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Porter Hypothesis vs. Pollution Haven Hypothesis: Can an environmental policy generate a win–win solution?

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ABSTRACT

This paper investigates the effects of environmental policy on firms' location and green innovation for a two-country, two-firm model. To address this issue, a two-stage game is solved. At the first stage, firms can choose between three actions: to stay in the home country and invest in a green technology; to stay in the home country and produce with the business-as-usual technology; or to move to a pollution haven. At the second stage, the firms compete in quantities while serving the demand in the home country. Despite the model is symmetric, our findings indicate that all market configurations – both symmetric and asymmetric – can be a subgame perfect Nash equilibrium of the game. This includes a "win–win" solution where both firms choose to stay in the home country and invest in green technology confirming the "weak" version of the Porter Hypothesis. Remarkably, this outcome can occur even in seemingly adverse conditions with relatively low setup costs of relocating to a pollution haven. The model predicts that a stricter environmental policy plays in favor of the Porter Hypothesis because the "win–win" solution becomes more likely to arise as an equilibrium of the game. Our analysis examines two policy scenarios – an emission tax and an emission standard – finding that the emission tax can induce firms to stay and invest in green technology under circumstances for which the standard cannot, confirming in this way the "narrow" version of the Porter Hypothesis.

1. Introduction

The interest in assessing the effects of environmental policy on firms' innovation, competitiveness and location received a great impulse in the early nineties with the publication of the papers by Porter (1991), Porter and van der Linde (1995) and Copeland and Taylor (1994). Copeland and Taylor (1994) formalized the idea that reducing trade barriers in an increasingly globalized world leads to the displacement of polluting firms or industries to countries with weaker environmental policies. This idea is known as the *Pollution Haven Hypothesis* (henceforth PHH). On the other hand, Porter (1991) and Porter and van der Linde (1995) challenged the traditional view that environmental policy will harm firms' profits and claimed that just the opposite may be true. According to the *Porter Hypothesis* (henceforth PH), stricter environmental regulations can stimulate investments in green technology to reduce the compliance costs of environmental policy. In the literature, the PH takes different forms (Jaffe and Palmer, 1997). While the "weak" version simply states that environmental regulations can incentivize firms to do certain types of technological innovation as a means to reduce the cost of complying with such regulations, the "strong" version states that these regulation-induced innovations can even exceed the regulatory costs and, in consequence, enhance the firm's competitiveness. Finally, there is a "narrow" version, which suggests that market-based regulations are better at fostering innovation than command-and-control type regulations.

Although the PHH and the PH tell different stories, they have important elements in common: they both address the expected reactions of firms to environmental policy pressures and their impacts on competitiveness. However, as far as we know, only a pair of papers, D'Agostino (2015) and Ranocchia and Lambertini (2021), have explored the interaction between the PH and the PHH focusing on how environmental regulations influence industrial relocation and innovation. D'Agostino (2015), integrating the PH and the PHH in an international business context, suggests that multinational firms can locate in countries with a weaker environmental regulation when they lack the capabilities to respond to environmental regulation in developed countries. Ranocchia and Lambertini (2021) explore the possibility that environmental policy yields a "win–win"solution with firms investing in green technology at the home country instead of moving to the pollution haven. The idea

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is that if the PH works, it could avoid the PHH. These authors resort to a Cournot duopoly supergame allowing for collusion between firms. Their results show that an emission tax rate lower than the per unit cost of transportation from the pollution haven to the home country is a necessary condition to induce firms to stay in the country and invest in the green technology, regardless of whether there is partial or full collusion, and of the punishment strategies used to support collusion. Thus, the authors show that under some conditions, the "win–win" solution is an equilibrium of the supergame.

In this paper, we wonder if the "win–win" solution is still an equilibrium when firms act non-cooperatively in a two-stage game, rather than engaging in collusion. To this end, we set a two-stage game in which two firms initially located in an industrialized country interact in terms of location, investment and production. In the first stage, firms can choose between three actions: (i) to remain in the home country and continue production using the business-as-usual (or "brown") technology, (ii) to stay in the home country and invest in a "green" technology, or (iii) to relocate to a pollution haven. Each action entails different costs: (i) staying with the brown technology involves bearing the costs associated to the compliance with the environmental regulation; (ii) the green technology option is assumed to be emissions-free and, thus, avoids any direct compliance expenses, but entails an investment cost; and (iii) relocation implies a fixed setup cost for relocation and an additional per-unit cost for shipping goods back to the home country. In the second stage, the firms compete in quantities while serving the demand in the home country. We examine this game under two distinct environmental policy scenarios: a price-based policy instrument, such as a tax on emissions, and a command-and-control instrument, as a standard, that forces the firms to cut-down production below the profit-maximizing level.

The main take-away messages of our analysis are as follows. First, despite the symmetry of the game, we obtain that, under suitable parameter values, all market configurations (both symmetric and asymmetric) can arise in a Subgame Perfect Nash Equilibrium in both environmental policy scenarios. Second, we show that the win–win solution is attainable in a non-cooperative game, even when the investment cost exceeds the setup costs to move to a pollution haven both for the tax and the standard. In the case of the tax, we also find that this will also occur when the tax is higher than the per-unit transportation cost. Thus, our results indicate that the "weak" version of the PH may hold, even under seemingly adverse conditions, something not occurring in Ranocchia and Lambertini (2021). Third, regarding the comparison between the two policy instruments, the tax option incentivizes firms to stay and invest in green technology under circumstances for which the (environmentally equivalent) standard cannot, thereby confirming the "narrow" version of the PH. And fourth, the scenario in which both firms relocate also turns out to be more likely under a tax-based policy than under a standard. The two last results suggest a trade-off in terms of policy design. While the tax is preferable in terms of inducing green investment, it may not be the most effective tool to prevent relocation.

1.1. Literature review

As the literature on the PH and the PHH is very abundant, a very prolific review is out of the aim of this paper. Instead, we will focus only on the papers that analyze imperfectly competitive markets. Looking first at the literature on the PH we could mention the papers by André et al. (2009), Constantatos and Herrmann (2011), Lambertini and Tampieri (2012), Buccella et al. (2021) and Agliardi and Lambertini (2024).[1]

André et al. (2009) study a duopoly model of vertical product differentiation in which firms simultaneously set the environmental quality at the first stage and engage in price competition at the purple second stage. They show that there always exists a range of environmental qualities such that the implementation of an environmental policy, consisting in a lump-sum tax on profits, may lead firms to choose the more environmental friendly variant and obtain larger profits. This result yields a theoretical foundation to the "strong" version of the PH. Our model cannot replicate this result because firms produce an homogeneous good, so that the consumers' willingness to pay does not depend on how clean the good is. On the other hand, while André et al. (2009) consider the binary choice for firms to either be green or not, we also consider a third alternative: relocating to avoid environmental pressure. Constantatos and Herrmann (2011) and Lambertini and Tampieri (2012) extend André et al. (2009) analysis assuming Cournot competition at the second stage of the game, to evaluate whether the same result can be derived in this setting. Constantatos and Herrmann (2011) also assume that consumers do not immediately perceive the environmental quality of a product, but realize the difference with a time lag of one period.

Buccella et al. (2021) consider a duopoly where at the first stage firms simultaneously choose whether to adopt an abatement technology that does not affect the production costs, then at the second stage the regulator selects the emission tax that maximizes social welfare and finally, at the third stage, firms compete in the market and choose the amount of abatement if they do abate. The authors analyze Cournot competition with an homogeneous good and with product differentiation and also in this case Bertrand competition. Their analysis shows that the Subgame Perfect Nash Equilibrium depends on parameter values, although the game only yields symmetric equilibria in both quantity-setting and price-setting duopolies. Under some conditions the equilibria are the Pareto efficient outcome of the game. We share with this paper the assumption that the adoption of a clean technology does not affect the production costs, but we identify conditions supporting asymmetric equilibria. The two main differences with this paper are that we assume that the environmental policy is given, as in André et al. (2009), and firms have a third option: moving to a pollution haven. Agliardi and Lambertini (2024) extend the Buccella et al. (2021) paper to consider time consistent emission taxation.[2]

Next, we look at the literature on the PHH. From this literature, we could point out a first group of papers formed by Motta and Thisse (1994), Markusen et al. (1995), Rauscher (1995), Hoel (1997), Petrakis and Xepapadeas (2003) and Sanna-Randaccio et al. (2017) that focus on a *single* firm's location decision. Motta and Thisse (1994) and Sanna-Randaccio et al. (2017) assume that the environmental policy is given and study the effects of a *unilateral* change in the environmental policy on the firm's location choice. In contrast, Markusen et al. (1995), Rauscher (1995) and Hoel (1997) endogenize environmental policies as a Subgame Perfect Nash Equilibrium of a policy game between governments. Our paper is close to Petrakis and Xepapadeas (2003) analysis of a monopoly that can use a pollution haven as an export platform to sell in the home country producing with the same costs. Their paper focuses on the effect of regulator's commitment on the firm's location decision. Their main finding is that the monopolist will relocate more often under time-consistent emission taxes.

There is a second group of papers consisting of Markusen et al. (1993), Hoel (1997), Ulph and Valentini (2001), Greaker (2003), Ikefuji et al. (2016) and more recently Elboghdadly and Finus (2022) that analyze the PHH in a two-firms, two country model considering the reaction of the two firms to the environmental policy.[3] All these papers,

[1] A review of the theoretical literature on the PH can be found, e.g., in André (2016) or Lambertini (2017).

[2] Qiu et al. (2018) revisit the PH under monopolistic competition in a general equilibrium setting with firm heterogeneity. They find that strict environmental regulations can encourage firm entry and exit, thereby improving the composition of firms in the regulated industry.

[3] Hoel (1997) looks at the case of monopolistic competition.

except Markusen et al. (1993) and Ikefuji et al. (2016), using different trade models, take into account the strategic nature of competition among governments in different countries in determining the optimal environmental policy. The paper by Ikefuji et al. (2016) is an extension of Petrakis and Xepapadeas (2003) to the case of a duopoly producing a homogeneous good, but without considering abatement as Petrakis and Xepapadeas (2003) do. They show that for a given tax, the three location equilibria, which include the asymmetric case with one firm in the pollution haven and the other firm in the home country, depend on the relocation cost. For the optimal tax, the type of equilibria the game yields can be written as a function of the damage coefficient and the relocation cost, and again the three market configurations can be an equilibrium. The authors find a lower bound for the damage coefficient such that for values of this coefficient higher than the lower bound, the two firms move to the pollution haven regardless of the level of the relocation costs. In this paper, we extend Ikefuji et al.'s (2016) model considering that firms can invest in a clean technology that completely eliminates emissions, although in our model the environmental policy is given as in Motta and Thisse (1994), André et al. (2009) and Sanna-Randaccio et al. (2017). We characterize the equilibria as a function of the investment costs and the relocation costs and show how the six types of equilibria the game can present, which include a "win–win"solution, depend on the value of these parameters.

Regarding the empirical motivation of our work, the literature has provided sound support for the weak version of the PH (see, for instance, the papers by Aghion et al. (2016); Calel and Dechezleprêtre (2016)) and mixed evidence for the PHH, particularly in the context of international trade.[4] The main argument against the latter hypothesis is that environmental policy alone cannot justify firms' relocation. Instead, there are additional pulling factors, such as countries' endowments and the availability of relevant production factors, such as labor. Nevertheless, there are examples in the literature of a pollution-haven effect of the environmental regulation we could mention. Kahn and Mansur (2013) have shown that, within the United States, polluting firms tend to locate in counties that apply the US Clean Air Act in a more lenient manner.[5] More recently, Tanaka et al. (2022) examine the effect of a stricter US air quality regulation for lead in 2009 on the relocation of battery recycling to Mexico and on infant health in Mexico. They find evidence that the tightening of the environmental regulation caused a displacement of the battery-recycling plants from the "North"(US) to the "South"(Mexico). As examples supporting both hypotheses can be found, our paper emphasizes the convenience of studying them together presenting under an integrated model to understand how environmental policy affects firms' decisions in terms of relocation and investment simultaneously. Our results establish conditions for the occurrence of both hypotheses under two different policy scenarios.

The rest of the paper is structured as follows. In Section 2 we present the main building blocks of the model. In Section 3 we discuss the possible equilibria of the game under a tax. Section 4 investigates how the results change when the environmental policy adopts the form of a standard instead of a tax. Finally, Section 5 compares both versions. The main conclusions are presented in Section 6.

[4] The reader interested in the empirical evidence on the PH could look at the reviews by Ambec et al. (2013) and Cohen and Tubb (2018). D'Agostino (20215) and Dechezleprêtre and Sato (2017) cover both the PH and the PHH.

[5] The Clean Air Act is an environmental regulation aimed at controlling and reducing air pollution in the United States. Initially enacted in 1963, it has been updated over time, with the most recent amendment being the 2022 Inflation Reduction Act, which promotes domestic clean energy production. For more information, see: https://www.epa.gov/laws-regulations/summary-clean-air-act.

2. The model

We consider a Cournot duopoly located in a country (the "home country") that faces a market demand represented by the decreasing inverse demand function $p = a - Q$, with $Q = q^1 + q^2$, where p is the market price, Q is total quantity sold in the market and q^i is the quantity produced by firm $i = 1, 2$. In the business as usual (BAU) situation, we assume that both firms have the same technology, summarized by the cost function $c(q^i) = cq^i$, where $c \in (0, a)$ is the constant marginal cost and there is no fixed cost. Moreover, the BAU production technology generates pollution emissions denoted by e^i, which after an appropriate choice of measurement units, are such that $e^i = q^i$.

We initially assume that the regulator in the home country imposes a tax of $\theta \geq 0$ monetary units per unit of emissions. The firm can respond to this tax in three different ways: first, it can stay in the home country with the BAU or brown technology and just pay the tax. We denote this action as B for "BAU". Second, it can avoid the tax by relocating to foreign country, which is a pollution haven, i.e. there is no environmental regulation. However, if it decides to move to the pollution haven, it incurs a transportation cost of $\tau \geq 0$ per unit of output for selling in the home country, and a relocation cost $k \geq 0$ for moving the production plant. We represent this action by M for "move". Third, it can stay at home and invest in a green technology that eliminates completely emissions. We denote this action as G for "green". For simplicity, we assume that the green technology has the same marginal cost as the BAU technology, but entails an investment cost $x \geq 0$.[6]

We consider a two-stage game with the following timing. In the first stage, both firms simultaneously decide about location and technology by choosing one of the three available actions, $\{G, B, M\}$. In the second stage they compete in quantities in the home country.[7] In the next section, we obtain the Subgame Perfect Nash Equilibrium (SPNE) solving the game by backward induction.

3. The equilibrium of the game with an emission tax

As the two firms can choose between three actions, in the second stage we can find nine different market configurations. Thus, to solve the game we need first to calculate the equilibrium quantities under each of these market configurations.

3.1. Second stage: Production and pollution

As a consequence of their decisions in the first stage, firms can face different cost functions. Specifically, the cost function of firm i takes the form $c(q^i) = cq^i + x$ if it stays in the home country and invest in the clean technology (i.e., it chooses action G in the first stage), $c(q^i) = (c + \theta)q^i$ if it stays, but does not invest (action B), and $c(q^i) = (c + \tau)q^i + k$ if it moves to the foreign country (action M). Computing the Cournot equilibrium for each market configuration we obtain the production of each firm $i = 1, 2$, that we represent by $q^i_{l,n}$, where the first subscript corresponding to the action taken by firm i in the first stage, and the

[6] Other authors as André et al. (2009) or Constantatos and Herrmann (2011) have considered that the green technology operates with a higher marginal cost. In order to facilitate the comparison with the paper by Ranocchia and Lambertini (2021) we assume that the green innovation does not affect the production costs. A similar approach has been adopted by Buccella et al. (2021) and Agliardi and Lambertini (2024). These authors consider an end-of-the-pipe abatement technology that does not affect the production costs.

[7] We do not consider the possibility that the firm also sells its output in the foreign country. The foreign country is used as an export platform. The focus of this paper is on explaining how a tax can avoid this type of relocation because firm can find more profitable invest in a green technology and stay at home.

Table 1
Equilibrium quantities for each market configuration.

Firm 1/Firm 2	G (green)	B (BAU)	M (move)
G (green)	$\frac{s}{3}, \frac{s}{3}$	$\frac{s+\theta}{3}, \frac{s-2\theta}{3}$	$\frac{s+\tau}{3}, \frac{s-2\tau}{3}$
B (BAU)	$\frac{s-2\theta}{3}, \frac{s+\theta}{3}$	$\frac{s-\theta}{3}, \frac{s-\theta}{3}$	$\frac{s-2\theta+\tau}{3}, \frac{s+\theta-2\tau}{3}$
M (move)	$\frac{s-2\tau}{3}, \frac{s+\tau}{3}$	$\frac{s+\theta-2\tau}{3}, \frac{s-2\theta+\tau}{3}$	$\frac{s-\tau}{3}, \frac{s-\tau}{3}$

Table 2
Total output and pollution for each market configuration.

Firm 1/Firm 2	G (green)	B (BAU)	M (move)
G (green)	$\frac{2s}{3}, 0$	$\frac{2s-\theta}{3}, \frac{s-2\theta}{3}$	$\frac{2s-\tau}{3}, \frac{s-2\tau}{3}$
B (BAU)	$\frac{2s-\theta}{3}, \frac{s-2\theta}{3}$	$\frac{2(s-\theta)}{3}, \frac{2(s-\theta)}{3}$	$\frac{2s-\theta-\tau}{3}, \frac{2s-\theta-\tau}{3}$
M (move)	$\frac{2s-\tau}{3}, \frac{s-2\tau}{3}$	$\frac{2s-\theta-\tau}{3}, \frac{2s-\theta-\tau}{3}$	$\frac{2(s-\tau)}{3}, \frac{2(s-\tau)}{3}$

second subscript to the action taken by its rival, with $l, n \in \{G, B, M\}$. These quantities are displayed in Table 1 where the market size is represented by $s \equiv a - c$. In each cell the first figure represents the equilibrium quantity produced by firm 1, whose actions in the first stage correspond to the rows of the table, and the second figure is the equilibrium quantity produced by firm j, whose actions in the first stage correspond to the columns of the table.

Assumption 1. $s > 0$ and $\theta, \tau \in (0, s/2)$.

It is easy to show that, if this assumption is satisfied, all equilibrium quantities are strictly positive. Notice that, if both firms adopt the same decision in stage 1, the model is completely symmetric, but this is not the case when those decisions are different. Nevertheless, the six asymmetric cases reduce to three because we observe that the market configuration (G, B) is totally equivalent to (B, G) just by interchanging the firms' indexes. The same can be said about (G, M) vs. (M, G) and (B, M) vs. (M, B).

