Information Sciences 448–449 (2018) 187–214

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

The value of information as a verification and regret-preventing mechanism in algorithmic search environments

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A R T I C L E I N F O

Article history:
Received 11 September 2017
Revised 15 January 2018
Accepted 11 March 2018
Available online 14 March 2018

Keywords:
Value of information
Regret
Sequential search
Choice verification
Decision algorithm
Risk

A B S T R A C T

Information is a fundamental asset in any organization and as such is assigned a value reflecting its importance. We define information to be valuable if it either confirms the choice that a decision maker (DM) would make based on the information acquired on a given alternative or prevents him from making a regrettable choice, or both. We introduce a novel information acquisition algorithm where the value of information is used as a verification and regret-prevention mechanism determining the behavior of the DM. The proposed algorithm shows how the incentives of the DM to continue acquiring information on a given alternative are determined by his attitude toward regret and the relative spread exhibited by the domains on which the characteristics of the alternative are defined. Moreover, our model proves the existence of relative spread scenarios where indecision arises, leading the DM to behave randomly. The generality and flexibility of the model allows to easily develop extensions and applications to decision theory, psychology, economics and operational research.

1. Introduction

1.1. Motivation and literature review

Information is a fundamental asset in any organization and as such is assigned a value reflecting its importance. Intuitively, the value of information could be defined as the price that a decision maker (DM) is willing to pay to reduce the uncertainty faced in a given decision environment. However, the value that a DM assigns to a piece of information, together with its subsequent effect on his information acquisition and decision processes, differs considerably across the different literature branches analyzing sequential search environments. Decision theorists [25], psychologists [15,28], behavioral

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https://doi.org/10.1016/j.ins.2018.03.032

economists [6] and operational researchers [2,21] each adapt the concept of value of information to their respective settings and define it based on their specific needs.

Moreover, the quantity and quality of information available to DMs in business organizations conditions the quality of their decisions [23]. From a knowledge management perspective, the way DMs use information determines the performance of an organization [4]. In particular, project managers tend to overestimate their decision-making abilities [19], and refuse considering any potential improvement in their quality [11], an attitude that can result in avoidable wrong judgments.

For example, Hallen and Pahnke [12] describe the importance that limited information and bounded rationality have for the misevaluation of potential partners when analyzing their track records. Indeed, the acquisition of information can be described as a strategic managerial process that requires the cooperation of managers and information specialists [40]. Managers must select the information considered to be more useful from the large amounts to which they have access [22]. This selection process is highly important from a strategic perspective and requires the addition of both reputation [10] and regret [39] considerations to the analysis.

The effects that large amounts of available information have on the information acquisition and choice behavior of DMs can also be observed in online shopping environments [5,33]. The paradox of choice identified by Schwartz [29] is the most relevant among these effects. Park et al. [24] perform several experiments to illustrate how DMs exert cognitive effort to minimize negative emotions such as regret in their online and offline decision making processes. Schwartz [30] argues that DMs should focus on robust satisficing, i.e. trying to guarantee a good enough outcome, following the basic precepts of bounded rationality defined by Simon [31]. In this regard, Zeelenberg [41] suggests that robust satisficing may take place via regret minimization in everyday decision making.

The interaction between regret and the value of information is particularly evident in medical environments. For example, Johnson [14] highlights the fact that DMs confronted with health problems engage in low levels of information seeking or even avoid seeking any information. Fels [9] elicits the value of information when DMs consider potential disappointments after receiving negative news and are loss averse to changes in their beliefs. The author shows that the emotional impact of information depends on whether or not it determines future decisions. That is, DMs account for the potential dynamic consequences derived from receiving a given piece of information when determining its value.

Lerner et al. [16] and Bagozzi et al. [1] review the literature on emotion and decision making and suggest that, given the regularities observed regarding the effects that emotions have on judgments and choices, decision theorists should include emotions as drivers of decision making. In particular, decision theoretical models should acknowledge the fact that DMs aim at anticipating potential regrettable choices when making a decision [1,27,37].

The remainder of the paper proceeds as follows. Section 2 describes the main contribution of the model introduced in the paper. Section 3 provides the basic notations. Section 4 describes the proposed information acquisition setting. Section 5 introduces the concept of value of information as a verification and regret-preventing mechanism, shows how the value of a new piece of information can be determined through two well-defined real-valued functions, and defines the selection criterion to identify the best information acquisition option. Section 6 introduces the concept of dynamic incentive and describes the stopping criterion of the algorithm determined by the beliefs of the DM. Section 7 presents several numerical examples relative to the main formal results. Section 8 implements the model to a sequential information acquisition environments similar to those generally found in online evaluation settings. Section 9concludes and suggests potential applications.

2. Contribution

The information acquisition model introduced in the current paper accounts for the main features outlined above regarding the value of information, the regret-based incentives determining the behavior of the DM and the role played by emotions on his judgments and choices. To maintain notational consistency, we refer to choice objects as products throughout the paper.

Two important aspects relative to the structure of the information acquisition process must be emphasized. First, similarly to the operational research literature [20,32,36], the information acquisition process will be defined through a sequential algorithmic structure. However

- we do not focus our attention exclusively on the importance of search costs but on incrementing the number of characteristics of the products considered by the DM;
- we endow the DM with memory capacity so as to analyze the effect that the information acquired at any point through the process has on his subsequent behavior;
- different reference products both partially and completely observed will be defined and modified through the information acquisition process.

Second, information will not be valued using a purely economic approach [17], where, together with the utility derived from a given choice, a regret-rejoice function is defined to account for the differences derived from having chosen a given alternative over another. In particular

- we do not only consider the prevention of regret when determining the value of information but also its capacity to verify the suitability (or unsuitability) of a given product;
- we incorporate to the value of information the subjective importance assigned by the DM to the prevention of a regrettable choice relative to verifying its suitability;

Table 1
Comparing the capabilities of DMs across formal environments.

Empirical capabilities	Formal environments				
	Tavana et al. [34]	Di Caprio et al. [7]	Di Caprio et al. [8]	Santos-Arteaga et al. [26]	Current paper
Verification	√	√	√		√
Domain width		√	√	√	√
Regret				√	√
Memory	√				√
Risk attitude	√	√	√	√	√
Regret attitude					√
Subjective beliefs	√	√	√	√	√
Extended capabilities					
Signal updates			√		
Forward looking capacity	√				
Intertemporal combinations		√			

- the value of information is determined within a dynamic environment where all the potential combinations of the characteristics defining the alternatives being analyzed must be considered.

It should be emphasized that the value of information must be redefined at each step of the sequential information acquisition process, due to the fact that the reference products determining the behavior of the DM will be modified as information is acquired. Thus, given the information available at each step of the proposed algorithm, a new piece of information will be considered valuable if it either confirms the choice that the DM would make based on the information acquired on a given product or prevents him from making a regrettable choice, or both.

In particular, information is considered to be valuable at each step of the proposed algorithm if

- it allows the DM to verify that a given product provides a higher or a lower utility than the best product observed and used as a reference;
- it prevents the DM from either choosing a product that turns out to be worse than the one being used as a reference or rejecting a product that turns out to be better than the reference one.

At the same time, before proceeding to the next step of the algorithm, the DM has to compare the expected value derived from continuing acquiring information with the cost required to fully observe a potentially suitable product. Our information acquisition model shows how the incentives of the DM to continue acquiring information on a given product are determined by his attitude toward regret and the relative spread exhibited by the domains on which the product characteristics are defined. Moreover, our model proves the existence of relative spread scenarios where indecision arises, leading the DM to behave randomly.

