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Abstract. We discuss an extended version of electromagnetism in which the usual gauge fixing term is promoted into a physical contribution that introduces a new scalar state in the theory. This new state can be generated from vacuum quantum fluctuations during an inflationary era and, on super-Hubble scales, gives rise to an effective cosmological constant. The value of such a cosmological constant coincides with the one inferred from observations as long as inflation took place at the electroweak scale. On the other hand, the new state also generates an effective electric charge density on sub-Hubble scales that produces both vorticity and magnetic fields with coherent lengths as large as the present Hubble horizon.

1. Introduction

Out of the four known fundamental interactions in nature, two of them are particularly interesting in cosmological contexts due to their long range action, namely: gravity and electromagnetism. In fact, the behaviour of these two interactions at large scales is far from clear. On the gravitational sector we find the intriguing problem of the cosmic acceleration. Although a cosmological constant provides a simple and accurate description of it, from a theoretical point of view it would be even more desirable to have a fundamental explanation for the tiny value of such a constant. In this sense, several modifications of the gravitational interaction on cosmological scales have been proposed in the literature [1]. Concerning the electromagnetic sector, the unknown origin of the μG magnetic fields observed in galaxies and clusters [2] and, more remarkably, the very recent claim of detection of extra galactic magnetic fields [3] still lacks of a satisfactory explanation.

In this work we will consider the potential role of a modified electromagnetic theory in the dark energy problem [5, 6, 8] and how this modified electromagnetism can generate magnetic fields at large scales. Thus, we shall explore the interesting possibility of finding a link between dark energy and the origin of cosmic magnetic field.

2. Extended electromagnetism without the Lorenz condition

In the covariant quantization of the electromagnetic field, the starting point is the modified Maxwell action including a gauge fixing term, that, in a curved spacetime, reads:

$$S = \int d^4x \sqrt{-g} \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\xi}{2} (\nabla_\mu A^\mu)^2 + J_\mu A^\mu)^2 \right] \tag{1}$$

which leads to the modified Maxwell equations:

$$\nabla_\nu F^{\mu\nu} + \xi\nabla^\mu(\nabla_\nu A^\nu) = J^\mu. \tag{2}$$

Taking the divergence of this equation, we obtain:

$$\Box(\nabla_\nu A^\nu) = 0 \tag{3}$$

The quantization in the Gupta-Bleuler formalism and in Minkowski spacetime proceeds as follows [4]: One works with the four polarizations of A_μ and then imposes the weak Lorenz condition on the physical states $|\phi\rangle$ so that $(\partial_\mu A^\mu)^{(+)}|\phi\rangle = 0$ to get rid of the unphysical polarizations. With this condition, the expected value of any physical observable only depends on the the transverse degrees of freedom, because the temporal and longitudinal polarizations contribute with opposite signs and, since the Lorenz condition imposes that there must be the same number of temporal and longitudinal photons in any physical state, they cancel each other. However, in an expanding universe, we see from (3) that $\nabla_\nu A^\nu$ can be excited from vacuum quantum fluctuations because it behaves as a massless scalar field and, therefore, the Lorenz condition is violated. The reason for such a violation is that the expansion excites a different number of temporal and longitudinal photons so the aforementioned cancellation does not occur anymore [5, 6].

In order to avoid the difficulties found when quantizing in the covariant formalism in an expanding universe, let us explore the possibility that the fundamental theory of electromagnetism is given by the modified action (1), where we allow the $\nabla_\mu A^\mu$ field to propagate. This theory still has the residual gauge symmetry $A_\mu \to A_\mu + \partial_\mu\theta$ provided $\Box\theta = 0$. Thus, having removed one constraint, the theory contains one additional degree of freedom and the general solution for the modified equations can be written as:

$$\mathcal{A}_\mu = \mathcal{A}_\mu^{(1)} + \mathcal{A}_\mu^{(2)} + \mathcal{A}_\mu^{(s)} + \partial_\mu\theta \tag{4}$$

where $\mathcal{A}_\mu^{(i)}$ with $i = 1, 2$ are the two transverse modes of the massless photon, $\mathcal{A}_\mu^{(s)}$ is the new scalar state, which is the mode that would have been eliminated if we had imposed the Lorenz condition and, finally, $\partial_\mu\theta$ is a purely residual gauge mode, which can be eliminated by means of a residual gauge transformation in the asymptotically free regions, in a completely analogous way to the elimination of the A_0 component in the Coulomb quantization. The fact that Maxwell's electromagnetism could contain an additional scalar mode decoupled from electromagnetic currents, but with non-vanishing gravitational interactions, was already noticed in a different context in [7].