If both firms choose action B in the first stage, a higher value of the tax rate, θ, reduces the firms' output/emissions, and if both choose M, a higher value of τ cost reduces the firms' output/emissions. However, the effect of the tax is asymmetric if one firm decides to pay the tax and the other invest in the green technology. In this case, the tax reduces the output/emissions of the firm that uses the polluting technology, but increases the output of the firm that has invested in the green technology. Something similar occurs when one firm decides to move and the other to invest. Now, the transportation cost decreases the output/emissions of the firm that has moved to the foreign country, but increases the output of the firm that has invested in the green technology. Finally, if one firm decides to stay and pay the tax whereas the other firm moves to the foreign country, we see that the tax increases the output/emissions of the firm that has decided to move, but reduces the output/emissions of the firm that pays the tax in the home country, whereas the effect of the transportation cost is just the contrary.

By adding up both quantities, we get total output for each market configuration. Using the fact that emissions are equal to output quantity when firms are producing with the BAU technology and equal to zero when they produce *green*, it is immediate to obtain the equilibrium amounts of emissions in each market configuration. The corresponding values of total output and total emissions in equilibrium can be compactly displayed in the following table. In each cell, the first entry represents total output and the second total emissions. Notice that when no firm invests in the green technology, total output of the industry is equal to total emissions. On the other hand, the (G, G) configuration yields the maximum output, and therefore, the minimum market price and the maximum consumer surplus, as well as the minimum level of emissions. Therefore, it is a particularly interesting market configuration, which we label as "win–win".

Table 3
Firms profit in equilibrium.

Firm i/Firm j	G (green)	B (BAU)	M (move)
G (green)	$\frac{s^2}{9} - x; \frac{s^2}{9} - x$	$\frac{(s+\theta)^2}{9} - x; \frac{(s-2\theta)^2}{9}$	$\frac{(s+\tau)^2}{9} - x; \frac{(s-2\tau)^2}{9} - k$
B (BAU)	$\frac{(s-2\theta)^2}{9}; \frac{(s+\theta)^2}{9} - x$	$\frac{(s-\theta)^2}{9}; \frac{(s-\theta)^2}{9}$	$\frac{(s-2\theta+\tau)^2}{9}; \frac{(s+\theta-2\tau)^2}{9} - k$
M (move)	$\frac{(s-2\tau)^2}{9} - k; \frac{(s+\tau)^2}{9} - x$	$\frac{(s+\theta-2\tau)^2}{9} - k; \frac{(s-2\theta+\tau)^2}{9}$	$\frac{(s-\tau)^2}{9} - k; \frac{(s-\tau)^2}{9} - k$

In the following section, we study under which conditions each market configuration can emerge in a SPNE of the game. As a previous step, we calculate the profits firms will get for all market configurations using the data in Tables 1 and 2. The results are shown in Table 3, where each cell contains the profit of firm i and firm j.

3.2. First stage: Location and investment decisions

As we show below, all market configurations can arise in a SPNE under suitable parameter values. Moreover, there are parameter configurations for which the equilibrium is not unique.

We begin studying under what conditions the "win–win" solution (i.e., the G, G market configuration) arises in a SPNE. This result requires $\pi^i_{GG} \geq \pi^i_{BG}$ and $\pi^i_{GG} \geq \pi^i_{MG}$ for $i = 1, 2$, where $\pi^i_{l,n}$ stands for firm i's profits when firm i chooses action l and firm j (with $j \neq i$) selects action n, with $l, n \in \{G, B, M\}$. In turn, these inequalities are satisfied if the following condition holds:[8]

$$x \leq \min\left\{x_0^\theta,\ f_0^\theta(k)\right\}, \tag{1}$$

$$\text{where } x_0^\theta \equiv \frac{4\theta(s-\theta)}{9},\ f_0^\theta(k) \equiv \frac{4\tau(s-\tau)}{9} + k \tag{2}$$

Given that $f_0^\theta(k)$ is increasing in k, which is assumed to be non-negative, we can find a lower bound at $f_0^\theta(0)$, and comparing this lower bound with x_0^θ we have

$$x_0^\theta - f_0^\theta(0) = \frac{4}{9}(\theta - \tau)(s - \theta - \tau),$$

the sign of which depends on $\theta - \tau$, since $s - \theta - \tau$ is positive by Assumption 1. Therefore, we can find two different cases.

Case 1 involves $\theta < \tau$, which implies $x_0^\theta < f_0^\theta(0)$ and, therefore, $x_0^\theta < f_0^\theta(k)$ for any admissible value of k. As a consequence, condition (1) reduces to $x \leq x_0^\theta$. In other words, when $\theta < \tau$ holds, (G, G) arises in a SPNE if the investment cost is small enough, regardless of the level of the relocation cost.[9] Another implication of this case is that, whenever $\theta < \tau$, no firm will relocate in equilibrium, i.e., the PHH never holds. In other words, setting up a tax rate lower than the per unit transportation cost is a sufficient condition to avoid the relocation of firms. The reason is that, when $\theta < \tau$, the marginal cost of producing in the foreign country is higher than the marginal cost of producing in the home country, and the relocation is never profitable regardless of the relocation cost. Therefore, the only market configurations that can arise in a SPNE are (G, G), (B, G) and (B, B). It can be shown that these cases arise in equilibrium for low, intermediate and high values of x respectively.

Case 2 involves $\theta > \tau$, which is more interesting in the sense that it gives rise to a wider range of possible equilibria and shows the possibility that firms may choose to invest in the green technology even when the tax rate is higher than the transportation cost. In the rest of the paper we will focus on this case. Fig. 1 shows the combinations of x and k that are compatible with the win–win solution (G, G) as an equilibrium, for given values of θ and τ. Specifically, these combinations belong to the area below x_0^θ and $f_0^\theta(k)$.

[8] The superscript θ denotes the policy scenario in which the regulator is using a tax to control emissions.

[9] According to Table 3, profits are non-negative if $x \leq s^2/9$. As $x_0^\theta < s^2/9$, we conclude that, in case 1, $x \leq x_0^\theta$ guarantees that both firms choose G and have positive profit in equilibrium.

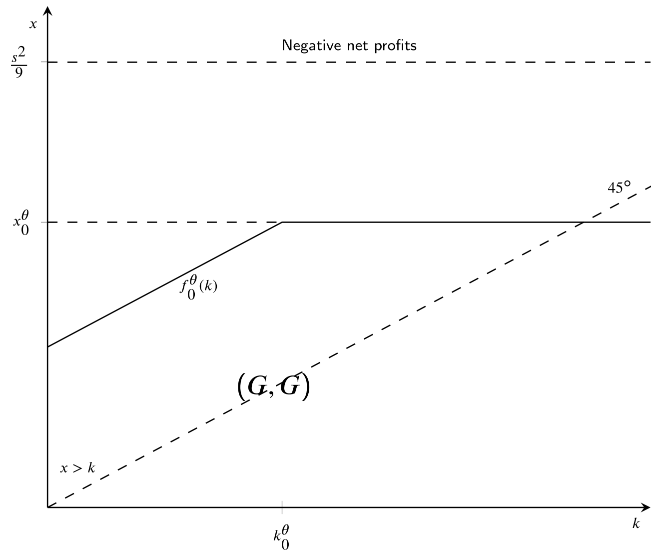

Fig. 1. Investment decisions as a function of k and x for $\theta > \tau$.

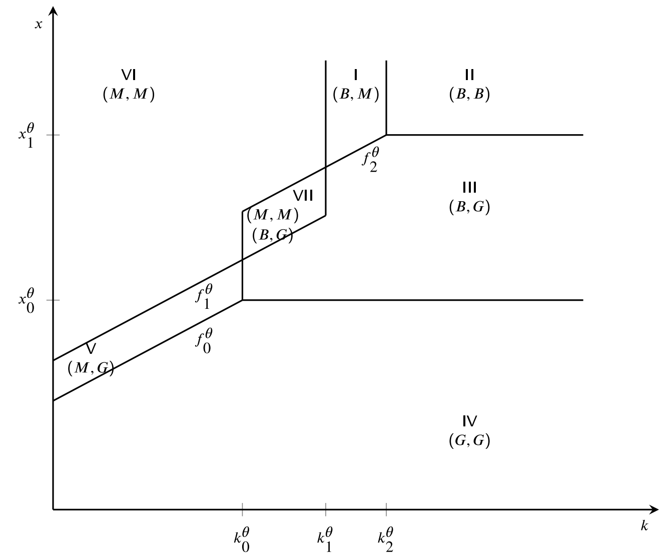

Fig. 2. Location and investment decisions for $\tau < \theta \leq s/2$.

It is interesting to notice that the win–win configuration may arise under apparently adverse conditions. Indeed, since function $f_0^\theta(k)$ lies above the 45^o line, there is an area below x_0^θ and $f_0^\theta(k)$ and above the 45^o line for which, in equilibrium, both firms decide to stay at home and invest in the green technology, even though the investment cost is higher than the relocation cost and the tax rate is higher than the transportation cost. This result shows that, under plausible conditions, a tax on emissions can avoid the relocation of firms and encourage them to invest in green technology. By producing with a green technology, firms can avoid to pay the tax and gain a cost advantage that make it unprofitable to move to the pollution haven, even if the relocation cost is lower than the investment cost. This result gives a theoretical foundation to the PH, as we conclude that both firms have an incentive to invest in green technology as a response to the environmental regulation.[10]

Regarding the shape of the win–win sector, note that, as far as the investment cost is large enough to make relocation unattractive (specifically, $k > k_0^\theta$), the threshold for the investment cost is independent of the relocation cost and condition (1) requires that the investment cost is low enough ($x < x_0^\theta$). But if the relocation cost is below a given threshold (specifically $k < k_0^\theta$), then the upper threshold for the investment cost is increasing in k.

Next, we study under which conditions the other market configurations can be an equilibrium. In order to characterize these conditions, we define the following auxiliary functions:

$$f_1^\theta(k) \equiv \frac{4s\tau}{9} + k, \qquad f_2^\theta(k) \equiv \frac{4\tau(s+\theta-\tau)}{9} + k, \tag{3}$$

and the following threshold values for k and x:[11]

$$k_0^\theta \equiv \frac{4}{9}(\theta-\tau)(s-\theta-\tau), \quad k_1^\theta \equiv \frac{4}{9}(\theta-\tau)(s-\theta), \quad k_2^\theta \equiv \frac{4}{9}(\theta-\tau)(s-\tau) \tag{4}$$

$$x_1^\theta \equiv \frac{4s\theta}{9}, \tag{5}$$

where, for any admissible parameter values in Case 2 ($\theta > \tau$), we have $f_1^\theta(k) < f_2^\theta(k)$, $k_0^\theta < k_1^\theta < k_2^\theta$ and $x_0^\theta < x_1^\theta$ as is showed in Appendix A.

The following proposition identifies all possible equilibria of the game depending on the parameter values.

Proposition 1. *For any (θ, τ, s) satisfying Assumption 1 with $\tau < \theta$, and provided that for each market configuration the firms produce their equilibrium quantity as in Table 1, the equilibrium in the first stage of the game is*

I. (B, M) if $f_2^\theta(k) \leq x$ and $k_1^\theta \leq k \leq k_2^\theta$.
II. (B, B) if $x_1^\theta \leq x$ and $k_2^\theta \leq k$.
III. (B, G) if $x_0^\theta \leq x \leq \min\{x_1^\theta, f_2^\theta(k)\}$ and $k_0^\theta \leq k$.
IV. (G, G) if $x \leq \min\{x_0^\theta, f_0^\theta(k)\}$ and $0 \leq k$.
V. (M, G) if $f_0^\theta(k) \leq x \leq f_1^\theta(k)$ and $k \leq k_0^\theta$.
VI. (M, M) if $f_1^\theta(k) \leq x$ and $k \leq k_1^\theta$.

Proof. See Appendix A

In Appendix B, it is showed that the conditions that support the different types of equilibria that appear in Proposition 1 also guarantee the non-negativity of profits.

Fig. 2 identifies the different parameter ranges (or "sectors") that are compatible with each equilibrium.[12] The win–win sector where both firms stay and invest in green technology is identified now as sector IV, where the investment cost x is low enough. In sectors III and V, x is such that only one firm invests and, in sectors I, II and VI, the investment cost is high enough to persuade both firms from investing. Finally, there is a region (VII) that is compatible with two different equilibria (see Corollary 1 below). Let us now have a closer look at each specific sector, starting with sector I and moving clockwise. Notice that, along this movement, there is always an asymmetric equilibrium between each pair of symmetric equilibria.

In sector I, along with a high investment cost, the relocation cost has an intermediate value which is high enough to persuade one of the firms, but not both, to relocate. The result is that one firm finds it profitable to move to the pollution haven (M), while the second prefers

[10] Since we do not model consumers' willingness to pay for a good produced with a greener technology, and investment does not reduce cost, but only emissions, our model cannot give support to the strong version of the PH, only to weak version. Therefore, unless otherwise stated, in the sequel, when we refer to the PH, we always mean the weak version.

[11] All these threshold values and auxiliary functions are calculated in Appendix A.

[12] The relationship between the different sectors is determined by the conditions of Proposition 1, but for determining their position and size we have used a numerical example respecting the conditions that τ and θ must satisfy. The same approach has been used for the rest of graphical representations of the paper.

to stay with the BAU technology (B) and bear the tax. In this sector (M, M) cannot be an equilibrium because, provided $\theta > \tau$, if a second firms moves to the pollution haven, the marginal cost of the industry would be lower, which would result in a reduction of the market price that would not allow the second firm to cover the relocation cost.

In sector II, we find a symmetric equilibrium where both firms stay at the home country with the BAU technology, since x is high enough to discourage investment (specifically, $x > x_1^{\theta}$) and k is high enough ($k > k_2^{\theta}$) to discourage relocation. Relocation is also discarded in sectors III (because k is high enough) and IV (because G becomes more attractive than M due to the low investment cost).

In sector III, both firms stay in the home country and the investment cost x is low enough to induce one of the firms, but not both to invest in green technology. The reason is similar to the one provided for sector I. If one firm is already investing, and the second one also does, no firm would pay the tax and the marginal cost of the industry would decrease, so that the revenue would not be enough to compensate for the additional cost associated to the investment. Sector IV is the "win–win" situation discussed above and illustrated in Fig. 1.

In sectors V and VI the investment cost is high enough to discourage one (sector V) or both firms (sector VI) from investing and the relocation cost is low enough so as to encourage one (sector V) or both firms (sector VI) to move to the foreign country. As a consequence, we find as SPNE (M, G) and (M, M) respectively.

Fig. 2 also shows that sector III intersects with sector VI, giving rise to a region with two equilibria, as stated in the corollary below. In the figure, we have called this intersection sector VII.

Corollary 1. *For any parameter combination satisfying Assumption 1 with $\tau < \theta$, $x \in [f_1^{\theta}(k), f_2^{\theta}(k)]$ and $k \in [k_0^{\theta}, k_1^{\theta}]$, the game has two equilibria: one asymmetric, with the two firms operating in the home country, but one investing in the green technology and the other paying the tax, and one symmetric with the two firms moving to the foreign country.*

Thus, for intermediate values of the investment cost and the relocation cost, firms could both stay at home or move to the pollution haven. However, if they decide to stay at home, they take different actions in equilibrium. Interestingly, we observe that in a model with symmetric firms if we give firms the possibility of taking different actions about location and investment, the model yields not only asymmetric solutions in terms of the decisions taken by firms, but also in some cases multiplicity of equilibria. Actually, as we have defined the SPNE using weak inequalities, all (k, x)-combinations in the borders separating the different sectors of Fig. 2 also support two equilibria. In other words, in the edges, where these inequalities hold with equality, two equilibria exist. For example, (G, G) and (B, G) are both equilibria for $x = x_0^{\theta}$ provided that $k \geq k_0^{\theta}$. However, in the sequel, we will refer to the multiplicity of equilibria to cases when multiple equilibria arise despite of strict inequalities, setting aside conditions of equality.

In order to provide further insight about the impact of the environmental policy on firms' decision on investment and location, we briefly discuss on the comparative statics of an increase in θ and a decrease in τ on firms' investment and location decisions. Qualitatively speaking, a tax increase and a transportation cost decrease have in common that both changes tend to make relocation more attractive. According to the comparative statics analysis developed in Appendix C, the actual impact of changing θ or τ depends on the specific parameter combination. If k is low enough to make relocation attractive enough versus the option of staying with the brown technology (left part of Fig. 2), the firms just chose between actions M and G. Under these circumstances, a change (upwards or downwards) of the tax rate is innocuous because firms are not paying the tax anyway, while a decrease in the transportation cost can induce firms to relocate instead of staying and investing, which would favor equilibria more aligned with the PHH. On the other hand, if k optimal alternative (right part of Fig. 2), the transportation cost is immaterial, because firms will stay in the home country anyway and the PHH will not arise. In this range, an increase in the tax rate can clearly incentivize investment and result in equilibria consistent with the PH.

Table 4
Firms output in equilibrium under a standard $\bar{e}$.

Firm i/Firm j	G (green)	B (BAU)	M (move)
G (green)	$\frac{s}{3}, \frac{s}{3}$	$\frac{s-\bar{e}}{2}, \bar{e}$	$\frac{s+\tau}{3}, \frac{s-2\tau}{3}$
B (BAU)	$\bar{e}, \frac{s-\bar{e}}{2}$	$\bar{e}, \bar{e}$	$\bar{e}, \frac{s-\tau-\bar{e}}{2}$
M (move)	$\frac{s-2\tau}{3}, \frac{s+\tau}{3}$	$\frac{s-\tau-\bar{e}}{2}, \bar{e}$	$\frac{s-\tau}{3}, \frac{s-\tau}{3}$

4. The equilibrium of the game with an emission standard

In this section we consider that the regulator, instead of a tax, imposes an emission standard $\bar{e} > 0$, such that $e^i \leq \bar{e}$ must hold for firms staying in the home country. Clearly, this legal requirement is trivially met by any firm producing with the green technology, which generates no emissions at all, and firms located outside the home country are not subject to it. For firms staying in the home country with the BAU technology, restricting emissions is equivalent to restricting output, but even in this case, the standard is binding if it has a lower value than the laissez-faire production level, $s/3$. We assume that this is always the case, i.e. $\bar{e} < s/3$, in order to avoid uninteresting situation in which the environmental policy is innocuous.

The model is still a two-stage game with firms simultaneously deciding between $\{G, B, M\}$ in the first stage and competing in quantities (*à la Cournot*) in the second stage. As above, the set of three qualitative actions by firms gives rise to nine market configurations, although only six (three symmetric and three asymmetric) are genuinely different. Again, we look for the SPNE's of the game starting by the equilibrium quantities for each market configuration.

4.1. Second stage: Production and pollution

Table 4 shows the Cournot equilibrium quantities for each market configuration, in the same fashion as in Table 1.

