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AN AXIOMATIC APPROACH TO FUZZY MULTICRITERIA ANALYSIS

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In this paper, an axiomatic approach to rational decision making is proposed. In particular, the successive aggregation into a global criterion of the judgements (about each alternative) with respect to all criteria is analyzed for a given multicriteria decision-making problem with weighted criteria. Some aggregation operations are axiomatically justified when such judgements are defined by a desirability function, and an analogous treatment can be developed when judgements are defined by a fuzzy outranking relation.

Keywords: Aggregation, Multicriteria Decision Making, Probabilistic Set.

1. INTRODUCTION

Most formal models of decision-making proceed from simple evaluation of the desirability of each feasible alternative. Such an evaluation summarizes the decision-maker's goal. Classical Decision Theory proposes to find an utility function which assigns an objective value for each alternative, and then the resulting optimization problem must be solved. But though this mathematical problem is well defined, it is not always representative of reality: on the one hand, comparision of alternatives is rarely made accorting on a single point of view, and even preferences on a single point of view are in many cases modelled with difficulty by an utility function (Roy-Vincke [1]); on the other hand, if we admit a complex problem with multiplicity of objectives, the idea of being the best alternative is not clear, and since we often use imprecise concepts in our analysis, criteria themselves may be of fuzzy nature (Blin [2]).

As pointed out by Saaty [3], the link between fuzziness and multicriteria problems is clear due to the fact that the complexity of experience acquired by our mind is fuzzy (fuzziness in perception, fuzziness in meaning). Moreover, the existence of a natural isomorphism between the class of fuzzy sets and the class of multicriteria decision-making procedures, as proved by Blin [2], shows the deepness of such a link.

In this paper, multiattribute decision-making problems are considered when criteria (or goals) are to be aggregated successively into a single global criterion, in such a way that at each moment we can obtain an aggregated criterion which summarizes the approval of each alternative under any family of criteria, by applying an adecuate operation aggregation.

Several authors have dealt with aggregation operations in different contexts (cf. [4,5,6,7,8]), though the point of departure is usually Bellman-Zadeh's paper [9]. In this article, logical connectives applied to probabilistic sets (Hirota [10]) are to be studied in a more general context than that investigated in Czogala et al. [4] and Czogala-Zimmerman [5].

In any case, aggregation operations will allow us to consider successive new criteria, by amalgamating them one by one. When a particular aggregation op[illegible] eration has been selected as rational for a given decision-making pr[illegible] classical methods in decision-making can be applied in order to cho[illegible] alternatives, by considering the distribution function of a probabil[illegible] (cf. [4,5,10,11]).

2. GENERALIZED AGGREGATION OPERATIONS

Let X be the set of feasible alternatives in a given decision-making [illegible] Each alternative can be analyzed under an infinite countable set of [illegible] D. If we denote by W the space of observable states, we shall suppose that [illegible] degree of approval of each alternative for any possible state is defined [illegible] each criterion. In other words, an approval function

$$\mu : X \times D \times W \longrightarrow [0,1]$$

assigns the degree of approval $\mu(x,c,w)$ of alternative x under criter[illegible] when w is the true state. If a probability space $(W,\mathcal{A},P)$ is defined on [illegible] the mapping $\mu(x,c,\cdot)$ is supposed to be measurable for any fixed altern[illegible] and any fixed criterion c (that is, $\{w\in W/\mu(x,c,w)\in B\}\in\mathcal{A}$ for any Borel [illegible] the unit interval), then it is clear that a probabilistic set in the [illegible] Hirota [10]

$$\mu_c: X \times W \longrightarrow [0,1]$$

has been associated to each criterion c, with $\mu_c(x,w)=\mu(x,c,w)$. It is [illegible] known that measurability of $\mu(x,c,\cdot)$ holds if and only if

$$\{w\in W/\mu(x,c,w)\leq r\}\in\mathcal{A} \quad \forall r\in[0,1]$$

and it must be pointed out that when the probability measure P is [illegible] the family of all subsets of W, $\mathcal{A}=\wp(D)$ (the usual situation in case [illegible] crete space of states), then any mapping $\mu(x,c,\cdot)$ is assured to be [illegible]

For our purpose, the following problem should be solved: from the given approval function, we must be able to build a mapping

$$\mu^+: X \times \wp(D) \times W \longrightarrow [0,1]$$

satisfying some logical conditions (as above, $\wp(D)$ represents the family of all subsets of D). In particular,

$$\mu^+(x,\{c\},w)=\mu(x,c,w) \quad \forall c\in D$$

and if we denote $\mu^+(x,A,w)=\mu^+_A(x,w)$, an aggregation operation ought to be defined in such a way that for any pair of disjoint subsets of criteria $A,B\in\wp(D)$, the probabilistic set $\mu^+_{A\cup B}$ is obtained from the probabilistic sets μ^+_A and μ^+_B.

