ symmetry. It turns out that this model is very special, as it allows an EFT expansion which is exclusively governed by inverse powers of the heavy mass. By contrast, and as we will show, a consistent EFT approach cannot be applied to a model like the 2HDM unless one makes further assumptions besides those related to the physical masses. This aspect is intimately related to the notions of decoupling and perturbativity, which shall be discussed in detail below.

We will focus on the tree-level scattering processes $WW \to hh$ and $hh \to hh$, where the HEFT and SMEFT may have potential differences when matched to the 2HDM. We pay particular attention to performing consistent expansions in the different EFTs, and investigate how accurately they reproduce the results of the 2HDM.

We perform a tree-level matching of the 2HDM to the SMEFT at dimension-6 and dimension-8 and also a tree-level matching of the 2HDM to the HEFT to $\mathcal{O}(p^2)$ in the HEFT expansion. We include all operators that are generated by this tree-level matching that contribute to the tree-level scattering processes $WW \to hh$ and $hh \to hh$.

This paper is organized as follows. We start by recapping the 2HDM in Sec. II. Section III is devoted to the notion of decoupling and to its consequences for an EFT expansion. That allows us to study the SMEFT and HEFT matchings to the 2HDM, which we do in Secs. IV and V, respectively. Finally, we present our results in Sec. VI and our conclusions in Sec. VII. We provide further details on the 2HDM and on the model of Ref. [19] in the appendices.

II. 2HDM

For this review of the 2HDM, we follow Ref. [29] closely (for more details, cf. Refs. [31,32]). The model adds an extra doublet Φ_2 to the SM scalar doublet Φ_1, and we define their vevs as $v_2/\sqrt{2}$ and $v_1/\sqrt{2}$, respectively (we take them to be real). We impose a softly broken $\mathbb{Z}_2$ symmetry, under which the scalar doublets transform as $\Phi_1 \to \Phi_1$ and $\Phi_2 \to -\Phi_2$, whereas the fermion fields can transform in four different ways (each one corresponds to a different type of 2HDM).[2] It is convenient to introduce an angle β such that $t_\beta = v_2/v_1$, which allows us to move to the Higgs basis [33–36] as[3]:

$$\begin{pmatrix} H_1 \\ H_2 \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix}\begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix}. \tag{4}$$

In the Higgs basis, the second doublet (H_2) has no vev, whereas H_1 has the vev $v/\sqrt{2}$, with $v \equiv \sqrt{v_1^2 + v_2^2} = 246$ GeV. Among the terms of the Lagrangian, we focus on just two, $\mathcal{L}_{\mathrm{2HDM}} \ni \mathcal{L}_{\mathrm{kin}} - V$, the former being the scalar kinetic piece and the latter the potential. In the Higgs basis, they read

$$\mathcal{L}_{\mathrm{kin}} = (D_\mu H_1)^\dagger (D^\mu H_1) + (D_\mu H_2)^\dagger (D^\mu H_2), \tag{5a}$$

$$V = Y_1 H_1^\dagger H_1 + Y_2 H_2^\dagger H_2 + (Y_3 H_1^\dagger H_2 + \mathrm{H.c.}) + \frac{Z_1}{2}(H_1^\dagger H_1)^2 + \frac{Z_2}{2}(H_2^\dagger H_2)^2 + Z_3 (H_1^\dagger H_1)(H_2^\dagger H_2) + Z_4 (H_1^\dagger H_2)(H_2^\dagger H_1)$$
$$+ \left\{\frac{Z_5}{2}(H_1^\dagger H_2)^2 + Z_6 (H_1^\dagger H_1)(H_1^\dagger H_2) + Z_7 (H_2^\dagger H_2)(H_1^\dagger H_2) + \mathrm{H.c.}\right\}, \tag{5b}$$

[2]Since the fermions will not be the focus of this paper, we refer the reader to Ref. [32] for details on the different types of 2HDMs.

[3]Here and in the following, it should be clear that, for any angle x, we use $c_x \equiv \cos(x)$, $s_x \equiv \sin(x)$, $t_x \equiv \tan(x)$.

in such a way that, on the one hand, the minimization equations imply

$$Y_1 = -\frac{Z_1}{2} v^2, \qquad Y_3 = -\frac{Z_6}{2} v^2, \tag{6}$$

and, on the other, the $\mathbb{Z}_2$ symmetry (which is only explicit in the basis of Φ_1, Φ_2) is manifested by the circumstance that only 5 of the 7 Z_i are independent. Although the parameters Y_3, Z_5, Z_6, Z_7 are, in general, complex (the remaining parameters are real by Hermiticity), we restrict ourselves to the solution in which they have real values.[4] CP symmetry is thus preserved at the leading order in the scalar sector, in which case H_1 and H_2 can be parametrized as

$$H_1 = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + h_1^{\rm H} + iG_0) \end{pmatrix}, \quad H_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(h_2^{\rm H} + iA) \end{pmatrix}, \tag{7}$$

with $h_1^{\rm H}$, $h_2^{\rm H}$, G_0 and A real fields, and G^+, H^+ complex ones. With the exception of $h_1^{\rm H}$, $h_2^{\rm H}$, all of these states are already mass states (G_0 and G^+ are the would-be Goldstone bosons, and A and H^+ are the pseudoscalar and the charged scalar bosons, respectively). The mass matrix for $h_1^{\rm H}$ and $h_2^{\rm H}$ can be diagonalized by introducing a mixing angle α such that

$$\begin{pmatrix} h \\ H \end{pmatrix} = \begin{pmatrix} s_{\beta-\alpha} & c_{\beta-\alpha} \\ c_{\beta-\alpha} & -s_{\beta-\alpha} \end{pmatrix} \begin{pmatrix} h_1^{\rm H} \\ h_2^{\rm H} \end{pmatrix}, \tag{8}$$

where h and H are the neutral scalar mass states, with h being the scalar that is observed at the LHC. Finally, defining the masses of h, H, A and $H^\pm$ to be m_h, m_H, m_A and $m_{H^\pm}$, respectively, we shall take the following parameters as independent:

$$c_{\beta-\alpha}, \beta, v, m_h, Y_2, m_H, m_A, m_{H^\pm}. \tag{9}$$

The expressions for the Z_i parameters in terms of the independent parameters can be found in Appendix A.

III. DECOUPLING AND PERTURBATIVITY

In the following sections, we shall derive EFTs for the model described in Sec. II, which is taken as our UV model. Such a derivation requires a separation of scales in the UV model. Let us focus on the UV model, and assume that it has two disparate mass scales, Λ and v, such that $\Lambda \gg v$. Intuition leads to the expectation that the physical effects of the particle(s) with mass of $\mathcal{O}(\Lambda)$ should be suppressed at low energies, i.e. at $\mathcal{O}(v)$. This is, in fact, the main idea of *decoupling*, which is formalized in the Appelquist-Carazzone decoupling theorem [38] (see also Ref. [7]).

Yet there is an important caveat here. The decoupling theorem was formulated in Ref. [38] for a model without SSB, where the masses are independent parameters in the Lagrangian; in particular, they are independent of interaction couplings. It follows that a given mass can be rendered very large [of $\mathcal{O}(\Lambda)$] without affecting the interaction couplings—and, in particular, without requiring these couplings to become very large. In this way, taking a particle to be very heavy in a model without SSB does not jeopardize *perturbativity*, which is an implicit assumption of the decoupling theorem.[5]

In models with SSB, the situation changes considerably [39–44]. The reason is that particles in models with SSB often get their masses from the product of a (fixed) vev and an interaction coupling. To obtain a very heavy mass for a particle, one would thus need to take the interaction coupling to be very large—which would, however, inevitably make perturbation theory invalid. Therefore, decoupling is not possible in this scenario: one cannot take a particle to be infinitely massive without violating perturbativity (see also the discussion in Ref. [7]).

It should be clear, on the other hand, that this does not mean that decoupling is impossible in a model with SSB. For it may happen that, in such theory, a particle gets at least *part* of its mass from a mass parameter of the Lagrangian—which, as mentioned above, is independent of the remaining Lagrangian parameters, and in particular of the interaction ones. Hence, by taking that mass parameter to be very heavy (while keeping the interaction parameters fixed), one renders the particle at stake to be very massive, without endangering the validity of a perturbative description.

We can apply this discussion to the 2HDM described in the previous section, which is the focus of this paper. Our goal is to make the particles which do not belong to the SM (H, A and $H^\pm$) very heavy, so that an EFT for the 2HDM can be built using solely the degrees of freedom of the SM. To that end, we must have

$$m_H \simeq m_A \simeq m_{H^+} \gg m_h = 125 \text{ GeV}. \tag{10}$$

To see how this can be obtained in a consistent way, it is convenient to write these masses in terms of v, $c_{\beta-\alpha}$ and parameters of the potential:

$$m_h^2 = \frac{c_{\beta-\alpha}^2}{2c_{\beta-\alpha}^2 - 1} Y_2 + \frac{2(c_{\beta-\alpha}^2 - 1)Z_1 + c_{\beta-\alpha}^2 Z_{345}}{4c_{\beta-\alpha}^2 - 2} v^2, \tag{11a}$$

$$m_H^2 = \frac{(c_{\beta-\alpha}^2 - 1)}{2c_{\beta-\alpha}^2 - 1} Y_2 + \frac{c_{\beta-\alpha}^2(2Z_1 + Z_{345}) - Z_{345}}{4c_{\beta-\alpha}^2 - 2} v^2, \tag{11b}$$

[4] As stressed in Ref. [37], though, one should keep in mind that those parameters are generally complex, since issues with renormalization would otherwise follow.