As shown in the table, given that the standard is assumed to be binding, any firm choosing action B will produce exactly $\bar{e}$ in equilibrium, whatever the rival's action in stage I. On the other hand, a firm (say i), which is not subject to the standard (because it chooses action G or M) and competes with a firm (say $j \neq i$) that is subject to it, will behave according to its reaction function, taking $q^j = \bar{e}$ as given. Specifically, $q_{GB}^i = \frac{s-\bar{e}}{2}$ and $q_{MB}^i = \frac{s-\tau-\bar{e}}{2}$. Trivially, we always have $q_{GB}^i > q_{BG}^i$, i.e., a firm producing with the green technology will always produce a larger amount of output than a rival that has played B, and is therefore constrained by the standard. On the other hand, when one firm has relocated (M) and the other has remained at home with the brown technology (B), we have $q_{MB}^i \geq q_{BM}^i$ if $\bar{e} \leq \frac{s-\tau}{3}$, i.e., the M-firm will produce more than the B-firm if the standard is tight enough. In those scenarios in which the standard is not involved (no firm is choosing action B), the equilibrium quantities are the same as in the tax case (Table 1). Inspecting Table 4, it is easy to check that if $\bar{e} \in (0, s/3)$ and $\tau \in (0, s/2)$ all equilibrium quantities displayed in this table are strictly positive. Before solving the first stage of the game, we want to make sure that the results under the standard are comparable to those under the tax (as we analyze in detail in Section 5). In the literature, a common way to make two environmental policy instruments comparable is to bound them to generate the same level of emissions.[13] Thus, for the sake comparison, we take the BAU market configuration, i.e., (B, B), as a reference point, and consider the amount of pollution that each firm would emit under a tax rate θ. Accordingly,

[13] For instance, Montero (2002) compares the environmental R&D incentives offered by four policy instruments in a Cournot duopoly assuming an exogenously given cap on aggregate emissions.

Table 5
Total output and pollution under a standard $\bar{e}$.

Firm i/Firm j	G (green)	B (BAU)	M (move)
G (green)	$\frac{2s}{3}, 0$	$\frac{s+\bar{e}}{2}, \bar{e}$	$\frac{2s-\tau}{3}, \frac{s-2\tau}{3}$
B (BAU)	$\frac{s+\bar{e}}{2}, \bar{e}$	$2\bar{e}, 2\bar{e}$	$\frac{s-\tau+\bar{e}}{2}, \frac{s-\tau+\bar{e}}{2}$
M (move)	$\frac{2s-\tau}{3}, \frac{s-2\tau}{3}$	$\frac{s-\tau+\bar{e}}{2}, \frac{s-\tau+\bar{e}}{2}$	$\frac{2(s-\tau)}{3}, \frac{2(s-\tau)}{3}$

Table 6
Firms profit in equilibrium under a standard $\bar{e}$.

Firm i/Firm j	G (green)	B (BAU)	M (move)
G (green)	$\frac{s^2}{9} - x; \frac{s^2}{9} - x$	$\frac{(s-\bar{e})^2}{4} - x; \frac{(s-\bar{e})\bar{e}}{2}$	$\frac{(s+\tau)^2}{9} - x; \frac{(s-2\tau)^2}{9} - k$
B (BAU)	$\frac{(s-\bar{e})\bar{e}}{2}; \frac{(s-\bar{e})^2}{4} - x$	$\bar{e}(s-2\bar{e}); \bar{e}(s-2\bar{e})$	$\frac{\bar{e}(s-\bar{e}+\tau)}{2}; \frac{(s-\bar{e}-\tau)^2}{4} - k$
M (move)	$\frac{(s-2\tau)^2}{9} - k; \frac{(s+\tau)^2}{9} - x$	$\frac{(s-\bar{e}-\tau)^2}{4} - k; \frac{\bar{e}(s-\bar{e}+\tau)}{2}$	$\frac{(s-\tau)^2}{9} - k; \frac{(s-\tau)^2}{9} - k$

when it comes to the policy comparison, we assume that the standard $\bar{e}$ is related to the tax rate θ in the following way:

$$\bar{e} = \frac{s-\theta}{3}. \tag{6}$$

In this case, the emissions under the standard are the same as those under the tax if both firms continue using purple the BAU technology at the home country when the environmental policy is applied. In the sequel we will refer to a standard $\bar{e}$ equivalent to a tax θ (and viceversa) when the relationship (6) is satisfied.

Assumption 1 imposes $\theta \in (0, s/2)$, which using (6), can be expressed in terms of the standard as $\bar{e} \in (s/6, s/3)$.[14] Even after applying these constraints, the parameter ranges that give rise to each equilibrium under the standard can adopt different configurations and, in some cases, the comparison with the tax is cumbersome. For this reason, we focus in the rest of the paper on a specific range of values for $\bar{e}$ that make the comparison more intuitive. To this end, we further restrict the standard and the transportation cost as specified in Assumption 2, which, as it will be shown later on in this section, results in a configuration of equilibria under the standard qualitatively similar to the one under the tax displayed in Fig. 2.

Assumption 2. $\tau \in (0, 0.08s)$ and $\bar{e} \in (s/6, e_1)$, where

$$e_1 = \frac{3(2s-\tau) - \sqrt{20s\tau + 37\tau^2 + s^2}}{21} < \frac{s}{3}. \tag{7}$$

Using (6), the standard range defined by this assumption can be translated into a tax rate in the interval $(\theta_1, s/2)$, where[15]

$$\theta_1 = \frac{s + 3\tau + \sqrt{20s\tau + 37\tau^2 + s^2}}{7}. \tag{8}$$

Thus, in this section we calculate the first stage of the game under Assumption 2 and in the next section we will develop the comparative analysis assuming that $\theta \in (\theta_1, s/2)$ and $\tau \in (0, 0.08s)$. These assumptions greatly simplify the comparison between the two environmental policies.

Table 5 displays total output and total emissions in equilibrium for each scenario, where total output is computed adding the individual quantities from Table 4 and emissions are calculated using the fact that firms producing with the green technology do not pollute at all.

As in the tax case, we consider the (G, G) market configuration a "win–win" outcome, in the sense that it yields zero emissions with both firms staying in the home country, which, apart, from the most desirable situation from the environmental point of view, also provides the maximum output, the minimum price and, thus, the maximum consumer surplus. In order to study under which conditions this and the rest of market configurations emerge in a SPNE, we first need to compute the equilibrium profits for each market configuration. The results are displayed in Table 6.

4.2. First stage: Location and investment decisions

Next, we calculate the equilibria of the game using the payoff matrix displayed in Table 6. We will show that all market configurations can be an equilibrium for some parameter combinations. Nevertheless, we will show that there are also some combinations for which there is no equilibrium. We begin studying under what conditions the "win–win" solution can be a SPNE, which requires $\pi^i_{GG} \geq \pi^i_{BG}$ and $\pi^i_{GG} \geq \pi^i_{MG}$ for $i = 1, 2$. These two inequalities are satisfied if the following condition holds[16]

$$x \leq \min\left\{x_0^{\bar{e}},\ f_0^{\bar{e}}(k)\right\}, \tag{9}$$

$$\text{where } x_0^{\bar{e}} \equiv \frac{(2s-3\bar{e})(s-3\bar{e})}{18}, \quad f_0^{\bar{e}}(k) \equiv \frac{4\tau(s-\tau)}{9} + k = f_0^{\theta}(k), \tag{10}$$

which has a similar interpretation as condition (1) in the tax scenario: if the investment cost is low enough, the "win–win" solution (G, G) is the SPNE of the game, where the meaning of "low enough" depends of the value of the parameters. As also occurs when a tax is applied, if $x_0^{\bar{e}} < f_0^{\bar{e}}(k)$ for all $k \geq 0$, the green-green equilibrium is only determined by the investment cost. Here, we focus on the more interesting case where firms' decision to invest in the green technology also depends on the relocation cost as we did in the previous section. This will occur if $x_0^{\bar{e}} = f_0^{\bar{e}}(k)$ yields a positive value for k that we designate by $k_0^{\bar{e}}$ and is given by

$$k_0^{\bar{e}} \equiv \frac{9\bar{e}^2 - 9s\bar{e} + 2(s-2\tau)^2}{18}. \tag{11}$$

It is easy to show that $k_0^{\bar{e}}$ is positive if Assumption 2 holds. Then, we can draw Fig. 3 where all (k, x)-combinations for given values of $\bar{e}$ and τ for which (G, G) is a SPNE of the game are represented when a standard is used to regulate emissions. The figure shows that when k is low enough, i.e. for $k < k_0^{\bar{e}}$, the firms may find relocation attractive enough, and will only choose action G when the investment cost x is low enough in terms of k.

Fig. 3 identifies the threshold value of k, such that the upper bound of x is equal to $f_0^{\bar{e}}(k)$ if $k \leq k_0^{\bar{e}}$, and equal to $x_0^{\bar{e}}$ if $k \geq k_0^{\bar{e}}$.

Notice that this figure is qualitatively similar to Fig. 1, but the value of $x_0^{\bar{e}}$ depends on the standard instead of the tax. Also, as it occurs in the tax scenario, there is a part of the "win–win" region that lies above the 45^o line, meaning that there is an area of (k, x)-combinations for which firms decide to invest in the green technology even when $x > k$, i.e., the investment cost is larger than the cost of moving to the foreign country. Nevertheless, despite these similarities, we will show in the next section that the two environmental policies are not completely equivalent.

In order to study under what conditions the other market configurations can arise in a SPNE, apart from $f_0^{\bar{e}}(k)$, which is defined in (10), we also define the following auxiliary functions:

$$f_1^{\bar{e}}(k) \equiv \frac{4s\tau}{9} + k, \qquad f_2^{\bar{e}}(k) \equiv \frac{\tau(2s-\tau-2\bar{e})}{4} + k, \tag{12}$$

[14] This condition is stronger than the one we need to guarantee that the equilibrium quantities for all market configurations are strictly positive when a standard is used as we have just seen. We could address the comparison between the two environmental policies assuming that $\bar{e} < s/6$, but in this case we would have corner solutions for some market configurations for the case of the tax. In this paper, we will assume that the environmental policy is not so demanding as to induce firms to leave the market if they decide to pay the tax. This requires to restrict the tax to take values in the interval $(0, s/2)$ according to Assumption 1 and consequently to restrict the equivalent standard to take values in the interval $(s/6, s/3)$..

[15] Note that $\tau < 0.08s$ implies $\tau < \theta_1$, and so $\theta \in (\theta_1, s/2)$ is consistent with $\theta > \tau$. Moreover, $\tau < 0.08s$ is also compatible with $\tau < s/2$ in Assumption 1.

[16] The superscript $\bar{e}$ denotes the scenario in which the regulator is using a standard to control emissions.

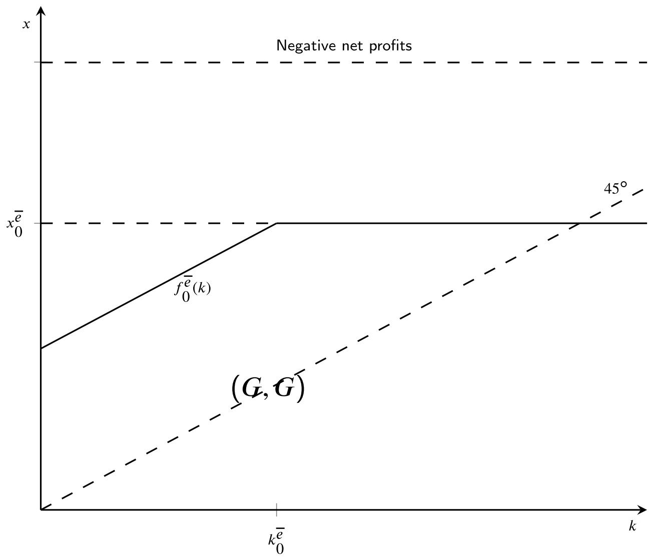

Fig. 3. Investment decisions as a function of k and x.

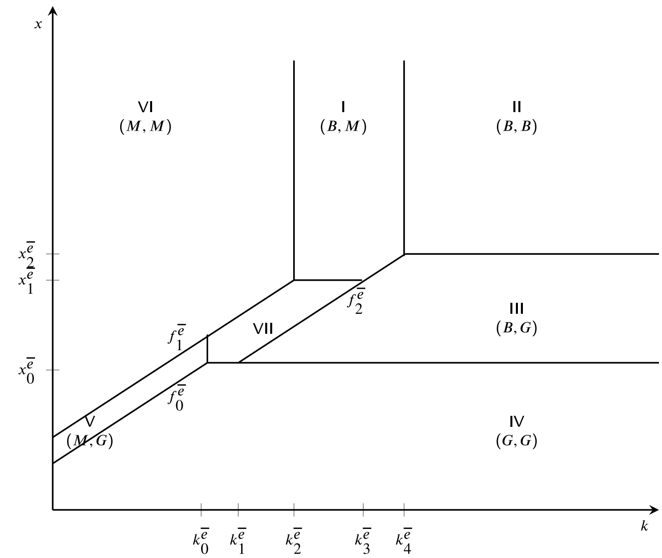

Fig. 4. Location and investment decisions for $\bar{e} < e_1$ and $\tau < 0.08s$.

where, provided $\bar{e} > s/6$, we have $f_2^{\bar{e}}(k) < f_0^{\bar{e}}(k) = f_0^{\theta}(k) < f_1^{\bar{e}}(k) = f_1^{\theta}(k)$. For convenience, in the sequel we will use the simplified notation $f_0(k)\left(= f_0^{\bar{e}}(k) = f_0^{\theta}(k)\right)$, and $f_1(k)\left(= f_0^{\bar{e}}(k) = f_0^{\theta}(k)\right)$.

Moreover, apart from $x_0^{\bar{e}}$ and $k_0^{\bar{e}}$, which are introduced in (10) and (11) respectively, we define the following thresholds for k and x:[17]

$$k_1^{\bar{e}} \equiv \frac{18\bar{e}^2 - 18(s-\tau)\bar{e} - 18s\tau + 9\tau^2 + 4s^2}{36} \qquad k_2^{\bar{e}} \equiv \frac{9\bar{e}^2 - 9(\tau+s)\bar{e} + 2(s-\tau)^2}{18}$$

$$k_3^{\bar{e}} \equiv \frac{18\bar{e}^2 - 18s\bar{e} - 10s\tau + 13\tau^2 + 4s^2}{36} \qquad k_4^{\bar{e}} \equiv \frac{9\bar{e}^2 - 2(3s-\tau)\bar{e} + (s-\tau)^2}{4} \tag{13}$$

$$x_1^{\bar{e}} \equiv \frac{9\bar{e}^2 - 9(\tau+s)\bar{e} + 2(s+\tau)^2}{18} \qquad x_2^{\bar{e}} \equiv \frac{(s-3\bar{e})^2}{4} \tag{14}$$

How these thresholds are ranked depend on the parameter values. We focus on a situation in which the thresholds are ranked in the same way as in the tax case, in order to make the analysis more comparable between both policy scenarios. In Appendix D is showed that if Assumption 2 holds, the thresholds for x and k satisfy the following inequalities:[18]

$$k_0^{\bar{e}} < k_1^{\bar{e}} < k_2^{\bar{e}} < k_3^{\bar{e}} < k_4^{\bar{e}}; \qquad x_0^{\bar{e}} < x_1^{\bar{e}} < x_2^{\bar{e}}. \tag{15}$$

The following proposition identifies the conditions for which each market configuration is a SPNE.

Proposition 2. *For any $(\bar{e}, \tau, s)$ satisfying Assumption 2, and provided that for each market configuration the firms produce their equilibrium quantity as in Table 4, the equilibrium in the first stage of the game is*

I. (B, M) if $x \geq \max\{x_1^{\bar{e}}, f_2^{\bar{e}}(k)\}$ and $k_2^{\bar{e}} \leq k \leq k_4^{\bar{e}}$.
II. (B, B) if $x \geq x_2^{\bar{e}}$ and $k \geq k_4^{\bar{e}}$.
III. (B, G) if $x_0^{\bar{e}} \leq x \leq \min\{x_2^{\bar{e}}, f_2^{\bar{e}}(k)\}$ and $k \geq k_1^{\bar{e}}$.
IV. (G, G) if $x \leq \min\{x_0^{\bar{e}}, f_0^{\bar{e}}(k)\}$.
V. (M, G) if $f_0^{\bar{e}}(k) \leq x \leq f_1^{\bar{e}}(k)$ and $k \leq k_0^{\bar{e}}$.
VI. (M, M) if $f_1^{\bar{e}}(k) \leq x$ and $k \leq k_2^{e}$.

Proof. See Appendix D

Moreover, in Appendix D, it is proved that the conditions that support the different types of equilibria that are shown in Proposition 2 also guarantee the non-negativity of profits.

Fig. 4 illustrates the profile of market configurations that arise in equilibrium under Assumption 2, which resembles Fig. 2 in the sense that the different equilibria are located, roughly, on the same positions of the map, specifically, the symmetric equilibria in sectors II, IV and VI, and the asymmetric equilibria in sectors I, III and V.

Although in Section 5 we present a detailed comparison between the equilibrium configuration under the tax and the standard and derive some policy implications, it is worth noticing here one qualitative difference between both policy scenarios. This difference is due to the position of the auxiliary function $f_2^{\bar{e}}(k)$ in the standard scenario, which is different from the position of $f_2^{\theta}(k)$ in the tax scenario. Indeed, it is the case that $f_2^{\theta}(k)$ lies above $f_1(k) > f_0(k)$ when a tax is applied, whereas $f_2^{\bar{e}}(k)$ lies below $f_0(k) < f_1(k)$ when the regulator uses a standard to control emissions. As a consequence of this divergence, there is a set of (k, x)-combinations for which there is no equilibrium. This result is formally stated in the following corollary:

Corollary 2. *For any $(\bar{e}, \tau, s)$ satisfying Assumption 2 and all (k, x) -combinations for which $\max\{x_0^{\bar{e}}, f_2^{\bar{e}}(k)\} \leq x \leq \min\{x_1^{\bar{e}}, f_1^{\bar{e}}(k)\}$ and $k \in [k_0^{\bar{e}}, k_3^{\bar{e}}]$, the game does not have a SPNE.*

In short, an equilibrium with the market configuration (M, M) -sector VI- requires $x \geq f_1(k)$, while (B, G) -sector III- requires $x \leq f_2^{\bar{e}}(k)$. In the tax case we have $f_2^{\bar{e}}(k) > f_1(k)$, meaning that sectors III and VI overlap, giving rise to a double equilibrium region. In the standard scenario, instead (under Assumption 2), $f_2^{\bar{e}}(k) < f_1(k)$, meaning that there is an "empty" region between sectors III and VI without equilibrium, where the parameter combinations do not satisfy the requirements to yield an equilibrium under any market configuration. Notice that, in both policy scenarios, the "problematic region" (double equilibrium under the tax and no equilibrium under the standard) arises for intermediate values of both k and x.