Following the sheme of [12], we can denote

$$F_W(X)=\{\mu:X\times W\longrightarrow[0,1]/\mu(x,\cdot) \text{ is measurable } \forall x\in X\}$$

and $M=F_W(X)\times\wp(D)$. Then the generalized approval aggregation operation will be defined as a correspondence

$$\circ : M\times M \longrightarrow M$$

$$(\mu_A,A)\circ(\mu_B,B)=(\mu_{A\cup B},A\cup B) \quad \forall A,B\neq\phi,\ A\cap B=\phi$$

which assigns the aggregated approval $\mu_{A\cup B}$ to each pair of approvals μ_A and μ_B from two disjoint and non-empty sets of criteria. Associativity and commutativity are supposed to be verified by definition:

$$((\mu_A,A)\circ(\mu_B,B))\circ(\mu_C,C)=(\mu_A,A)\circ((\mu_B,B)\circ(\mu_C,C))$$

$$(\mu_A,A)\circ(\mu_B,B)=(\mu_B,B)\circ(\mu_A,A)$$

and other ethical conditions can be required in adition:

i) $\mu_A(x,w)=\mu_B(x,w)=\mu(x,w) \implies \mu_{A\cup B}(x,w)=\mu(x,w)$

ii) $\mu_A(x,w)\geq\mu'_A(x,w),\ \mu_B(x,w)\geq\mu'_B(x,w) \implies \mu_{A\cup B}(x,w)\geq\mu'_{A\cup B}(x,w)$

In section 4 the problem with unequal criteria will be considered. But previously we shall study how their relative importance can be aggregated.

3. AMALGAMATING CRITERIA WEIGHTS

Let us suppose that the set of criteria is defined by a membership function

$$\beta : D \longrightarrow [0,1]$$

$\beta(c)$ meaning the weight of criterion c. Following [13], we must be able to define the weight of any subset of criteria, in such a way that we can expect that approval function only depends on criteria through their weights.

If we denote $N=[0,1]\times\wp(D)$, each pair $(b,A)\in N$ represents a set of criteria A with relative weight b. The weight aggregation operation

$$\S : N\times N \longrightarrow N$$

$$(b(A),A)\S(b(B),B)=(b(A\cup B),A\cup B) \quad \forall A,B\neq\phi,\ A\cap B=\phi$$

assigns the aggregated weight $b(A\cup B)$ to each pair of weights $b(A)$ and $b(B)$ from two disjoint and non-empty sets of criteria. Analogously to the approval aggregation operation, commutativity and associativity are supposed by definition, and both aggregation operations must be connected in a rational way. In this section, some weight aggregation operations are characterized.

We shall suppose that $b(\{c\})=B(c)$, and under the following conditions:

$$\text{i')}\ b(A)=b(B)=b \implies b(A\cup B)=b$$

$$\text{ii')}\ b(A)\geq b'(A),\ b(B)\geq b'(B) \implies b(A\cup B)\geq b'(A\cup B)$$

it makes sense to say that a set of criteria A with weight $b(A)$ is d-decisive over a set of criteria B ($A\cap B=\phi$) with weight $b(B)$ ($b(A)\neq b(B)$) if and only

$b(A\cup B)=d.b(A)+(1-d).b(B)'$.

THEOREM 1.- Let us consider weight aggregation operations verifying

i'), ii') and

$$\text{iii')}\ \left.\begin{array}{l}\text{card}(A)=\text{card}(A'),\ b(A)=b(A')\\ \text{card}(B)=\text{card}(B'),\ b(B)=b(B')\end{array}\right\} \implies b(A\cup B)=b(A'\cup B')$$

Then the aggregation rule defined as

$$\bar{b}(A\cup B)=(\text{card}(A).\bar{b}(A)+\text{card}(B).\bar{b}(B))/\text{card}(A\cup B) \quad \forall A\cap B=\phi$$

makes maximum the minimum decisiveness d of criteria.