[5] In this paper, we assume that decoupling requires perturbativity, and we do not consider the scenario in which the UV model violates perturbativity.

$$m_A^2 = Y_2 + \frac{Z_{345} - 2Z_5}{2} v^2, \tag{11c}$$

$$m_{H^+}^2 = Y_2 + \frac{Z_3}{2} v^2, \tag{11d}$$

with $Z_{345} \equiv Z_3 + Z_4 + Z_5$. As suggested above, each of the squared masses (m_h^2 included) contains two parts: one of them proportional to a mass parameter of the Lagrangian (Y_2), the other one proportional to the product between interaction couplings (Z_i) and the squared vev (v^2). This means that Y_2 plays a fundamental role in decoupling, as it can be used to render m_H, m_A and m_{H^+} very large without compromising the validity of the perturbation theory.[6] It is also clear that, if Eq. (10) is to be obeyed, and if $c_{\beta-\alpha}$ is chosen as an independent parameter, then taking Y_2 to be very large is not enough; more than that, $c_{\beta-\alpha}$ must behave so as to ensure that m_h stays fixed as Y_2 is increased. Another way to realize this is to consider Eq. (A1), which shows the Z_i parameters written in terms of the parameters of Eq. (9). From those equations [in particular Eq. (A1a)], it is clear that the only way Eq. (10) can hold without having large Z_i (i.e. without violating perturbativity) is to require that $c_{\beta-\alpha}$ scales with Λ^{-2}.

All of this leads us to define the *decoupling limit* of the 2HDM [45–48]—which ensures Eq. (10) while complying with $Z_i/(4\pi) \lesssim \mathcal{O}(1)$—as[7]

$$Y_2 = \Lambda^2, \qquad m_H^2 = \Lambda^2 + \Delta m_H^2,$$
$$m_A^2 = \Lambda^2 + \Delta m_A^2, \qquad m_{H^+}^2 = \Lambda^2 + \Delta m_{H^+}^2, \tag{12a}$$

$$\Lambda^2 \gg v^2, \quad m_h^2 \sim \mathcal{O}(v^2), \quad \Delta m_H^2, \Delta m_A^2, \Delta m_{H^+}^2 \sim \mathcal{O}(v^2), \tag{12b}$$

$$c_{\beta-\alpha} \sim \mathcal{O}(v^2/\Lambda^2), \tag{12c}$$

with Δm_H^2, Δm_A^2 and $\Delta m_{H^+}^2$ real parameters. In the following sections, the decoupling limit defined in Eq. (12) will be used to build *expansions*, corresponding to either the SMEFT or the HEFT. This can be more easily done if we introduce an auxiliary dimensionless parameter ξ, which acts as the *de facto* expansion parameter. Then, assuming Eq. (12a), we implement the scaling $v^2/\Lambda^2, c_{\beta-\alpha} \sim \mathcal{O}(\xi)$ of Eqs. (12b) and (12c) at the practical level (in our codes) through

$$\frac{1}{\Lambda^2} \to \frac{\xi}{\Lambda^2}, \qquad c_{\beta-\alpha} \to \xi c_{\beta-\alpha}, \tag{13}$$

while all the other scales and parameters are $\mathcal{O}(\xi^0)$ and are left untouched.[8] In this way, the expansion will correspond to a series of positive powers of ξ. The trivial order—$\mathcal{O}(\xi^0)$—implies the alignment limit, which is defined by $c_{\beta-\alpha} \to 0$ and corresponds to the scenario in which the h couplings are exactly those of the SM. In this way, decoupling implies alignment.[9]

Several aspects are worth mentioning here. The first one is that Eq. (10) is found *if and only if* we have both $Y_2 \gg v^2$ and perturbativity. That is, assuming Eq. (10) or assuming $Y_2 \gg v^2$ and $Z_i/(4\pi) \lesssim \mathcal{O}(1)$ are just two equivalent ways to describe the same physical scenario corresponding to the decoupling limit. Equation (12) is yet another equivalent way, which specifies how the parameters of Eq. (9) behave in that physical scenario.

This implies that the power counting of the SMEFT and the HEFT matchings are going to be equivalent. It is true that, as shall be seen in detail, the SMEFT performs the expansion before SSB and the HEFT after it—such that the former uses the Lagrangian parameter Y_2 as an expansion parameter, whereas the latter uses physical masses m_H, m_A, m_{H^+}. Yet, since those physical masses can be made large if and only if $Y_2 \gg v^2$ and $Z_i/(4\pi) \lesssim \mathcal{O}(1)$, the two expansions are the same, in the sense that they follow the same power counting. Given the set of independent parameters of Eq. (9), that power counting is organized by powers of ξ, as defined in Eq. (13).

This leads to another aspect, related to the role of $c_{\beta-\alpha}$. The special scaling of $c_{\beta-\alpha}$ in Eq. (12c), as well as the subsequent ξ power counting introduced in Eq. (13), both follow from the choice of $c_{\beta-\alpha}$ as an independent parameter. If, instead of $c_{\beta-\alpha}$, one of the Z_i were chosen as independent—say, Z_6—one would simply need to require Z_6 to obey $Z_6/(4\pi) \lesssim \mathcal{O}(1)$, in which case the expansion would simply be in inverse powers of Λ^2.[10] The two scenarios—the one in which $c_{\beta-\alpha}$ is independent, and the one in which Z_6 is independent—are perfectly equivalent. Note also that, if Z_6 were independent, we would find

[6]This also shows that, in a 2HDM with an exact (i.e. not softly broken) $\mathbb{Z}_2$ symmetry, it is not possible to decouple H, A and H^+. The reason is that, in that case, $Y_2 \sim \mathcal{O}(Z_i v^2)$, so that Y_2 cannot be taken to be very large without violating perturbativity [45].

[7]Although Y_2 could be written $Y_2 = \Lambda^2 + \Delta Y_2$, with ΔY_2 a real parameter, the latter can be set to zero without loss of generality.

[8]Reference [49] followed a similar procedure. One might wonder whether it would be possible to have alternatives to Eq. (13) for which β might also scale in a nontrivial way. Yet, by considering Eqs. (A1b) and (A1g), it is easy to conclude that any scaling of β would violate perturbativity.

[9]The reverse is in general not true: it is possible to have alignment without decoupling [47,50,51]. An EFT approach to the 2HDM in general does not work in this case [20].

[10]Just as $c_{\beta-\alpha} \sim \mathcal{O}(v^2/\Lambda^2)$, the scaling $Z_6/(4\pi) \lesssim \mathcal{O}(1)$ would ensure not only perturbativity, but also that m_h would be fixed. This last aspect can be seen by considering Eq. (A3a), which is equivalent to E. (11a), but with $c_{\beta-\alpha}$ replaced by Z_6. It is clear that, as long as perturbativity is ensured [all Z_i obeying $Z_i/(4\pi) \lesssim \mathcal{O}(1)$], the scenario of very large Y_2 will imply the cancellation of Y_2 in the expression (A3a).

$c_{\beta-\alpha} = Z_6 v^2/Y_2 + \mathcal{O}(v^4/Y_2^2) \sim \mathcal{O}(\xi^1)$, so that the scaling of the $c_{\beta-\alpha}$ mixing in Eq. (13) would show up in a natural way.

Also relevant is an aspect concerning the mass states. As suggested above, the extreme case of the decoupling limit—namely, $Y_2 \to \infty$ taken in a way consistent with perturbativity—implies $c_{\beta-\alpha} \to 0$. This, in turn, implies $h_1^{\rm H} \to h$ and $h_2^{\rm H} \to -H$ by Eq. (8). It follows that $h_1^{\rm H}$ and $h_2^{\rm H}$ effectively correspond to mass states in the extreme decoupling. In the case in which Y_2 is very large but finite, there will be differences between $h_2^{\rm H}$ and the mass state $-H$ which are proportional to $c_{\beta-\alpha} \sim \mathcal{O}(v^2/\Lambda^2)$.

Finally, we have been discussing how Eq. (10) can be obtained without spoiling perturbativity. We should keep in mind, however, that the latter (perturbativity) is not restricted to that equation. Put another way, there are issues concerning perturbativity which are independent of the limit of heavy scalar masses. A simple example is provided by β; even though this parameter is independent of Eq. (10), its values can be such that perturbativity is violated.[11] Note that this feature is already present in the full 2HDM, so that it is not specific to an EFT expansion. This also means that the expansion of Eq. (13) does not ensure that perturbativity will be obeyed order by order in ξ; it only ensures that Eq. (10) can be obtained without violating perturbativity.

IV. SMEFT

As referred to in the Introduction, the starting point of the SMEFT is the SM before SSB (to which higher-dimensional operators are added). Therefore, the SMEFT matching to the 2HDM must be done in such a way that the integration out of the heavy degrees of freedom of the 2HDM happens before SSB. Yet, here we are faced with a problem: not all the mass states of the 2HDM are defined before SSB. In fact, as seen above, the states $h_1^{\rm H}$ and $h_2^{\rm H}$ mix after SSB, and their mass matrix is diagonalized to yield the mass states h and H. In that case, how can the heavy state H be integrated out before SSB, if it is not even defined by then?

The answer has to do with the decoupling limit. We saw above that, in the extreme decoupling limit ($Y_2 \to \infty$), $h_2^{\rm H}$ becomes a mass state. In that case, the doublet H_2 of the Higgs basis can be integrated out before SSB: on the one hand, the fact that H_2 is a doublet of $\mathrm{SU_L}(2)$ means that the states contained in it can be integrated out as a whole (without violating the symmetries of the theory before SSB). On the other hand, by Eq. (7), all its states become very heavy in that extreme decoupling scenario.