In terms of comparative statics, reducing the value of the standard $\bar{e}$ (i.e., making it tighter) has qualitatively similar implications as increasing the tax rate in the first policy scenario, in the sense of inducing firms to reduce further their output and make less profit. Such a policy movement makes the option of staying with the BAU technology (the only one that makes firms directly bear the burden of the environmental policy) less attractive. In terms of Fig. 4, the relevant thresholds will

[17] All these threshold values and auxiliary functions are calculated in Appendix D where the proof of Proposition 2 is presented.

[18] The upper bound e_1 in Assumption 2 is required to get that $x_1^{\bar{e}} < x_2^{\bar{e}}$ and $\tau < 0.08s$ is required to obtain that $k_0^{\bar{e}}$ is positive when $e < e_1$ as we have just pointed above.

displace in such a way that the (B,B) region shrinks, making more space for the remaining ones. Nevertheless, the implications in terms of the changes to obtain the PHH or the PH are not clear-cut and depend on the parameter values. In the most favorable scenario in which the investment cost is relatively low and the relocation cost relatively high, a more demanding policy clearly enhances the incentives for firms (one or both) to invest, providing support for the PH. In the opposite case (high x and low k) a tighter standard induces firms to leave the home country as predicted by the PHH. In the case in which both x and k are low enough to make option B irrelevant, the degree of tightness of the standard is immaterial. For more details about the comparative statics analysis refer to Appendix F.

5. Comparison between emission tax and standard. Policy implications

In this section we compare the two environmental policy instruments. Our main goal is to determine which one is more likely to be effective in inducing results compatible with the (weak version of the) PH. We do so by comparing the size of the parameter region that gives rise to this market configuration as an equilibrium of the game under both policy scenarios. As explained in the previous section, and in order to ease the comparison between both scenarios, we use the comparison pattern (6) and assume that $\theta \in (\theta_1, s/2)$ and $\tau \in (0, 0.08s)$ where θ_1 is given by (8) according to Assumption 2.

We begin the analysis comparing the size of the win–win region under both policy alternatives. According to Proposition 1, when an emission tax is used, the market equilibrium with the two firms investing in the green technology occurs for any k such that $x < \min\{x_0^\theta, f_0(k)\}$. However, when a standard is applied, according to Proposition 2, the market equilibrium with the two firms investing in the green technology occurs for any k such that $x < \min\{x_0^{\bar{e}}, f_0(k)\}$. Thus, any difference between both "win–win" regions must be due to differences between x_0^θ and $x_0^{\bar{e}}$. Using the comparison pattern (6) in (10) we can write $x_0^{\bar{e}}$ in terms of the tax rate as

$$x_0^{\bar{e}}\Big|_{\bar{e}=\frac{(s-\theta)}{3}} = \frac{(2s-3\bar{e})(s-3\bar{e})}{18} = \frac{1}{18}\left(2s - 3\frac{(s-\theta)}{3}\right)\left(s - 3\frac{(s-\theta)}{3}\right) = \frac{(s+\theta)\theta}{18}$$

and using (2) we get

$$x_0^{\bar{e}}\Big|_{\bar{e}=\frac{(s-\theta)}{3}} - x_0^\theta = \frac{(s+\theta)\theta}{18} - \frac{4\theta(s-\theta)}{9} = \frac{\theta(9\theta - 7s)}{18} < 0,$$

where the latter inequality derives from the fact that $\theta < s/2$.[19] Therefore, we have the following result:

Proposition 3. *Using the comparison pattern (6), and for any admissible values of the parameters under Assumption 2, given a pair (θ, s), the set of (k,x)-combinations for which a SPNE of the game involves the market configuration (G,G) is larger under an emission tax than under an emission standard.*

Fig. 5 illustrates this result. In the figure it can be seen that for all (k,x)-combinations for which the win–win solution arises in a SPNE of the game under the standard it will also arise under an equivalent tax. Conversely, there are (k,x)-combinations (corresponding to the shaded area in Fig. 5) such that the win–win solution arises under a tax and not under the equivalent standard. Thus, we find that the emission tax can induce firms to stay and invest in green technology under circumstances for which the standard cannot. This result is consistent with the narrow version of the PH, which states that price-based instruments outperform command-and-control policy instruments in terms of incentivizing green investment.

[19] And, obviously, assuming that we are in the "general" case in which k matters, this inequality also implies $k_0^\theta > k_0^{\bar{e}}$ (see Figs. 1 and 3).

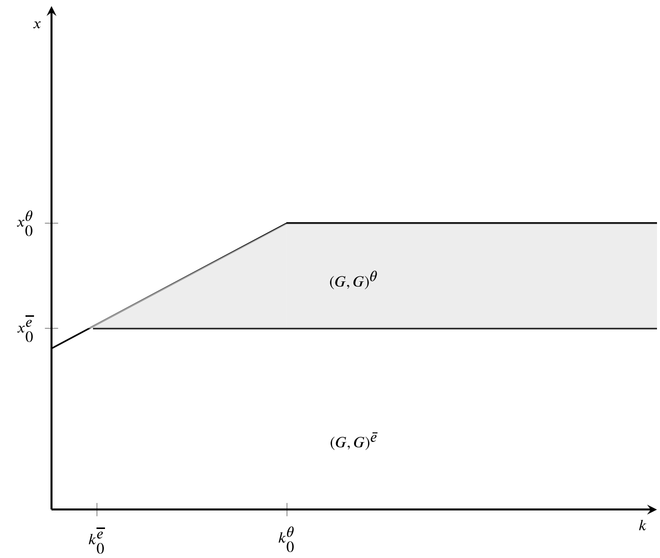

Fig. 5. Taxes vs standards for the green-green equilibrium.

For the sake of completeness, in what follows we compare the parameter regions that give rise to all market configurations under both policy scenarios. A full-fledged comparison would be extremely cumbersome and dependent on the parameter values. However, under Assumption 2 the characterization of equilibria under the standard have a similar shape than under the tax, which greatly facilities the comparison between both environmental policies as already noted in Section 4.1. The following Lemma establishes the order that Assumption 2 yields for all threshold values defined in Propositions 1 and 2.

Lemma 1. *Under Assumption 2, the threshold values for k and x for both environmental policy are ordered in the following way*

$$k_0^{\bar{e}} < k_1^{\bar{e}} < k_2^{\bar{e}} < k_3^{\bar{e}} < k_4^{\bar{e}} < k_0^\theta < k_1^\theta < k_2^\theta, \tag{16}$$

$$x_0^{\bar{e}} < x_1^{\bar{e}} < x_2^{\bar{e}} < x_0^\theta < x_1^\theta. \tag{17}$$

Proof. See Appendix G

Thus, taking into account that $f_2^{\bar{e}}(k) < f_0(k) < f_1(k) < f_2^\theta(x)$, we can display in the same graph the sectors that correspond to each market configuration under both policy scenarios.

The red lines delimit the equilibrium sectors in the tax policy scenario (as in Fig. 2) and the black ones the equilibrium sectors in the standard policy scenario (as in Fig. 4). The dashed red lines correspond to functions $f_0(k)$ and $f_1(k)$, which are common to both scenarios. As a result, Fig. 6 includes sixteen regions, and for each one we identify the prevailing market configuration in equilibrium under both policy scenarios.[20] In other words, for a given (k,x)-combination, the graph indicates what is the SPNE of the game depending of the policy instrument used by the regulator. In this way, we can localize when both policy instruments yield the same type of equilibrium or when they support a different one. Notice that in sector A three types of equilibria are possible because with the tax there is a duplicity of equilibria. However, in sectors F and H, only one type of equilibrium is possible, the one supported by the tax because with the standard there does not exist an equilibrium for this set of (k,x)-combinations.

[20] The superscripts θ and $\bar{e}$ are used to represent the equilibria under the tax and the standard policy scenarios respectively.

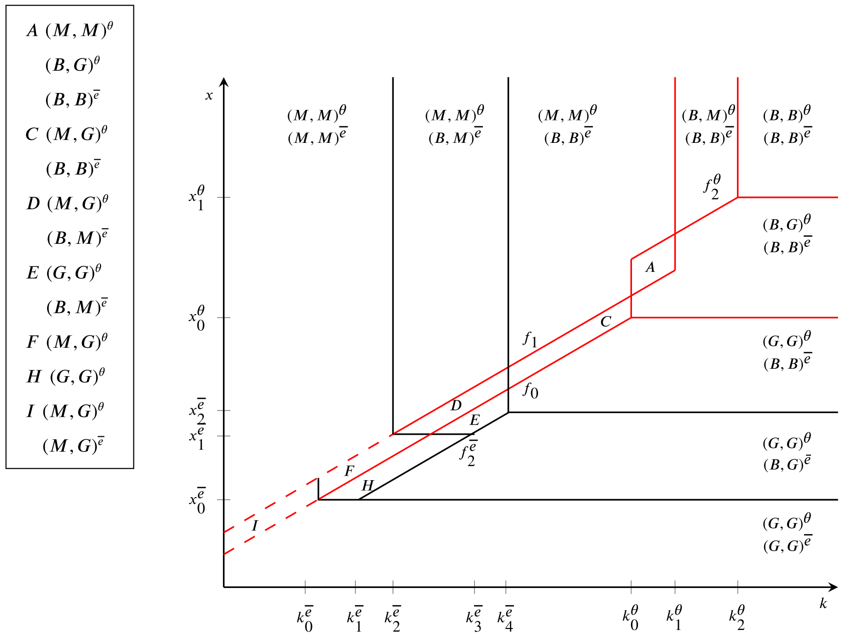

Fig. 6. Taxes vs standards.

From the figure we can get clear-cut conclusions about the relative size of sectors II, IV, V and VI under both scenarios, but not about sectors I and III.

Focusing on the symmetric equilibria, we have already stated that the (G,G) region is larger under the tax than under the standard. From Fig. 6 we confirm that this is also the case for the (M,M) region and it is the other way around -larger under the standard than under the tax- for the (B,B) region. We can get some interesting policy lessons from these results.

If we compare the (M,M) regions that give support to the PHH, we find that the tax can lead the two firms to relocate for parameter values for which the standard does not yield the relocation of the two firms. When an emission tax is used, according to Proposition 1, a market equilibrium with the two firms moving to the foreign country emerges for any value of $k < k_1^{\theta}$ provided that $x > f_1(k)$ as Figs. 2 and 6 show. However, when a standard is applied, according to Proposition 2, a market equilibrium with the two firms moving to the foreign country occurs for any value of $k < k_2^{\bar{e}}$ provided that $x > f_1(k)$ as can be checked in Figs. 4 and 6. Then, as (16) in Lemma 1 established that $k_2^{\bar{e}} < k_1^{\theta}$ we can conclude that

Proposition 4. *Under the comparison pattern (6), and for any admissible values of the parameters satisfying Assumption 2, the set of (k,x) combinations for which a SPNE of the game involves both firms relocating is larger under an emission tax than under an emission standard.*

These propositions suggest a trade-off in terms of policy design: while a tax is more effective than a standard to induce investment and generate results more consistent with the narrow version of the PH, it also increases the risk of pushing firms towards pollution havens as predicted by the PHH.

Proposition 3 compares sectors IV for which the (G,G) equilibrium emerges and Proposition 4 sectors VI for which the combination of actions (M,M) is the equilibrium of the game. To end the comparative analysis, we use our model to investigate the theoretical support that the model gives to the PH and the PHH considering that the investment cost x and the relocation cost k respectively, are high enough to avoid green investment in one case and relocation in the other case. Proceeding in this way we can focus, in the first case, on the study of the PHH and in the second case on the study of the PH. Fig. 6 shows that when the values of k and x are high, in particular if $k > k_2^{\theta}$ and $x > x_1^{\theta}$, firms decide to stay at the home country and use the BAU technology in one case paying the tax and in the other case producing at the level fixed by the standard. Then, if we consider a reduction on the investment cost for a constant level of the cost of relocating the plant in the foreign country higher than k_2^{θ}, we can evaluate how the firms' decisions to invest in the green technology are affected by the cost of the technology, but also by the policy instruments. We observe in Fig. 6 that for values of x between x_0^{θ} and x_1^{θ}, one of the firms will decide to invest in the clean technology with the tax, but not with the standard. Moreover, once x is lower than x_0^{θ} both firms invest in the clean technology with the tax, but not necessarily with the standard. In fact, for values of x between $x_2^{\bar{e}}$ and x_0^{θ} the green–green equilibrium is supported by the tax whereas for the standard firms still select the brown–brown equilibrium. If x is lower than $x_2^{\bar{e}}$ but higher than $x_0^{\bar{e}}$, one of the firm chooses to invest in the clean technology but still the other firm uses the brown technology if a standard is used to control emissions. Only, when x is lower than $x_0^{\bar{e}}$, we find that the green-green equilibrium is supported by the two policy instruments. The following proposition summarizes these results:

Proposition 5. *If $k > k_2^{\theta}$, no relocation occurs in equilibrium whatever the environmental policy. Then, for all $x < x_0^{\theta}$ both firms invest in green technology in equilibrium under a tax. If an equivalent standard is implemented instead, at least one firm will keep producing with the BAU technology if $x > x_0^{\bar{e}}$, and both firms if $x > x_2^{\bar{e}}$, where $x_0^{\theta} > x_2^{\bar{e}} > x_0^{\bar{e}}$.*

Thus, if relocation is avoided by a high cost of k, the analysis establishes that the PH works with an emission tax for values of the investment cost for which it does not work with an emission standard. Taxation would be a better option to promote green innovation than standards. This result is explained by the difference in profits that firms get depending on the policy instrument used by the regulator. The difference in profits affects the incentives that firms have to choose the

BAU technology when the other firm is selecting the green technology. The incentives of firm i to deviate from (G,G) to (B,G) when a tax is applied are:[21]

$$\pi^{i\theta}_{GG} - \pi^{i\theta}_{BG} = \frac{s^2}{9} - x - \frac{(s-2\theta)^2}{9} = x^{\theta}_0 - x,$$

where

$$\pi^{i\theta}_{GG} = \frac{s^2}{9} - x \text{ and } \pi^{i\theta}_{BG} = \frac{(s-2\theta)^2}{9}.$$

Observe that the incentives increase with the investment cost and that there is a threshold value for x, given by x^{θ}_0, such that if $x > x^{\theta}_0$, $\pi^{i}_{GG} < \pi^{i}_{BG}$ and (G,G) cannot be an equilibrium of the game. However, for a standard, the incentives are

$$\pi^{i\bar{e}}_{GG} - \pi^{i\bar{e}}_{BG} = \frac{s^2}{9} - x - \frac{(s-\bar{e})\bar{e}}{9} = x^{\bar{e}}_0 - x, \text{ where } \pi^{i\bar{e}}_{BG} = \frac{(s-\bar{e})\bar{e}}{9}.$$

The difference between the standard and the tax is that we have a different threshold value for the investment cost. In the proof of Proposition 3 we show that $x^{\bar{e}}_0 < x^{\theta}_0$, which implies that $\pi^{i\theta}_{BG} < \pi^{i\bar{e}}_{BG}$, i.e., the profits that firm i obtain when firms play (B,G) are lower when a tax is used to regulate emissions. The consequence is that $\pi^{i\theta}_{GG} - \pi^{i\theta}_{BG}$ is positive for $x \in [x^{\bar{e}}_0, x^{\theta}_0]$ whereas $\pi^{i\bar{e}}_{GG} - \pi^{i\bar{e}}_{BG}$ is negative, which explains that a tax leads firms to invest when a standard cannot do that. Tables 1 and 4 allows us to compare the quantities using the comparison criterion (6). We find that firm i produces more under a standard when firms choose (B,G), and total output is also higher. Moreover, firm i has to pay the tax. Nevertheless, the net effect is that profits are higher for firm i when a standard is applied. Thus, we have that for a standard firm i chooses the brown technology when the firm j selects the green technology if $x > x^{\bar{e}}_0$. However, to conclude that (B,G) is an equilibrium, we have to check that the firm j has no incentives to deviate from choosing the green technology. The incentives that firm j has to choose the green technology instead of using the BAU technology are given by

$$\pi^{j\bar{e}}_{GB} - \pi^{j\bar{e}}_{BB} = \frac{(s-\bar{e})^2}{4} - x - \bar{e}(s-2\bar{e}) = x^{\bar{e}}_2 - x,$$

where

$$\pi^{j\bar{e}}_{GB} = \frac{(s-\bar{e})^2}{4} - x \text{ and } \pi^{j\bar{e}}_{BB} = \bar{e}(s-2\bar{e}).$$

Then, for $x < x^{\bar{e}}_2$, $\pi^{j\bar{e}}_{GB} > \pi^{j\bar{e}}_{BB}$ and (B,G) is the equilibrium of the game. Lemma 1 establishes that $x^{\bar{e}}_0 < x^{\bar{e}}_2 < x^{\theta}_0$, which implies that for $x \in (x^{\bar{e}}_0, x^{\bar{e}}_2)$, (B,G) is the equilibrium of the game for a standard, but not for a tax. For a tax the equilibrium will be (G,G). Finally, when $x > x^{\bar{e}}_2$, firm j will find more profitable to use the BAU technology and the equilibrium will be (B,B) when a standard is used to regulate emissions, but still the equilibrium will be (G,G) for a tax.

