



## Research Report

# Electrophysiological connectivity of logical deduction: Early cortical MEG study



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## ABSTRACT

Complex human reasoning involves minimal abilities to extract conclusions implied in the available information. These abilities are considered “deductive” because they exemplify certain abstract relations among propositions or probabilities called deductive arguments. However, the electrophysiological dynamics which supports such complex cognitive processes has not been addressed yet. In this work we consider typically deductive logico-probabilistically valid inferences and aim to verify or refute their electrophysiological functional connectivity differences from invalid inferences with the same content (same relational variables, same stimuli, same relevant and salient features). We recorded the brain electrophysiological activity of 20 participants (age =  $20.35 \pm 3.23$ ) by means of an MEG system during two consecutive reasoning tasks: a search task (invalid condition) without any specific deductive rules to follow, and a logically valid deductive task (valid condition) with explicit deductive rules as instructions. We calculated the functional connectivity (FC) for each condition and conducted a seed-based analysis in a set of cortical regions of interest. Finally, we used a cluster-based permutation test to compare the differences between logically valid and invalid conditions in terms of FC. As a first novel result we found higher FC for valid condition in beta band between regions of interest and left prefrontal, temporal, parietal, and cingulate structures. FC analysis allows a second novel result which is the definition of a propositional network with operculo-cingular, parietal and medial nodes, specifically including disputed medial deductive “core” areas. The experiment discloses measurable cortical processes which do not depend on content but

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on truth-functional propositional operators. These experimental novelties may contribute to understand the cortical bases of deductive processes.

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## 1. Introduction

Learning, reasoning, deciding, communicating involves minimal deductive abilities in order to extract conclusions implied in the already available information. The eventual neural substrate of deductive phenomena has been studied by the spatial and temporal analysis of the cerebral dynamics of typically deductive arguments. They are presented in several formats (visual, linguistic, agentive) and with several inferential profiles (propositional, categorical, relational, spatial, ...). Variability in these both dimensions make the neural study of inference highly complex (Shin & Jeon, 2021) as manifested in several meta studies (Prado et al., 2011; Wendelken, 2015; Zhang et al., 2020). The usual strategy has taken clear examples of deductive arguments, like Modus Ponens (from A and, if A then B, deduce B) or Modus Tollens (from not B and, if A then B, deduce not-A) and assume they correspond to (or are realized in) deductive inferences as psychological or neural events. This assumption is conceptually ungrounded and limited, since deductive arguments are abstract relations among propositions (Harman, 1984) or probabilities (Oaksford & Chater, 2020), while deductive inferences are eventual time-consuming events in the nervous system.

On the other hand, bivalent truth-functional arguments are arguments in which logical validity (in classical sense) coincides with probabilistic p-validity (in Adam's sense) (Kleiter, 2018). These arguments are paradigmatic examples of deductions, but there are sparse psychometrical or cortical evidence showing that they are distinctive inferential processes. Some results on reasoning research suggest that in deductive inferences the balance between the role of content and logical validity (Heit & Rotello, 2010; Singmann & Klauer, 2011), and between content and probabilistic validity (Markovits et al., 2018; Singmann et al., 2016) is different from non-deductive inferences. However, it is far from being proved that the normative difference between deductive and inductive arguments corresponds to a factual difference between deductive and inductive inferences. Available experimental results offer weak psychometrical (Stephens et al., 2018, 2020), psychological (Evans & Over, 2013) and cortical (Goel, 2007; I. A. Noveck et al., 2004; Prado, 2018; Ruff et al., 2003; Salto et al., 2021) differences between deductive and non-deductive inferences. In fact, since reasoning and fluid intelligence constructs are measured by inductive matrices, deductivity is scarcely significant in reasoning measurements (Wilhelm, 2005; Álvarez-Merino et al., 2019a; 2019b).

A key point in the neuroscience of deduction is to determine the neural balance between content (semantical or visosemantical) and logical structure in inferences eventually corresponding to deductive arguments. Resonance research

has verified the neural effects of visual and semantic content both in logically valid and invalid inferences, for example its familiarity (Goel, 2019), abstraction (I. A. Noveck et al., 2004) (I. Noveck, 2018), and the number of its variables (Holyoak & Monti, 2021) cerebrally determine the neural processing of both logically valid and invalid inferences (Reverberi et al., 2007, 2012). Moreover, topographical brain studies on deductive vs inductive arguments are heavily dependant on the format (visual, linguistic) (Prado et al., 2010; Zhu et al., 2019) and also on the contents of premises and conclusions (Goel, 2019; Reverberi et al., 2007, 2009), but not specifically on logical structure (Bonnefond & Henst, 2013; Salto et al., 2021). For example, EEG research on deductive inferences has identified longer processing periods for logically complex inferences as opposed to simpler ones (Li et al., 2014; Álvarez-Merino et al., 2019a; 2019b), but has not identified specific deductive potentials. Also, it has been demonstrated that sharing literal contents among premises and conclusions both accelerates and favors the cerebral processing of inferences. Moreover, it is commonly accepted the existence of an integration phase in the reasoning process (Fangmeier et al., 2006; I. A. Noveck et al., 2004; Reverberi et al., 2009) that may be extended until 300 ms or 400 ms (Bonnefond & Van der Henst, 2009). Remarkably, both visual and linguistic propositional inferences mainly activate left operculo-fronto-parietal areas, particularly sylvan regions with remarkable differences with non-propositional deductive inferences, such as spatial and categorical (Knauff, 2007). There are however late neural features in deductive processes (at about 600 ms) which are not obviously content-related, such as inhibitory neural processes associated with formal features of logically complex inferences (Houdé & Borst, 2015; Li et al., 2014; Rosenblum & Kurths, 1996; Zhang et al., 2020), pragmatically counterintuitive elements (Prado, 2018; Reverberi et al., 2012), or logical training (Álvarez Merino et al., 2018; Álvarez-Merino et al., 2019a; 2019b; Mackey et al., 2013). As time passes, meta-studies show increasing consistency among results, with prefrontal (medial and lateral) and parietal cortical brain areas associated with propositional inferences (Prado et al., 2020; Wendelken, 2015; Zhang et al., 2020).