H_2 is then integrated out at tree level. This means (a) assuming H_2 can be expressed as an expansion in inverse powers of Y_2, (b) deriving a truncated solution for H_2 using equations of motion (EoM) and (c) plugging that solution back in the original Lagrangian. The resulting Lagrangian will thus be itself an expansion in inverse powers of Y_2.[12] This parameter is identified with the squared UV scale Λ^2, and the resulting EFT can be written in the format of the SMEFT. This exercise has been performed up to $\mathcal{O}(1/\Lambda^4)$ in Ref. [29]; in terms of the operators of dimension-6 and dimension-8 of the bases of Refs. [54,55], respectively, the result reads

$$\mathcal{L}_{\rm SMEFT} = \mathcal{L}_{\rm SM} + \frac{C_{\mathcal{H}}}{\Lambda^2}(\mathcal{H}^\dagger\mathcal{H})^3 + \frac{C_{\mathcal{H}^8}}{\Lambda^4}(\mathcal{H}^\dagger\mathcal{H})^4 + \frac{C_{\mathcal{H}^6}^{(1)}}{\Lambda^4}(\mathcal{H}^\dagger\mathcal{H})^2(D_\mu\mathcal{H})^\dagger(D^\mu\mathcal{H}) + \ldots + \mathcal{O}(1/\Lambda^6), \tag{14}$$

where $\mathcal{L}_{\rm SM}$ is the SM Lagrangian, $\mathcal{H}$ is the Higgs doublet of the SMEFT expansion and the ellipses represent terms with fermions.[13] The expressions for the WCs read [29]

$$\frac{C_{\mathcal{H}}}{\Lambda^2} = \frac{Z_6^2}{\Lambda^2} + \frac{2}{\Lambda^4}(Y_3 Z_1 Z_6 - Y_3 Z_{345} Z_6 + Y_1 Z_6^2), \tag{15a}$$

$$\frac{C_{\mathcal{H}^8}}{\Lambda^4} = \frac{1}{\Lambda^4}(2 Z_1 Z_6^2 - Z_{345} Z_6{}^2), \tag{15b}$$

$$\frac{C_{\mathcal{H}^6}^{(1)}}{\Lambda^4} = -\frac{Z_6{}^2}{\Lambda^4}. \tag{15c}$$

We can rewrite these matching relations in terms of the parameters of Eq. (9). To that end, we use Eq. (A1) and, after assuming Eq. (12a), we consider the scaling of Eq. (13) and expand up to $\mathcal{O}(\xi^2)$. The result is

$$\frac{C_{\mathcal{H}}}{\Lambda^2} = c_{\beta-\alpha}^2(\sqrt{2}G_F)^2[\Lambda^2 - 4(m_h^2 - \Delta m_H^2)], \tag{16a}$$

$$\frac{C_{\mathcal{H}^8}}{\Lambda^4} = 2c_{\beta-\alpha}^2(\sqrt{2}G_F)^3(m_h^2 - \Delta m_H^2), \tag{16b}$$

$$\frac{C_{\mathcal{H}^6}^{(1)}}{\Lambda^4} = -c_{\beta-\alpha}^2(\sqrt{2}G_F)^2, \tag{16c}$$

where G_F is the Fermi constant.[14] The appearance of Λ in the numerator of the right-hand side of Eq. (16a) is a

[11] For example, via the interactions of between h and fermions (cf. e.g. Ref. [52]), or via Z_2 and Z_7 [cf. Eq. (A1)]. Finally, in some of the four types of 2HDM, β can also cause a delayed decoupling [47,52,53].

[12] Recall that we assumed the extreme decoupling scenario. Relaxing this assumption (i.e. taking Y_2 to be not so large) corresponds to considering higher powers in $1/Y_2$.

[13] Again, fermions are not relevant for our purposes. $\mathcal{H}$ is related to H_1 by a normalization factor; cf. Ref. [29] for details.

[14] Reference [29] considered the scenario in which $\Delta m_H = \Delta m_A = \Delta m_{H^+} = 0$, which is stronger than what is required by the decoupling limit of Eq. (12). We write the expressions in terms of G_F instead of the vev, since the relation between the two gets corrections in the SMEFT; see Ref. [29] for details.

consequence of our choice of $c_{\beta-\alpha}$ as independent parameter. Note also that there is no information about β or asymmetry in $c_{\beta-\alpha}$ (i.e. odd powers of $c_{\beta-\alpha}$). Finally, among the Δm^2 parameters introduced in Eq. (12a), only Δm_H^2 shows up, and always in the form $m_h^2 - \Delta m_H^2$.

The coefficients of Eq. (16) are evaluated at the scale Λ, which we typically take to be $\sim$1 TeV. The renormalization group equations at dimension-6 [56–58] and dimension-8 [59–63] can be used to evolve the coefficients to a lower scale (in Sec. VI, we take $\sqrt{s} = 260$ GeV). The numerical effect of the running is expected to be $\mathcal{O}(\log(\frac{\Lambda^2}{s})/(16\pi^2)) \sim .02$ and can be neglected for our purpose.

V. HEFT

We saw in the Introduction that the HEFT considers the SM Higgs field h to be a gauge singlet. This means that the HEFT matching to the 2HDM can only be accomplished if the heavy degrees of freedom of the 2HDM are integrated out after SSB. Contrary to the SMEFT approach, then, the HEFT matching to the 2HDM starts with physical states, i.e. states with well defined masses, without mixing terms in the propagator. As a result, one can directly integrate out the heavy mass states H, A and $H^\pm$. Consistent with the discussion of Sec. III, however, we will show that such an operation cannot be done by considering an expansion simply in inverse powers of m_H, m_A and m_{H^+}. More than that, the scaling of $c_{\beta-\alpha}$ must be taken into account, or else there will be no consistent decoupling, since perturbativity is lost.

This danger can be illustrated by considering the cubic self-interaction of h. As with any three-point function, this interaction is not affected by the integration out of the heavy states at tree level.[15] Thus, the cubic self-interaction of h of the HEFT Lagrangian is obtained by considering the same interaction in the 2HDM Lagrangian and simply applying the EFT expansion. The Feynman rule for the cubic self-interaction of h in the 2HDM reads

$$\begin{aligned}\frac{3i\csc^2(2\beta)}{2v}\Big\{&s_{\beta-\alpha}\cos(4\beta)\big[-3c_{\beta-\alpha}^4 m_H^2 - 2c_{\beta-\alpha}^2 Y_2 + (3c_{\beta-\alpha}^4 + c_{\beta-\alpha}^2 + 1)m_h^2\big]\\ &+ c_{\beta-\alpha}^3\sin(4\beta)\big[(1-3c_{\beta-\alpha}^2)m_h^2 + (3c_{\beta-\alpha}^2 - 2)m_H^2 + 2Y_2\big]\\ &+ s_{\beta-\alpha}\big[2c_{\beta-\alpha}^2 Y_2 - c_{\beta-\alpha}^4 m_H^2 + (c_{\beta-\alpha}^4 - c_{\beta-\alpha}^2 - 1)m_h^2\big]\Big\}, \qquad (17)\end{aligned}$$

with $s_{\beta-\alpha} = \sqrt{1 - c_{\beta-\alpha}^2}$. From this expression, we realize that an EFT expansion that considers simply inverse powers of m_H, m_A and m_{H^+} is doomed to inconsistency. This is because Eq. (17) contains positive powers of those heavy masses, so that the final HEFT Lagrangian can never be simply an expansion in inverse powers of those masses.[16] Note that this has physical consequences, since observables like $WW \to hh$ would suffer the same inconsistency. On the other hand, an expansion according to the ξ scaling in Eqs. (12) and (13) leads to a well-behaved cubic self-interaction of h.

The conclusion is then clear: the HEFT Lagrangian cannot be obtained from the 2HDM simply by performing an expansion in inverse powers of the heavy masses. Decoupling and perturbativity in the UV theory require the consistent scaling in Eqs. (12) and (13), which leads to a well-defined expansion. The heavy states H, A and H^+ can then be integrated out. As mentioned above, this operation cannot affect three-point functions, which are thus trivially derived from the equivalent function in the UV model simply by applying Eq. (12a) and expanding according to Eq. (13). By contrast, vertices with four particles or more receive contributions from integrating out the heavy states.

It is to this procedure that we now turn. To that end, and as described in the Introduction, we treat h and the ω^a separately, such that the latter are embedded into a unitary matrix U. The scalar doublets of the Higgs basis then take the form[17]

[15]The reason is that the solution of the EoM for a given heavy particle will always depend on terms which contain at least two light fields (since there are no bilinear terms in the Lagrangian which depend on two different fields, by definition of mass eigenstates). This means that, when replacing this heavy-particle EoM solution in the original UV Lagrangian, the two-point functions containing the heavy field will yield effective operators with four or more light fields. UV interaction terms with only one heavy field must also contain at least another two light fields and, hence, the corresponding effective operator has at least four light particles when the heavy scalar EoM solution is substituted. The same thing happens for UV interaction terms with two or more heavy fields, which give place to low-energy operators with four or more light fields for identical reasons [64].

[16]This is also true for the quartic self-interaction of h. As discussed in Appendix B, the singlet model of Ref. [19] is very special, since the cubic light-Higgs interaction does not scale with the heavy mass.

[17]The inclusion of the U matrix in the second doublet H_2 removes the Goldstone bosons from the potential in Eq. (5). This was also noted in Ref. [65], but a different parametrization was used to eliminate the problem.

$$\mathcal{H}_1 = \frac{v+h_1^H}{\sqrt{2}} U(\omega) \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \mathcal{H}_2 = U(\omega) \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(h_2^{\rm H} + iA) \end{pmatrix}. \tag{18}$$

We choose the unitary gauge, where the Goldstone bosons are eliminated from the theory, i.e. $U = 1$ (our results were checked in an arbitrary R_ξ gauge). This has the advantage that there are no interactions with more than four fields.[18] Following Ref. [19], we write the terms of $\mathcal{L}_{\rm 2HDM}$ involving scalars in such a way that we isolate the heavy scalars:

$$\mathcal{L}_{\rm 2HDM} \supset \frac{1}{2}(\partial_\mu H^a)^2 - \frac{1}{2}(M^2)^{ab} H^a H^b + J_0 + J_1^a H^a + J_2^{ab} H^a H^b + J_3^{abc} H^a H^b H^c + J_4^{abcd} H^a H^b H^c H^d, \tag{19}$$

where $(M^2)^{ab}$ is a diagonal matrix, $H^a = (H, A, H_3, H_4)$, with $H^\pm \equiv (H_3 \mp iH_4)/\sqrt{2}$, and the J_k contain only light fields.[19] The expressions for the J_k are given in Appendix A.