Proof: Let us consider any aggregation rule b verifying such conditions, let us denote $d=d_{1,n}(p,q)$ the decisiveness of a criterion with weight p a set of n criteria with weight q. On the one hand, since $d_{1,1}(p,q)=1-d_{1,1}$ due to commutativity, it is clear that $d_{1,1}(p,q)\leq\frac{1}{2}$ for some $p\neq q$. On the hand, assuming $d_{1,n-1}(p,q)\leq 1/n$ for some $p\neq q$, let us consider a set of $n-1$ teria B and $A=\{a\}$, $C=\{c\}$, with a and c two distinct criteria not belongin B. Taking $b(A)=p$, $b(B)=q$ and $b(C)=q$, then

$$\begin{aligned} b(A\cup(B\cup C)) &= d_{1,n}(p,q).p+(1-d_{1,n}(p,q)).q= \\ &= b((A\cup B)\cup C)=(1-d_{1,n}(q,r)).r+d_{1,n}(q,r).q \end{aligned}$$

where $r=b(A\cup B)$. Since

$$b(A\cup B)=d_{1,n-1}(p,q).p+(1-d_{1,n-1}(p,q)).q$$

it must be

$$d_{1,n}(p,q)=(1-d_{1,n}(q,r)).d_{1,n-1}(p,q)$$

and therefore

$$d_{1,n}(p,q)\leq(1-d_{1,n}(q,r))/n$$

due to induction hypothesis. If $d_{1,n}(q,r)\leq 1/(n+1)$, the result holds; $d_{1,n}(q,r)>1/(n+1)$, then $d_{1,n}(p,q)\leq 1/(n+1)$, and in any case

$$\inf_{p\neq q} d_{1,n}(p,q) \leq 1/(n+1)$$

Since the proposed rule is characterized by the fact that its decisi takes the value

$$d_{1,n}(p,q)=1/(n+1) \quad \forall p\neq q$$

then the theorem is proved. Moreover,

$$\bar{b}(A)=\Sigma_{c\in A}\,B(c)/\text{card}(A)$$

It is clear that if we modify condition iii') in order to get independence of the sizes of criteria sets, then the weight aggregation operation can be analyzed through a classical fuzzy connective: any aggregation operation such that

$$\text{iv')}\ b(A)=b(A'),\ b(B)=b(B') \implies b(A\cup B)=b(A'\cup B')$$

is characterized by a mapping

$$h:\ [0,1]\times[0,1] \longrightarrow [0,1]$$

verifying commutativity and associativity, in such a way that

$$b(A\cup B)=h(b(A),b(B)) \quad \forall A,B\neq\phi,\ A\cap B=\phi$$

Therefore, under condition iv'), classical results on fuzzy connectives can be translated here. For example:

THEOREM 2.- Let h be a fuzzy connective verifying continuity, commutativity, associativity, idempotent law and non-decreasingness. Then we can define an associated weight aggregation rule, and it must exist an $a\in[0,1]$ such that

$$h(p,q)=\max(p,q) \quad \text{if} \quad p,q\leq a$$

$$h(p,q)=\min(p,q) \quad \text{if} \quad p,q\geq a$$

and $h(p,q)=a$ otherwise.

Proof: see Fung-Fu [7], taking into account that condition iv') is equivalent to Fung-Fu's law of independent components. When $a=1$ or $a=0$, they talk about an "optimistic" rule and a "pessimistic" rule, respectively. When $a\neq 0,1$ they say that the rule is a "mixed" rule. Moreover, $a=h(0,1)$.

Analogously to Fung-Fu's approach [7], in Dubois-Prade's article [6] the condition of independent components is supposed to be verified. Under some conditions ($h(1,1)=1$, $h(0,0)=0$, commutativity, associativity, continuity and a compatibility condition with respect to the Pareto approach), they define the "conjunction operators" (if $h(p,q)\leq\min(p,q)$ $\forall p,q$), the "disjunction operators" (if $h(p,q)\geq\max(p,q)$ $\forall p,q$) and the "averaging operators" (if $\min(p,q)\leq h(p,q)\leq\max(p,q)$ $\forall p,q$). As expected, similar results to those of Fung-Fu [7] are easily found. In particular, they prove that the only associative averaging operators are the pessimistic, optimistic and mixed rules.