The evolution of the new mode is given by (3), so that on super-Hubble scales, $|\nabla_\mu \mathcal{A}_k^{(s)\mu}| = const.$ which, as shown in [5], implies that the field contributes as a cosmological constant in (1). Notice that, as seen in (3), the new scalar mode is a massless free field and it is possible to calculate the corresponding power spectrum generated during inflation, $P_{\nabla A}(k) = 4\pi k^3 |\nabla_\mu \mathcal{A}_k^{(s)\mu}|^2$. In the super-Hubble limit, we get in a quasi-de Sitter inflationary phase characterized by a slow-roll parameter ϵ:

$$P_{\nabla A}(k) = \frac{9H_{k_0}^4}{16\pi^2}\left(\frac{k}{k_0}\right)^{-4\epsilon} \tag{5}$$

where H_{k_0} is the Hubble parameter when the k_0 mode left the horizon [5]. Notice that this result implies that $\rho_A \sim (H_{k_0})^4$. The measured value of the cosmological constant then requires $H_{k_0} \sim 10^{-3}$ eV, which corresponds to an inflationary scale $M_I \sim 1$ TeV. Thus we see that the cosmological constant scale can be naturally explained in terms of physics at the electroweak

scale. This is one of the most relevant aspects of the present model in which, unlike existing dark energy theories based on scalar fields, dark energy can be generated without including any potential term or dimensional constant.

Since the field amplitude of the scalar state remains frozen on super-Hubble scales, there is no modification of Maxwell's equation on those scales. However, as the amplitude starts decaying once the mode enters the horizon in the radiation or matter eras, the ξ-term in (2) generates an effective current which can produce magnetic fields on cosmological scales, as we will show in the following.

By passing, we notice that, in Minkowski spacetime, the theory (1) is completely equivalent to standard QED because, although non-gauge invariant, the corresponding effective action is equivalent to the standard BRST invariant effective action of QED [6]. This prevents from potential unobserved effects in accelerators because both theories lead to the same phenomenology in flat spacetime, being distinguishable only in curved backgrounds.

On the other hand, despite the fact that the homogeneous evolution in the present case is the same as in ΛCDM, the effective cosmological constant generated by the new scalar state fluctuates so that the evolution of metric perturbations could be different. We have calculated the evolution of metric, matter density and electromagnetic perturbations [8]. The propagation speeds of scalar, vector and tensor perturbations are found to be real and equal to the speed of light, so that the theory is classically stable. Moreover, the three physical states carry positive energy so it is also quantum-mechanically stable. On the other hand, it is possible to see that all the post-Newtonian parameters [9] agree with those of General Relativity, i.e. the theory is compatible with all the local gravity constraints for any value of the homogeneous background electromagnetic field at the same level of accuracy as General Relativity [5, 10]. Concerning the evolution of scalar perturbations, we find that the only relevant deviations with respect to ΛCDM appear on large scales $k \sim H_0$ and that they depend on the primordial spectrum of electromagnetic fluctuations. However, the effects on the CMB temperature and matter power spectra are compatible with observations except for very large primordial fluctuations [8].

3. Generation of cosmic magnetic fields

It is interesting to note that the ξ-term can be seen, at the equations of motion level, as a conserved current acting as a source of the usual Maxwell field. To see this, we can write $-\xi \nabla^\mu (\nabla_\nu A^\nu) \equiv J^\mu_{\nabla \cdot A}$ which, according to (3), satisfies the conservation equation $\nabla_\mu J^\mu_{\nabla \cdot A} = 0$ and we can express (2) as:

$$\nabla_\nu F^{\mu\nu} = J^\mu_T \qquad (6)$$

with $J^\mu_T = J^\mu + J^\mu_{\nabla \cdot A}$ and $\nabla_\mu J^\mu_T = 0$. Physically, this means that, while the new scalar mode can only be excited gravitationally, once it is produced it will generally behave as a source of electromagnetic fields. Therefore, the modified theory is described by ordinary Maxwell equations with an additional "external" current. For an observer with four-velocity u^μ moving with the cosmic plasma, it is possible to decompose the Faraday tensor in its electric and magnetic parts as: $F_{\mu\nu} = 2E_{[\mu}u_{\nu]} + \frac{\epsilon_{\mu\nu\rho\sigma}}{\sqrt{g}}B^\rho u^\sigma$, where $E^\mu = F^{\mu\nu}u_\nu$ and $B^\mu = \epsilon^{\mu\nu\rho\sigma}/(2\sqrt{g})F_{\rho\sigma}u_\nu$. Due to the infinite conductivity of the plasma, Ohm's law $J^\mu - u^\mu u_\nu J^\nu = \sigma F^{\mu\nu}u_\nu$ implies $E^\mu = 0$. Therefore, in that case the only contribution would come from the magnetic part. Thus, from Maxwell's equations, we get, for comoving observers in a FLRW metric (see also [11]):