Each heavy scalar H^a is integrated out at tree level by solving its EoM:

$$J_1^a + (-\partial^2 - M^2 + 2J_2)^{ab} H^b + 3J_3^{abc} H^b H^c + 4J_4^{abcd} H^b H^c H^d = 0. \tag{20}$$

As mentioned before, the auxiliary parameter ξ will act as the *de facto* parameter of the expansion, as in the SMEFT case. This means that Eq. (20) will be solved iteratively in powers of ξ, so that the solution for the heavy fields will itself be given as a series in ξ. As can be anticipated, even the lowest orders contain a long a list of terms. We present only those which are relevant for the tree-level scattering processes we are interested in: $WW \to hh$ and $hh \to hh$.[20] Other processes such as $WW \to WW$ depend solely on one EFT parameter, i.e the a HEFT coupling that does not receive a modification from integrating out a heavy field at lowest order in HEFT and is the same as in the 2HDM. This is in contrast to $WW \to hh$ and $hh \to hh$ that involve corrections to b and d_4, respectively. For this reason we will focus on these processes for our comparison.[21] The heavy state A does not play any role in these scatterings, so that it will be ignored in the following. We then have

$$H = \sum_{i=0}^{\infty} H_{(\xi^i)}, \qquad H^+ = \sum_{i=0}^{\infty} H^+_{(\xi^i)}, \tag{21}$$

where the lowest orders are

$$H_{(\xi^0)} = H^+_{(\xi^0)} = 0, \tag{22a}$$

$$H_{(\xi^1)} = -\frac{3c_{\beta-\alpha}h^2}{2v}, \tag{22b}$$

$$H^+_{(\xi^1)} = 0, \tag{22c}$$

$$H_{(\xi^2)} = \frac{2c_{\beta-\alpha}}{v\Lambda^2}\Delta m_H^2 h^2 + \frac{2c_{\beta-\alpha}}{v\Lambda^2} m_W^2 W_\mu W^{\dagger\mu} + \frac{3c_{\beta-\alpha}}{v\Lambda^2}[(\partial^\mu h \partial_\mu h) + h(\partial^2 h)], \tag{22d}$$

$$H^+_{(\xi^2)} = -\frac{ic_{\alpha-\beta}M_W}{2v\Lambda^2}[h(\partial_\mu W^\mu) + 2W_\mu(\partial^\mu h)], \tag{22e}$$

[18] Alternative parametrizations of U are common, such as the spherical one ($U = \sqrt{1-\omega^a\omega^a/v^2} + i\omega^a\sigma^a/v$) or the exponential one ($U = \exp\{i\omega^a\sigma^a/v\}$). In general, one would need to expand U up to the desired order.

[19] In particular, the part of the Lagrangian without heavy fields is encoded in J_0. Note that the derivation could also be done for H^+ and H^- (instead of H_3 and H_4), but the expressions would not be as symmetric and simple as those presented here. The two bases are related by $H_3 J^{H_3} + H_4 J^{H_4} = H^+ J^{H^-} + H^- J^{H^+}$, with $J^{H^\pm} = (J^{H_3} \mp iJ^{H_4})/\sqrt{2}$. Also note that the Lagrangian terms quadratic in H^a have been split in the form: terms without light fields are provided by $\frac{1}{2}(\partial_\mu H^a)^2 - \frac{1}{2}(M^2)^{ab}H^a H^b$; terms with light fields have been placed in $J_2^{ab} H^a H^b$.

[20] Since the process $ZZ \to hh$ would allow us to find the same matching as $WW \to hh$, and the comparison between the 2HDM and the EFT yield similar results for both processes, we have chosen $WW \to hh$ to assess the accuracy of the EFT fit.

[21] The general solution for H and H^+ (containing all terms up to $\mathcal{O}(\xi^3)$, up to interactions with four particles) will be provided as Supplemental material with this manuscript [66].

$$H_{(\xi^3)} = \frac{c_{\beta-\alpha}h^2}{4t_\beta^2 v\Lambda^4}[c_{\beta-\alpha}^2(3t_\beta^4 - 2t_\beta^2 + 3)\Lambda^4 - 3c_{\beta-\alpha}(t_\beta^2 - 1)t_\beta\Lambda^2 2(2\Delta m_H^2 - m_h^2) - 8\Delta m_H^2{}^2 t_\beta^2]$$
$$- \frac{7\Delta m_H^2 c_{\beta-\alpha}}{\Lambda^4 v}[(\partial^\mu h)(\partial_\mu h) + h(\partial^2 h)] - \frac{3c_{\beta-\alpha}}{\Lambda^4 v}[(\partial^2 h)(\partial^2 h) + h(\partial^2\partial^2 h)]$$
$$- \frac{6c_{\beta-\alpha}}{\Lambda^4 v}[(\partial^\mu h\partial^\nu h)(\partial_\mu h\partial_\nu h) + (\partial_\mu h)(\partial^\mu\partial^2 h) + (\partial_\nu h)(\partial^2\partial^\nu h)]$$
$$- \frac{2c_{\beta-\alpha}m_W^2}{v\Lambda^4}[W^{\dagger\nu}(\Delta m_H^2 W_\nu + \partial^2 W_\nu) + 2(\partial^\mu W^\nu)(\partial_\mu W_\nu^\dagger) + W_\nu(\partial^2 W^{\dagger\nu})], \quad (22f)$$

$$H^+_{(\xi^3)} = -\frac{im_W c_{\beta-\alpha}}{v\Lambda^4}[h(\partial^2\partial^\nu W_\nu) + (\partial^2 h)(\partial^\nu W_\nu)]$$
$$+ \frac{ic_{\beta-\alpha}m_W}{v\Lambda^4}[h(\partial^2\partial^\nu W_\nu) + 2W^\nu(\partial^2\partial_\nu h)] - \frac{4im_W c_{\beta-\alpha}}{v\Lambda^4}(\partial^\mu\partial^\nu h)(\partial_\mu W_\nu)$$
$$- \frac{ic_{\beta-\alpha}\Delta m_{H_+}^2 m_W}{v\Lambda^4}[h(\partial^\nu W_\nu) + 2W^\nu(\partial_\nu h)]. \quad (22g)$$

Note that the first nonvanishing solution starts at $\mathcal{O}(\xi)$ for H, and at $\mathcal{O}(\xi^2)$ for H^+. As a consequence, the integration out of H and H^+ will contribute to $WW \to hh$ at order $\mathcal{O}(\xi^2)$ and $\mathcal{O}(\xi^3)$, respectively. Finally, contrary to what was done in the SMEFT, we performed the expansion in the HEFT up to $\mathcal{O}(\xi^3)$. We justify this difference of truncations between the SMEFT and the HEFT matchings at the end of this section.

By substituting the solutions for the heavy fields of Eq. (21) in $\mathcal{L}_{2HDM}$, we obtain the effective HEFT Lagrangian.[22] Note that, for the two-to-two tree-level scatterings discussed in this article ($WW \to hh$ and $hh \to hh$), only the first line in Eq. (19) is required—at any order in ξ. Since the H^a EoM solutions contain at least two light fields, the effective operators in the second line of Eq. (19) will contain five or more light fields. Comparing the effective HEFT Lagrangian with that of Eqs. (2) and (3) results in the following matching equations:

$$\Delta a^2 \equiv a^2 - 1 = -c_{\beta-\alpha}^2, \quad (23a)$$

$$\Delta b \equiv b - 1 = -3c_{\beta-\alpha}^2 + 4c_{\beta-\alpha}^2\frac{\Delta m_H^2}{\Lambda^2} + \mathcal{O}(\xi^4), \quad (23b)$$

[22]This contains in general terms that cannot be written in the form of Eqs. (2) and (3), since they would require terms in the HEFT Lagrangian with additional derivatives. In the expressions for the HEFT matching in this paper, we will not be presenting such terms.

$$\Delta d_3 \equiv d_3 - 1 = -2c_{\beta-\alpha}^2\frac{\Lambda^2}{m_h^2} + \frac{1}{2}c_{\beta-\alpha}^2 + c_{\beta-\alpha}^3\left[-\cot(2\beta)\left(1 - \frac{2\Delta m_H^2}{m_h^2}\right) + 2c_{\beta-\alpha}\cot^2(2\beta)\frac{\Lambda^2}{m_h^2}\right] + \mathcal{O}(\xi^4), \quad (23c)$$

$$\Delta d_4 \equiv d_4 - 1 = -12c_{\beta-\alpha}^2\frac{\Lambda^2}{m_h^2} + c_{\beta-\alpha}^2\left(\frac{16\Delta m_H^2}{m_h^2} - 11\right) + c_{\beta-\alpha}^2\left[2c_{\beta-\alpha}^2\frac{\Lambda^2}{m_h^2}(22\cot^2(2\beta) - 17) - 22c_{\beta-\alpha}\cot(2\beta)\left(1 - \frac{2\Delta m_H^2}{m_h^2}\right) + 16\frac{\Delta m_H^2}{\Lambda^2}\left(\frac{2 - \Delta m_H^2}{m_h^2}\right)\right] + \mathcal{O}(\xi^4). \quad (23d)$$

In order to more easily compare with the SM, we introduced the quantities with Δ; from Eqs. (12) and (13), it is easy to see that the SM limit ($\Delta a^2 = \Delta b = \Delta d_3 = \Delta d_4 = 0$) is recovered at $\mathcal{O}(\xi^0)$. For both a^2 and b, the first deviation from the SM occurs at $\mathcal{O}(\xi^2)$; this happens in such a way that a^2 has no additional contributions. Note also that the β dependence appears for the first time at $\mathcal{O}(\xi^3)$ for Δd_3 and Δd_4 [second lines of Eqs. (23c) and (23d), respectively]. Finally, the factors $\cot(2\beta) = (1 - \tan^2\beta)/(2\tan\beta)$ become large for $\beta \sim 0$ or $\beta \sim \pi/2$ (i.e., when $\tan\beta \to 0$ or ∞, respectively), and vanish for $\theta = \pi/4$ (i.e., when $\tan\beta = 1$).