The line of research proposed in this work is focused exclusively in propositional logically valid inferences which demonstrably preserve bivalent truth (Boolos et al., 2007) and probability in Adam's sense (Adams, 1996) instead of presupposing that inferences exemplifying deductive arguments are eo ipso deductive inferences. The class of arguments we are interested in are truth functional and contain only truth functional operators such as conjunction, disjunction, material conditional and negation. This class of truth functional logico-probabilistically valid arguments demonstrably has computational procedures able to determine their validity and consistency only with their logical operators. The

experimental paradigm is designed to contrast the cortical activity generated by inferences with the same content: same non-logical predicates, relational variables, number of variables, relevant properties and stimuli. The difference between the two conditions lies in the logical complexity of the task in the valid condition. Methodologically, the neural difference between the conditions grounds a “forward” inference (Heit, 2015a) verifying or refuting if cortical inference processes reproduce deductive arguments or on the contrary, they are not genuinely different inferential processes. The paradigm is based on the SET game, and it is described in detail in (Salto et al., 2021) with MEG.

The objective of this research is to identify brain connectivity patterns of logically valid inferences distinct from those of logically invalid inferences. If these patterns are identified, then it is feasible to acknowledge new electrophysiological patterns of valid deduction. If they are not found, this would be a clue that logical validity is a purely normative phenomenon.

## 2. Methods

In this study, we detail the methodology used to determine our sample size, as well as the criteria for inclusion and exclusion of data. We also report any data exclusions and all manipulations and measures employed in the study. All data, materials and codes used in this research are available in open-source repositories.

- Raw MEG data, source-space reconstruction, MATLAB analysis codes and experimental task: [https://osf.io/m47ex/?view\\_only=19f57b61e3f641b99ab2fc4e7cf5846d](https://osf.io/m47ex/?view_only=19f57b61e3f641b99ab2fc4e7cf5846d).
- MEG processing scripts for MEG raw data: [https://github.com/rbruna/meeg\\_analysis/tree/main/script](https://github.com/rbruna/meeg_analysis/tree/main/script).

### 2.1. Participants

For this experiment, twenty-three young right-handed subjects (12 males and 11 females) were registered for the MEG database. After data preprocessing, three subjects were ruled out from sample due a poor quality of the signal and/or source reconstruction. The final dataset after data processing was composed of 20 subjects (10 males and 10 females), with a mean age of 20.35 years ( $SD = \pm 3.23$ ). Participants did not report any significant neurological or psychopathological conditions or any psychoactive drug intake. Each participant went through two experimental tasks sequentially. First, we recorded electrophysiological brain activity while they performed an INVALID LOGICAL DEDUCTION paradigm task (the control task), whereupon they performed a VALID LOGICAL DEDUCTION paradigm. The responding hand for each condition was counterbalanced across subjects. All participants signed an informed consent form before their participation in this study, following the guidelines in the Declaration of Helsinki. The project was approved by the University of León and received the approval of the Ethics Committee (the code of ethics for research is 0–181, dated 11-06-2019). No part of the study procedure nor analyses was pre-registered prior to the research.

### 2.2. Stimuli and experimental design

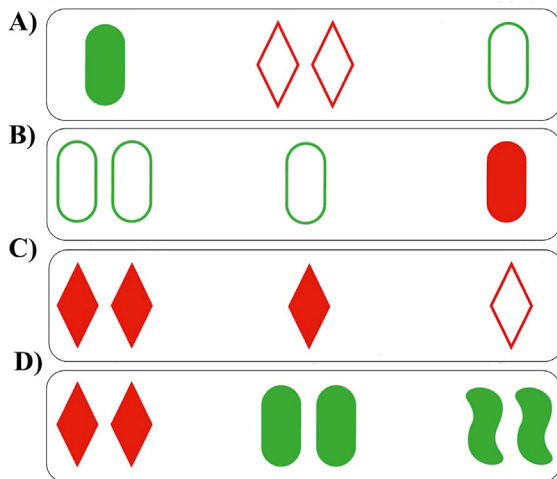
We used identical stimulation paradigm and procedure as reported in (Salto et al., 2021). The experiment compares inferences with the same content but different logical complexity. In both conditions (logically valid and invalid), the subject manipulates the same visual stimuli under the same categories and with the same description. Only in the logically valid condition the task involves logical complexity as given by the SET definition (See Complementary Materials and (Salto et al., 2021). The items in the study were trios of cards from the game SET (Set Enterprise, 2019). Each card has a variation of the following four features: figure (diamond, ovoid, squiggle), color (green, red, black), cardinality (1 or 2), filling (filled, empty). See Fig. 1 for an example of each possible case.