Following the latest developments from the psychology-related literature outlined in the previous section, we endow the DM with a battery of abilities, including the capacity to recall previous observations and anticipate emotions, which determine his information acquisition and choice behavior. The resulting attitudes of the DM toward choice verification and regret prevention and their effect in determining his behavior are completely novel to the current paper.

Table 1 summarizes the main empirical-based properties considered by the closest information acquisition and evaluation models provided in the literature. These properties include the capacity of the DM to verify information for different widths of the domains on which the characteristics are defined while accounting for his subjective beliefs regarding their distribution. The ability of the DM to perform comparisons with previously observed products and his attitude toward risk are also generally considered. Moreover, several models have extended the cognitive abilities of the DM beyond the standard empirical-based ones and allowed for the assimilation of signals regarding the distribution of characteristics or the capacity to forecast any potential stream of future interdependent realizations and combine it with current ones before making a decision.

3. Preliminaries and basic notations

Let D be a DM and $\Gamma = \{G_i : i = 1, 2, \ldots, N\}$ denote the set of all products or alternatives. We assume that:

- the sub-index of each product coincides with the order in which it will be checked by D;
- each product is described by two main characteristics or attributes;
- D checks the characteristics of a product based on the order of dominance he establishes between them; that is, the first characteristic is more important to D than the second one.

We denote by X and Y the sets of all possible values that can be assigned to the first and second characteristic, respectively. Thus, the i-th product G_i in Γ corresponds to a pair (x_i, y_i) of the Cartesian product $X \times Y$, where x_i and y_i are the values of the first and second characteristic of G_i.

The symbols $\geq$ and $>$ denote the standard partial and linear order on the set R of all the real numbers. Also, given $a,\ b \in R$, with $a \neq b$, the symbols $(a,\ b)$, $(a,\ b]$, $[a,\ b)$, $[a,\ b]$ denote the open, the left half-open, the right half-open and the closed nonempty real interval of end-points a and b, respectively.

Following the classical economic approach to consumer information demand [38], we identify both X and Y with two closed and bounded real subintervals of $[0, +\infty)$, that is:

$$X = [x_m,\ x_M] \quad \text{and} \quad Y = [y_m,\ y_M] \tag{1}$$

where $x_m,\ x_M, y_m, y_M \in R$, $x_m \neq x_M$, $y_m \neq y_M$.

Thus, we assume the topology and the preference relation defined by D on both X and Y to be those induced by the standard Euclidean topology and the standard linear order $<$, respectively. As a consequence, D can define two strictly increasing continuous utility functions $u : X \to R$ and $v : Y \to R$ to represent his preferences on X and Y, respectively. That is:

$$\begin{aligned} &\forall x', x'' \in X, \quad x' > x'' \quad \Leftrightarrow \quad u(x') > u(x'') \\ &\forall y', y'' \in Y, \quad y' > y'' \quad \Leftrightarrow \quad v(y') > v(y'') \end{aligned} \tag{2}$$

Note that the ranges of u and v are closed real intervals since they are continuous images of closed real intervals.

Moreover, the sum function $U : X \times Y \to R$, defined by $U(x, y) = u(x) + v(y)$, $\forall (x, y) \in X \times Y$, allows D to define a preference relation $\succeq_U$ on $X \times Y$ as follows [8]:

$$\forall (x_1, y_1),\ (x_2, y_2) \in X \times Y,\ (x_1, y_1) \succeq_U (x_2, y_2) \overset{def}{\longleftrightarrow} U(x_1, y_1) \geq U(x_2, y_2) \tag{3}$$

We will refer to U as the "total utility function" . We also assume D to be endowed with a subjective probability density function over each factor space.

Abusing notation, both X and Y can be considered as two absolutely continuous random variables with probability density functions $\mu : X \to [0,\ 1]$ and $\eta : Y \to [0,\ 1]$, respectively. The functions μ and η can be interpreted as the subjective "beliefs" of D. More precisely, $\mu(Z)$ [resp. $\eta(Z)$] is the subjective probability that a randomly observed product displays as first [second] characteristic an element $x \in Z \subseteq X$ [resp. $y \in Z \subseteq Y$].

Without loss of generality, we can assume that the supports of the probability density functions correspond to the set of truly available values for the characteristics when the information gathering process starts. Thus, we assume that $Support(\mu) = X$ and $Support(\eta) = Y$.

Finally, following the standard economic theory of choice under uncertainty [18], we assume that D assigns to each unknown characteristic of a product, either the first or the second, the certainty equivalent value induced by the corresponding subjective probability density function.

We denote by c_X and c_Y the certainty equivalents associated with $(\mu,\ u)$ and $(\eta,\ v)$, respectively. That is, c_X is the element of X whose utility equals the expected utility value induced by μ:

$$u(c_X) = E_X, \quad \text{where } E_X = \int_X \mu(x) u(x) dx \tag{4}$$

Similarly for c_Y:

$$v(c_Y) = E_Y, \quad \text{where} \quad E_Y = \int_Y \eta(y) v(y) dy \tag{5}$$

The existence and uniqueness of the certainty equivalent values c_X and c_Y are guaranteed by the continuity and strict increasingness of u and v, respectively. The use of certainty equivalent values implies that if the known characteristics deliver a higher (lower) utility than the corresponding subjective certainty equivalent value, then D prefers the product defined by the former (latter) one.

The sum $U_{rand} = E_X + E_Y$ is the utility value that D associates to any randomly chosen product in Γ different from those that he has already checked completely or partially. This product will be denoted by *Rand*.

The main notations introduced in this section and those that will be introduced through the following ones are summarized in Table 2.

4. Proposed information acquisition setting

The algorithm that we propose for D to optimally collect information on the products composing the set Γ in a sequential way builds on the non-recursive setting of Tavana et al. [34,35].

At each step of the algorithm, D can collect one piece of information. The algorithm initializes with D checking the first characteristic x_1 of the first product, G_1. Since the products are described by two characteristics, D needs to decide at each step whether it is better to finish collecting information on a partially observed product or to start checking a new product. Moreover, before actually pursuing either one of these two options, D must consider whether the costs from acquiring additional information (and move on to the next step of the algorithm) are higher than the value expected to be obtained from this information. In other words, at each step, D needs:

Table 2
Notations and nomenclature.

D	decision maker (DM)
Γ	set of all products or alternatives
G_i	ith product to be checked by D; $i = 1, 2, ..., N$
x_i	value of the first characteristic of G_i; $i = 1, 2, ..., N$
y_i	value of the second characteristic of G_i; $i = 1, 2, ..., N$
$X = [x_m, x_M]$	set of all possible values that can be taken by the first characteristic
$Y = [y_m, y_M]$	set of all possible values that can be taken by the second characteristic
u	strictly increasing continuous utility functions on X
v	strictly increasing continuous utility functions on Y
U	total utility function (i.e. the sum of u and v)
$\succeq_U$	preference relation on $X \times Y$ induced by U
μ	subjective "beliefs" of D on X
η	subjective "beliefs" of D on Y
c_X	certainty equivalent associated with (μ, u)
c_Y	certainty equivalent associated with (η, v)
$E_X = u(c_X)$	expected utility value induced by μ
$E_Y = v(c_Y)$	expected utility value induced by η
Rand	random product different from those checked completely or partially
$U_{rand} = E_X + E_Y$	total utility of *Rand*
$k = k\left(j; \begin{array}{l}\text{"info sequence from}\\ \text{Step 1 to Step } j-1\text{"}\end{array}\right)$	total number of products observed at Step j
$\bar{x}_k$	highest 1st characteristic value of the products partially observed at Step j
$\bar{G}_k$	product whose 1st characteristic takes the value $\bar{x}_k$
$h = h(j)$	total number of products completely observed at Step j
Γ_j	set of all products completely observed at Step j
$\hat{\Gamma}_j$	set of all $G_i \in \Gamma_j$ such that $u(x_i) + v(y_i) \geq U_{rand}$
$G_j^{opt} = (x_j^{opt}, y_j^{opt})$	product in $\Gamma_j \cup \{Rand\}$ with the highest total utility
$U_j^{opt} = u(x_j^{opt}) + v(y_j^{opt})$	total utility of G_j^{opt} at Step j
Option (I) at Step j	*Continue with* $\bar{G}_k$: check the value $\bar{y}_k$ of the 2nd characteristic of $\bar{G}_k$
Option (II) at Step j	*Start with* G_{k+1}: check the value x_{k+1} of the 1st characteristic of G_{k+1}
B O at Step j	best option between Option (I) and Option (II) at Step j
$P_j^+(\bar{x}_k)$; $P_j^-(\bar{x}_k)$	reference intervals to evaluate Option (I) at Step j
$Q_j^+(\bar{x}_k)$; $Q_j^-(\bar{x}_k)$	reference intervals to evaluate Option (II) at Step j
$V_j^I(\bar{x}_k)$	expect information value of Option (I) at Step j
$V_j^{II}(\bar{x}_k)$	expect information value of Option (II) at Step j
σ_j	dynamic incentive at Step j