We can compare the analytical results obtained in this section with the ones from SMEFT. We start by realizing that, up to $\mathcal{O}(\xi^2)$, and just as in the SMEFT matching, there is no information about β or odd powers in $c_{\beta-\alpha}$, and the only Δm^2 parameter present is Δm_H^2. We also observe that the relation found between Eqs. (23a) and (23b), $\Delta b = 3\Delta a^2 + \mathcal{O}(\xi^3)$, is not compatible with the usual dimension-6 SMEFT constraint $\Delta b = 2\Delta a^2$ [11]. That is, the HEFT matching to the 2HDM cannot be described by means of a SMEFT Lagrangian that starts with dimension-6 operators. On the other hand, if one assumes that the contributions from the dimension-6 SMEFT operators to a and b vanish (as is indeed the case in the SMEFT matching to the 2HDM, see Ref. [29]), one obtains a dimension-8 constraint, which is precisely $\Delta b = 3\Delta a^2$ [11,67]. Regarding the Higgs potential term, a SMEFT Lagrangian starting at dimension-6 also requires the relation $\Delta d_4 = 3\Delta d_3^2 - 2\Delta a^2/3$ between the HEFT couplings [11,12], where the latter Δa^2 comes from a finite Higgs field redefinition. It is easy to observe that the values of Δd_3 and Δd_4 in the 2HDM obey this relation at $\mathcal{O}(\xi)$, as $\Delta a = 0$ at that order.

We end this section with a remark about the difficulty of implementation of the two EFT approaches to the 2HDM. The HEFT approach is considerably simpler to implement than the SMEFT one for the processes considered here. First of all, the higher orders terms in SMEFT in general contain the SM Higgs doublet, which contains the SM vev. This means that two-point functions are in general affected; in particular, kinetic terms and the relations between masses and Lagrangian parameters need to be redefined. In the HEFT approach, this never happens, since the integration out of the heavy states only affects four-point functions or higher, as discussed above. This is related to a second advantage, which is that the three-point functions in the HEFT approach at tree level are trivially obtained from the corresponding functions in the 2HDM, which is not the case in the SMEFT approach. Finally, for the processes considered here, the HEFT approach at tree level does not require the formal procedure of integrating out heavy states. The same results can be obtained simply by considering the amplitudes of the 2HDM contributing to the process at stake, and applying the expansion of Eqs. (12) and (13) directly to them. All of this allows us to easily derive the $\mathcal{O}(\xi^3)$ results in the HEFT expansion (Appendix A). The derivation of the same order results in the SMEFT (which involve dimension-10 operators) is beyond the scope of this work.

VI. RESULTS

We now turn to our numerical results. We assume that H, A and H^+ are all degenerate and we define the quantity $\Delta\Lambda$, such that[23]

$$m_H = m_A = m_{H^+} = \Lambda + \Delta\Lambda. \tag{24}$$

Comparing with Eqs. (11c) and (11d), and recalling that $Y_2 = \Lambda^2$ [Eq. (12a)], we realize that $\Delta\Lambda$ measures the amount of mass in m_A and m_{H^+} which is not generated by the Lagrangian parameter Y_2. In other words, $\Delta\Lambda = 0$ implies that m_A and m_{H^+} are entirely generated by Y_2, whereas larger and larger values of $\Delta\Lambda$ imply larger and larger contributions from the vev.[24] Equation (24) implies that the quantities defined in Eq. (12a) obey

$$\Delta m_H^2 = \Delta m_A^2 = \Delta m_{H^+}^2 = 2\Lambda\Delta\Lambda + (\Delta\Lambda)^2. \tag{25}$$

Accordingly, the new parameter scales as $\Delta\Lambda \sim \mathcal{O}(v^2/\Lambda) \sim \mathcal{O}(\xi^{1/2})$.

Naively, $\Delta\Lambda$ is expected to control the increase of accuracy of the HEFT matching over the SMEFT one. The reason is that the heavy mass parameter in the SMEFT matching is Y_2 (which is set equal to Λ^2), whereas in the HEFT the heavy mass parameters are the heavy masses [which are given by Eq. (24)]. The HEFT thus contains information about $\Delta\Lambda$, so that, for large values of $\Delta\Lambda$, the agreement of the HEFT matching to the 2HDM is expected to be better than that of the SMEFT matching. A similar reasoning motivated the v-improved matching proposed in Ref. [17].

However, two aspects should not be neglected. First, the numerators of the expressions of the SMEFT matching to the 2HDM in general depend on the masses. Therefore, they will in general depend on $\Delta\Lambda$ [and they indeed do: see Eq. (16)]. Second, even if the HEFT heavy mass parameters are the heavy masses of the 2HDM, these are constrained to follow Eq. (12). It follows that the scaling $\Delta\Lambda \sim \mathcal{O}(v^2/\Lambda)$ implies a suppressed dependence of the HEFT matching on $\Delta\Lambda$. All in all, then, it is to be seen if a correlation exists between $\Delta\Lambda$ and an increase in accuracy of the HEFT matching over the SMEFT one.

For the numerical results that follow, we require our 2HDM results to comply with the theoretical constraints of perturbative unitarity and boundedness from below [68–71], as well as electroweak precision measurements

[23] For the processes considered here, A does not play any role, so that the results are independent of m_A.

[24] Negative values of $\Delta\Lambda$ are in principle also possible, but they are generally ruled out by theoretical constraints.

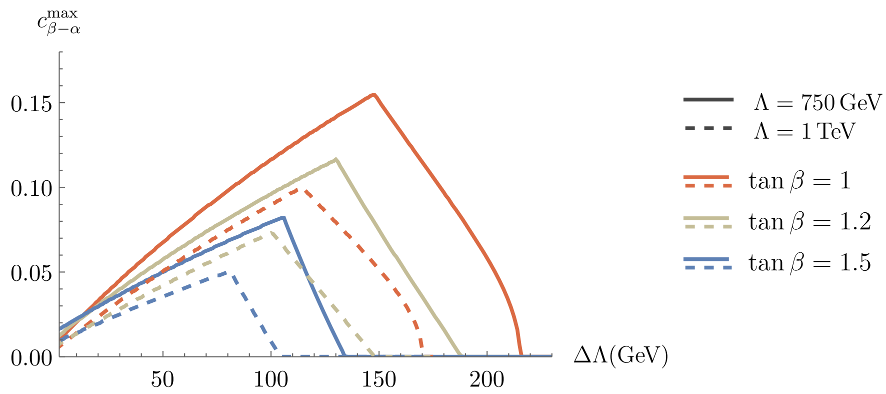

FIG. 1. Maximum value of $c_{\beta-\alpha}$ allowed by the theoretical constraints of the 2HDM, as a function of $\Delta\Lambda$. For each curve, the maximum value of $c_{\beta-\alpha}$ is determined by boundedness from below in the region where the curve has positive slope, and by perturbative unitarity in the region where the curve has negative slope.

via the oblique parameters S, T and U [32]. We start by ascertaining the relevance of these contraints on the parameter space. This can be seen in Fig. 1, where the maximum value of $c_{\beta-\alpha}$ allowed ($c^{\max}_{\beta-\alpha}$) is shown versus $\Delta\Lambda$, for different values of Λ and $\tan\beta$. Each curve shows an abrupt inflexion point; in all cases, the values of $\Delta\Lambda$ below that point are such that $c^{\max}_{\beta-\alpha}$ is determined by boundedness from below, whereas those above it have $c^{\max}_{\beta-\alpha}$ determined by perturbative unitarity.[25] The figure also shows that the window of allowed values of $c_{\beta-\alpha}$ becomes narrower with both increasing Λ and increasing $\tan\beta$. We checked, in particular, that scenarios with $\tan\beta \sim 1$ and $\Lambda \gg 1$ TeV have an extremely narrow allowed window, as do also scenarios with $\Lambda \sim 1$ TeV and $\tan\beta \gg 1$. Finally, for the (large) values of Λ shown, the largest value of $c^{\max}_{\beta-\alpha}$ allowed is around 0.15. The result is that one is restricted to be very close to the exact alignment limit $c_{\beta-\alpha} = 0$. Still, interesting results can be found inside that narrow window.

The 2HDM is limited by numerous experimental results, of which the most stringent are Higgs coupling measurements, b meson decays and searches for heavy Higgs bosons. These limits depend on the couplings of the fermions to the Higgs doublets, and we assume that the couplings respect a $\mathbb{Z}_2$ symmetry. The limits from Higgs couplings typically require that $c_{\beta-\alpha}$ be close to the alignment limit, and all of the values considered below are currently allowed [72,73]. The charged Higgs boson that is present in the 2HDM contributes to the decay $b \to s\gamma$ and current experimental results require that $\tan\beta > 1.2$ [74]. Additionally, ATLAS and CMS have searched for heavy neutral scalars with the couplings of the 2HDM and for $\tan\beta > 1.2$, the limit is quite weak, $m_H > 400$ GeV [75,76]. In the following, we shall take $\tan\beta = 1.2$ since, from Fig. 1, this gives the largest theoretically allowed region that is consistent with experiment. The results that follow were obtained independently via FeynMaster [77,78] (and its accompanying software [79–84]) and FeynArts [85].

Before considering our numerical analysis of the SMEFT and the HEFT matchings to the 2HDM, we highlight that both approaches end up using the same expansion [in powers of ξ, defined in Eq. (13)], since the decoupling limit of Eq. (13) needs to be obeyed by both in order to have a weakly interacting perturbative 2HDM. Hence, even if they are structurally different—the SMEFT matching complying with the symmetries of the SM before SSB, the HEFT one with those after SSB—some of their results are very similar. For example, both the three-point

[25]Given the assumed degeneracy of H, A and H^+, the oblique parameters play no relevant role in our analyses. Boundedness from below requires that none of the elements of specific combinations of quartic parameters of the potential (usually in the original basis of the doublets Φ_1, Φ_2) take negative values (see e.g. Ref. [68]). For the values of $\tan\beta$ considered in the figure, the most important element is λ_2. When written in terms of the parameters involved in the plot, and when expanded to first order in $c_{\beta-\alpha}$, this quartic parameter is of the form $c_1 - c_2 c_{\beta-\alpha}$. Here, c_1 and c_2 are real numbers which depend on $\Delta\Lambda$ and which, for the values of $\Delta\Lambda$ involved, are both positive. The requirement that $c_1 - c_2 c_{\beta-\alpha}$ is non-negative thus imposes an upper limit on the value of $c_{\beta-\alpha}$. Moreover, c_1 turns out to grow with $\Delta\Lambda$ twice as quickly as c_2, which explains the linear character of the positive-slope branch of the curves. As for the negative-slope branch, it is determined by perturbative unitarity, which requires all the elements of another combination of quartic parameters of the potential to be smaller than a certain limit. For the values at stake here, the decisive element is $\left|\frac{3(\lambda_1+\lambda_2)+\sqrt{9(\lambda_1-\lambda_2)^2+4(2\lambda_3+\lambda_4)^2}}{2}\right| \leq 8\pi$. Just as before, we can write it in terms of the parameters involved in the plot and expand it to second order in $c_{\beta-\alpha}$, in which case it acquires the form $c_3 + c_4 c^2_{\beta-\alpha} \leq 8\pi$. Just as c_1 and c_2, also c_3 and c_4 are real positive numbers (in the range of values at stake), such that c_4 grows with $\Delta\Lambda$. This happens in such a way that, from a certain value of $\Delta\Lambda$, the maximum allowed value of $c_{\beta-\alpha}$ is no longer determined by boundedness from below, but rather from perturbative unitarity. The inflexion point in each curve (where the negative slope and the positive slope unite) is thus a nontrivial combination of these two theoretical constraints.