The same trios of cards are presented in both conditions as premises for the inferences, and the set of relevant properties (sharing figure, color, number or filling) is also the same in both conditions, as it is the set of relevant predicates describing cards in both conditions. The point in this paradigm is that not only the premises, but also the conclusions of both valid and invalid inferences manipulate the same content, namely, the same viso-semantic properties and the same predicates and categories describing the cards. Both tasks are deductive in the sense that no new or additional information external to the premises is involved in the inference. The valid condition consists in a logically valid task (“is the trio a SET?”) and the invalid condition a logically invalid task (“does the trio follow a rule?”). Both SET and rules are defined for the subject in terms of cards and their four relevant features. Notice that “being SET” is defined in terms of elementary logic with propositional operators.

A detailed description of experimental paradigm is depicted in supplementary materials and in (Salto et al., 2021). The focus is here on the methodological adequacy of the paradigm. Both task conditions have fundamentally the same viso-semantic content and the same relevant perceptual categories. Following Friston et al. (Friston et al., 2017), the content-complexity of a neural task (which they also call “cost”) is formalized as mutual information between hidden states (i.e., perceptual categories) and sensory outcomes (stimuli) under each task condition. A formal proof is not needed to see that both conditions have broadly the same informational content. However, only in the valid condition there is also certain logical complexity given by the propositional operators defining SET. The paradigm does not assume that there is any specific measurable neural activity related to the logical processing but tries to experimentally verify or refute this fact.

#### 2.2.1. MEG acquisition

Electrophysiological activity was acquired by means of an MEG system of 306 channels, Elekta Neuromag system located in the Center for Biomedical Technology (Madrid, Spain). We used an online anti-alias filter between .1 and 330 Hz and a 1000 Hz sampling rate. Environmental noise was reduced offline using the temporal extension of the signal space separation method (Taulu & Hari, 2009), and subject movements were compensated using the same algorithm. Signal data was semi-automatically processed and visually inspected by an



**Fig. 1 – Example of items of SET in both conditions. A) No feature in common; B) One feature in common (figure); C) Two features in common (figure and color); D) Three features in common (number, filling, and color).**

expert. We used an independent component analysis based on SOBI (Belouchrani et al., 1997) to remove eye-blink and cardiac activity. Lastly, the data was segmented in 1-s epochs of artifact-free data. Additionally, trials with a response time (i.e., time elapsed between the stimulus onset and the response) of less than 1000 ms were discarded to minimize the influence of motor response over interest time windows. After trial filtering, the mean trial count per condition were 137 trials for VALID condition and 99 trials for INVALID condition.

### 2.3. Source-space reconstruction

We used a realistic single shell (Nolte, 2003), generated from the Montreal Neurological Institute (MNI) template-based T1 images (ICBM 152), as forward model. As inverse model, we employed a linearly constrain minimum variance (LCMV) beamformer (Veen et al., 1997) beamformer (Antón-Toro et al., 2021; Nakamura et al., 2018; Pusic et al., 2019). MEG data were reconstructed into source space for each classical frequency band: theta (4–8 Hz), alpha (8–12 Hz), beta (12–30 Hz), and low gamma (30–45 Hz). Finally, we used the epoch-averaged covariance matrix to build the adaptive spatial filter, obtaining a source-space model of 2459 source positions located inside the cranial cavity. To quality-check the reconstruction of the estimated source-signal, we calculated the well-known P100 visual component in source space, as the mean power-spectrum in the time window from 80 ms to 120 ms. Peak activity was adequately reconstructed in the bilateral calcarine fissure for both conditions. This procedure is extensively depicted in supplementary material “Source-space reconstruction of visual potential”. The result of this analysis is shown in supplementary figure S3.

### 2.4. Functional connectivity analysis

Functional connectivity (FC) was calculated under the hypothesis of phase synchronization (Rosenblum & Kurths,