- a criterion to identify what information to acquire (selection criterion);
- a criterion to decide whether to acquire the selected information or stop (stopping criterion).

The structure of the proposed algorithm is described below. The decision and stopping criteria that allow D to decide at each step which characteristics he should check and whether or not it deserves to check them are introduced in the next section.

Step 1. Initialization
D checks the value x_1 of the 1st characteristic of G_1.
Step 2. D knows x_1
D has two options:
Option (I). *Continue with* G_1: check the value y_1 of the 2nd characteristic of G_1.
Option (II). *Start with* G_2: check the value x_2 of the 1st characteristic of G_2.
Selection Criterion:
D applies the selection criterion to compare Options (I) and (II)
D identifies the best option **BO**
Stopping Criterion:
D applies the stopping criterion to decide whether to pursue **BO**
If the stopping criterion is satisfied,
$>>$ D stops
Else,
$>>$ D **acquires the information described by BO**
$>>$ D **goes to the next step.**
Step 3. There are two cases to consider
Step 3.1. D knows $G_1 = (x_1, y_1)$
D has only one option:
Option (II). *Start with* G_2: check the value x_2 of the 1st characteristic of G_2.

Stopping Criterion:
D applies the stopping criterion to decide whether to pursue Opt (II)
If the stopping criterion is satisfied,
>> D stops
Else,
>> D **gathers the information described by Option (II)**
>> D **goes to next step**

Step 3.2. D knows x_1 and x_2
Let $x_2^* = \max\{x_1, x_2\}$ and G_2^* be the corresponding product.
Note that $x_2^* = \max\{\{x_1, x_2\}\backslash Z_3\}$ where $Z_3 \stackrel{def}{=} \{x_i : G_i$ is completely known at Step 3 $\} = \emptyset$.
D has two options:
Option (I). *Continue with* G_2^*: check the value y_2^* of the 2nd characteristic of G_2^*.
Option (II). *Start with* G_3: check the value x_3 of the 1st characteristic of G_3.
>> D applies the selection criterion.
>> D applies the stopping criterion.

Step 4. There are three cases to consider

Step 4.1. D knows $G_1 = (x_1, y_1)$ and x_2
D has two options:
Option (I). *Continue with* G_2: check the value y_2 of the 2nd characteristic of G_2.
Option (II). *Start with* G_3: check the value x_3 of the 1st characteristic of G_3.
>> D applies the selection criterion.
>> D applies the stopping criterion.

Step 4.2. D knows $G_2^* = (x_2^*, y_2^*)$ and the value of the characteristic in $\{x_1, x_2\}\backslash\{x_2^*\}$.
Let $\bar{x}_2$ be the value in $\{x_1, x_2\}\backslash\{x_2^*\}$ and $\bar{G}_2$ be the corresponding product.
Note that $\bar{x}_2 = \max\{\{x_1, x_2\}\backslash Z_4\}$ where $Z_4 \stackrel{def}{=} \{x_i : G_i$ is completely known at Step 4 $\} = \{x_2^*\}$.
D has two options:
Option (I). *Continue with* $\bar{G}_2$: check the value $\bar{y}_2$ of the 2nd characteristic of $\bar{G}_2$.
Option (II). *Start with* G_3: check the value x_3 of the 1st characteristic of G_3.
>> D applies the selection criterion.
>> D applies the stopping criterion.

Step 4.3. D knows x_1, x_2 and x_3
Let $x_3^* = \max\{x_1, x_2, x_3\}$ and G_3^* be the corresponding product.
Note that $x_3^* = \max\{\{x_1, x_2, x_3\}\backslash Z_4\}$ where $Z_4 \stackrel{def}{=} \{x_i : G_i$ is completely known at Step 4 $\} = \emptyset$.
D has two options:
Option (I). *Continue with* G_3^*: check the value y_3^* of the 2nd characteristic of G_3^*.
Option (II). *Start with* G_4: check the value x_4 of the 1st characteristic of G_4.
>> D applies the selection criterion.
>> D applies the stopping criterion.

Step 5. There are six cases to consider

Step 5.1. D knows $G_1 = (x_1, y_1)$ and $G_2 = (x_2, y_2)$
D has only one option:
Option (II). *Start with* G_3: check the value x_3 of the 1st characteristic of G_3.
>> D applies the stopping criterion.

Step 5.2. D knows $G_1 = (x_1, y_1)$, x_2 and x_3.
Let $\bar{x}_3 = \max\{x_2, x_3\}$ and $\bar{G}_3$ be the corresponding product.
Note that $\bar{x}_3 = \max\{\{x_1, x_2, x_3\}\backslash Z_5\}$ where $Z_5 \stackrel{def}{=} \{x_i : G_i$ is completely known at Step 5 $\} = \{x_1\}$.
D has two options:
Option (I). *Continue with* $\bar{G}_3$: check the value $\bar{y}_3$ of the 2nd characteristic of $\bar{G}_3$.
Option (II). *Start with* G_4: check the value x_4 of the 1st characteristic of G_4.
>> D applies the selection criterion.
>> D applies the stopping criterion.

Step 5.3. D knows $G_2^* = (x_2^*, y_2^*)$ and $\bar{G}_2 = (\bar{x}_2, \bar{y}_2)$
Then, D knows $G_1 = (x_1, y_1)$ and $G_2 = (x_2, y_2)$
>> **see Step 5.1.**

Step 5.4. D knows $G_2^* = (x_2^*, y_2^*)$, $\bar{x}_2$ and x_3
Then, either D knows $G_1 = (x_1, y_1)$, x_2 and x_3, or D knows x_1, $G_2 = (x_2, y_2)$ and x_3
If D knows $G_1 = (x_1, y_1)$, x_2 and x_3
>> **see Step 5.2.**
If D knows x_1, $G_2 = (x_2, y_2)$ and x_3

Let $\bar{x}_3 = \max\{x_1, x_3\}$ and $\bar{G}_3$ be the corresponding product.
D has two options:
Option (I). *Continue with* $\bar{G}_3$: check the value $\bar{y}_3$ of the 2nd characteristic of $\bar{G}_3$.
Option (II). *Start with* G_4: check the value x_4 of the 1st characteristic of G_4.
>> D applies the selection criterion.
>> D applies the stopping criterion.