It must be pointed out that many interesting fuzzy connectives are special cases of t-norms or t-conorms (cf. Klement [14]): any commutative and associative fuzzy connective h is called t-norm if non-decreasingness holds and

$h(p,1)=p$; when $h(p,0)=p$ holds instead of this last condition, such a fuzzy connective is called t-conorm. Examples of important t-norms and t-conorms can be seen in Czogala-Zimmermann [5] and Weber [15].

4. AMALGAMATING UNEQUAL CRITERIA

The concept of generalized aggregation operation has been introduced in section 2. Let us consider now a fixed weight aggregation operation, in such a way that the weights of all subsets of criteria are defined. We shall suppose $b(A)\neq 0$ for each non-empty set of criteria A.

It is clear that the above considerations on fuzzy connectives can be applied here under some conditions: any aggregation operation such that

$$iv)\ \mu_A(x,w)=\mu_{A'}(x,w),\ \mu_B(x,w)=\mu_{B'}(x,w) \implies \mu_{A\cup B}(x,w)=\mu_{A'\cup B'}(x,w)$$

is characterized by a family of mappings

$$h_{x,w}: [0,1]\times[0,1] \longrightarrow [0,1] \quad \forall x\in X,\ \forall w\in W$$

verifying commutativity and associativity, in such a way that

$$\mu_{A\cup B}(x,w)=h_{x,w}(\mu_A(x,w),\mu_B(x,w)) \quad \forall A,B\neq\phi,\ A\cap B=\phi$$

and therefore the following result holds:

THEOREM 3.- Let us suppose a family of fuzzy connectives $\{h_{x,w}\}$ verifying [illegible] one continuity, associativity, idempotent law and non-decreasingness. Then we can define an associated generalized aggregation rule, and it must be such that

$$h_{x,w}(p,q)=\max(p,q) \quad \text{if } p,q\leq a_{x,w}$$
$$h_{x,w}(p,q)=\min(p,q) \quad \text{if } p,q\geq a_{x,w}$$

and $h_{x,w}(p,q)=a_{x,w}$ otherwise, for some $a_{x,w}\in[0,1]$.
Proof: analogous to theorem 2.

In many cases we can suppose that there exists a mapping h such that $h_{x,w}=h$ $\forall x\in X$, $\forall w\in W$. But in general the rule of aggregation for each alternative [illegible] depend upon the possible states (the rule of evaluation may change with the state of environment).

Dubois-Prade [6] discuss the amalgamating of more than two criteria by [illegible] recursivity, and therefore the necessity of associative law. If associativity [illegible] no longer holds, they define, not in an axiomatic way, symmetrical [illegible] through simultaneous combination of consistent values, when criteria [illegible] importance. When dealing with unequal criteria, they propose a weighted [illegible] of criteria, but they themselves emphasize the lack of generality of such [illegible]

aggregation. The following result will characterize a linear combination, by considering that under conditions i) and ii) it makes sense to say that a set of criteria A with approval μ_A is π-decisive over a set of criteria B ($A\cap B=\phi$) with approval μ_B relative to a pair (x,w) such that $\mu_A(x,w)\neq\mu_B(x,w)$ if and only if $\mu_{A\cup B}(x,w)=\pi.\mu_A(x,w)+(1-\pi).\mu_B(x,w)$.

THEOREM 4.- Let us consider criteria aggregations verifying conditions i) and ii). Then the aggregation rule such that

$$\bar{\mu}_{A\cup B}(x,w)=(card(A).\delta(A).\bar{\mu}_A(x,w)+card(B).\delta(B).\bar{\mu}_B(x,w))/(card(A\cup B).\delta(A\cup B))$$

makes maximum the minimum ratio decisiveness $\pi/\beta(c)$ of criteria.
Proof: let us denote $\pi=\pi^{x,w}_{A,B}(p,q)$ the decisiveness of a set of criteria A with $\mu_A(x,w)=p$ over a disjoint set of criteria B with $\mu_B(x,w)=q$ ($p\neq q$) for an arbitrary aggregation rule verifying such conditions. We shall prove that