1996) and evaluated using the Phase Locking Value (PLV) Lachaux et al. (1999). This metric is based on the study of the distribution of the instantaneous phase difference of two time-depending signals. PLV was calculated separately for each classical frequency in a time window from 100 ms to 550 ms, capturing the initial stages of cognitive processing. Source-space PLV were calculated for each source positions. Since MEG are not suitable for measuring deep brain regions, subcortical areas were discarded from the analysis, resulting in a PLV matrix of 1188 x 1188 cortical sources for each frequency band. This source-level FC matrix was the starting point for all the subsequent analyses. We performed two complementary approaches of analysis. First, we conducted a seed-based analysis over a set of eight regions of interest as seeds. Those regions were selected from previous literature (Coetzee & Monti, 2018; Zhang et al., 2020) as important for logical reasoning processes: Left inferior frontal gyrus (IIFG, formed by the IIFG orbital, opercular and triangular), Left Middle Frontal Gyrus (IMFG), Left Medial Superior frontal gyrus (ISFGm), Anterior Cingulate Cortex (ACC), Middle Cingulate Cortex (MCC), Left Insular Cortex (IIns), Left Superior Parietal gyrus (ISPG) and Left Inferior Parietal Gyrus (IIPG). Each seed was formed by cortical sources defined in the Automated Anatomical Labeling atlas (AAL) (Tzourio-Mazoyer et al., 2002). Supplementary figure S4 shows the cortical location of each seed of interest. PLV value of each seed was calculated as the mean of PLV values of each source opposition within. Subsequently, we obtained for a vector of 1 x 1188 for each seed and frequency band, representing the FC of the seed with the rest of cortical sources in an atlas-unconstrained way. In a second approach (Network-based), we built a functional network using these cortical seeds of interest as nodes, representing the FC of all seeds with each other. Additionally, in order to test the laterality of logical reasoning process we included in the network the contra-hemispheric regions of seeds of interest, resulting in a network of 14 x 14 functional nodes.

### 2.5. Statistical analysis

We tested FC differences of logical validity conditions with two complementary approaches of analysis: a seed-based analysis, and a network-based analysis.

In the first approach, we calculated the FC differences between each seed of interest and the rest of cortical sources, for each frequency band. We assess FC differences between conditions by means of a t-test contrast for dependent samples, using a cluster-based permutation test (CBPT) method (Oostenveld et al., 2011). This method identifies the presence of clusters of cortical sources connected statistically different between conditions, without predefined atlas constrictions. We corrected the number of significant clusters per band and seed using a Bonferroni's stepwise method (each successive threshold for *P-value* is equal to .05 fractioned by the number of significant clusters). Only those cluster which survived this correction were reported as significant.

For our second approach, the network-based analysis, we tested the FC differences between the regions of interest defined according to the atlas AAL. We performed a permutation-based t-test for related samples between each

pair of nodes, for each frequency band. This method identifies the PLV differences between condition for each pair of brain regions. We corrected for multiple comparisons using FDR method, and only those connections below FDR threshold ( $P < .001$ ) were reported as significant.

PLV analysis is known to be potentially affected by source leakage and volume conduction biases. We addressed these biases by means of the direct estimation of both effects and controlling them in our statistical analysis as additional quality check. This procedure is depicted in supplementary material.

### 3. Results

#### 3.1. Behavioral results

RT measure was obtained for each subject and each condition. It was calculated from the onset of the items, within a temporal window of 1000 ms. Higher RTs were observed in response to the Valid deductive condition than to the Invalid condition (Table 1).

#### 3.2. Seed-based results

We performed a seed-based analysis using CBPT method for each predefined seed of interest and each frequency band. Using as seed the IIFG, results showed one cluster of higher FC for VALID condition in beta band ( $P = .0031$ ). The cluster was formed by 82 cortical sources located predominantly in the MCC (see Fig. 2A). For the IMFG seed, we found one significant cluster with higher FC in beta band for the VALID condition ( $P = .0011$ ), formed by 34 sources located in the dorsal PFC and IIFG region. Additionally, we found one cluster with higher connectivity for VALID condition in theta band ( $P = .0351$ ) which did not survive correction for multiple comparisons (see Fig. 2B). Using ACC as seed we found a cluster in beta band ( $P = .0372$ ) with higher FC for VALID condition, formed by 17 sources and encompassing left parietal regions, which did not survive multiple comparison corrections (see Fig. 2C). Regarding ISFGm, results showed one significant cluster of higher FC for VALID condition in beta band ( $P = .0187$ ) composed by 21 cortical sources in the opercular part of IIFG. An additional cluster in theta band was found in the posterior part of the left dorsolateral cortex with higher connectivity for the VALID condition ( $P = .0326$ ) but did not survive multiple comparison correction (see Fig. 2D). Finally, LSPG seed shows a significant cluster with higher connectivity for VALID condition in beta band ( $P = .0185$ ), formed by 23 cortical sources in the orbital and triangular parts of the IIFG and left temporal pole (see Fig. 2E). Results in MCC, IIns and IIPG does not show significant results.

#### 3.3. Network-based results

For this analysis we tested the differences in the connectivity between regions of interest. We built a network using those regions as nodes, including the contrahemispheric regions in the network to test the lateralization of the reasoning process.

Results showed significant differences between the FC of both conditions for beta band ( $P < .001$ ). Such differences

**Table 1 – Descriptive data of RT of the conditions.**

RTs	Mean	Median	SD
Valid condition	2667.89	2273.27	1766.47
Invalid condition	1450.53	1160.22	783.13

Note Behavioral results of VALID and INVALID condition in MEG study.

revealed higher FC for VALID condition in 6 connectivity links, predominantly located in the left hemisphere. This network engaged mainly left medial and dorsal prefrontal regions (IIFG, IMFG, ISFGm), cingulate structures (left ACC and bilateral MCC), and left superior parietal regions (LSPG). Fig. 3 shows the cortical distribution of the significant links. Complementarily, we conducted same network-based analysis using all cortical regions (78 regions). Results shows a similar network distribution in beta band, engaging left-lateralized frontoparietal regions (see Fig. 4). Regarding theta band, we did not find any significant result in this analysis.