Step 5.5. D knows $G_3^* = (x_3^*, y_3^*)$ and the value of the characteristics in $\{x_1, x_2, x_3\}\backslash\{x_3^*\}$
Let $\bar{x}_3 = \max\{\{x_1, x_2, x_3\}\backslash\{x_3^*\}\}$ and $\bar{G}_3$ be the corresponding product.
Note that $\bar{x}_3 = \max\{\{x_1, x_2, x_3\}\backslash Z_5\}$ where $Z_5 \overset{def}{=} \{x_i : G_i \text{ is completely known at Step 5 }\} = \{x_3^*\}$.
D has two options:
Option (I). *Continue with* $\bar{G}_3$: check the value $\bar{y}_3$ of the 2nd characteristic of $\bar{G}_3$
Option (II). *Start with* G_4: check the value x_4 of the 1st characteristic of G_4
>> D applies the selection criterion.
>> D applies the stopping criterion.

Step 5.6. D knows x_1, x_2, x_3 and x_4
Let $x_4^* = \max\{x_1, x_2, x_3, x_4\}$ and G_4^* be the corresponding product.
Note that $x_4^* = \max\{\{x_1, x_2, x_3, x_4\}\backslash Z_5\}$ where $Z_5 \overset{def}{=} \{x_i : G_i \text{ is completely known at Step 5 }\} = \emptyset$.
D has two options:
Option (I). *Continue with* G_4^*: check the value y_4^* of the 2nd characteristic of G_4^*.
Option (II). *Start with* G_5: check the value x_5 of the 1st characteristic of G_5.
>> D applies the selection criterion.
>> D applies the stopping criterion.

Fig. 1 provides a graphical representation of Steps 1–5 of the algorithm.

Remark 1. The structure of the algorithm implies that, at Step j, the maximum number of products about which information has been acquired equals $j-1$ while the minimum number corresponds to the ceiling value $\lceil \frac{j-1}{2} \rceil$. That is, when D reaches the generic Step j, he has observed at most $j-1$ products partially and at least $\lceil \frac{j-1}{2} \rceil$ products completely. In other words, he has checked either the first or both characteristics of the first k products $G_1, G_2, ..., G_k$, where $k \le j-1$. □

Remark 2. Suppose that D is in Step j after having acquired at least one piece of information about each of the first k products $G_1, G_2, ..., G_k$. The products G_k^* and $\bar{G}_k$ do not need to coincide with the optimal product available when D reaches Step j. They represent the products with the maximal first characteristic among those that D has checked completely (both characteristics) and partially (only the first characteristic), respectively. These products coincide when only the first characteristic has been checked for all products. In the algorithm, they are used to identify the continuation and starting options. They are not used as a reference point to optimize the utility of D. The goal of the algorithm at each step does not consist of choosing the best product, but selecting the product that deserves to be checked next. □

The total number of products, k, that D has observed at a certain Step j depends not only on the number of steps that has been performed but also on the specific sequence of information that has been acquired. That is, k depends both on j and the sequence of information acquired from Step 1 to Step $j-1$, and should be denoted as $k(j;$ "info sequence from Step 1 to Step $j-1$"). However, to simplify notations we will keep on using k.

At a generic Step j, D can face different potential situations but they all fall in one of the two general subcases described below.

Step *j*.
D knows both characteristics of h products among $G_1, G_2, ..., G_k$, where $k < j$, and only the first characteristic of the remaining $k-h$ products.
Step *j*.1. if $h = k$
D has only one option:
Option (II). *Start with* G_{k+1}: check the value x_{k+1} of the 1st characteristic of G_{k+1}.
>> D applies the stopping criterion.
Step *j*.2. if $h < k$
let $\Gamma_j \overset{def}{=} \{G_i : x_i \text{ and } y_i \text{ are both known at Step j }\}$, $Z_j \overset{def}{=} \{x_i : G_i \in \Gamma_j\}$,
$\bar{x}_k = \max\{\{x_1, x_2, ..., x_k\}\backslash Z_j\}$ and $\bar{G}_k$ be the corresponding product.
D has two options:
Option (I). *Continue with* $\bar{G}_k$: check the value $\bar{y}_k$ of the 2nd characteristic of $\bar{G}_k$.
Option (II). *Start with* G_{k+1}: check the value x_{k+1} of the 1st characteristic of G_{k+1}.
>> D applies the selection criterion.
>> D applies the stopping criterion.

Remark 3. Note that, in the generic Step j, the following chain of equivalences holds:

$$h = 0 \quad \leftrightarrow \quad \Gamma_j = \emptyset \leftrightarrow \quad Z_j = \emptyset \leftrightarrow$$
$$x_k^* = \max\{x_1, x_2, ..., x_k\} = \max\left\{\{x_1, x_2, ..., x_k\}\backslash Z_j\right\} = \bar{x}_k \quad \leftrightarrow \quad G_k^* = \bar{G}_k \quad \square$$