$$\inf_{p\neq q}\ \pi^{x,w}_{\{c\},A-\{c\}}(p,q) \leq \beta(c)/\Sigma_{e\in A}\beta(e) \quad \forall A,\ \forall c\in A$$

and since the proposed rule is characterized by the fact that

$$\pi^{x,w}_{\{c\},A-\{c\}}(p,q) = \beta(c)/\Sigma_{e\in A}\beta(e) \quad \forall c\in A,\ \forall p\neq q$$

the theorem will be proved. On the one hand, if we suppose

$$\pi^{x,w}_{\{c\},\{e\}}(p,q) > \beta(c)/(\beta(c)+\beta(e))$$

for any $c\neq e$, then

$$\pi^{x,w}_{\{e\},\{c\}}(q,p) < \beta(e)/(\beta(c)+\beta(e))$$

due to

$$\pi^{x,w}_{A,B}(p,q) = 1-\pi^{x,w}_{B,A}(q,p) \quad \forall p\neq q,\ \forall A,B$$

On the other hand, if we suppose that for a set A of n criteria and $c\notin A$

$$\pi^{x,w}_{\{c\},A}(p,q) \leq \beta(c)/(\beta(c)+\Sigma_{e\in A}\beta(e))$$

for some $p\neq q$, then we can define

$$\mu_{\{c\}}(x,w)=p\ ,\ \mu_A(x,w)=\mu_{\{e\}}(x,w)=q$$

for a criterion $e\in A$ ($e\neq c$). Analogously to theorem 1, it is easy to prove that

$$\pi^{x,w}_{\{c\},A\cup\{e\}}(p,q) = (1-\pi^{x,w}_{\{e\},A\cup\{c\}}(q,r)).\pi^{x,w}_{\{c\},A}(p,q)$$

where $r=\mu_{A\cup\{c\}}(x,w)$, and in any case there exists some criterion verifying the required relation against a set of n+1 criteria.

Two questions arise at this point: 1) fixed a weight aggregation, when is it consistent, in a sense to be specified, with a criteria aggregation?, and 2) fixed a weight aggregation, can a criteria aggregation be defined depending on the aggregated criteria only through their weights and sizes? The second question has positive answer when the following condition is verified:

$$iii)\ \left.\begin{array}{l} card(A)=card(A'),\ b(A)=b(A'),\ \mu_A=\mu_{A'} \\ card(B)=card(B'),\ b(B)=b(B'),\ \mu_B=\mu_{B'} \end{array}\right\} \longrightarrow \mu_{A\cup B}=\mu_{A'\cup B'}$$

and it is clear, for example, that criteria aggregation $\bar{\mu}$ of theorem 4 verifies such a condition when the weight aggregation δ of theorem 1 is given. But not any criteria aggregation under iii) can be defined through any weight aggregation. For example, $\bar{\mu}$ is not compatible with neither pessimistic, optimistic or mixed rules. Moreover, it seems reasonable to impose that criteria decisiveness increases with weight and size:

$$v)\ card(A)\geq card(B),\ b(A)\geq b(B)\ \Longrightarrow\ \pi_{A,B}^{x,w}(p,q)\geq\pi_{B,A}^{x,w}(q,p)$$

The following results are related to the first question:

THEOREM 5.- Given a mixed weight aggregation, there is no criteria aggregation verifying condition i) and

$$v')\ b(A\cup B)=b(A)\neq b(B)\ \Longrightarrow\ \mu_{A\cup B}=\mu_A\quad \forall A\cap B=\phi$$

Proof: Let us consider $a\in(0,1)$ and A, A', B, B' disjoint sets of criteria such that $\mu_A=\mu_{B'}\neq\mu_B=\mu_{A'}$ and $b(A')\geq b(A)<a<b(B)\leq b(B')$. Since $b(A'\cup B)=b(A\cup B\cup A')=a$, it must be $\mu_{A\cup B\cup A'}=\mu_{B\cup A'}=\mu_B$ due to unanimity of condition i). Analogously it is proved that $\mu_{B\cup A\cup B'}=\mu_A$ in contradiction with the previous consideration.

THEOREM 6.- Pessimistic weight aggregation is not consistent with a criteria aggregation verifying conditions i), ii), v) and v').

Proof: Trivial, since $\pi_{A,B}^{x,w}(p,q)=1$ should hold for any pair of disjoint sets of criteria A and B such that $b(A)<B(B)$.