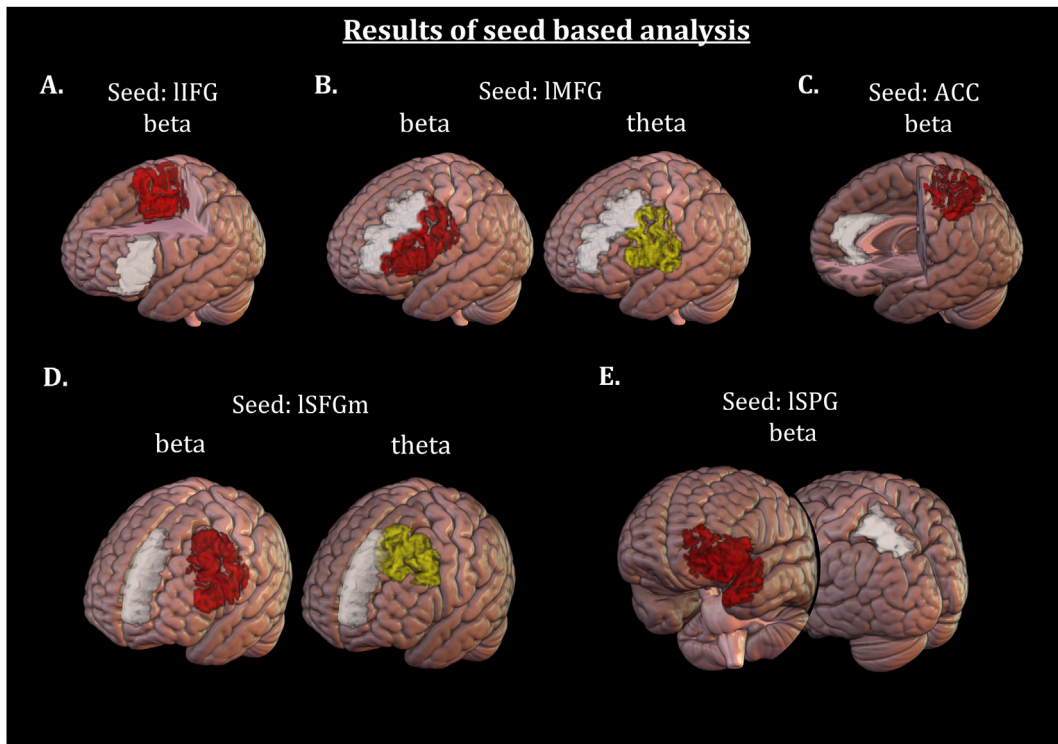
### 4. Discussion

This work has tried to uncover the differences in neural MEG connectivity between logico-probabilistically valid visual inferences and invalid ones with the same content. Logically valid inferences show higher connectivity with specific spatio-temporal patterns. Cluster permutation analysis in the temporal window between 100 and 550 ms after stimulus shows significant neuroelectric connectivity differences between logically valid and invalid inferences in beta band (see Fig. 2A–E), and locally in theta band (see Fig. 2B and D). The network-based analysis shows a distributed propositional deductive network including left cingulo-opercular and left frontoparietal areas (see Fig. 3) already identified in previous studies and metastudies (Prado et al., 2011; Wendelken, 2015; Zhang et al., 2020), and core deductive medial areas (Coetzee et al., 2022; Coetzee & Monti, 2018; M et al., 2009) whose role has been disputed in the literature. The frequential specificity of logical computations and their deductive network are two main experimental contributions of this study. Forward methodology analysis of the neuroelectric differences suggests that logical complexity processing is a distinctive cognitive process which is not only determined by content but also by logical form. This is the main cognitive result of this experiment which suggests the hypothesis that logical inference processing is recursive and involves beta-2 frequencies.

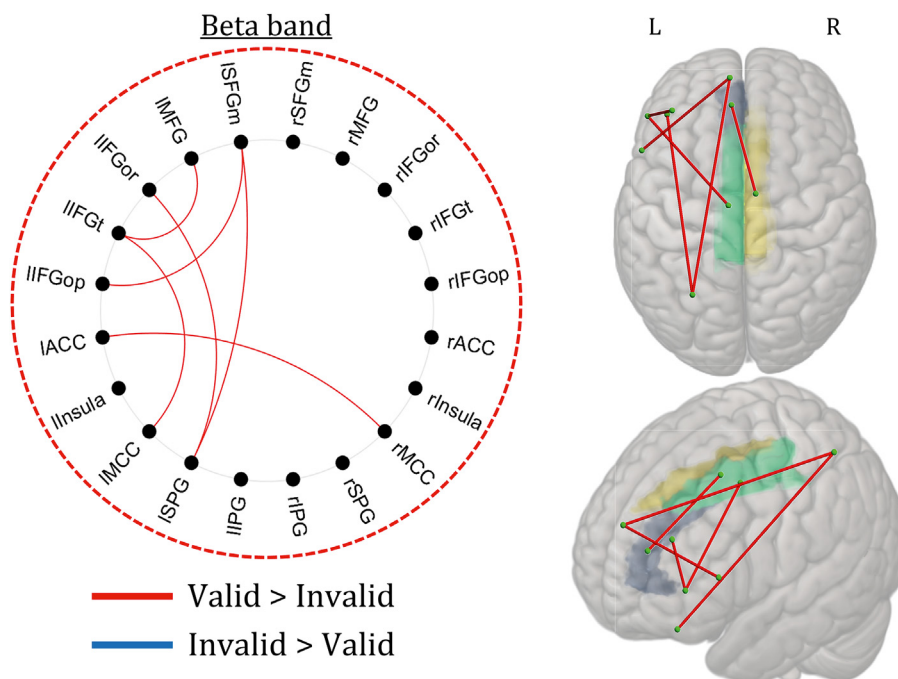
#### 4.1. Description of the logically valid propositional network