$$\vec{\omega} \cdot \vec{B} = \rho^0_g \qquad (7)$$

where $\vec{v} = d\vec{x}/d\eta$ is the conformal time fluid velocity, $\vec{\omega} = \vec{\nabla} \times \vec{v}$ is the fluid vorticity, $\rho^0_g = -\xi \partial_0 (\nabla_\mu A^\mu)$ is the effective charge density today whose power spectrum can be obtained
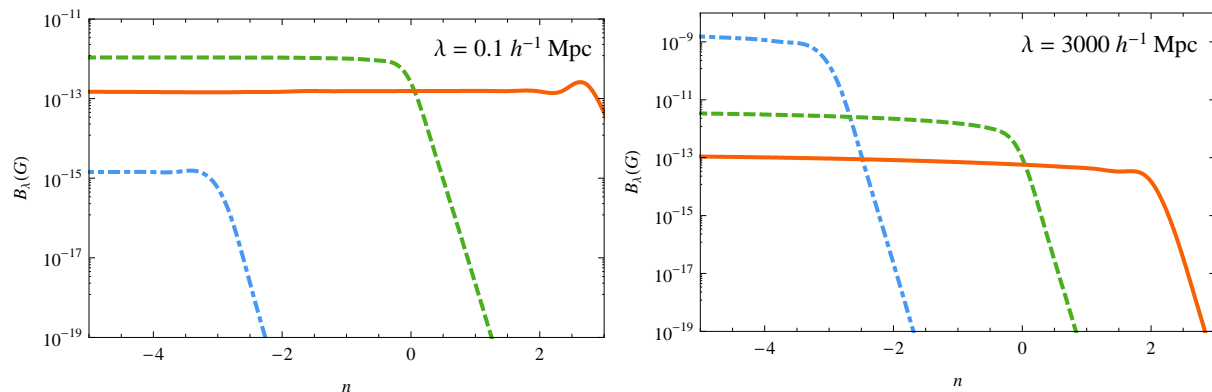
Figure 1. Lower limits on the magnetic fields generated on galactic scales (left panel) and Hubble horizon scales (right panel) in terms of the magnetic spectral index n for different values of the vorticity spectral index m. Dot-dashed blue for $m = 0$, dashed green for $m \simeq -3$ and full red for $m \simeq -5$.

from (5) (see [12] for more details), and the $\vec{B}$ components scale as $B_i \propto 1/a$ as can be easily obtained from $\epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma;\nu} = 0$ to the lowest order in $|\vec{v}|$. Thus, the presence of the non-vanishing cosmic effective charge density necessarily creates both magnetic field and vorticity. Due to the presence of the effective current, we find that vorticity grows as $|\vec{\omega}| \propto a$, from radiation era until present.

Using (7), it is possible to translate the existing upper limits on vorticity coming from CMB anisotropies [11] into *lower* limits on the amplitude of the magnetic fields generated by this mechanism [12]. In Fig. 1 we show the corresponding limits on the magnetic field at galactic and Hubble horizon scales for different spectral indices of magnetic field (n) and vorticity (m). We see that this mechanism allows to generate relatively strong magnetic fields on scales as large as the Hubble horizon and act as seeds for a galactic dynamo or even play the role of primordial fields and account for observations just by amplification due to the collapse and differential rotation of the protogalactic cloud.

4. Discussion
We have discussed and extended electromagnetic theory in which we do not need to impose the Lorenz condition. In order to quantise the theory we introduce an additional scalar state which can be excited by gravitational fields. Indeed, fluctuations of such a state during an inflationary era at the electroweak scale generates an effective cosmological constant on super-Hubble scales with the correct value. This theory is free form both classical and quantum instabilities and is consistent with all local gravity tests at the same level as General Relativity, CMB and large scale structure observations. On the other hand, the sub-Hubble modes of the new state generated during inflation acts as an effective electromagnetic current so it can produce cosmological magnetic fields all the way to the Hubble horizon (but not beyond). This allows to establish an important link between the problems of dark energy and cosmic magnetic fields. In fact, also non-minimal couplings have been considered in [13] where a potential relationship between angular momentum and magnetic fields has been explored.

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