Now, we look at a reduction in k for a constant value of x for values of $x > x^{\theta}_1$. In this case as Fig. 6 shows, once k is lower than k^{θ}_2 at least one firm decides to move to the foreign country if a tax is used to regulate emissions. However, with a standard this occurs for a lower value of k, for $k^{\bar{e}}_4$. In fact, for k lower than k^{θ}_1, already both firms relocate with a tax, whereas with a standard is needed a value of k lower than $k^{\bar{e}}_2$. The following proposition formalizes these results:

Proposition 6. *If $x > x^{\theta}_1$ no firm invests in equilibrium whatever the environmental policy. Then, for all $k > k^{\bar{e}}_4$, no relocation occurs under an emission standard if a given level of environmental protection is targeted using an emission standard, whereas if the same target is implemented with a tax, at least one firm relocates if $k < k^{\theta}_2$, and both firms relocate if $k < k^{\theta}_1$, where $k^{\bar{e}}_4 < k^{\theta}_1 < k^{\theta}_2$.*

Thus, if investment is avoided by a high investment cost, our results show that the PHH operates with an emission tax for values of the cost of relocation for which it does not operate with an emission standard. Standards would be a better option to avoid firms relocation than taxation. As occurred with Proposition 5, this result can be also explained by the difference in profits caused by the policy instruments. We follow the same kind of argument we provided for Proposition 4. The incentive of firm i to stay in the home country and choose the BAU technology when the other firm moves to the foreign country if a tax is applied is:[22]

$$\pi^{i\theta}_{MM} - \pi^{i\theta}_{BM} = \frac{(s-\tau)^2}{9} - k - \frac{(s-2\theta+\tau)^2}{9} = k^{\theta}_1 - k,$$

where

$$\pi^{i\theta}_{MM} = \frac{(s-\tau)^2}{9} - k \text{ and } \pi^{i\theta}_{BM} = \frac{(s-2\theta+\tau)^2}{9}.$$

In this case, the incentives increase with the relocation cost. The threshold value k^{θ}_1 defines when this difference is positive or negative. If $k > k^{\theta}_1$, $\pi^{i\theta}_{MM} < \pi^{i\theta}_{BM}$ and (M,M) cannot be an equilibrium of the game. However, for a standard, the difference in profits is:

$$\pi^{i\bar{e}}_{MM} - \pi^{i\bar{e}}_{BM} = \frac{(s-\tau)^2}{9} - k - \frac{\bar{e}(s-\bar{e}+\tau)}{2} = k^{\bar{e}}_2 - k \text{ where } \pi^{i\bar{e}}_{BM} = \frac{\bar{e}(s-\bar{e}+\tau)}{2}.$$

Comparing these two expressions, we see that with a standard the threshold value for the relocation cost is different. Lemma 1 establishes that $k^{\bar{e}}_2 < k^{\theta}_1$ so that we can conclude that $\pi^{i\theta}_{BM} < \pi^{i\bar{e}}_{BM}$, i.e., the profits that firm i obtains when firms choose (B,G) are lower when a tax is used to control emissions. The consequence is that $\pi^{i\theta}_{MM} - \pi^{i\theta}_{BM}$ is positive for $k \in (k^{\bar{e}}_2, k^{\theta}_1)$ whereas $\pi^{i\bar{e}}_{MM} - \pi^{i\bar{e}}_{BM}$ is negative. The standard induces firm i to stay at home using the BAU technology in this range of values of the relocation cost when the tax cannot avoid the relocation. Comparing the quantities using Tables 1 and 4 and the comparison criterion (6), we find that the output of firm i is higher when firms choose (B,M) and a standard is used that when a tax is applied. Total output is also higher. Nevertheless, the net effect is that profits are larger for firm i when a standard is applied. Thus, we obtain that firm i prefers to stay at home and use the BAU technology when firm j moves to the foreign country if $k > k^{\bar{e}}_2$ and a standard is used in the home country to control emissions. However, to conclude that (B,M) is an equilibrium, we need to check that firm j has no incentives to deviate from choosing M. The incentives that firm j has to move to the pollution haven instead of staying home and use BAU technology are given by

$$\pi^{j\bar{e}}_{MB} - \pi^{j\bar{e}}_{BB} = \frac{(s-\bar{e}-\tau)^2}{4} - k - \bar{e}(s-2\bar{e}) = k^{\bar{e}}_4 - k,$$

where

$$\pi^{j\bar{e}}_{MB} = \frac{(s-\bar{e}-\tau)^2}{4} - k \text{ and } \pi^{j\bar{e}}_{BB} = \bar{e}(s-2\bar{e}).$$

Then, for $k < k^{\bar{e}}_4$, $\pi^{j\bar{e}}_{MB} > \pi^{j\bar{e}}_{BB}$ and (B,M) is the equilibrium of the game. Lemma 1 says that $k^{\bar{e}}_2 < k^{\bar{e}}_4 < k^{\theta}_1$ what, implies that for $k \in (k^{\bar{e}}_2, k^{\bar{e}}_4)$, (B,M) is the equilibrium of the game for a standard, while, under a tax, it will be (M,M). Finally, when $k > k^{\bar{e}}_4$, firm j will find more profitable to stay at the home country and use the BAU technology and the equilibrium will be (B,B) when a standard is applied to control emissions, however still the equilibrium will be (M,M) for a tax.

Summarizing, when the ratio k/x is very low or very high or, when both k and x are extremely high, the two policy instruments yield the same equilibrium. This is represented in Fig. 6 by the intersection of sectors II, IV, V and VI corresponding to Figs. 2 and 4.[23] However, it seems that the emission tax is a better policy to promote the green

[21] We do not look at the incentives to deviate from (G,G) to (M,G) because for $k > k^{\theta}_2$ this is not a profitable action.

[22] We do not look at the incentives to deviate from (M,M) to (G,M) because for $x > x^{\theta}_1$ this is not a profitable action.

innovation, as has been established by Proposition 3, whereas an emission standard is better to avoid the relocation of firms as has been showed in Proposition 4. Finally, we could point out that for (k, x) -combinations for which there does not exist an equilibrium with the standard, sectors F and H in Fig. 6, the tax supports an equilibrium with at least one firm investing in the green technology.

6. Concluding remarks

This paper explores the impact on environmental policy on firms' decisions in an oligopoly setting where firms produce an homogeneous good and compete in quantities. We focus on decisions related to location and investment, in order to explore the implications regarding the weak version of the Porter Hypothesis (PH) and the Pollution Haven Hypothesis (PHH) under different policy scenarios. We compare the incentives introduced by an emission tax and a standard. Our investigation defines the conditions for which the "win–win" solution (i.e., the one fully consistent with the PH, in which both firms stay at the home country and invest in the green technology) is a subgame perfect Nash equilibrium of the game.

We conclude that a PH-like result is more prone to arise under a tax than under a standard. This type of result can hold in a seemingly unfavorable situation: a tax higher than the per unit transportation cost and an investment cost higher than the relocation costs. Moreover, under some conditions, the likelihood of achieving a win–win situation increases by tightening the environmental tax. But, on the other hand, the tax may induce firms to relocate, as predicted by the PHH, and this event is more likely to arise under a tax than under a standard. Relocation tends to happen if the investment cost is high as compared to the relocation cost. In this unfavorable situation, if the regulator aims to reduce emissions, she should opt for a standard to at least prevent firms from moving abroad.

There are several paths that could be followed to extend the analysis presented in this paper that point out avenues for future research. Instead of a duopoly producing a homogeneous good, we could consider a duopoly with product differentiation and compare Cournot and Bertrand competition as Buccella et al. (2021) propose. André et al. (2009) also consider product differentiation by allowing for green vs. brown goods. In our setting, consumers are not assumed to be environmentally concerned, and thus they do not change their consumption decisions as a response to firms' changes in their production technology. This implies that the strong version of PH cannot emerge by construction because investing in green technology can only alleviate the costs associated to policy compliance, but cannot generate induced benefits in the form of a price premium in the market. This limitation suggest an avenue for further investigation, which seems particularly relevant when one considers the current environmental challenges and concerns.

Another interesting extension would involve the consideration that green innovation affects the production costs. André et al. (2009), among others, assume that producing the high quality variant implies higher production costs. This would be a case to study, to assume that the green technology operates with a higher marginal cost, but our analysis suggests that although the set of combinations of parameter values for which the "win–win" solution will shrink, it would not be an empty set. Finally, following Ikefuji et al. (2016), we could introduce a third player of the game, the regulator, and solve a three-stage game with the regulator choosing the optimal environmental policy. In this setting, we could study how the regulator's degree of commitment could affect the location and investment decisions of the firms.

[23] The area for which (B, B) is the equilibrium for both policies is the intersection of sectors II in Figs. 2 and 4. The area for which (G, G) is the equilibrium for both policies is the intersection of sectors IV. The area for which (M, G) is the equilibrium for both policies is the intersection of sectors V. Finally, the area for which (M, M) is the equilibrium for both policies is the intersection of sectors VI in Figs. 2 and 4.

CRediT authorship contribution statement

F.J. André: Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Software, Resources, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization. **C. Ranocchia:** Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Software, Resources, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization. **S.J. Rubio:** Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Software, Resources, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization.

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Appendix A. Proof of Proposition 1

We proceed for this proof as we did in the main text to obtain (1) for the green-green equilibrium. Before moving to derive the conditions that characterize the other market configurations as an equilibrium, we define the threshold value k_0^θ as the value of k in the intersection $f_0^\theta(k_0^\theta) = x_0^\theta = 4\theta(s-\theta)/9$. This threshold value is $k_0^\theta = 4(\theta-\tau)(s-\theta-\tau)/9$, which is clearly positive for $\theta > \tau$ and $\theta, \tau \le s/2$. On the other hand, firm i moving to the foreign country and firm j investing in the green technology is an equilibrium in the first stage provided that for firm i is satisfied that

$$\pi_{MG}^i = \frac{(s-2\tau)^2}{9} - k \ge \pi_{BG}^i = \frac{(s-2\theta)^2}{9} \rightarrow k_0^\theta \ge k, \tag{A.18}$$

$$\pi_{MG}^i = \frac{(s-2\tau)^2}{9} - k \ge \pi_{GG}^i = \frac{s^2}{9} - x \rightarrow x \ge f_0^\theta(k), \tag{A.19}$$

and that for firm 2 is satisfied that

$$\pi_{GM}^i = \frac{(s+\tau)^2}{9} - x \ge \pi_{BM}^i = \frac{(s-2\theta+\tau)^2}{9} \rightarrow \frac{4}{9}\theta(s-\theta+\tau) \ge x, \tag{A.20}$$

$$\pi_{GM}^i = \frac{(s+\tau)^2}{9} - x \ge \pi_{MM}^i = \frac{(s-\tau)^2}{9} - k \rightarrow \ f_1^\theta(k) = \frac{4}{9}s\tau + k \ge x. \tag{A.21}$$

The same conditions would be obtained if firm j moves to the foreign country and firm i invests. Then, we would have an equilibrium when the following conditions hold

$$f_0^k \le x \le \min\left\{\frac{4}{9}\theta(s-\theta+\tau), f_1^\theta(k)\right\} \text{ and } k \le k_0^\theta. \tag{A.22}$$

Now, we have to show that these two conditions do not define an empty set of parameter values. It is straightforward that $f_0^\theta(k) < f_1^\theta(k)$ for all $k \ge 0$. Moreover, it is easy to show that the value of $f_1^\theta(k)$ for $k = k_0^\theta$ is lower than $4\theta(s-\theta+\tau)/9$ so that $4\theta(s-\theta+\tau)/9$ is higher than $f_1^\theta(k)$ for all $k \le k_0^\theta$ since $f_1(k)$ is an increasing function. Notice that $f_1^\theta(k_0^\theta) = 4((s-\theta)\theta+\tau^2)/9$ and the difference $4\theta(s-\theta+\tau)/9 - 4((s-\theta)\theta+\tau^2)/9 = 4\tau(\theta-\tau)$ is positive for $\theta > \tau$. Then the two conditions in (A.22) simplify to $f_0^k \le x \le f_1^\theta(k)$ and $k \le k_0^\theta$, which are the conditions that appear in Proposition 1, section V.

Both firms moving to the foreign country is an equilibrium in the first stage of the game provided that for firm i,

$$\pi^i_{MM} = \frac{(s-\tau)^2}{9} - k \ge \pi^i_{BM} = \frac{(s-2\theta+\tau)^2}{9} \to k_1^\theta = \frac{4}{9}(s-\theta)(\theta-\tau) \ge k, \tag{A.23}$$

and

$$\pi^i_{MM} = \frac{(s-\tau)^2}{9} - k \ge \pi^i_{GM} = \frac{(s+\tau)^2}{9} - x \to x \ge f_1^\theta(k). \tag{A.24}$$

The same conditions would be obtained for firm j. These two conditions are in Proposition 1, section VI. It is easy to check that $k_0^\theta < k_1^\theta$:

$$k_1^\theta - k_0^\theta = \frac{4}{9}(s-\theta)(\theta-\tau) - \frac{4}{9}(\theta-\tau)(s-\theta-\tau) = \frac{4}{9}\tau(\theta-\tau) > 0 \text{ for } \theta > \tau.$$

Firm i using the brown technology in the home country and firm j investing in the green technology is an equilibrium in the first stage provided that for firm i is satisfied that

$$\pi^i_{BG} = \frac{(s-2\theta)^2}{9} \ge \pi^i_{GG} = \frac{s^2}{9} - x \to x \ge x_0^\theta, \tag{A.25}$$

$$\pi^i_{BG} = \frac{(s-2\theta)^2}{9} \ge \pi^i_{MG} = \frac{(s-2\tau)^2}{9} - k \to k \ge k_0^\theta, \tag{A.26}$$

and that for firm 2 is satisfied that

$$\pi^i_{GB} = \frac{(s+\theta)^2}{9} - x \ge \pi^i_{BB} = \frac{(s-\theta)^2}{9} \to x_1^\theta = \frac{4}{9}s\theta \ge x, \tag{A.27}$$

$$\pi^i_{GB} = \frac{(s+\theta)^2}{9} - x \ge \pi^i_{MB} = \frac{(s+\theta-2\tau)^2}{9} - k \tag{A.28}$$

$$\to \quad f_2^\theta(k) = \frac{4}{9}\tau(s+\theta-\tau) \ge x. \tag{A.29}$$

These conditions can be summarized in the following ones

$$x_0^\theta = \frac{4\theta(s-\theta)}{9} \le x \le \min\left\{x_1^\theta, f_2^\theta(k)\right\}, \quad k \ge k_0^\theta. \tag{A.30}$$

The same conditions would be obtained if firm i invests and firm j does not invest and pays the tax. Now, we have to show that these conditions do not define an empty set of parameter values. For that first we calculate the intersection of $f_2^\theta(k)$ with x_1^θ. This intersection yields $k_2^\theta = 4(\theta-\tau)(s-\tau)/9$, a value that is higher than k_1^θ and consequently higher than k_0^θ :

$$k_2^\theta - k_1^\theta = \frac{4}{9}(\theta-\tau)(s-\tau) - \frac{4}{9}(s-\theta)(\theta-\tau) = \frac{4}{9}(\theta-\tau)^2 > 0.$$

Then, as $f_2^\theta(k)$ is an increasing function we can conclude that $f_2^\theta(k)$ is lower than x_1^θ for $k \in [k_0^\theta, k_2^\theta)$, it is equal to x_1^θ for $k = k_2^\theta$ and $f_2^\theta(k)$ is larger than x_1^θ for $k > k_2^\theta$. Finally, we should check that $f_2^\theta(k_0^\theta)$, the lowest value of $f_2^\theta(k)$, is greater than x_0^θ in (A.30). $f_2^\theta(k_0^\theta)$ is equal to $4\theta(s-\theta+\tau)/9$, which is clearly larger than x_0^θ. Thus, we can conclude that the two conditions that appear in (A.30) do not define an empty set for parameter values. The two conditions in (A.30) are those that appear in section III of Proposition 1.

Both firms staying at home and using the brown technology is an equilibrium of the first stage provided that for firm i,

$$\pi^i_{BB} = \frac{(s-\theta)^2}{9} \ge \pi^i_{GB} = \frac{(s+\theta)^2}{9} - x \to x \ge x_1^\theta, \tag{A.31}$$

and

$$\pi^i_{BB} = \frac{(s-\theta)^2}{9} \ge \pi^i_{MB} = \frac{(s+\theta-2\tau)^2}{9} - k \to k \ge k_2^\theta. \tag{A.32}$$

For firm j, we would derive the same conditions. These two conditions are the conditions that appear in section II of Proposition 1.

Firm i moving to the foreign country and firm j staying in the home country and using the brown technology is an equilibrium of the first stage provided that for firm i is satisfied that

$$\pi^i_{MB} = \frac{(s+\theta-2\tau)^2}{9} - k \ge \pi^i_{BB} = \frac{(s-\theta)^2}{9} \to k_2^\theta \ge k, \tag{A.33}$$

$$\pi^i_{MB} = \frac{(s+\theta-2\tau)^2}{9} - k \ge \pi^i_{GB} = \frac{(s+\theta)^2}{9} - x \to x \ge f_2^\theta(k), \tag{A.34}$$

and that for firm 2 is satisfied that

$$\pi^i_{BM} = \frac{(s-2\theta+\tau)^2}{9} \ge \pi^i_{GM} = \frac{(s+\tau)^2}{9} - x \to x \ge f_2^\theta(k_0^\theta), \tag{A.35}$$

$$\pi^i_{BM} = \frac{(s-2\theta+\tau)^2}{9} \ge \pi^i_{MM} = \frac{(s-\tau)^2}{9} - k \to k > k_1^\theta. \tag{A.36}$$

These conditions can be summarized in the following ones

$$\max\left\{f_2^\theta(k_0^\theta), f_2^\theta(k)\right\} \le x, \quad k_1^\theta \le k \le k_2^\theta. \tag{A.37}$$

The same conditions would be obtained if firm i stays at home and does not invest in the green technology and firm j moves to the foreign country. As $f_2^\theta(k)$ is an increasing function it is immediate that $f_2^\theta(k)$ is greater than $f_2^\theta(k_0^\theta)$ for $k \in [k_1^\theta, k_2^\theta]$ and the two conditions in (A.37) simplify to $f_2^\theta(k) \le x$ and $k \in [k_1^\theta, k_2^\theta]$, which are the conditions that appear in section I of Proposition 1. This completes the calculation of all conditions that can be found in Proposition 1 and characterize the different equilibria that the game can present. In this proof, we have obtained three threshold values for k:

$$k_0^\theta = \frac{4}{9}(\theta-\tau)(s-\theta-\tau) < k_1^\theta = \frac{4}{9}(\theta-\tau)(s-\theta) < k_2^\theta = \frac{4}{9}(\theta-\tau)(s-\tau),$$

two threshold values for x : $x_0^\theta < x_1^\theta$

$$x_0^\theta = \frac{4}{9}\theta(s-\theta) < x_1^\theta = \frac{4}{9}s\theta,$$

and three auxiliary functions:

$$f_0^\theta(k) = \frac{4\tau(s-\tau)}{9} + k < f_1^\theta(k) = \frac{4s\tau}{9} + k < f_2^\theta = \frac{4\tau(s+\theta-\tau)}{9} + k.$$

All the relationships we have just indicated hold for $\theta > \tau$ and are also presented in the main text. Finally, we would like to point out that we find that $f_2^\theta(k_0^\theta) = f_1^\theta(k_1^\theta)$, a property that has not played any role in the proof we have just concluded, but helps to draw Fig. 2.

Appendix B. Non-negativity constraint for profits with an emission tax

In this appendix we show that conditions in Proposition 1 guarantee that all profits in Table 2 are non negative when the corresponding market configuration is an equilibrium of the game. Observe that profits are always positive when k and x are not involved. For this reason, we look only at profits that include one of these two parameters.