Three families of neural connections in beta band stand out differentially in logically valid propositional inferences and not in logically invalid inferences with the same content: (a) opercular, (b) medial, (c) parietal.

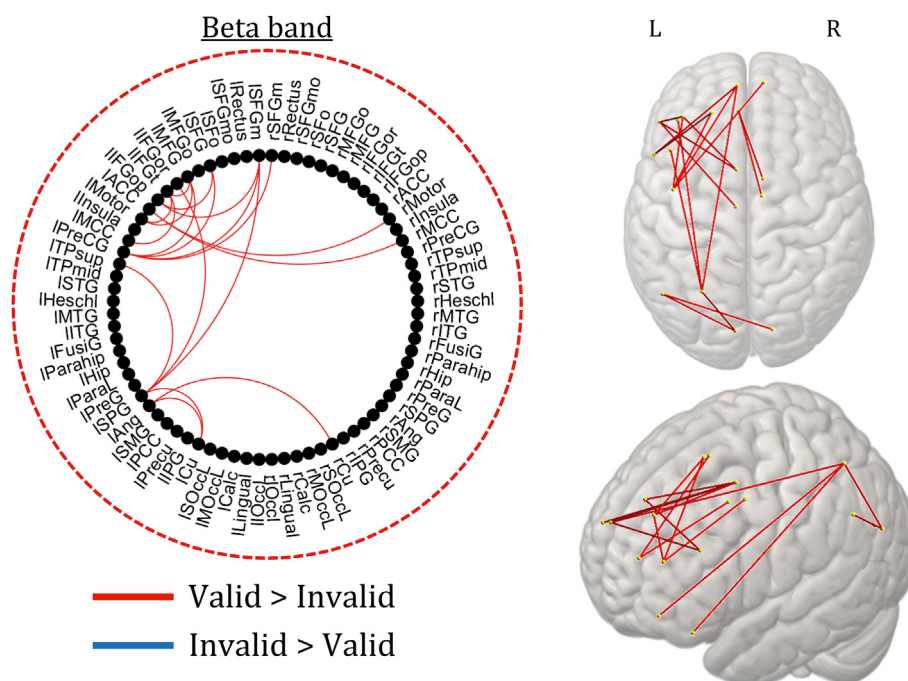
- (a) Two opercular connections between the left prefrontal area IIFG and IMFG (a1) see Fig. 2B) and the middle cingulate cortex (MCC) (a2) (see Fig. 2A), are in valid



**Fig. 2** – This figure shows the location of significant connectivity clusters for each seed and frequency band. White colored areas represent the seeds. Red (beta) and orange (theta) represents clusters of sources with higher connectivity for VALID condition. A) Seed = IIFG; B) Seed = IMFG; C) Seed = ACC; D) Seed = ISFGm; E) Seed = ISPG.



**Fig. 3** – This figure shows the distribution of significant connectivity links between each pair of regions of interest in beta band. Left: red links represents higher connectivity between the pair of connected regions in the VALID condition. Right: Colored regions represents nodes of the network with significant FC differences between conditions.



**Fig. 4** – This figure shows the distribution of significant connectivity links between all pairs of cortical regions in beta band. Left: red links represents higher connectivity between the pair of connected regions in the VALID condition. Right: Distribution of the links in the cortical scalp.

inferences significantly more connected compared with invalid ones. These connections are confirmed in the network-based analysis (see Fig. 3). They plausibly correspond to the electrical correlates of the opercular and cingulo-opercular circuits described with fMRI in the literature on propositional reasoning (Wang et al., 2020). Since all inferences in the experiment are integrable, no delays or additional activation of other areas due to semantical or viso-semantical content are produced, as demonstrably happens in fMRI (Goel, 2007, 2019) and MEG (Bonnefond & Van der Henst, 2009) studies. Moreover, the experiments that have measured the cortical processing of negationless truth functional operators (conditional, disjunction, conjunction) also confirm the same left prefrontal areas (Prado, 2018).