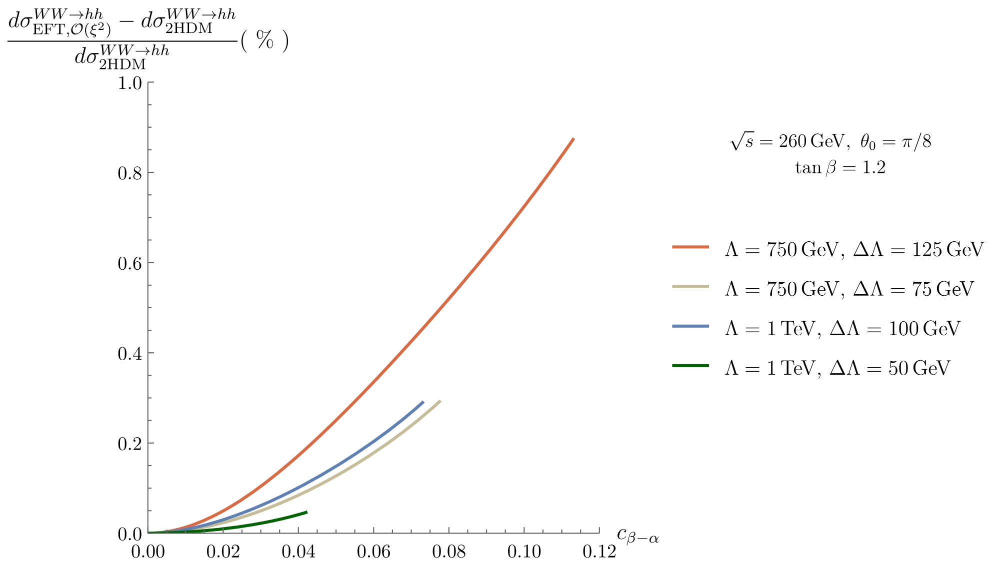

FIG. 2. Relative difference between the differential tree-level cross sections for $WW \to hh$ in the 2HDM and in the EFT matching to the 2HDM at $\mathcal{O}(\xi^2)$ with $d\sigma \equiv \frac{d\sigma}{d\theta}|_{\theta=\theta_0}$. Four pairs of values of Λ and $\Delta\Lambda$ are considered, according to the labels. For each curve, only the range of (positive values of) $c_{\beta-\alpha}$ allowed by the theoretical constraints is shown. All results assume a center-of-mass energy $\sqrt{s} = 260$ GeV, a scattering angle $\theta_0 = \pi/8$ and $\tan\beta = 1.2$.

tree-level interactions between h and fermions and between h and gauge bosons are exactly the same in the two effective Lagrangians at $\mathcal{O}(\xi^2)$. This implies, in particular, that the fits to global Higgs signal strengths performed in Ref. [27] will be the same in the SMEFT and in the HEFT matchings at that order.[26]

It turns out that the tree-level scatterings $WW \to hh$ and $hh \to hh$ are also identical at $\mathcal{O}(\xi^2)$. This result does not appear obvious to us, since the individual Feynman rules contributing to the processes are different at $\mathcal{O}(\xi^2)$. Specifically, the h^3 coupling—which contributes to both $WW \to hh$ and $hh \to hh$—involves derivatives in the SMEFT matching [recall Eq. (14)], whereas in the HEFT matching it does not [as can be seen by applying Eq. (14) and to Eq. (13) to Eq. (14)]. But the fact that the local 4-point interactions ($WWhh$ in $WW \to hh$, and h^4 in $hh \to hh$) are also different exactly compensates for the difference in h^3 to $\mathcal{O}(\xi^2)$.

Note that this conclusion holds even before the assumption of degenerate heavy masses, Eq. (24). That it holds in the case of degenerate heavy masses implies that it holds for all $\Delta\Lambda$. In other words, the parameter $\Delta\Lambda$ is irrelevant to compare the SMEFT and the HEFT matchings in $WW \to hh$ and $hh \to hh$ at tree level at $\mathcal{O}(\xi^2)$, since the two approaches are analytically identical. In the following, we refer to the two identical matchings at $\mathcal{O}(\xi^2)$ simply as the EFT matching, and we investigate how accurately it describes the 2HDM results.

We start by illustrating the case $WW \to hh$, depicted in Fig. 2. The plot shows the relative differential cross section between the 2HDM and the EFT matching at $\mathcal{O}(\xi^2)$, for different values of Λ and $\Delta\Lambda$, and for a center-of-mass energy $\sqrt{s} = 260$ GeV and a scattering angle $\theta_0 = \pi/8$.[27] The plot only shows positive values of $c_{\beta-\alpha}$, and each curve is shown only up to the value of $c_{\beta-\alpha}$ where the theoretical constraints start being violated (cf. Fig. 1). It is manifest that the EFT matching reproduces quite well the 2HDM, with relative differences smaller than 1%.

This is to be contrasted to what is shown in Fig. 3, which considers the same as in Fig. 2, but now for $hh \to hh$. The EFT matching no longer faithfully reproduces the 2HDM result, allowing differences larger than 40% for $\Lambda = 750$ GeV, $\Delta\Lambda = 125$ GeV, $c_{\beta-\alpha} \sim 0.08$. These large values demonstrate that, in the region of parameter space considered, $\mathcal{O}(\xi^2)$ is not enough in the EFT expansion. This means that, to accurately reproduce the 2HDM result, one would need a matching to dimension-10 operators in SMEFT, and to operators beyond the leading order in the derivative expansion in HEFT.

Figure 4 displays again $WW \to hh$ and $hh \to hh$, but with three main differences: first, it shows the absolute

[26]We refer to the fits which do not include the effects of the Higgs trilinear coupling, Fig. 6 of Ref. [27]. Note that even one-loop processes such as $gg \to h$ or $h \to \gamma\gamma$ are the same in both EFT approaches [at $\mathcal{O}(\xi^2)$].

[27]The general features of the plot are not sensitive to the specific values of $\sqrt{s}$ and θ_0. Moreover, the expressions for $d\sigma^{WW\to hh}_{\mathrm{HEFT},\mathcal{O}(\xi^2)}$ and $d\sigma^{WW\to hh}_{\mathrm{SMEFT},\mathcal{O}(\xi^2)}$ are consistently of $\mathcal{O}(\xi^2)$, in the sense that higher order effects resulting from squaring the amplitude were excluded.

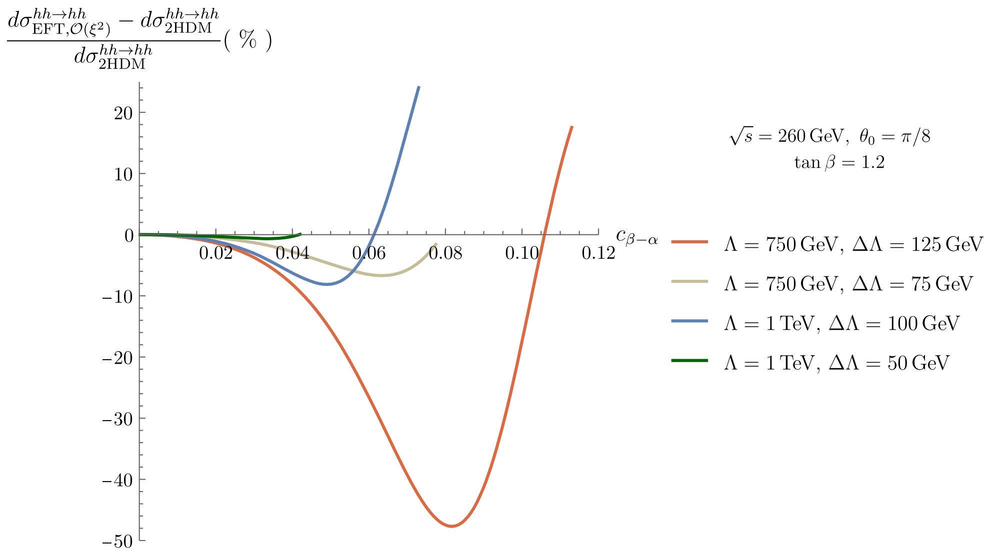

FIG. 3. The same as in Fig. 2, but for $hh \to hh$.