For section I of Proposition 1, profits are positive if

$$\pi^i_{BM} = \frac{(s+\theta-2\tau)^2}{9} - k \ge 0 \Leftrightarrow k \le \frac{(s+\theta-2\tau)^2}{9}.$$

According to the proposition, one firm paying the tax and the other firm moving to foreign country is a SPNE provided that $k_1^\theta \le k \le k_2^\theta$. Comparing the two upper bounds for k we obtain that

$$k_2^\theta - \frac{(s+\theta-2\tau)^2}{9} = \frac{4}{9}(\theta-\tau)(s-\tau) - \frac{(s+\theta-2\tau)^2}{9} = -\frac{1}{9}(s-\theta)^2 < 0.$$

Thus, if $k \le k_2^\theta$ then $k < (s+\theta-2\tau)^2/9$ and profits are positive in this case. For section II when both firms paying the tax is a SPNE, no constraints on k and x apply.

For section III of Proposition 1, profits are positive if

$$\pi^i_{BG} = \frac{(s+\theta)^2}{9} - x \ge 0 \Leftrightarrow x \le \frac{(s+\theta)^2}{9}.$$

According to the proposition, one firm paying the tax and the other investing in the green technology is a SPNE provided that $x_0^\theta \le x \le \min\{x_1^\theta, f_2^\theta(k)\}$ and $k \ge k_0^\theta$. Comparing the two upper bounds on x yields

$$\frac{(s+\theta)^2}{9} - x_1^\theta = \frac{(s+\theta)^2}{9} - \frac{4s\theta}{9} = \frac{1}{9}(s-\theta)^2 > 0.$$

Thus, if the conditions for which this market configuration is an equilibrium are satisfied, profits are non-negative. Notice that $\min\{x_1^\theta, f_2^\theta(k)\} =$

x_1^θ for $k \geq k_2^\theta$. The non-negativity of profits when the two firms invest in the green technology has already been checked in the main text.

For section V, profits are positive if

$$\pi^i_{MG} = \frac{(s-2\tau)^2}{9} - k \geq 0 \Leftrightarrow k \leq \frac{(s-2\tau)^2}{9},$$

$$\pi^i_{GM} = \frac{(s+\tau)^2}{9} - x \Leftrightarrow x \leq \frac{(s+\tau)^2}{9}.$$

According to Proposition 1, one firm moving to foreign country and the other firm investing in the green technology is a SPNE if $f_0^\theta(k) \leq x \leq f_1^\theta(k)$ and $k \leq k_0^\theta$. Comparing first the two upper bounds on k, we obtain the following expression

$$\frac{(s-2\tau)^2}{9} - k_0^\theta = \frac{(s-2\tau)^2}{9} - \frac{4}{9}(\theta-\tau)(s-\theta-\tau) = \frac{1}{9}(s-2\theta)^2 > 0.$$

In this case, if $k \leq k_0^\theta$ then $k \leq (s-2\tau)^2/9$ and π^i_{MG} is positive. Now, we define $\tilde{x} = f_1^\theta(k_0^\theta)$ that is the maximum value that $f_1^\theta(k)$ can take for $k \leq k_0^\theta$. Remember that $f_0^\theta(k) < f_1^\theta(k)$ and that both are increasing functions.

$$\tilde{x} = \frac{4s\tau}{9} + \frac{4}{9}(\theta-\tau)(s-\theta-\tau) = \frac{4}{9}\left((s-\theta)\theta + \tau^2\right).$$

Comparing this threshold value for x with $(s+\tau)^2/9$ gives

$$\frac{(s+\tau)^2}{9} - \tilde{x} = \frac{(s+\tau)^2}{9} - \frac{4}{9}\left((s-\theta)\theta + \tau^2\right)$$

$$= \frac{1}{9}\left(-4(s-\theta)\theta + (2s-3\tau)\tau + s^2\right) > 0 \text{ for } \tau, \theta < \frac{s}{2}.$$

Notice that the second term is positive for $\tau < s/2$ and that the maximum absolute value of the first term is exactly s^2 for $\theta = s/2$ so that $-4(s-\theta)\theta + s^2$ is positive for $\theta < s/2$. Thus, if $x \leq f_1^\theta(k)$ and $k \leq k_0^\theta$ then $x \leq (s+\tau)^2/9$ and π^i_{GM} are positive.

For section VI, profits are positive if

$$\pi^i_{MM} = \frac{(s-\tau)^2}{9} - k \geq 0 \Leftrightarrow k \leq \frac{(s-\tau)^2}{9}.$$

According to Proposition 1, both firms moving to the foreign country is a SPNE provided that $k \leq k_1^\theta$. Comparing both upper bounds we obtain that

$$\frac{(s-\tau)^2}{9} - k_1^\theta = \frac{(s-\tau)^2}{9} - \frac{4}{9}(\theta-\tau)(s-\theta) = \frac{1}{9}(s-2\theta+\tau)^2 > 0.$$

In this case, if $k \leq k_1^\theta$ then $k < (s-\tau)^2/9$ and profits are positive. This completes the proof.

Appendix C. Comparative statics analysis with an emission tax

Figs. 1 and 2 in the main text illustrate the configuration of equilibria for each combination of x and k, given values of the tax rate θ and the transportation cost τ. In order to complete this analysis in this appendix we analyze the impact of θ and τ on firm's decisions.

Increasing the tax rate

We begin evaluating the effect of a variation in the tax rate over the relevant thresholds and the auxiliary functions used to characterize the equilibria. First, note that $f_0^\theta(k)$ and $f_1^\theta(k)$ do not depend on θ, while $\partial f_2^\theta/\partial\theta = 4\tau/9 > 0$, meaning that an increase in the tax rate will move up function $f_2^\theta(k)$. On the other hand, given $\tau < \theta < s/2$, we have

$$\frac{\partial k_0^\theta}{\partial\theta} = \frac{4}{9}(s-2\theta) > 0, \quad \frac{\partial k_1^\theta}{\partial\theta} = \frac{4}{9}(s-2\theta+\tau) > 0, \quad \frac{\partial k_2^\theta}{\partial\theta} = \frac{4}{9}(s-\tau) > 0,$$
$$\frac{\partial x_0^\theta}{\partial\theta} = \frac{4}{9}(s-2\theta) > 0, \quad \frac{\partial x_1^\theta}{\partial\theta} = \frac{4s}{9} > 0,$$

meaning that an increase in the tax rate augments the value of all the relevant threshold values of k and x.

These effects of increasing the tax rate are illustrated in Fig. 7, from which we can infer the implications on the size of the different sectors. Starting with the "win–win" solution, since x_0^θ increases and $f_0^\theta(k)$ does not change, sector IV expands. This means that, when the tax range increases, the range of (k, x) combinations that are compatible with both firms staying and investing gets larger, which we can interpret as the "win–win" solution becoming more likely to arise as an equilibrium. Nevertheless, it is worthwhile to stress that this effect is only relevant if the relocation cost is large enough so as to discourage relocation. In such a case, firms would stay anyway and the only relevant decision is between investing or not. If, on the other hand, the relocation cost is low enough, producing with the brown technology is discarded anyway and the relevant decision is between staying and investing or moving. In these circumstances, the tax rate is irrelevant for the firms' decision.

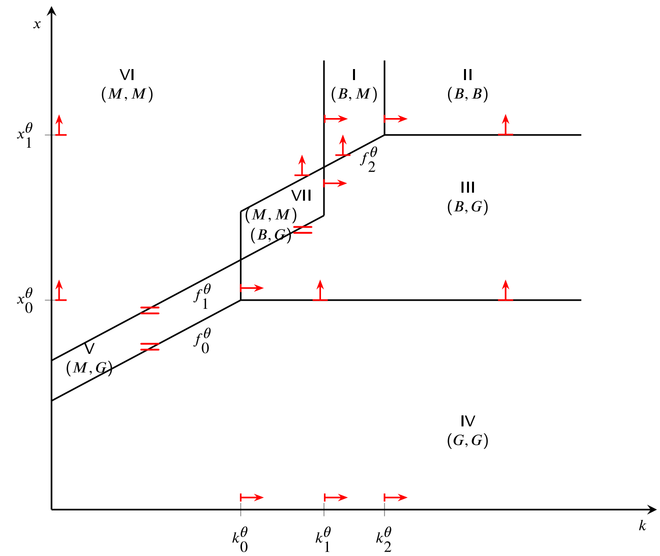

Fig. 7. Effects of an increase in the emission tax.

Regarding the rest of the sectors, we find that increasing θ makes sector II smaller and sectors V and VI larger (including in VI the part that overlaps with sector III), whereas the effect on the size of sectors I and III is undetermined. Notice that sectors V and VI involve at least one firm relocating, which means that an increase in the emission tax may encourage relocation. For instance, there are (k, x)-combinations that belong to sector I before increasing θ and to sector VI afterwards. In this region, the SPNE changes from (B, M) to (M, M), implying that both, instead of only one would relocate. There are also (k, x)-combinations that belong to sector III, with equilibrium (B, G), and after the increase in the tax are in sector V -(M, G)-, meaning that one firm producing brown will decide to relocate, or even to the multiple-equilibria sector VII, where both firms may simultaneously decide to move.

Summing up, the impact of increasing the tax rate on the location and investment decisions of the firms is heavily dependent on the parameter values. In some cases it is innocuous, mainly when x and k are very low or very high or, in general, when the tax increase is not large enough so as to make the prevailing parameter combination shift from one sector to another.[24] Under some circumstances (low enough investment cost and large relocation cost), an increase in the tax rate may increase the likelihood of firms staying and going green, but it may also encourage firms to relocate. Most likely, this moving incentive will affect firms that are currently producing with the BAU technology,

[24] When some (one or both) of the firms is playing B, even in an increase in tax rate is innocuous in terms of the qualitative decisions (location and investment), it will not be so in terms of the quantitative decision (output), and thus on prices, emissions, profit, consumers surplus and social welfare.

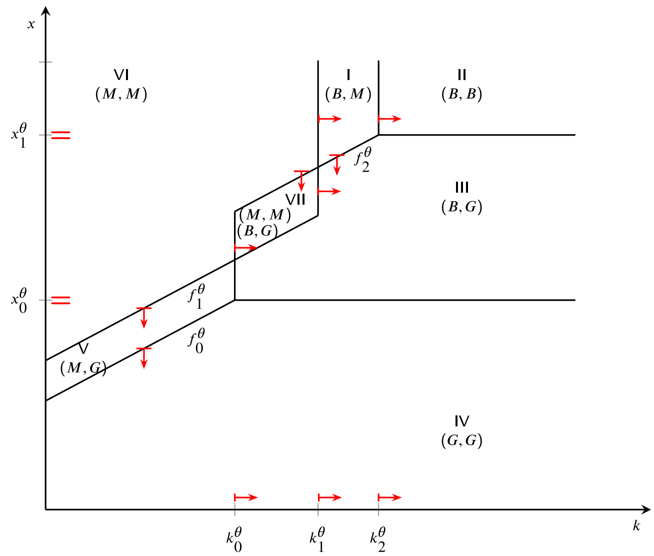

Fig. 8. Effects of a decrease in the transportation cost.

although under some conditions, it may also push out currently green firms.

Reducing the transportation cost

Next, we study the effect of a reduction in transportation costs on firms' decisions about location and investment. As we did with the tax, we first evaluate the effects on the relevant thresholds and auxiliary functions that delimit the different sectors. Specifically, we get

$$\frac{\partial f_0^k}{\partial \tau} = \frac{4}{9}(s-2\tau) > 0, \quad \frac{\partial f_1^k}{\partial \tau} = \frac{4s}{9} > 0, \quad \frac{\partial f_2^k}{\partial \tau} = \frac{4}{9}(s+\theta-2\tau) > 0,$$
$$\frac{\partial k_0^\theta}{\partial \tau} = -\frac{4}{9}(s-2\tau) < 0, \quad \frac{\partial k_1^\theta}{\partial \tau} = -\frac{4}{9}(s-\theta) < 0, \quad \frac{\partial k_2^\theta}{\partial \tau} = -\frac{4}{9}(s+\theta-2\tau) < 0,$$
$$\frac{\partial x_0^\theta}{\partial \tau} = \frac{\partial x_1^\theta}{\partial \tau} = 0,$$

i.e., when the transportation cost decreases, the auxiliary $f_t^\theta(k)$ functions ($t = 0, 1, 2$) shift downwards, all the relevant threshold values for k increase and the relevant thresholds for x remain unchanged. These effects are illustrated in Fig. 8, where we can study which are the effects of a decrease in τ on the characterization of all the equilibria of the game.

Since the x_0^θ threshold remains unchanged and the $f_0^k(k)$ auxiliary function shifts downwards, the "win–win" sector shrinks. As a consequence, there are (k, x)-combinations for which the SPNE initially involves the market configuration (G, G) -sector IV- and, after a decrease in τ, the prevailing market configuration in equilibrium is (M, G) -sector V- and it could be even (M, M), meaning that, in view of a lower transportation cost, ceteris paribus, some firms that originally found it profitable to remain in the home country and invest in green technology may change its decision to relocating. Notice that this event may only happen for situations in which $x > k$. The reason is that, for any admissible value of τ, $f_0^\theta(k)$ is always above the 45 ^{o} line, as illustrated in Fig. 1.[25] Therefore, for any combination in sector IV such that $x < k$, a reduction in τ will not affect the firms' decisions. An important implication of this fact is that the win–win region will never become empty due to a reduction in the transportation cost.

Regarding the rest of sectors identified in Proposition 1, we conclude that a decrease in τ makes sectors II, III and IV become smaller, the effect on the size of sectors I and V are undetermined and only sector VI becomes clearly bigger. In terms of policy implications, the market configuration in which both firms move to the pollution haven becomes more likely, meaning that, in terms of incentives for relocation, a reduction in the transportation cost is similar to an increase in the emission tax. Also, as it happens with the emission tax, the effect of τ on the equilibrium actions of the firms depends on the parameter values. For intermediate values of k and x, firms that were initially inclined to stay at home (either playing G or B) may decide to move in view of a lower transportation cost. On the other hand, when the k/x ratio is very low or very high or when both k and x are extremely high or extremely low the transportation cost may be innocuous in terms of location and investment decisions, although it will always have an effect on output when at least one of the firm operates in the foreign country.

[25] According to (1), $f_0^\theta(k) \geq x$ for any value of τ satisfying Assumption 1.

Appendix D. Proof of Proposition 2

We proceed with the proof as we did in the main text for deriving condition (9) for the green-green equilibrium. firm i moving to the foreign country and firm j investing in the green technology is an equilibrium in the first stage of the game provided that for firm i is satisfied that

$$\pi_{MG}^i = \frac{(s-2\tau)^2}{9} - k \geq \pi_{BG}^i = \frac{\bar{e}(s-\bar{e})}{2} \to k_0^e \geq k, \tag{D.1}$$
$$\pi_{MG}^i = \frac{(s-2\tau)^2}{9} - k \geq \pi_{GG}^i = \frac{s^2}{9} - x \to x \geq f_0^{\bar{e}}(k) = \frac{4\tau(s-\tau)}{9} + k, \tag{D.2}$$

and that for firm 2 is satisfied that

$$\pi_{GM}^i = \frac{(s+\tau)^2}{9} - x \geq \pi_{BM}^i = \frac{\bar{e}(s-\bar{e}+\tau)}{2} \to$$

$$x_1^{\bar{e}} = \frac{9\bar{e}^2 - 9(\tau+s)\bar{e} + 2(s+\tau)^2}{18} \geq x, \tag{D.3}$$

$$\pi_{GM}^i = \frac{(s+\tau)^2}{9} - x \geq \pi_{MM}^i = \frac{(s-\tau)^2}{9} - k \to f_1^{\bar{e}}(k) = \frac{4s\tau}{9} + k \geq x. \tag{D.4}$$

The same conditions would be obtained if firm j moves to the foreign country and firm i invests. Then, we would have an equilibrium of this type when the following conditions hold

$$f_0^{\bar{e}}(k) \leq x \leq \min\left\{x_1^{\bar{e}}, f_1^{\bar{e}}(k)\right\} \text{ and } k \leq k_0^{\bar{e}}. \tag{D.5}$$

Notice that $f_0^{\bar{e}}(k) = f_0^\theta(k)$ and $f_1^{\bar{e}}(k) = f_1^\theta(k)$. Now, we must show that these two conditions do not define an empty set of parameter values. We know from the analysis of the tax that $f_0^{\bar{e}}(k) < f_1^{\bar{e}}(k)$ for all $k \geq 0$. Next, we define $k_2^{\bar{e}}$ as the value of k that satisfies $f_1^{\bar{e}}(k_2^{\bar{e}}) = x_1^{\bar{e}}$,

$$k_2^{\bar{e}} = \frac{9\bar{e}^2 - 9(\tau+s)\bar{e} + 2(s-\tau)^2}{18}, \tag{D.6}$$

and show that $k_0^{\bar{e}} < k_2^{\bar{e}}$. The difference between these two threshold values is

$$k_2^{\bar{e}} - k_0^{\bar{e}} = \frac{9\bar{e}^2 - 9(\tau+s)\bar{e} + 2(s-\tau)^2}{18} - \frac{9\bar{e}^2 - 9s\bar{e} + 2(s-2\tau)^2}{18}$$
$$= -\frac{\tau(2(3\tau-2s) + 9\bar{e})}{18},$$

with $3\tau - 2s < 0$ for $\tau < 0.08s$. Then, we have a positive threshold value for $\bar{e}$, $\tilde{e} = 2(2s-3\tau)/9$ such that if $\bar{e} < \tilde{e}$, then $k_2^e > k_0^e$. But, we have established in Proposition 2 that $\bar{e} < e_1$ so that if $e_1 < \tilde{e}$ we could conclude that $k_2^e > k_0^e$. Suppose the contrary

$$e_1 = \frac{3(2s-\tau) - \left(20s\tau + 37\tau^2 + s^2\right)^{0.5}}{21} \geq \tilde{e} = \frac{2(2s-3\tau)}{9},$$

that implies that

$$-3\,(10s - 33\tau) - 9\left(20s\tau + 37\tau^2 + s^2\right)^{0.5} \geq 0,$$

which is a contradiction for $\tau < 0.08s$. Thus, we can conclude that $k_0^{\bar{e}} < k_2^{\bar{e}}$. In this case, we will have that $\min\left\{x_1^{\bar{e}}, f_1^{\bar{e}}(k)\right\} = f_1^{\bar{e}}(k)$ for $k \leq$

$k_0^{\bar{e}}$ since $f_1^{\bar{e}}(k)$ is an increasing function of k. Then the two conditions defined by (D.5) reduces to $f_0^{\bar{e}}(k) \le x \le f_1^{\bar{e}}(k)$ and $k \le k_0^{\bar{e}}$ that are the conditions that appear in section V of Proposition 2. Moreover, it is straightforward to establish that $x_0^{\bar{e}} < x_1^{\bar{e}}$ since $f_0^{\bar{e}}(k)$ and $f_1^{\bar{e}}(k)$ are two increasing functions with $f_0^{\bar{e}}(k) < f_1^{\bar{e}}(k)$ for all $k \ge 0$ and we have that $f_0^{\bar{e}}(k_0^{\bar{e}}) = x_0^{\bar{e}}$ and $f_1^{\bar{e}}(k_2^{\bar{e}}) = x_1^{\bar{e}}$ with $k_0^{\bar{e}} < k_2^{\bar{e}}$.