- (b) The medial prefrontal cluster in SFGm is differentially connected in valid inferences with opercular areas (IIFGo) (b2) (see Fig. 2D) The network-based analysis confirms (see Fig. 3) the involvement of medial bilateral, specially left regions. Remarkably, the connection is present even without recalcitrant increase in the logical complexity of the valid task, as it is the case in other deductive studies (Coetzee et al., 2022; Coetzee & Monti, 2018; Monti et al., 2012).
- (c) Connections between the anterior cingulate cortex (ACC) and left parietal areas (IIPG, ISPG) (see Fig. 2E) are presumably the electrical left correlates of bi-hemispheric cingulo-parietal connections identified in the reasoning literature in spatial and relational contexts

(Holyoak & Monti, 2021; Wendelken, 2015; Wertheim & Ragni, 2018).

A subset of two (a) and (b) circuits connected in the positive beta cluster is also connected in theta band, with minor statistical significance (see Fig. 2). Both theta connections are, according to the literature, linked with the semantical or viso-semantical processing of content (Schmidt et al., 2019) and their higher connectivity in the valid condition is consistent with the magnetical (Reverberi et al., 2012) and electrical (Bonnefond et al., 2014; Bonnefond & Henst, 2013; Reverberi et al., 2012) correlates of premises integration. Therefore, differences in content neural processing between valid and invalid neural processes are residual and the experimental paradigm successfully isolates frequential differences between logically valid and invalid processes.

The early literature already associated the beta band with logical tasks (Saul, 1966) and the ulterior research has verified its presence in top-down cognitive control (Ridderinkhof et al., 2004; Stoll et al., 2016), cognitive load (Schapkin et al., 2020), grammar (Beltrán et al., 2019) and false reasoning (Payumo, 2021). Limanowski, Litvak & Friston (Limanowski; Friston, 2020) found sensory/cognitive differential interaction effects in beta also related to rule-following instructions and recent MEG (Salto et al., 2021) and EEG (Álvarez-Merino et al., 2019a; 2019b) research associate beta-2 band activity with logical reasoning in the early premise integration phase (300–350 ms) and again later in the re-processing phase (600–650 ms). In this research, we focus exclusively in the early neuroelectric activity in order to avoid motor processing interferences.

#### 4.2. Forward methodological interpretation: neuroelectrically mapping content and logical form of visual inferences

Forward methodologies as introduced by Hemson ((Henson, 2006)) define explicit conditions in which measurable variables map different cognitive processes. They have been used and discussed in neural contexts (Poldrack & Poldrack, 2011) (Heit, 2015b), to test mapping patterns of brain activity with cognitive processes as in Schoemann et al. (2019). In the context of neuroreasoning research, forward methodologies have also been fruitfully applied in premise integration studies ((Goel, 2019) (Reverberi et al., 2009), (Heit, 2015a)) but have found difficulties disclosing systematic links between deductive inferential processes and neural events (Prado, 2018; Prado et al., 2020; Wang et al., 2020). The methodological proposal in this paper is to give up the assumption that deductive arguments are cortically realized in specifically deductive inferences in order to prove or refute the specific cortical processing of logical complexity. The neural frequential differences found in circuits (a), (b), (c) (see above) shows differences in cost or processing complexity (Friston, K. FitzGerald, T., Rigoli, F., Schwartenbeck, P., & Pezzulo, 2017) between logically valid and invalid conditions. Since the viso-semantic contents of premises and conclusions in both conditions are the same, these differences can only be attributed to differences in logical complexity processing (logical complexity is measured by the number of occurrences of logical operators in the SET definition). Thus, we conclude that behind these frequential differences there must be differentiated inferential cognitive processes that are temporally slower (see Table 1) and more connected in beta (see Figs. 2 and 3).

The review of other neuroelectrical results on logical inference offers a consistent picture in which logically valid inferences show slower RT and generate less intense electrical fields than invalid inferences with the same content (Salto et al., 2021, 2023). The specific temporal location of beta-2 activity verifies the adequacy and sensibility of the experimental design, since only a strong coherence among time-dependent MEG trials explains the specific and coherent temporal and spatial location of beta-2 band activity (Lally et al., 2014). Moreover, only logically valid inferences are massively beta-2 connected in the early integration phase (see Fig. 3 and (Salto et al., 2023)). The differential neuroelectric answer in the valid condition is homogeneous in all items, in the sense that valid inferences follow a discernible temporal and frequential pattern in all applications of the SET definition even if the subject uses different propositional operators in any order. In this regard, a strength of the current paradigm is that frequential differences between valid/invalid conditions are interpreted as inferential differences between cognitive processes. However, the paradigm is unsensible to eventual logically valid inferential processes also present in the invalid condition (neuroelectric processes eventually present in both conditions).

#### 4.3. Electrical confirmation of “core” deductive features

The cortical relationship between deduction and language is a major open problem in cognitive neuroscience (Prado, 2018)