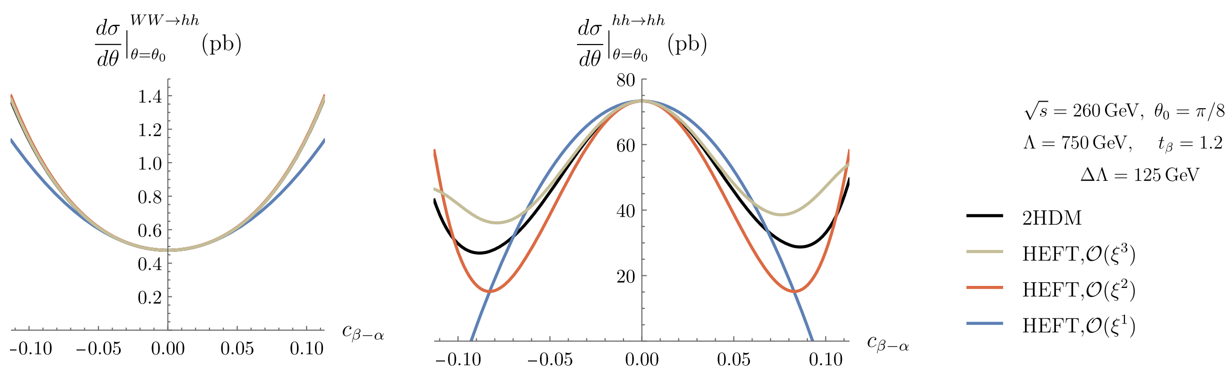

FIG. 4. Left: differential cross section for $WW \to hh$ at tree level, both for the 2HDM (black), as well as for three different truncations of the HEFT matching (the black and the red are behind the beige). Right: the same, but for $hh \to hh$. Both panels take $\sqrt{s} = 260$ GeV, $\theta_0 = \pi/8$, $\Lambda = 750$ GeV, $\Delta\Lambda = 125$ GeV and $\tan\beta = 1.2$. The region of values of $c_{\beta-\alpha}$ shown is allowed by the theoretical constraints.

values of the differential cross sections; second, it includes negative values of $c_{\beta-\alpha}$; finally, it separately shows the different orders in the HEFT expansion, up to $\mathcal{O}(\xi^3)$.[28] Several aspects are worth mentioning here. First, we stress that the plots show the HEFT matching, which we performed up to $\mathcal{O}(\xi^3)$, but which we are only assured of being identical to the SMEFT matching up to $\mathcal{O}(\xi^2)$. Then, the right panel shows that the 2HDM result is slightly asymmetric in $c_{\beta-\alpha}$, even though the EFT matchings at $\mathcal{O}(\xi^2)$ do not contain this information, as discussed above.[29]

Concerning the different truncations, the right plot of Fig. 4 illustrates that, while the lowest truncation in enough to reproduce the 2HDM for values of $c_{\beta-\alpha}$ very close to zero, the $\mathcal{O}(\xi^3)$ truncation is clearly the most appropriate one for the whole range of $c_{\beta-\alpha}$ shown. On the other hand, even that truncation is far from an exact reproduction of the 2HDM result, which indicates that the next order would be relevant. In other words, the convergence of the EFT expansion is quite slow for $hh \to hh$ for larger values of $c_{\beta-\alpha}$. This is to be

[28]For the values of t_β, Λ and $\Delta\Lambda$ considered, some values of $c_{\beta-\alpha}$ more negative than the ones shown in the plots are still allowed by theoretical constraints. Moreover, even if we are not showing all the terms $\mathcal{O}(\xi^3)$ in Eq. (22), we are including them in these plots. Finally, the $\mathcal{O}(\xi^1)$ curve yields negative values for $d\sigma^{hh\to hh}$ for $|c_{\beta-\alpha}| > 0.07$. These are unphysical (and thus not shown), and result from neglecting the higher order terms when taking the square of the amplitude.

[29]As we also noted, the EFT matching at $\mathcal{O}(\xi^2)$ does not have information about $\tan\beta$. This suggests that the two approaches will poorly reproduce the 2HDM whenever the latter shows a strong dependence on that parameter. On the other hand, and as discussed in the context of Fig. 1, a scenario with large Λ and $\tan\beta$ significantly different from 1 will lead to the alignment limit $c_{\beta-\alpha}$, where the EFT matching coincides with the 2HDM.

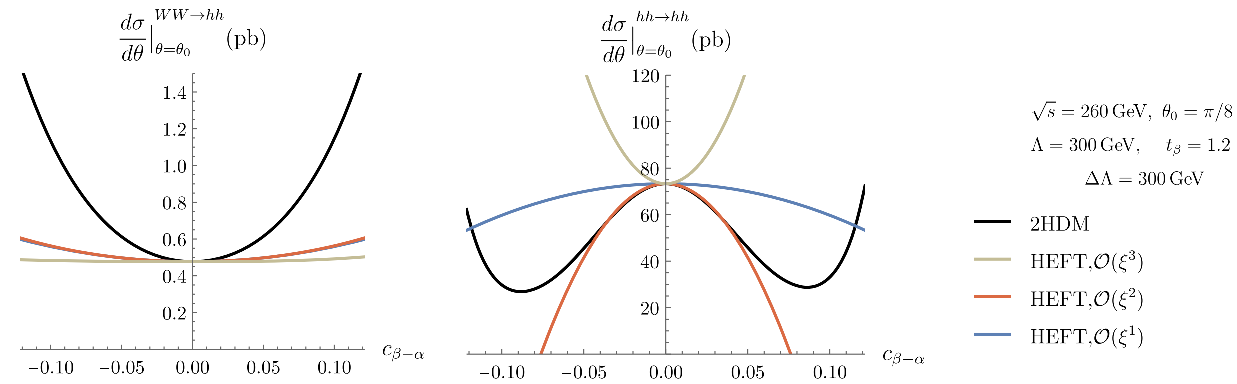

FIG. 5. The same as in Fig. 4, but for $\Lambda = \Delta\Lambda = 300$ GeV (on the left plot, the blue is behind the red). As before, the region of values of $c_{\beta-\alpha}$ shown is allowed by the theoretical constraints.

contrasted with the left panel, which shows the equivalent plot for $WW \to hh$. There, a faithful reproduction of the 2HDM results is obtained immediately at $\mathcal{O}(\xi^2)$, in which case higher orders are not needed. Nevertheless, both panels also show that, again for larger values of $c_{\beta-\alpha}$, the $\mathcal{O}(\xi^1)$ truncation clearly fails to reproduce the UV model.

In Fig. 5, we investigate the scenario in which the decoupling is lost. These plots are equivalent to those of Fig. 4, but with $\Lambda = \Delta\Lambda = 300$ GeV. Note that, even if this means $m_H = m_A = m_{H^+} = 600$ GeV, the choice $\Lambda = 300$ GeV is a blatant violation of the assumptions of Eq. (12). Indeed, both plots of Fig. 5 clearly show that the EFT is no longer valid according to the expansion of Eqs. (12) and (13): the different orders do not improve the convergence to the 2HDM results. We verified that the same conclusion holds for even smaller values of Λ.

VII. CONCLUSIONS

In this work, we presented two tree-level EFT matchings to the 2HDM: the SMEFT and the HEFT. We began with the 2HDM as our UV complete theory and imposed decoupling and perturbativity on the model. This implies that in the large mass limit of the heavy Higgs masses, the mixing angle $c_{\beta-\alpha}$ must obey the scaling $c_{\beta-\alpha} \sim \xi$, where ξ parametrizes the approach to the alignment limit, $c_{\beta-\alpha} \to 0$. We organized our studies of the SMEFT and HEFT matching in terms of an expansion in powers of ξ.

We discussed the matching of the HEFT to the 2HDM at $\mathcal{O}(\xi^2)$ (the matching of the SMEFT to that order was discussed in a previous paper, [29]) and used the unitary gauge to simplify the results, which were checked in an arbitrary R_ξ gauge. The matching equations for the parameters of the HEFT Lagrangian relevant for the processes discussed in this paper were given analytically.

We found that the SMEFT and the HEFT matchings to the 2HDM were identical to $\mathcal{O}(\xi^2)$ when the UV theory is required to obey decoupling and perturbativity. This holds for the fits to global Higgs signal strength, as well as the tree-level scatterings $WW \to hh$ and $hh \to hh$. We investigated how accurately the EFT matching at $\mathcal{O}(\xi^2)$ reproduces the 2HDM results in both these scatterings. In $WW \to hh$, the EFT matching accurately reproduces the 2HDM result, with differences smaller than the percent level. In the case of $hh \to hh$, by contrast, it fails to properly reproduce the 2HDM result in some regions of the parameter space. In this case, therefore, even the second order of the SMEFT (HEFT) expansion is not enough, and one should in principle consider dimension-10 operators (next-to-leading order operators in p^2). We further showed that the convergence to the 2HDM could be improved if $\mathcal{O}(\xi^3)$ effects are included in the HEFT. Finally, we probed the case without decoupling, and concluded that the EFT expansion in powers of ξ does not converge in that case.

This paper is a first tree-level exploration of the matchings of a UV model to both the SMEFT and the HEFT in a way consistent with decoupling and perturbativity. Several directions of future work are open. It would be particularly interesting to ascertain if the similarities between the two approaches found here will also hold for higher orders in the EFT expansion, as well for other processes. UV models other than the 2HDM could also be explored, with the purpose of ascertaining the consequences of perturbativity for the matchings in those cases.

It would be interesting to study the effects of loop corrections and their impact on the comparison between the SMEFT and the HEFT matchings to a UV model. The matching could be done using automated tools for one-loop matching [86–90], and the renormalization group evolution of the dimension-6 coefficients could be automated using the tools of Refs. [91,92]. Such a study would require careful attention to the interplay between the loop expansion and the expansion in $1/\Lambda^2$ [93].

Digital data pertaining to the HEFT matching is contained in the auxiliary file submitted with this paper [66].

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Note added.—Recently, Ref. [94] was made publicly available. It focuses on the SMEFT matching to the 2HDM and proposes a basis alternative to the Higgs basis. That reference is an interesting complement to our paper.