Both firms moving to the foreign country is an equilibrium in the first stage provided that for firm i

$$\pi^i_{MM} = \frac{(s-\tau)^2}{9} - k \ge \pi^i_{BM} = \frac{\bar{e}(s-\bar{e}+\tau)}{2} \rightarrow k_2^{\bar{e}} \ge k, \tag{D.7}$$

$$\pi^i_{MM} = \frac{(s-\tau)^2}{9} - k \ge \pi^i_{GM} = \frac{(s+\tau)^2}{9} - x \rightarrow x \ge f_1^{\bar{e}}(k). \tag{D.8}$$

The same conditions are obtained for firm j. These two conditions are in Proposition 2, section VI. Next, we compare $x_0^{\bar{e}}$ with $f_1^{\bar{e}}(0) = 4s\tau/9$. We know that x_0^e is decreasing with $\bar{e}$, so that the maximum value is reached when $\bar{e} = 0$ and this value is $x_0^{\bar{e}}(\bar{e}=0) = s^2/9$. Comparing it with $f_1^{\bar{e}}(0)$ yields

$$x_0^{\bar{e}}(\bar{e}=0) - f_1^{\bar{e}}(0) = \frac{1}{9}s(s-4\tau),$$

that is positive for $\tau < 0.08s$, and we can conclude that $x_0^{\bar{e}}$ is higher than $f_1^{\bar{e}}(0)$.

Firm i staying at home and using the brown technology and firm j investing in the green technology is an equilibrium in the first stage provided that for firm i is satisfied that

$$\pi^i_{BG} = \frac{\bar{e}(s-\bar{e})}{2} \ge \pi^i_{GG} = \frac{s^2}{9} - x \rightarrow x \ge x_0^{\bar{e}}, \tag{D.9}$$

$$\pi^i_{BG} = \frac{\bar{e}(s-\bar{e})}{2} \ge \pi^i_{MG} = \frac{(s-2\tau)^2}{9} - k \rightarrow k \ge k_0^{\bar{e}}, \tag{D.10}$$

and that for firm j the following conditions hold

$$\pi^i_{GB} = \frac{(s-\bar{e})^2}{4} - x \ge \pi^i_{BB} = (s-2\bar{e})\bar{e} \rightarrow x_2^{\bar{e}} = \frac{(s-3\bar{e})^2}{4} \ge x, \tag{D.11}$$

$$\pi^i_{GB} = \frac{(s-\bar{e})^2}{4} - x \ge \pi^i_{MB} = \frac{(s-\bar{e}-\tau)^2}{4} - k \rightarrow$$

$$f_2^{\bar{e}}(k) = \frac{\tau(2s-\tau-2\bar{e})}{4} + k \ge x. \tag{D.12}$$

These conditions can be summarized as follows

$$x_0^{\bar{e}} \le x \le \min\left\{x_2^{\bar{e}}, f_2^{\bar{e}}(k)\right\}, \ k_0^{\bar{e}} \le k. \tag{D.13}$$

The same conditions would be obtained if firm i invests and firm j decides to stay in the home country and use the brown technology. Now we have to check that these conditions do not define an empty set of parameter values. For this, we first investigate if $x_2^{\bar{e}}$ intersects $f_2^{\bar{e}}(k)$ for a positive value of k. Using (D.11) and (D.12) we obtain that $x_2^{\bar{e}}$ and $f_2^{\bar{e}}(k)$ cuts for

$$k_4^{\bar{e}} = \frac{9\bar{e}^2 - 2(3s-\tau)\bar{e} + (s-\tau)^2}{4}. \tag{D.14}$$

This threshold value is zero for the positive roots of the polynomial equation $9\bar{e}^2 - 2(3s-\tau)\bar{e} + (s-\tau)^2 = 0$ that are given by

$$\bar{e} = \frac{3s-\tau \pm 2(\tau(3s-2\tau))^{0.5}}{9},$$

and consequently it takes positive values for $\bar{e}$ lower than the lowest root and for values of $\bar{e}$ higher than the highest root. However, it is easy to check that e_1 is lower than the lowest root if $\tau < 0.08s$ so that we can conclude that under Assumption 2, $k_4^{\bar{e}} > 0$. Let us suppose that e_1 is higher than or equal to the lowest root that we call e_2. In this case,

$$e_1 = \frac{3(2s-\tau) - \left(20s\tau + 37\tau^2 + s^2\right)^{0.5}}{21} \ge e_2 = \frac{3s-\tau-2(\tau(3s-2\tau))^{0.5}}{9}.$$

Reordering terms and squaring in both sides of the inequality, we obtain the following expression

$$81\left(44s\tau - 81\tau^2 - 2s^2\right) \ge 54(3s+2\tau)\left(20s\tau + 37\tau^2 + s^2\right)^{0.5} > 0.$$

Squaring again in both sides of the inequality and reordering terms yields

$$-35\,721\tau\left(48s^3 - 368s^2\tau + 1352s\tau^2 - 1193\tau^3\right) \ge 0. \tag{D.15}$$

For $\tau = xs$, the expression between parenthesis is $-s^3 1193x^3 - 1352x^2 + 368x - 48$ where the polynomial equation $1193x^3 - 1352x^2 + 368x - 48 = 0$ has only one positive real root $x = 0.8155$ so that the polynomial in x takes negative values for $x < 0.8155$. This implies that if $\tau < 0.08s$, (D.15) gives a contradiction and we can claim that $e_1 > e_2$ that implies that $k_4^{\bar{e}} > 0$.

Next, we define the intersection point of $x_0^{\bar{e}}$ with $f_2^{\bar{e}}(k)$:

$$f_2^{\bar{e}}(k) - x_0^{\bar{e}} = \frac{\tau(2s-\tau-2\bar{e})}{4} + k - \frac{(2s-3\bar{e})(s-3\bar{e})}{18} = 0,$$

that yields

$$k_1^{\bar{e}} = \frac{18\bar{e}^2 - 18(s-\tau)\bar{e} - 18s\tau + 9\tau^2 + 4s^2}{36}. \tag{D.16}$$

Comparing k_2^e with k_1^e:

$$k_2^{\bar{e}} - k_1^{\bar{e}} = \frac{9\bar{e}^2 - 9(\tau+s)\bar{e} + 2(s-\tau)^2}{18} - \frac{18\bar{e}^2 - 18(s-\tau)\bar{e} - 18s\tau + 9\tau^2 + 4s^2}{36}$$

$$= \frac{\tau(5(2s-\tau) - 36\bar{e})}{36}. \tag{D.17}$$

For $\bar{e} = s/6$, the difference $k_2^e - k_1^e$ is positive provided that $\tau < 0.08s$. In this case, $k_2^e - k_1^e$ is positive for $\bar{e}$ between $s/6$ and the root of $5(2s-\tau) - 36\bar{e} = 0$. This root is

$$e_3 = \frac{5(2s-\tau)}{36}. \tag{D.18}$$

Is this root higher than or lower than e_1? Let us suppose that $e_1 \ge e_3$.

$$e_1 = \frac{3(2s-\tau) - \left(20s\tau + 37\tau^2 + s^2\right)^{0.5}}{21} \ge e_3 = \frac{5(2s-\tau)}{36},$$

that yields

$$6s - 3\tau \ge 36\left(20s\tau + 37\tau^2 + s^2\right)^{0.5} > 0,$$

squaring in both sides of the inequality and reordering terms we obtain that

$$-1260s^2 - 25\,956s\tau - 47\,943\tau^2 \ge 0,$$

resulting in a contradiction. The result is that $e_1 < e_3$ and then we can conclude that $k_2^{\bar{e}} - k_1^{\bar{e}}$ is positive for $\bar{e} \in (s/6, e_1)$.

Next, we compare $k_1^{\bar{e}}$ with $k_0^{\bar{e}}$

$$k_1^{\bar{e}} - k_0^{\bar{e}} = \frac{18\bar{e}^2 - 18(s-\tau)\bar{e} - 18s\tau + 9\tau^2 + 4s^2}{36} - \frac{9\bar{e}^2 - 9s\bar{e} + 2(s-2\tau)^2}{18},$$

$$k_1^{\bar{e}} - k_0^{\bar{e}} = \frac{\tau(18\bar{e} - 2s - 7\tau)}{36}. \tag{D.19}$$

This difference is positive for $\bar{e} = s/6$ if $\tau < 0.08s$. But, as $e_1 > s/6$ we have that $k_1^{\bar{e}} - k_0^{\bar{e}}$ is positive for $\bar{e} \in (s/6, e_1)$ since the difference $k_1^{\bar{e}} - k_0^{\bar{e}}$ is increasing with $\bar{e}$ and we can conclude that $f_2^{\bar{e}}(k) < f_0^{\bar{e}}(k)$ since $k_0^{\bar{e}} < k_1^{\bar{e}}$ and $f_2^{\bar{e}}(k_1^{\bar{e}}) = x_0^{\bar{e}}$ and $f_0^{\bar{e}}(k_0^{\bar{e}}) = x_0^{\bar{e}}$.

Next, we compare $x_1^{\bar{e}}$ with $x_2^{\bar{e}}$.

$$x_2^{\bar{e}} - x_1^{\bar{e}} = \frac{1}{4}(s-3\bar{e})^2 - \frac{9\bar{e}^2 - 9(\tau+s)\bar{e} + 2(s+\tau)^2}{18}$$

$$= \frac{63\bar{e}^2 - 18(2s-\tau)\bar{e} + (s-2\tau)(5s+2\tau)}{36}. \tag{D.20}$$

It is easy to show that this difference is positive for $\bar{e} = s/6$ for $\tau < 0.08s$. Thus, the difference will be positive for all $\bar{e}$ between $s/6$ and the lowest root of the numerator of (D.20). But, the lowest root is just e_1. Thus, if $\bar{e} \in (s/6, e_1)$, $x_2^{\bar{e}} > x_1^{\bar{e}}$. Then, as $f_2^{\bar{e}}(k_1^{\bar{e}}) = x_0^{\bar{e}}$ and $f_2^{\bar{e}}(k_4^{\bar{e}}) = x_2^{\bar{e}}$ we can conclude that $k_1^{\bar{e}} < k_4^{\bar{e}}$ since $x_2^{\bar{e}} > x_1^{\bar{e}} > x_0^{\bar{e}}$. Summarizing, we have established that $k_0^{\bar{e}} < k_1^{\bar{e}} < k_2^{\bar{e}}$, $k_1^{\bar{e}} < k_4^{\bar{e}}$, $x_0^{\bar{e}} < x_1^{\bar{e}} < x_2^{\bar{e}}$ and that $f_2^{\bar{e}}(k) < f_0^{\bar{e}}(k) < f_1^{\bar{e}}(k)$. Taking into account these results we obtain that $x_0^{\bar{e}} \leq x \leq \min\left\{x_2^{\bar{e}}, f_2^{\bar{e}}(k)\right\}$ in (D.13) is only satisfied when $k \geq k_1^{\bar{e}}$ that is Condition III in Proposition 2.

Both firms staying in the home country and using the brown technology is an equilibrium of the first stage provided that for firm i

$$\pi^i_{BB} = (s-2\bar{e})\bar{e} \geq \pi^i_{GB} = \frac{(s-\bar{e})^2}{4} - x \rightarrow x \geq x_2^{\bar{e}}, \tag{D.21}$$

$$\pi^i_{BB} = (s-2\bar{e})\bar{e} \geq \pi^i_{MB} = \frac{(s-\bar{e}-\tau)^2}{4} - k \rightarrow k \geq k_4^{\bar{e}}. \tag{D.22}$$

The same conditions are obtained for firm j. These two conditions are the conditions that appear in section II of Proposition 2.

Firm i moving to the foreign country and firm j staying in the home country and using the brown technology is an equilibrium provided that for firm i holds

$$\pi^i_{MB} = \frac{(s-\bar{e}-\tau)^2}{4} - k \geq \pi^i_{BB} = (s-2\bar{e})\bar{e} \rightarrow k_4^{\bar{e}} \geq k, \tag{D.23}$$

$$\pi^i_{MB} = \frac{(s-\bar{e}-\tau)^2}{4} - k \geq \pi^i_{GB} = \frac{(s-\bar{e})^2}{4} - x \rightarrow x \geq f_2^{\bar{e}}(k), \tag{D.24}$$

and for firm j it is satisfied that

$$\pi^i_{BM} = \frac{\bar{e}(s-\bar{e}+\tau)}{2} \geq \pi^i_{GM} = \frac{(s+\tau)^2}{9} - x \rightarrow x \geq x_1^{\bar{e}}, \tag{D.25}$$

$$\pi^i_{BM} = \frac{\bar{e}(s-\bar{e}+\tau)}{2} \geq \pi^i_{MM} = \frac{(s-\tau)^2}{9} - k \rightarrow k \geq k_2^{\bar{e}}. \tag{D.26}$$

These conditions can be summarized as follows

$$\max\left\{x_1^{\bar{e}}, f_2^{\bar{e}}(k)\right\} \leq x,\ k_2^{\bar{e}} \leq k \leq k_4^{\bar{e}}. \tag{D.27}$$

The same conditions would be obtained if firm j moves to the foreign country and firm i stays in the home country and uses the brown technology. Notice that as $x_0^{\bar{e}} < x_1^{\bar{e}} < x_2^{\bar{e}}$, $x_1^{\bar{e}} = f_2^{\bar{e}}(k)$ defines another threshold value for k:

$$x_1^{\bar{e}} = \frac{9\bar{e}^2 - 9(\tau+s)\bar{e} + 2(s+\tau)^2}{18} = \frac{\tau(2s-\tau-2\bar{e})}{4} + k = f_2^{\bar{e}}(k),$$

$$k_3^{\bar{e}} = \frac{18\bar{e}^2 - 18s\bar{e} - 10s\tau + 13\tau^2 + 4s^2}{36}. \tag{D.28}$$

It is easy to show that $k_3^{\bar{e}} > k_2^{\bar{e}}$ for $\bar{e} \in (s/6, e_1)$ and as $x_2^{\bar{e}} > x_1^{\bar{e}}$ we obtain that $k_3^{\bar{e}} < k_4^{\bar{e}}$, the result is that $k_0^{\bar{e}} < k_1^{\bar{e}} < k_2^{\bar{e}} < k_3^{\bar{e}} < k_4^{\bar{e}}$.

Conditions in (D.27) are the conditions that appear in section I of Proposition 2. This completes the calculation of all conditions that can be found in Proposition 2. In this proof, we have derived three auxiliary functions, two of them identical to the ones we obtained in the proof of Proposition 1, and established the relationship between them

$$f_2^{\bar{e}}(k) = \frac{\tau(2s-\tau-2\bar{e})}{4} + k < f_0^{\bar{e}}(k) = \frac{4\tau(s-\tau)}{9} + k < f_1^{\bar{e}}(k) = \frac{4s\tau}{9} + k.$$

We also find four threshold values for k and established the relationship between these values

$$k_0^{\bar{e}} = \frac{9\bar{e}^2 - 9s\bar{e} + 2(s-2\tau)^2}{18} < k_2^{e} = \frac{6\bar{e}^2 - 2(7s-9\tau)\bar{e} - 18s\tau + 9\tau^2 + 4s^2}{36}$$

$$< k_2^{\bar{e}} = \frac{9\bar{e}^2 - 9(\tau+s)\bar{e} + 2(s-\tau)^2}{18} < k_3^{\bar{e}} = \frac{18\bar{e}^2 - 18s\bar{e} - 10s\tau + 13\tau^2 + 4s^2}{36}$$

$$< k_4^{\bar{e}} = \frac{9\bar{e}^2 - 2(3s-\tau)\bar{e} + (s-\tau)^2}{4}$$

and three threshold values for x

$$x_0^{\bar{e}} = \frac{(2s-3\bar{e})(s-3\bar{e})}{18} < x_1^{\bar{e}} = \frac{9\bar{e}^2 - 9(\tau+s)\bar{e} + 2(s+\tau)^2}{18} < x_2^{\bar{e}} = \frac{(s-3\bar{e})^2}{4}.$$

Appendix E. Non-negativity constraint for profits with an emission standard

For section I of Proposition 2, profits are positive if

$$\pi^i_{BM} = \frac{(s-\bar{e}-\tau)^2}{4} - k \geq 0 \Leftrightarrow k \leq \frac{(s-\bar{e}-\tau)^2}{4}.$$

According to the proposition, one firm staying at the home country and using the brown technology and the other firm moving to the foreign country is a SPNE provided that $k_2^{\bar{e}} \leq k \leq k_4^{\bar{e}}$. Comparing the two upper bounds for k we obtain that

$$k_4^{\bar{e}} - \frac{(s-\bar{e}-\tau)^2}{4} = \frac{9\bar{e}^2 - 2(3s-\tau)\bar{e} + (s-\tau)^2}{4} - \frac{(s-\bar{e}-\tau)^2}{4}$$
$$= -(s-2\bar{e})\bar{e} < 0 \text{ for } \bar{e} < e_1.$$

Thus, if $k \leq k_4^{\bar{e}}$, then $k < (s-\bar{e}-\tau)^2/4$ and profits are positive in this case. For section II when both firms staying at the home country and using the brown technology is a SPNE, no constraints on k and x apply, and moreover $s - 2\bar{e} > 0$ for $\bar{e} < e_1$.

For section III of Proposition 2, profits are positive if

$$\pi^i_{BG} = \frac{(s-\bar{e})^2}{4} - x \geq 0 \Leftrightarrow x \leq \frac{(s-\bar{e})^2}{4}.$$

According to the proposition, one firm staying at the home country and using the brown technology and the other firm investing in the green technology is a SPNE provided that $x_0^{\bar{e}} \leq x \leq \min\{x_2^{\bar{e}}, f_2^{\bar{e}}(k)\}$. Comparing the two upper bounds on x yields

$$\frac{(s-\bar{e})^2}{4} - x_2^{e} = \frac{(s-\bar{e})^2}{4} - \frac{(s-3\bar{e})^2}{4} = \bar{e}(s-2\bar{e}) > 0 \text{ for } \bar{e} < e_1.$$

Thus, if the conditions for which this market configuration is an equilibrium are satisfied, profits are non-negative. Notice that $\min\{x_2^{\bar{e}}, f_2^{\bar{e}}(k)\} = x_2^{\bar{e}}$ for $k \geq k_4^{\bar{e}}$. The non-negativity of profits when the two firms invest in the green technology has already checked in the main text.