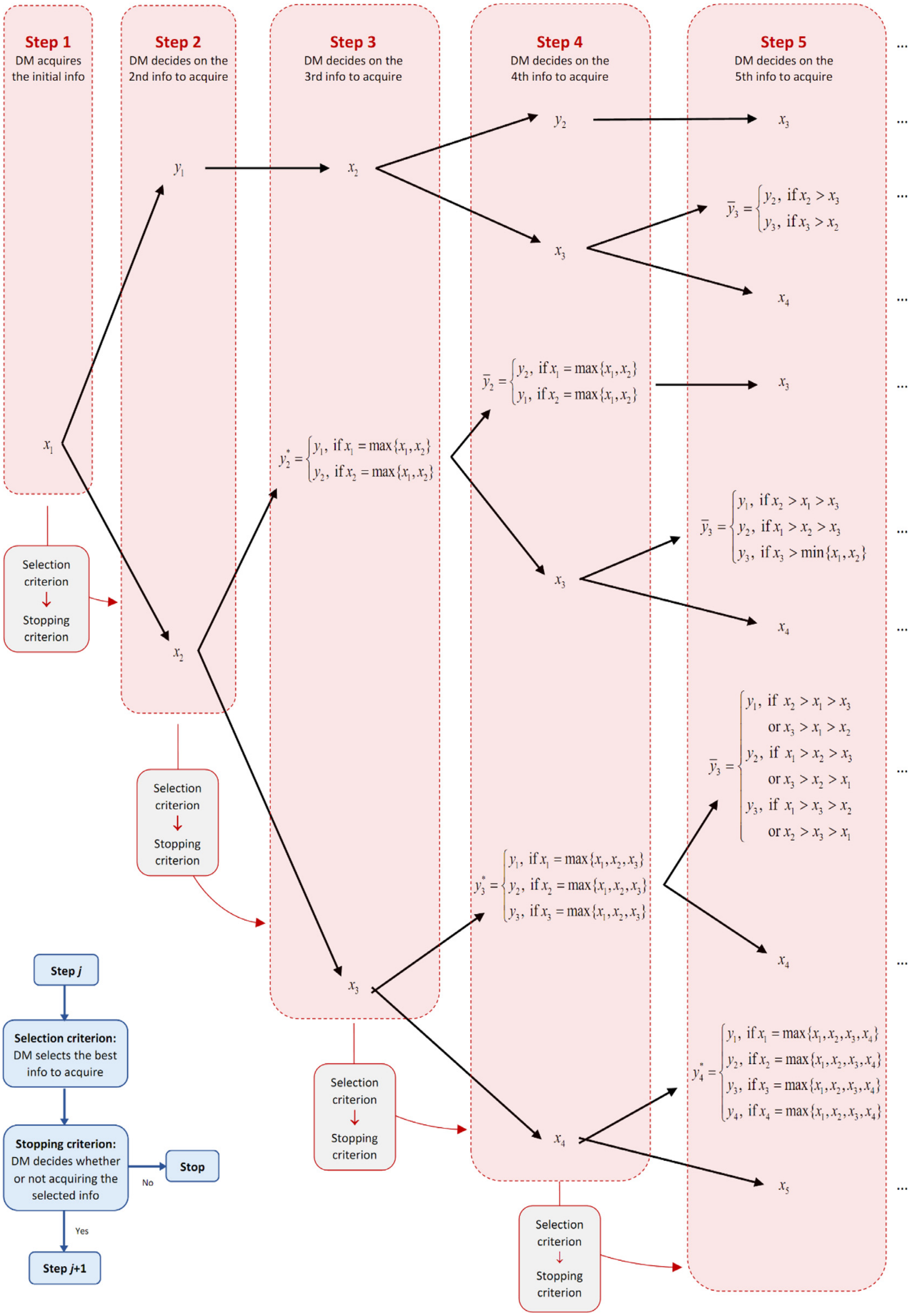


Fig. 1. The structure of the proposed algorithm: Steps 1 to 5.

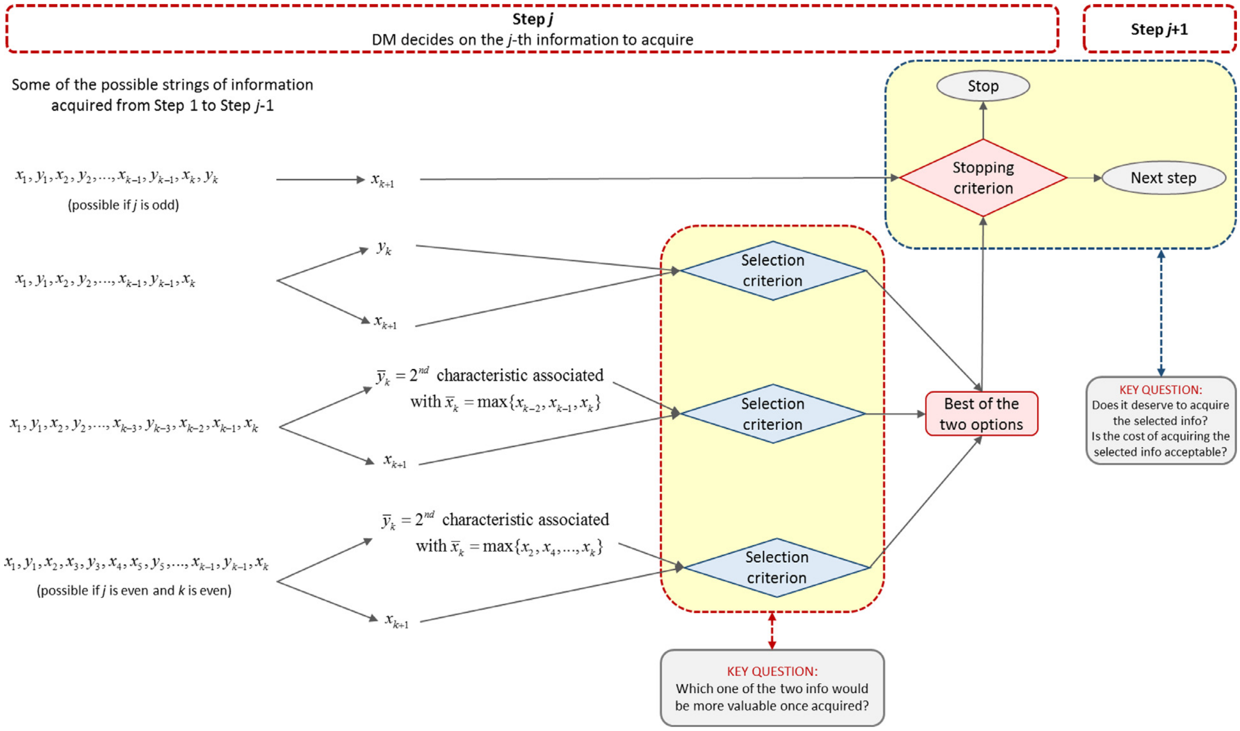


Fig. 2. The structure of the proposed algorithm: from Step j to Step $j+1$.

Fig. 2 illustrates some of the possible situations that D could face at a generic Step j, together with the corresponding options and the criteria to apply in order to decide whether to stop or move on to the next step.

To add further intuition, Fig. 3 provides three examples of possible situations that D could face after having acquired information on the first five products. Fig. 3.a shows an instance of Step 11 of the algorithm corresponding to the case with $h = k = 5$. If $h = k = 5$, D has checked both characteristics from the first five products, that is, he has acquired a total of 10 pieces of information and reached Step 11, where he needs to decide on the 11-th information to acquire. Similarly, Fig. 3.b depicts Step 6 when $h = 0$ and $k = 5$. The fact that $h = 0$ implies that D has checked only the first characteristic of the first five products, that is, he has acquired a total of 5 pieces of information and is in Step 6. Finally, Fig. 3.c refers to Step 9 in the case where $h = 3$ and $k = 5$. In this case, $h = 3$ means that D has checked both characteristics from three products out of the first five ones, acquiring a total of 8 pieces of information.

Remark 4. The examples presented in Fig. 3 emphasize two important facts concerning the products already observed at Step j:

(a) G_k^* and $\bar{G}_k$ coincide when $Z_j = \emptyset$ (see also Remark 3);
(b) G_k^* does not need to coincide with G_j^{opt}, that is, the optimal product available at Step j (see also Remark 2). □

Remark 5. We would like to emphasize that the information acquisition environment proposed in this study cannot be formalized as an optimization problem. The proposed algorithm does not merely search for the product with the highest utility value. What the algorithm does at each step is to:

(a) systematically identify the new pieces of information that should be checked;
(b) determine the value that acquiring each of the new pieces of information would have for D *before* he actually acquires them;
(c) consider the cost that the acquisition of the new information would impose on D so as to allow for a stopping mechanism. □

5. Selection criterion to decide between option (I) and option (II)

In this section, we design a selection criterion for D to compare Option (I) with Option (II) whenever facing both within a given step of the algorithm described in Section 4. To do so, we build on the expected information value maximization approach introduced by Santos-Arteaga et al. [26].