It must be pointed out that condition v'), introduced in [13], means that approval remains the same when importance does not change. It is equivalent to impose

$$d_{A,B}(a,b)=1\ \Longrightarrow\ \pi_{A,B}^{x,w}(p,q)=1$$

when conditions i) and ii) are supposed to be verified. Moreover, it is easy to see that condition v') can be relaxed in theorem 6, for example by assuming

$$d_{A,B}(a,b)=1\ \Longrightarrow\ \pi_{A,B}^{x,w}(p,q)>\tfrac{1}{2}$$

Optimistic weight aggregation can be consistent with criteria aggregations verifying conditions of theorem 6, and under these conditions the set

$$A=\{c\in D\ /\ B(c)=\max_{d\in D} B(d)\}$$

verifies, in case of being non-empty, that $\mu_{A\cup B}=\mu_A$ for any disjoint set B of criteria (A is said to be absolutely decisive). Since A contains criteria with the same relative importance, aggregation operations for equal criteria can be applied in order to get the aggregated approval.

Other interesting aggregation rules for unequal criteria, verifying conditions i) and ii), are the following:

$$\hat{\mu}_{A\cup B}(x,w)=[\hat{\mu}_A(x,w)]^{card(A).\bar{b}(A)/card(A\cup B).\bar{b}(A\cup B)}.[\hat{\mu}_B(x,w)]^{card(B).\bar{b}(B)/card(A\cup B).\bar{b}(A\cup B)}$$

and

$$\hat{\mu}'_{A\cup B}(x,w)=1-[1-\hat{\mu}'_A(x,w)]^{card(A).\bar{b}(A)/card(A\cup B).\bar{b}(A\cup B)}.[1-\hat{\mu}'_B(x,w)]^{card(B).\bar{b}(B)/card(A\cup B).\bar{b}(A\cup B)}$$

which are types of weighted pessimistic rules and weighted optimistic rules, respectively. But they do not hold condition v): aggregated approval can be fixed by criteria with low relative weight. Their axiomatic justification can be obtained by considering a multiplicative model, instead of the aditive model developed here.

5. CONCLUDING REMARKS

In this paper, criteria have been supposed to be represented by probabilistic sets, to be amalgamated by using generalized aggregation operations, which must take into account that those criteria relevant for the evaluation are of different importance for the final judgement. The set of criteria has been defined as a fuzzy set which allows us to build some valid system of relative weights for each subset of criteria, and the amalgamated approval represents how well each alternative satisfies a given family of criteria. But two things must be pointed out: on the one hand, a purely mathematical justification of an aggregation operation is not sufficient, and it must be tested empirically in order to stablish the adequacy of the model to human behavior (a first analysis in this sense can be seen in [16]). On the other hand, though decision analysis depends on the state of the environment, weights of each criterion has been supposed to remain constant, whatever the state or degree of approval, and non-interaction of criteria (something that may be difficult to justify) has been assumed (see [17] for a discussion). Another kind of models must be developed in order to avoid this problem.

In any case, in this context it makes sense to look for alternatives with highest degree of expected approval, for a fixed probability distribution on the space of states (see [5]): given a set A of criteria we must solve

$$\max_{x\in X} E_p[\mu_A(x,\cdot)]$$

where $\mu_A(x,w)=\mu(x,A,w)$ is the aggregated approval of alternative x when w is the state of the environment, and E_p means the mathematical expectation with respect to the probability measure P.

Following Hirota [10], other classical methods in Decision Theory, like moment analysis, can be applied.

Finally, it must be pointed out that here we have considered approval function taking values on the unit interval. Analogous treatment must be tried in a more general topological framework. Moreover, here we have developed an aggregation approach instead of an order-focussed approach (see Zimmermann [18] for a study of these two distinct families of approaches). But results obtained in this paper are easily translated to the aggregation of Roy's outranking relations [19], by considering approval functions

$$\mu : (X \times X) \times D \times W \text{ -----> } [0,1]$$

in such a way that each value $\mu(x,y,c,w)$ represents the degree to which alternative x outranks alternative y under criteria c and state w. In this context, Saaty's procedure (see [3]) for obtaining a ratio scale of importance can be useful. In any case, general considerations about decision-making in management (see [20], for example) must be taken into account under both approaches.

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