For section V, profits are positive if

$$\pi^i_{MG} = \frac{(s-2\tau)^2}{9} - k \geq 0 \Leftrightarrow k \leq \frac{(s-2\tau)^2}{9},$$

$$\pi^i_{GM} = \frac{(s+\tau)^2}{9} - x \geq 0 \Leftrightarrow x \leq \frac{(s+\tau)^2}{9}.$$

According to Proposition 2, one firm moving to country II and the other firm investing in the green technology is a SPNE if $f_0^{\bar{e}}(k) \leq x \leq f_1^{\bar{e}}(k)$ and $k \leq k_0^{\bar{e}}$. Comparing first the two upper bounds on k, we obtain the following expression

$$\frac{(s-2\tau)^2}{9} - k_0^{\bar{e}} = \frac{(s-2\tau)^2}{9} - \frac{9\bar{e}^2 - 9s\bar{e} + 2(s-2\tau)^2}{18}$$
$$= \frac{1}{2}(s-\bar{e})\bar{e} > 0 \text{ for } \bar{e} < e_1.$$

In this case, if $k \leq k_0^{\bar{e}}$, then $k \leq (s-2\tau)^2/9$ and π^i_{MG} is positive. Now, we define $\tilde{x} = f_1^{\bar{e}}(k_0^{\bar{e}})$ that is the maximum value that $f_1^{\bar{e}}(k)$ can take for $k \leq k_0^{\bar{e}}$. Remember that $f_0^{\bar{e}}(k) < f_1^{\bar{e}}(k)$ and that both functions are increasing.

$$\tilde{x} = \frac{4s\tau}{9} + \frac{9\bar{e}^2 - 9s\bar{e} + 2(s-2\tau)^2}{18} = \frac{9\bar{e}^2 - 9s\bar{e} + 8\tau^2 + 2s^2}{18}.$$

Comparing this threshold value for x with $(s+\tau)^2/9$ gives

$$\frac{(s+\tau)^2}{9} - \tilde{x} = \frac{(s+\tau)^2}{9} - \frac{9\bar{e}^2 - 9s\bar{e} + 8\tau^2 + 2s^2}{18}$$

$$= \frac{1}{18}(9(s-\bar{e})\bar{e} + 2(2s-3\tau)\tau) > 0 \text{ for } \bar{e} < e_1 \text{ and } \tau < 0.08s.$$

Thus, if $x \leq f_1^{\bar{e}}(k)$ and $k \leq k_0^{\bar{e}}$, then $x \leq (s+\tau)^2/9$ and π^i_{GM} are positive.

For section VI, profits are positive if

$$\pi^i_{MM} = \frac{(s-\tau)^2}{9} - k \Leftrightarrow k \leq \frac{(s-\tau)^2}{9}.$$

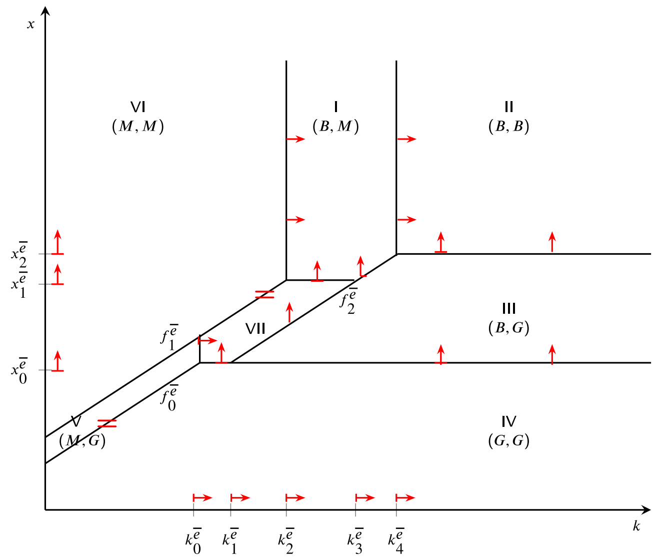

Fig. 9. Effects of a decrease in the emission standard.

According to Proposition 2, both firms moving to the foreign country is a SPNE provided that $k \le k_1^{\bar{e}}$. Comparing both upper bounds we obtain that

$$\frac{(s-\tau)^2}{9} - k_1^{\bar{e}} = \frac{(s-\tau)^2}{9} - \frac{9\bar{e}^2 - 9(s+\tau)\bar{e} + 2(s-\tau)^2}{18}$$
$$= \frac{1}{2}\left((s-\bar{e})\bar{e} + \tau\bar{e}\right) > 0,$$

for $\bar{e} < e_1$. In this case, if $k \le k_1^{\bar{e}}$, then $k < (s-\tau)^2/9$ and profits are positive. This completes the proof.

Appendix F. Comparative statics analysis for the standard

In this Appendix, we give a formal proof of the results on the comparative statics for the standard outlined in Section 4.2. In comparative terms, we stick to the effects of a stricter environmental regulation on firms' decision on investment and location. We begin evaluating the effect of a variation in the standard over auxiliary functions and the threshold values for k and x. As $f_0(k)$ and $f_1(k)$ do not depend on the standard, these functions will not change with a change in the standard. However, for $f_2^{\bar{e}}(k)$ we have that $\partial f_2^{\bar{e}}(x)/\partial \bar{e} = -\tau/2$, and a decrease in the standard will shift function $f_2^{\bar{e}}(x)$ upwards. On the other hand, under Assumption 2, the effects of the standard on the threshold values of k are:

$$\frac{\partial k_0^{\bar{e}}}{\partial \bar{e}} = -\frac{s-2\bar{e}}{2} < 0,\quad \frac{\partial k_1^{\bar{e}}}{\partial \bar{e}} = -\frac{s-\tau-2\bar{e}}{2} < 0,\quad \frac{\partial k_2^{\bar{e}}}{\partial \bar{e}} = \frac{2\bar{e}-(\tau+s)}{2} < 0,$$

$$\frac{\partial k_3^{\bar{e}}}{\partial \bar{e}} = -\frac{s-2\bar{e}}{2} < 0,\quad \frac{\partial k_4^{\bar{e}}}{\partial \bar{e}} = -\frac{3s-\tau-9\bar{e}}{2} < 0.$$

Thus, a decrease in the emission standard augments all threshold values of k. Finally, we evaluate the effect on the threshold values of x

$$\frac{\partial x_0^{\bar{e}}}{\partial \bar{e}} = -\frac{s-2\bar{e}}{2} < 0,\quad \frac{\partial x_1^{\bar{e}}}{\partial \bar{e}} = -\frac{\tau+s-2\bar{e}}{2} < 0,\quad \frac{\partial x_2^{\bar{e}}}{\partial \bar{e}} = -\frac{3(s-3\bar{e})}{2} < 0,$$

resulting in the same result, a decrease in the standard increases the three threshold values of x. Fig. 9 shows how the different lines that define the sectors of Fig. 4 moves with a decrease in the emission standard.

Comparing this figure with Fig. 8, we conclude that a decrease in $\bar{e}$ under the standard scenario has the same qualitative effects as an increase in the tax rate θ under the tax scenario. Specifically, in both cases, function $f_2(k)$ moves up and the threshold values k_0^{θ}, k_1^{θ} and k_2^{θ} in Fig. 8, that correspond in Fig. 9 to the threshold values $k_0^{\bar{e}}$, $k_2^{\bar{e}}$ and $k_4^{\bar{e}}$, increase. The same occurs with the threshold values of x. Threshold values x_0^{θ} and x_1^{θ} in Fig. 8, that correspond in Fig. 9 to the threshold values $x_0^{\bar{e}}$ and $x_2^{\bar{e}}$, also increase. Thus, we can claim that the size of the sectors in which every market configuration arises change in the same direction under both scenarios. The same can be said about the effects of a decrease in the transportation cost, which we omit for the sake of brevity.[26]

Appendix G. Proof of Lemma 1

We begin comparing k_0^{θ} with $k_4^{\bar{e}}$. Using the comparison criterion of section V, $k_4^{\bar{e}}$ can be written as follows

$$k_4^{\bar{e}} = \frac{3\theta^2 - 2\tau\theta + \tau(-4s+3\tau)}{12}.$$

Then, the difference between $k_4^{\bar{e}}$ and k_0^{θ} is

$$k_4^{\bar{e}} - k_0^{\theta} = \frac{3\theta^2 - 2\tau\theta + \tau(-4s+3\tau)}{12} - \frac{4(\theta-\tau)(s-\theta-\tau)}{9}$$

$$= \frac{25\theta^2 - (16s+6\tau)\theta + 4s\tau - 7\tau^2}{36}, \tag{G.1}$$

where $4s\tau - 7\tau^2$ is positive for $\tau < 0.08s$. We should prove that this difference is negative for $\theta \in (\theta_1, s/2)$. For $\theta = s/2$, the numerator of (G.1) is $-\frac{1}{4}\left(-4s\tau + 28\tau^2 + 7s^2\right)$. Substituting τ by xs in the parenthesis, we obtain the following expression $s^2\left(28x^2 - 4x + 7\right)$, where $28x^2-4x+7$ is positive for all $x > 0$. This means that $s^2\left(28x^2 - 4x + 7\right)$ is positive for all x so that $-\frac{1}{4}\left(-4s\tau + 28\tau^2 + 7s^2\right)$ is negative for $\tau < 0.08s$. Thus, the difference $k_4^{\bar{e}} - k_0^{\theta}$ is negative for $\theta = s/2$. Now, we compare θ_1 with the lowest root of polynomial equation $25\theta^2 - (16s+6\tau)\theta + 4s\tau - 7\tau^2 = 0$. This root is

$$\frac{8s + 3\tau - 2\left(-13s\tau + 46\tau^2 + 16s^2\right)^{0.5}}{25}. \tag{G.2}$$

Let us suppose that

$$\theta_1 \le \frac{8s + 3\tau - 2\left(-13s\tau + 46\tau^2 + 16s^2\right)^{0.5}}{25},$$

which implies that

$$\theta_1 = \frac{s + 3\tau + \left(20s\tau + 37\tau^2 + s^2\right)^{0.5}}{7} \le \frac{8s + 3\tau - 2\left(-13s\tau + 46\tau^2 + 16s^2\right)^{0.5}}{25},$$

that reordering terms yields the following inequality

$$0 < 25\left(20s\tau + 37\tau^2 + s^2\right)^{0.5} \le 31s - 54\tau - 14\left(-13s\tau + 46\tau^2 + 16s^2\right)^{0.5},$$

squaring in both sides of the inequality, reordering terms and multiplying by -1 yields

$$\left(-2628s\tau - 1599\tau^2 + 496s^2\right) \ge 4(31s - 54\tau)\left(-13s\tau + 46\tau^2 + 16s^2\right)^{0.5} > 0,$$

squaring again in both sides of the inequality and reordering terms we obtain that

$$625\tau\left(-2480s^3 + 5072s^2\tau + 18\,360s\tau^2 + 657\tau^3\right) \ge 0. \tag{G.3}$$

Doing $\tau = xs$ is the parenthesis of the LHS of (G.3) gives

$$s^3(657x^3 + 18\,360x^2 + 5072x - 2480), \tag{G.4}$$

where the polynomial equation

$$657x^3 + 18\,360x^2 + 5072x - 2480 = 0,$$

[26] Details available upon request.

has only one positive real root equal to 0.2537. Thus, we obtain that (G.4) is negative for $x < 0.08$ and then the LHS of (G.3) is also negative for $\tau < 0.08s$ resulting in a contradiction. The conclusion is that

$$\theta_1 > \frac{8s + 3\tau - 2\left(-13s\tau + 46\tau^2 + 16s^2\right)^{0.5}}{25},$$

and the difference $k_4^{\bar{e}} - k_0^{\theta}$ is negative for $\theta \in (\theta_1, s/2)$ since θ_1 is higher than the lowest root of polynomial equation $25\theta^2 - (16s + 6\tau)\theta + 4s\tau - 7\tau^2 = 0$ and $s/2$ is lower than the largest root. As in the proof of Proposition 2 we showed that $k_4^{\bar{e}}$ is the highest threshold value for k when a standard is applied, and in the proof of Proposition 1 we obtained that k_0^{θ} is the lowest root of the threshold values for k when a tax is used, we can conclude that

$$k_0^{\bar{e}} < k_1^{\bar{e}} < k_2^{\bar{e}} < k_3^{\bar{e}} < k_4^{\bar{e}} < k_0^{\theta} < k_1^{\theta} < k_2^{\theta}.$$

Next, we compare x_0^{θ} with $x_2^{\bar{e}}$. Using the comparison criterion of section V, $x_2^{\bar{e}} = \theta^2/4$, and the difference between these two threshold values is

$$x_2^{\bar{e}} - x_0^{\theta} = \frac{1}{4}\theta^2 - \frac{4\theta(s-\theta)}{9} = \frac{1}{36}\theta\,(25\theta - 16s)\,,$$

that is an increasing function in θ. For $\theta = s/2$, the difference is $-7s^2/144$. Then as $\theta_1 < s/2$ we obtain that $x_2^{\bar{e}} - x_0^{\theta}$ is negative for $\theta \in (\theta_1, s/2)$. In the proof of Proposition 2 we obtained that $x_2^{\bar{e}}$ is the highest threshold value for x when a standard is used to control emissions, and in the proof of Proposition 1 we showed that x_0^{θ} is the lowest threshold value of x when a tax is applied. This implies that the order for the threshold values of x is

$$x_0^{\bar{e}} < x_1^{\bar{e}} < x_2^{\bar{e}} < x_0^{\theta} < x_1^{\theta}.$$

References

Aghion, P., Dechezleprêtre, A., Hemous, D., Martin, R., Van Reenen, J., 2016. Carbon taxes, path dependency, and technical change: Evidence from the auto industry. J. Political Econ. 124 (1), 1–51.

Agliardi, E., Lambertini, L., 2024. To abate, or not to abate? The arising of the win-win solution under time consistent emission taxation. Environ. Resour. Econ. 87, 1398–1405.

Ambec, S., Cohen, M.A., Elgie, S., Lanoie, P., 2013. The Porter hypothesis at 20: Can environmental regulation enhance innovation and competitiveness? Rev. Environ. Econ. Policy 7, 2–22.

André, F.J., 2016. Strategic behavior and the Porter hypothesis. In: Dinar, A. (Ed.), In: The WSPC Reference on Natural Resource and Environmental Policy in the Era of Global Change, vol. 1, World Scientific, pp. 231–262.

André, F.J., González, P., Porteiro, N., 2009. Strategic quality competition and the Porter hypothesis. J. Environ. Econ. Manag. 57, 182–194.

Buccella, D., Fanti, L., Gori, L., 2021. To abate, or not to abate? A strategic approach on green production in Cournot and Bertrand duopolies. Energy Econ. 96, 105164.

Calel, R., Dechezleprêtre, A., 2016. Environmental policy and directed technological change: Evidence from the European carbon market. Rev. Econ. Stat. 98 (1), 173–191.

Cohen, M.A., Tubb, A., 2018. The impact of environmental regulation on firm and country competitiveness: A meta-analysis of the Porter hypothesis. J. Assoc. Environ. Resour. Econ. 5, 371–399.

Constantatos, C., Herrmann, M., 2011. Market inertia and the introduction of green products: Can strategic effects justify the Porter hypothesis? Environ. Resour. Econ. 50, 267–284.

Copeland, B.R., Taylor, M.S., 1994. North-South trade and the environment. Q. J. Econ. 109, 755–787.

D'Agostino, L.M., 2015. How MNEs respond to environmental regulation: Integrating the Porter hypothesis and the pollution haven hypothesis. Econ. Politica 32, 245–269.

Dechezleprêtre, A., Sato, M., 2017. The impacts of environmental regulations on competitiveness. Rev. Environ. Econ. Policy 11 (2), 183–206.

Elboghdadly, N., Finus, M., 2022. Strategic climate policy with endogenous plant location: The role of border carbon adjustments. J. Public Econ. Theory 24, 1266–1309.

Greaker, M., 2003. Strategic environmental policy when the governments are threatened by relocation. Resour. Energy Econ. 25, 141–154.

Hoel, M., 1997. Environmental policy with endogenous plant locations. Scand. J. Econ. 99, 241–259.

Ikefuji, M., Itaya, J.-i., Okamura, M., 2016. Optimal emission tax with endogenous location choice of duopolistic firms. Environ. Resour. Econ. 65, 463–485.

Jaffe, A.B., Palmer, K., 1997. Environmental regulation and innovation: A panel data study. Rev. Econ. Stat. 79, 610–619.

Kahn, M.E., Mansur, E.T., 2013. Do local energy prices and regulation affect the geographic concentration of employment? J. Public Econ. 101, 105–114.

Lambertini, L., 2017. Green innovation and market power. Annu. Rev. Resour. Econ. 9, 231–252.

Lambertini, L., Tampieri, A., 2012. Vertical differentiation in a Cournot industry: The Porter hypothesis and beyond. Resour. Energy Econ. 34, 374–380.

Markusen, J.R., Morey, E.R., Olewiler, N.D., 1993. Environmental policy when market structure and plant locations are endogenous. J. Environ. Econ. Manag. 24, 69–86.

Markusen, J.R., Morey, E.R., Olewiler, N.D., 1995. Competition in regional environmental policies when plant locations are endogenous. J. Public Econ. 56, 55–77.

Montero, J.P., 2002. Permits, standards, and technology innovation. J. Environ. Econ. Manag. 44, 23–44.

Motta, M., Thisse, J.-F., 1994. Does environmental dumping lead to delocation? Eur. Econ. Rev. 38, 563–576.

Petrakis, E., Xepapadeas, A., 2003. Location decisions of a polluting firm and the time consistency of environmental policy. Resour. Energy Econ. 25, 197–214.

Porter, M.E., 1991. America's green strategy. Sci. Am. 264 (168).

Porter, M.E., van der Linde, C., 1995. Toward a new conception of the environment-competitiveness relationship. J. Econ. Perspect. 9, 97–118.

Qiu, L.D., Zhou, M., Wei, X., 2018. Regulation, innovation, and firm selection: The porter hypothesis under monopolistic competition. J. Environ. Econ. Manag. 92, 638–658.

Ranocchia, C., Lambertini, L., 2021. Porter hypothesis vs. pollution haven hypothesis: Can there be environmental policies getting two eggs in one basket? Environ. Resour. Econ. 78, 177–199.

Rauscher, M., 1995. Environmental regulation and the location of polluting industries. Int. Tax Public Financ. 2, 229–244.

Sanna-Randaccio, F., Sestini, R., Tarola, O., 2017. Unilateral climate policy and foreign direct investment with firm and country heterogeneity. Environ. Resour. Econ. 67, 379–401.

Tanaka, S., Teshima, K., Verhoogen, E., 2022. North-South displacement effects of environmental regulation: The case of battery recycling. Am. Econ. Rev.: Insights 4 (3), 271–288.

Ulph, A., Valentini, L., 2001. Is environmental dumping greater when plant are footloose? Scand. J. Econ. 103, 673–688.