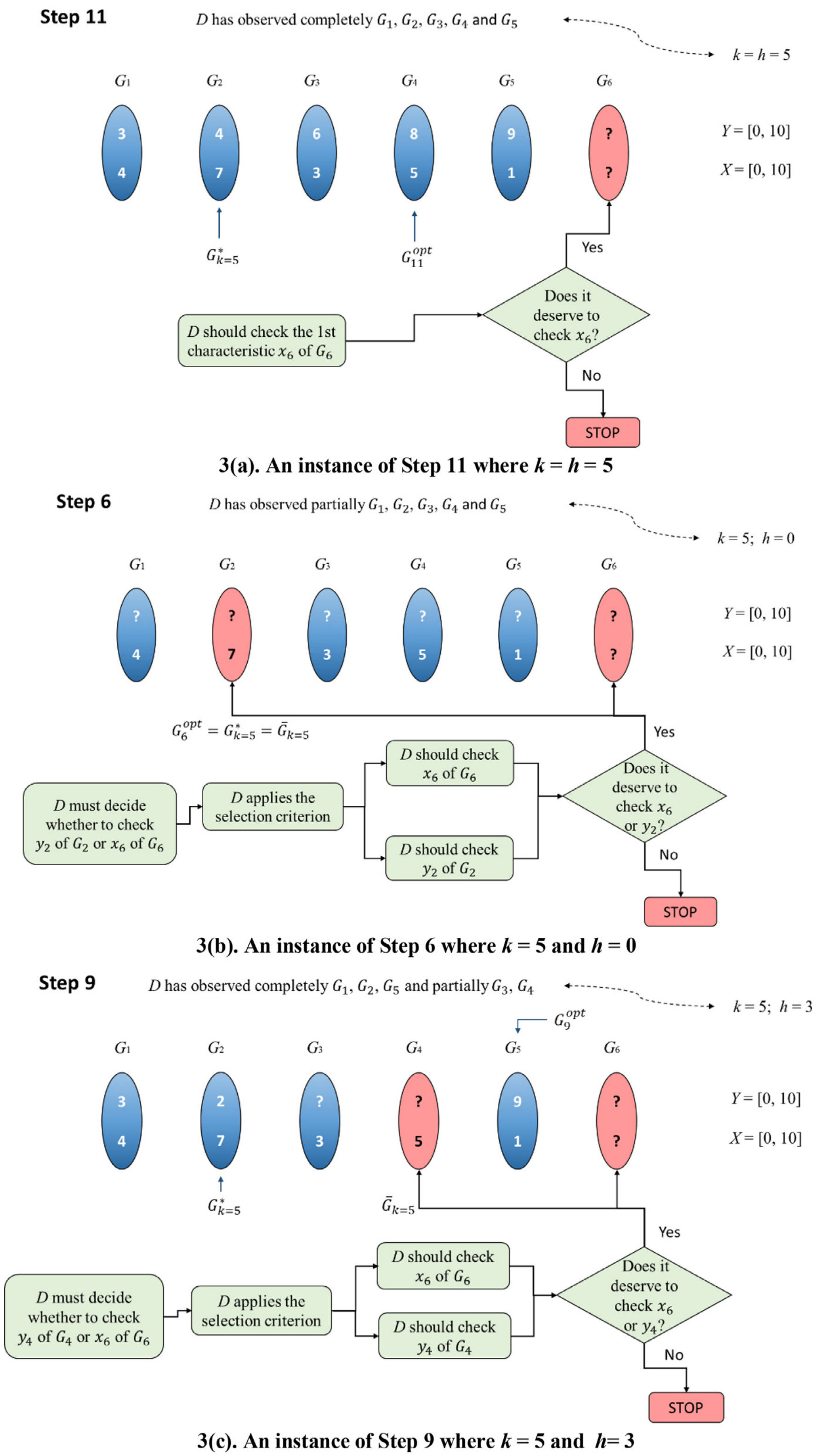


Fig. 3. Three examples of possible situations after acquiring information on the first five products.

5.1. Defining the expected information value of option (I) and option (II)

To fix the ideas, consider the generic Step j. Assume that D has acquired $j-1$ pieces of information and knows the values of both characteristics of h among the first k products $G_1, G_2, ..., G_k$, where $h < k < j$, and only the first characteristic of the remaining $k-h$ products.

Recall that $Z_j \overset{def}{=} \{x_i : G_i \in \Gamma_j\}$ and $\Gamma_j \overset{def}{=} \{G_i :\ x_i$ and y_i are both known at Step j $\}$.

D has the following two options:

- **Option (I).** *Continue with* $\bar{G}_k$: check the value $\bar{y}_k$ of the 2nd characteristic of $\bar{G}_k$
- **Option (II).** *Start with* G_{k+1}: check the value x_{k+1} of the 1st characteristic of G_{k+1}

where $\bar{G}_k$ is the product among $G_1, G_2, ..., G_k$ whose first characteristic takes the value $\bar{x}_k = \max\{\{x_1, x_2, ..., x_k\}\backslash Z_j\}$.

5.1.1. Introducing reference intervals

To define the functions that will allow D to evaluate the expected value of the information to acquire, we need to introduce some additional notations.

- $\hat{\Gamma}_j \overset{def}{=} \{G_i \in \Gamma_j : u(x_i) + v(y_i) \geq U_{rand}\}$
- $G_j^{opt} \overset{def}{=} \begin{cases} \text{a product in } \hat{\Gamma}_j \text{ with the highest total utility}, & \text{if } \hat{\Gamma}_j \neq \emptyset \\ Rand, & \text{otherwise} \end{cases}$
- x_j^{opt} and y_j^{opt} are the first and second characteristic of G_j^{opt}, respectively.
- $U_j^{opt} = u(x_j^{opt}) + v(y_j^{opt})$ is the total utility of G_j^{opt}.

That is, G_j^{opt} is the product with the highest total utility among those completely checked at Step j and *Rand*. Given the dynamic character of the proposed algorithm, D has to adapt his reference improvement values as observations are acquired. Thus, G_j^{opt} will be used as a reference product by D to determine the value of any new information at Step j.

- $S_j \overset{def}{=} \begin{cases} \{x \in X : \exists y^{|x} \text{such that} \quad v(y_j^{|x}) = U_{rand} - u(x)\}, & if\, j = 2 \\ \{x \in X : \exists y_j^{|x} \text{such that} \quad v(y_j^{|x}) = U_j^{opt} - u(x)\}, & if\, j > 2 \end{cases}$

S_j is the set of all values x of the first characteristic for which there exists a value $y_j^{|x}$ of the second characteristic such that D is indifferent between the product $G = (x, y_j^{|x})$ and *Rand*, if he is in Step 2, or between $G = (x, y_j^{|x})$ and G_j^{opt}, if he is in Step j, with $j > 2$.

To simplify notations, we will write $y^{|x}$ in place of $y_2^{|x}$ whenever it is clear that we refer to Step 2 (i.e. to the case $j = 2$).

Trivially, $c_X \in S_j$ if $j = 2$, while $x_j^{opt} \in S_j$ if $j > 2$. The fact that S_j is non-empty guarantees that D can use *Rand* and G_j^{opt} as reference products when measuring the value of information associated with Option (I) in Step 2 and Step j, respectively.

Proposition 1. The set S_j is a closed interval of X. That is, there exist $s_{jm},\ s_{jM} \in X$ such that $S_j = [s_{jm},\ s_{jM}]$.

Proof. We prove the statement for the case $j = 2$. The case $j > 2$ can be proved in a similar way by replacing U_{Rand} with U_j^{opt}.

First, we show that S_2 is an interval. Let $x',\ x''$ be two elements of S_2. It suffices to show that if $x \in (x',\ x'')$, then $x \in S_2$. Note that $x' < x < x'' \Leftrightarrow u(x') < u(x) < u(x'') \Leftrightarrow \exists\, y^{|x'},\ y^{|x''} \in Y$ such that $v(y^{|x''}) = U_{rand} - u(x'') < U_{rand} - u(x) < U_{rand} - u(x') = v(y^{|x'})$. Also, note that the range of v is a closed interval, since it is the continuous image of a closed interval. Hence, there exists y such that $v(y) = U_{rand} - u(x)$ which proves that $x \in S_2$.