APPENDIX A: FURTHER DETAILS ON THE 2HDM

The quartic couplings of Eq. (5b) can be written in terms of the parameters of Eq. (9) as

$$Z_1 = \frac{(1-c^2_{\beta-\alpha})m_h^2 + c^2_{\beta-\alpha}m_H^2}{v^2}, \tag{A1a}$$

$$\begin{aligned} Z_2 = \frac{1}{2v^2 t_\beta^3}[&c^2_{\beta-\alpha}t_\beta(3t_\beta^4 - 8t_\beta^2 + 3)(m_h^2 - m_H^2) \\ &+ \sqrt{1-c^2_{\beta-\alpha}}c_{\beta-\alpha}(t_\beta^6 - 7t_\beta^4 + 7t_\beta^2 \\ &- 1)(m_h^2 - m_H^2) - m_h^2(t_\beta^5 - 4t_\beta^3 + t_\beta) \\ &+ 2t_\beta(t_\beta^2-1)^2(m_H^2 - Y_2)], \end{aligned} \tag{A1b}$$

$$Z_3 = \frac{2}{v^2}(m^2_{H^\pm} - Y_2), \tag{A1c}$$

$$Z_4 = \frac{c^2_{\beta-\alpha}(m_h^2 - m_H^2) + m_A^2 + m_H^2 - 2m^2_{H^\pm}}{v^2}, \tag{A1d}$$

$$Z_5 = \frac{c^2_{\beta-\alpha}(m_h^2 - m_H^2) - m_A^2 + m_H^2}{v^2}, \tag{A1e}$$

$$Z_6 = \frac{c_{\beta-\alpha}\sqrt{1-c^2_{\beta-\alpha}}(m_h^2 - m_H^2)}{v^2}, \tag{A1f}$$

$$\begin{aligned} Z_7 = \frac{1}{2v^2t_\beta^2}[&-3c^2_{\beta-\alpha}t_\beta(t_\beta^2-1)(m_h^2-m_H^2) \\ &- \sqrt{1-c^2_{\beta-\alpha}}c_{\beta-\alpha}(t_\beta^4 - 4t_\beta^2 + 1)(m_h^2 - m_H^2) \\ &+ t_\beta(t_\beta^2-1)(m_h^2 - 2m_H^2 + 2Y_2)]. \end{aligned} \tag{A1g}$$

As mentioned in Sec. II, the $\mathbb{Z}_2$ symmetry implies that not all the Z's are independent. The two dependence relations read [20]

$$Z_2 - Z_1 = \frac{1-2s_\beta^2}{s_\beta c_\beta}(Z_6 + Z_7), \tag{A2a}$$

$$Z_{345} - Z_1 = \frac{1-2s_\beta^2}{s_\beta c_\beta}Z_6 - \frac{2s_\beta c_\beta}{1-2s_\beta^2}(Z_6 - Z_7). \tag{A2b}$$

Equations (11a) and (11b) can be rewritten by replacing the dependence on $c_{\beta-\alpha}$ by Z_i parameters as

$$m_h^2 = \frac{2Y_2 + v^2(2Z_1 + Z_{345}) - \sqrt{[2Y_2 + v^2(Z_{345} - 2Z_1)]^2 + 16v^4Z_6^2}}{4}, \tag{A3a}$$

$$m_H^2 = \frac{2Y_2 + v^2(2Z_1 + Z_{345}) + \sqrt{[2Y_2 + v^2(Z_{345} - 2Z_1)]^2 + 16v^4Z_6^2}}{4}. \tag{A3b}$$

In what follows, we present further details concerning the integration out of H, A and H^+ in the HEFT. As we saw in Sec. V, the use of the unitary gauge implies a maximum of four fields in each term of the 2HDM Lagrangian. Then, from Eq. (19), it is clear that J_0, J_1, J_2, J_3, and J_4 will contain only light fields with a maximum number of four, three, two, one, and zero, respectively. For the tree-level scattering processes $WW \to hh$ and $hh \to hh$, only the following J's are needed: J_0 (up to four light fields) and J_1^a, with $a = H, H^+$ (up to two light fields). They read

$$\begin{aligned} J_0 = &\frac{1}{2}\partial_\mu(h)^2 - \frac{1}{2}h^2m_h^2 + \left(\frac{1}{2}m_Z^2Z^\mu Z_\mu + m_W^2W^\mu W_\mu^\dagger\right)\left(1 + \frac{2s_{\beta-\alpha}h}{v} + \frac{h^2}{v^2}\right) \\ &+ \frac{1}{4vt_\beta^2}h^3\{(t_\beta^4 - 4t_\beta^2 + 1)c^4_{\beta-\alpha}(m_h^2 - m_H^2)s_{\beta-\alpha} + 3t_\beta(t_\beta^2-1)c^5_{\beta-\alpha}(m_h^2 - m_H^2) \\ &- t_\beta(t_\beta^2-1)c^3_{\beta-\alpha}(m_h^2 - 2m_H^2 + 2Y_2) - 2t_\beta^2c^2_{\beta-\alpha}(m_h^2 - 2Y_2)s_{\beta-\alpha} - 2m_h^2t_\beta^2s_{\beta-\alpha}\} \end{aligned}$$

$$
\begin{aligned}
&-\frac{1}{16v^2t_\beta^3}h^4\{[t_\beta^6-19t_\beta^4+19t_\beta^2-1]c_{\beta-\alpha}^5(m_H^2-m_h^2)s_{\beta-\alpha}\\
&-4t_\beta^2(t_\beta^2)c_{\beta-\alpha}^3s_{\beta-\alpha}(m_h^2-2m_H^2+2Y_2)+t_\beta(7t_\beta^4-26t_\beta^2+7)c_{\beta-\alpha}^6(m_h^2-m_H^2)\\
&+t_\beta c_{\beta-\alpha}^4[m_h^2(-5t_\beta^4+18t_\beta^2-5)+6m_H^2(t_\beta^4-4t_\beta^2+1)-2Y_2(t_\beta^4-6t_\beta^2+1)]\\
&+2t_\beta^3c_{\beta-\alpha}^2(m_h^2+m_H^2-4Y_2)+2m_h^2t_\beta^3\},
\end{aligned} \tag{A4}
$$

$$
\begin{aligned}
J_1^H=&\frac{2c_{\beta-\alpha}}{v}\left(m_W^2W^\mu W_\mu^\dagger+\frac{1}{2}m_Z^2Z^\mu Z_\mu\right)+\frac{c_{\beta-\alpha}}{4vt_\beta^2}h^2\Big\{9t_\beta(t_\beta^2-1)c_{\beta-\alpha}^3(m_h^2-m_H^2)s_{\beta-\alpha}\\
&-3t_\beta(t_\beta^2-1)c_{\beta-\alpha}s_{\beta-\alpha}(m_h^2-2m_H^2+2Y_2)+3(t_\beta^4-4t_\beta^2+1)c_{\beta-\alpha}^4(m_h^2-m_H^2)\\
&+c_{\beta-\alpha}^2\big[m_h^2(-3t_\beta^4+8t_\beta^2-3)+m_H^2(3t_\beta^4-14t_\beta^2+3)+12Y_2t_\beta^2\big]+2t_\beta^2(m_H^2-4Y_2)\Big\}+\mathcal{O}(h^3),
\end{aligned} \tag{A5}
$$

$$
J_1^{H^+}=(J_1^{H^-})^\dagger=\frac{im_Wc_{\beta-\alpha}}{v}[h(\partial^\mu W_\mu)+2W^\mu(\partial_\mu h)], \tag{A6}
$$

where we express e, c_W and s_W by means of $g=e/s_W=2m_W/v$ and $c_W=m_W/m_Z$. The case $a=A$, with $J_1^A=-c_{\beta-\alpha}[h(\partial^\mu Z_\mu)+2Z^\mu(\partial_\mu h)](m_W^2+m_Z^2s_W^2)/(vm_Z)$ would contribute to the $ZZ\to hh$ process that is not studied here.

APPENDIX B: A NOTE ON THE $\mathbb{Z}_2$ SYMMETRIC SINGLET EXTENSION OF THE SM

We briefly review the $\mathbb{Z}_2$ symmetric real singlet extension of the SM discussed in Ref. [19] in the context of the HEFT matching. Our purpose is to illustrate the crucial differences between that model and the 2HDM. The potential in terms of a real singlet, S, and the usual $SU_L(2)$ doublet, ϕ, is

$$
V=-\frac{\mu_1^2}{2}\phi^\dagger\phi-\frac{\mu_2^2}{2}S^2+\frac{\lambda_1}{4}(\phi^\dagger\phi)^2+\frac{\lambda_2}{4}S^4+\frac{\lambda_3}{2}\phi^\dagger\phi S^2. \tag{B1}
$$

After SSB, ϕ and S get vevs $v/\sqrt{2}$ and $v_s/\sqrt{2}$, respectively. The physical states h and H have masses m and M, respectively (m is assumed to be light and M heavy). These are determined by minimizing the potential and diagonalizing the mass matrix with the mixing angle χ. This happens such that the Feynman rule for the cubic self-interaction of h reads

$$
i\frac{m^2}{2vv_s}(s_\chi^3v-c_\chi^3v_s). \tag{B2}
$$

Therefore, in stark contrast with what happens in the 2HDM [recall Eq. (17)], the cubic self-interaction of h in the model of Ref. [19] does not scale with positive powers of heavy masses (in this case, just M). This allows the authors to perform a HEFT matching as an expansion in inverse powers of the heavy mass M. On the other hand, such an expansion does not comply with decoupling and perturbativity.[30] To see this, note that the quartic couplings of the potential can be written in terms of the masses, the vevs and the mixing angle as

$$
\begin{aligned}
\lambda_1&=\frac{2}{v^2}[M^2s_\chi^2-m^2(s_\chi^2-1)],\\
\lambda_2&=\frac{2}{v_s^2}[m^2s_\chi^2-M^2(s_\chi^2-1)],\\
\lambda_3&=\frac{2c_\chi s_\chi}{vv_s}(M^2-m^2).
\end{aligned} \tag{B3}
$$

This clearly shows that, if M is taken to be very large and no other assumption is made, perturbativity is violated. As a consequence, even if no inconsistency is found in the cubic self-interaction of h, an expansion that simply assumes M to be very large and uses $1/M$ as an expansion parameter does not comply with perturbativity. Such compliance thus requires a different expansion, with more assumptions—specifically, assumptions about v_s and χ. Along the lines of Eq. (13), the scalings $1/M^2\sim O(\xi)$, $1/v_s^2\sim O(\xi)$ and $s_\chi^2\sim O(\xi)$ would lead to well-behaved quartic couplings.

[30]This does not mean that the different orders in the $1/M$ expansion performed in Ref. [19] violate perturbativity. The problem, rather, is that the expansion *itself* does not respect perturbativity for a very large M, if no other assumption is made.

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