since both logical and semantical tasks share key neural left hemispheric substrates with several functionalities (Wendelken et al., 2017). Both the seed-based analysis (see Fig. 2D) and the subsequent network analysis (see Fig. 3) shows the implication in logically valid propositional reasoning of the medial area of the superior frontal gyrus (SFGm, BA8), with a left medial activation manifest in the functional connectivity analysis (see Fig. 2D) and a less significant activation in bilateral medial parts (see Fig. 3). This area does not appear significantly in all deductive reasoning fMRI studies and metastudies (Prado et al., 2011; Zhang et al., 2020). However it does appear in experimental paradigms (M et al., 2009) contrasting trials with different logical complexity (different number of occurrences of logical operators) evoking this area in BA8, which is a deductive “core area” identified by Monti et al. (Coetzee & Monti, 2018; M et al., 2009; Monti et al., 2007, 2012). While Monti's paradigm uses hefty differences in logical complexity requiring complex alternative cognitive processes to evidence the difference with non-deductive processes, the current paradigm evokes BA8 in the valid condition even with minor differences in logical complexity. These medial areas are significantly connected both in beta and theta with opercular areas (see Fig. 2D) within the here identified deductive propositional network. The fact that opercular areas play such a central role in valid propositional inferences had previously raised the question of the cortical relationships between linguistic and deductive processes. The deductive propositional network (see Fig. 3) partially overlaps but does not coincide frequentially nor spatially with semantical processing, confirming lesions studies (Ivanova et al., 2021) and results on the multiple functionality of opercular areas (Fedorenko & Blank, 2020; Prado, 2018).

#### 4.4. Hypothesis on the recursive computational nature of cortical correlates of logically valid inferences

When it is said that the brain computes, it is often just a metaphorical expression. But when the brain makes logical deductions, it may compute in a literal sense since -with certain standard assumptions (Boalos et al., 2007), (Sieg, 2009), the metamathematical definition of recursion extensionally coincides with computation and in fact also with deduction. Therefore, since recursivity is physically present in many computational processes, it is a reasonable candidate to explain why logical validity leaves physical traits in inferential cortical activities. In fact, recursivity is suggested by key physical properties present only in the valid condition, since logically valid inferences are slower (see Table 1), hypoactive (see Salto et al., 2021), and recursivity is consistent with the role of beta oscillations in frontal, parietal and medial circuits. Even if a comprehensive theoretical explanation of the origin and role of beta-2 is still lacking (Engel & Fries, 2010; Schmidt et al., 2019) neurocomputational research has shown its role in the maintenance or repetition of abstract properties in the same information (Bernhard Spitzer et al., 2014; Spitzer & Haegens, 2017) and in top-down motor control tasks (Friston et al., 2017). Beta activations have been systematically found in the recognition of repeated stimuli as the same stimulus (Baggio & Hagoort, 2011; Martin & Baggio, 2020) and in the

generation of motor and visual sequences and sequences of sequences (Fischmeister et al., 2017). In propositional information contexts, the recursive nature of operators such as repetition, order and identity has been verified in an EEG experiment (Álvarez-Merino et al., 2019a; 2019b) in which beta oscillations were also identified. The role of beta oscillations in ACC and PFG has been studied by (HajiHosseini et al., 2020), attributing to them a cognitive anticipatory and control role consistent with the oscillatory activity accompanying non-vegetative minimal brain activity. These data don't proof the recursive nature of cortical realizations of deductive phenomena but suggest that the neural processing of iteratively self-applied rules (namely logical operators in SET) is a slow automatism which is more connected only in the deductive network.

This work also has some limitations to consider. First, experimental paradigm compared two types of reasoning processes which showed quite different reaction times responses. This may lead to a differential contamination of the motor-planning activity ('Bereitschafts potential') on the time-windows of interest. To overcome this limitation, we limited our analysis to trial with RT above 1000 ms. However, this factor should be taken into account in future experiments. Secondly, we use a LCMV beamforming method as inverse model in our source reconstruction. This approach is potentially biases in cases of highly correlated sources, due to correlated source cancellation phenomenon. Further inverse models should be explored in future researches to overcome this limitation.

## 5. Conclusions

Brain electrical activity during deductive inferences is mostly determined by content, but the experiment shows measurable cortical processes which don't depend on content but on the processing of logical operators, in particular truth-functional propositional operators (and, or, if). Logically valid inferences are topographically very similar to invalid ones and don't show specific potentials, but the cluster permutation analysis shows clear connectivity differences in beta band. On the other hand, the connectivity analysis allows the definition of a propositional network with operculo-cingular, parietal and medial nodes. This network offers electrical evidence for "core" deductive medial elements, even if it also stresses the spatio-temporal overlap of deductive, semantical and visosemantical functions. The neuroelectric properties of the inference network (and not the normative properties of the corresponding arguments) explain why the network is described as propositional deductive. The results are limited to visual propositional inferences, even if the methodology can be extended in the future to verbal-analytic formats and to other kinds of non-propositional inferences.

## Author statement

L. F. Antón Toro: Data curation, Formal analysis, Validation, Review and editing. F. Salto: Conceptualization, Writing, Funding acquisition. C. Requena: Validation, investigation,

Visualization, Supervision. F. Maestú: Formal analysis, Resources, Project administration.

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## Open practices section

The study in this article earned Open Data and Open Materials badges for transparent practices. All data, materials and codes used in this research are available in open-source repositories: [https://osf.io/m47ex/?view\\_only=19f57b61e3f641b99ab2fc4e7cf5846d](https://osf.io/m47ex/?view_only=19f57b61e3f641b99ab2fc4e7cf5846d)

## Declaration of competing interest

Authors do not have any conflict of interest to declare.

## Supplementary data

Supplementary data to this article can be found online at <https://doi.org/10.1016/j.cortex.2023.06.004>.

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