Now, we show that S_2 is closed. Let $\{x_n\}_{n\in N}$ be a sequence in S_2 converging to an element x. It suffices to show that $x \in S_2$. Recall that continuous functions map converging sequences into converging sequences. Thus, since u is continuous, the sequence $\{u(x_n)\}_{n\in N}$ must converge to $u(x)$. It follows that $\{U_{rand} - u(x_n)\}_{n\in N}$ converges to $U_{rand} - u(x)$. At the same time, for each x_n there exists $y^{|x_n}$ such that $v(y^{|x_n}) = U_{rand} - u(x_n)$. Hence, we obtain a sequence of elements of Y whose image sequence $\{v(y^{|x_n})\}_{n\in N}$ converges to $U_{rand} - u(x)$. Since v is strictly increasing and continuous, so is its inverse v^{-1}. It follows that there exists $\hat{y} \in Y$ such that $\{y^{|x_n}\}_{n\in N}$ converges to $\hat{y}$ and $v(\hat{y}) = U_{rand} - u(x)$. Therefore, $x \in S_2$. □

Proposition 2. (a) If $x \in S_2$, there exists a unique $y^{|x} \in Y$ such that $v(y^{|x}) = U_{rand} - u(x)$.

(b) If $x \in S_j$, with $j > 2$, there exists a unique $y_j^{|x} \in Y$ such that $v(y_j^{|x}) = U_j^{opt} - u(x)$.

Proof. If follows from the fact that u and v are strictly increasing. □

Thus, Proposition 2 yields the following.

Proposition 3. The function $\phi_j : X \to Y$ such that for every $x \in X$,

$$\phi_j(x) = \begin{cases} y_M & \text{if} \quad x \in [x_m,\ s_{jm}) \\ y_j^{|x} & \text{if} \quad x \in S_j \\ y_m & \text{if} \quad x \in (s_{jM},\ x_M] \end{cases} \tag{6}$$

is well-defined at Step j. □

It is trivial to see that S_j does not necessarily coincide with X, that is, it might exist x such that for all $y \in Y$, $u(x) + v(y) \neq U_{rand}$ or $u(x) + v(y) \neq U_j^{opt}$. The definition of ϕ_j accounts for this case by associating to such x either the lowest or highest value in Y by default. The reason for this choice is explained below.

Suppose, for instance, that in Step 2, $x_1 \in [x_m,\ s_{2m})$. Then, for every $y \in Y$, $u(x_1) + v(y) < U_{rand}$, that is, no value of the second characteristic suffices for G_1 to deliver the same total utility as *Rand*. In particular, $u(x_1) + v(y_M) < U_{rand}$. Thus, the product closer in utility terms to *Rand* would be the one whose second characteristic value is y_M. Similarly, suppose that, in Step 2, $x_1 \in (s_{2M},\ x_M]$. Then, for every $y \in Y$, $u(x_1) + v(y) > U_{rand}$. It follows that $u(x_1) + v(y_m) > U_{rand}$ and, hence, the product closer in utility terms to *Rand* would be the one whose second characteristic value is y_m.

At Step j of the algorithm, the set S_j and the function ϕ_j allow us to define the following "reference intervals" .

$$P_j^+(\bar{x}_k) \overset{def}{=} (\phi_j(\bar{x}_k),\ y_M] = \begin{cases} \emptyset, & \text{if} \quad \bar{x}_k \in [x_m,\ s_{jm}) \\ (y_j^{|\bar{x}_k},\ y_M], & \text{if} \quad \bar{x}_k \in S_j \\ (y_m,\ y_M], & \text{if} \quad \bar{x}_k \in (s_{jM},\ x_M] \end{cases} \tag{7}$$

and

$$P_j^-(\bar{x}_k) \overset{def}{=} [y_m, \phi_j(\bar{x}_k)) = \begin{cases} [y_m,\ y_M), & \text{if } \bar{x}_k \in [x_m,\ s_{jm}) \\ [y_m,\ y_j^{\bar{x}_k}), & \text{if } \bar{x}_k \in S_j \\ \emptyset, & \text{if } \bar{x}_k \in (s_{jM},\ x_M] \end{cases} \tag{8}$$

In particular, at Step 2 of the algorithm, we have $\bar{x}_k = x_1$. That is, $P_2^+(x_1) = (\phi_2(x_1),\ y_M]$ and $P_2^-(x_1) = [y_m,\ \phi_2(x_1))$.

The intervals $P_j^+(\bar{x}_k)$ and $P_j^-(\bar{x}_k)$ contain all the values y of the second characteristic of $\bar{G}_k$ that D should observe for the total utility $U(\bar{x}_k,\ y) = u(\bar{x}_k) + v(y)$ of $\bar{G}_k$ to be respectively higher or lower than the total utility of the reference product at Step j (i.e. U_{Rand}, if D is in Step 2, and U_j^{opt} if D is in Step j, with $j > 2$). Clearly, $P_j^+(\bar{x}_k)$ and $P_j^-(\bar{x}_k)$ form a partition of Y. Intuitively speaking, $P_j^+(\bar{x}_k)$ and $P_j^-(\bar{x}_k)$ provide D with the reference intervals required to evaluate Option (I).

Finally, at Step j of the algorithm, let:

$$Q_j^+(\bar{x}_k) \overset{def}{=} (\max\{\bar{x}_k,\ c_X\},\ x_M] \tag{9}$$

and

$$Q_j^-(\bar{x}_k) \overset{def}{=} [x_m,\ \max\{\bar{x}_k,\ c_X\}). \tag{10}$$

In particular, at Step 2, we have $\bar{x}_k = x_1$. That is, $Q_2^+(x_1) = (\max\{x_1,\ c_X\},\ x_M]$ and $Q_2^-(x_1) = [x_m,\ \max\{x_1,\ c_X\})$.

The intervals $Q_j^+(\bar{x}_k)$ and $Q_j^-(\bar{x}_k)$ contain all the values x of the first characteristic of the next product, G_{k+1}, that provide a utility respectively higher or lower than the reference value for the first characteristic at Step j (i.e. $\max\{\bar{x}_k,\ c_X\}$). $Q_j^+(\bar{x}_k)$ and $Q_j^-(\bar{x}_k)$ form a partition of X and provide D with the reference intervals required in order to evaluate Option (II).

Fig. 4 represents the reference intervals $P_j^+(\bar{x}_k)$, $P_j^-(\bar{x}_k)$, $Q_j^+(\bar{x}_k)$ and $Q_j^-(\bar{x}_k)$ at the generic Step j both when $\bar{x}_k < c_X$ (Fig. 4.a) and when $\bar{x}_k > c_X$ (Fig. 4.b).

5.1.2. Measuring the expected value of new information

To establish whether or not a piece of information is valuable and measure its value we build on the criterion introduced by Santos-Arteaga et al. [26]. According to their criterion, a new piece of information has value only if, by acquiring it, D modifies the decision that he would have made without it. We extend this concept of valuable information as follows.

Definition 1. At each step of the proposed algorithm, a new piece of information is valuable if it either confirms the choice that D would make based on the information acquired on the first characteristic of the product being considered (verification) or prevents D from making a regrettable choice (prevention), or both. □

Thus, information is considered valuable at the generic Step j, $j \geq 2$, of the algorithm if

- *Verification*: it allows D to confirm that a given product provides a higher or a lower utility than the reference product, G_j^{opt};
- *Prevention*: it prevents D from either choosing a product that turns out to be worse than the reference product, G_j^{opt}, or rejecting a product that turns out to be better than G_j^{opt}.

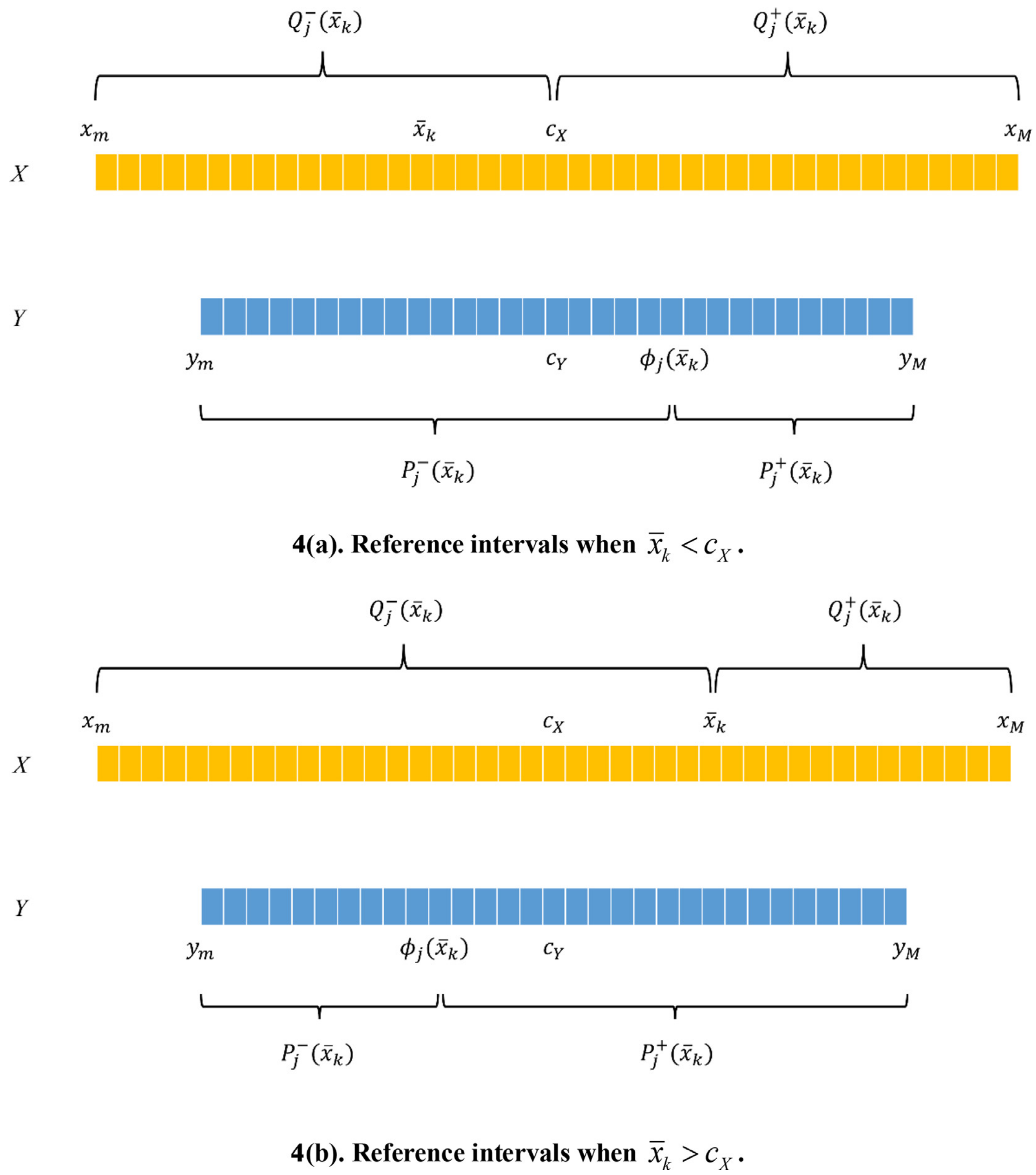


Fig. 4. Reference intervals to evaluate Option (I) and Option (II) at Step j.

Therefore, we propose the following definition for the value of information when acquiring a new piece of information.

Definition 2. The expected information value of Option (I) at the generic Step j, $j \geq 2$, is defined as follows.

$$V_j^I(\bar{x}_k) = \begin{cases} \underbrace{\alpha \int_{P_j^+(\bar{x}_k)} \eta(y)\left(u(\bar{x}_k) + v(y) - U_j^{opt}\right) dy}_{\text{Prevention}} + \underbrace{(1-\alpha) \int_{P_j^-(\bar{x}_k)} \eta(y)\left(U_j^{opt} - u(\bar{x}_k) - v(y)\right) dy}_{\text{Verification}}, & \text{if } \bar{x}_k < x_j^{opt} \\ \underbrace{\alpha \int_{P_j^-(\bar{x}_k)} \eta(y)\left(U_j^{opt} - u(\bar{x}_k) - v(y)\right) dy}_{\text{Prevention}} + \underbrace{(1-\alpha) \int_{P_j^+(\bar{x}_k)} \eta(y)\left(u(\bar{x}_k) + v(y) - U_j^{opt}\right) dy}_{\text{Verification}}, & \text{if } \bar{x}_k \geq x_j^{opt} \end{cases} \quad (11)$$

The expected information value of Option (II) at the generic Step j is defined as follows.

$$V_j^{II}(\bar{x}_k) = \begin{cases} \underbrace{\alpha \int_{Q_j^+(\bar{x}_k)} \mu(x)(u(x) - E_X)\,dx}_{\text{Prevention}} + \underbrace{(1-\alpha)\int_{Q_j^-(\bar{x}_k)} \mu(x)(E_X - u(x))\,dx}_{\text{Verification}}, & \text{if } \bar{x}_k < c_X \\ \underbrace{\alpha \int_{Q_j^+(\bar{x}_k)} \mu(x)(u(x) - u(\bar{x}_k))\,dx}_{\text{Prevention}} + \underbrace{(1-\alpha)\int_{Q_j^-(\bar{x}_k)} \mu(x)(u(\bar{x}_k) - u(x))\,dx}_{\text{Verification}}, & \text{if } \bar{x}_k \geq c_X \end{cases} \tag{12}$$

In Eqs. (11) and (12), $\alpha \in [0, 1]$ is subjectively assigned by D and reflects the importance that D gives to preventing regrettable choices (and, hence, to reversing his originally intended choice). □

Clearly, once the parameter α is assigned, $(1-\alpha)$ corresponds to the relative importance assigned to the verification of a potential choice. It should be remarked that for $\alpha = 1$, $V_j^I(\bar{x}_k)$ and $V_j^{II}(\bar{x}_k)$ describe the value of information in the particular case where D considers an information valuable only if it allows him to reverse his choice at a given step of the algorithm. This particular case has been considered in Santos-Arteaga et al. [26], where a self-regulating algorithm was introduced to model the information acquisition behavior of DMs. It is no difficult to see that the expected information value approach proposed in the current study generalizes the one introduced by Santos-Arteaga et al. [26].

5.1.3. Intuition behind the proposed definition of expected information value

Recall that at Step j, Option (I) means to continue with $\bar{G}_k$, while Option (II) means to start with G_{k+1}. Thus, $V_j^I(\bar{x}_k)$ measures the expected value derived from checking the second characteristic of the product $\bar{G}_k$ whose first characteristic is $\bar{x}_k$, while $V_j^{II}(\bar{x}_k)$ measures the expected value derived from checking the first characteristic of a new product. Moreover, recall that due to the dynamic adaptive characterization of the current algorithm, D must adapt his reference values as he acquires information and observes the characteristics of different products.

Assume that** D **proceeds with Option (I):

There are two cases to consider. In both cases, acquiring information about the second characteristic of $\bar{G}_k$ is valuable from both a verification and prevention viewpoint.

Case (I.a): Suppose that $\bar{x}_k < x_j^{opt}$.

- If the second characteristic $\bar{y}_k$ of $\bar{G}_k$ is such that $u(\bar{x}_k) + v(\bar{y}_k) > U_j^{opt}$, then rejecting $\bar{G}_k$ would be regrettable at Step $j+1$ of the algorithm and should be prevented, since it provides a higher utility than the reference product, G_j^{opt}.
- If the second characteristic $\bar{y}_k$ of $\bar{G}_k$ is such that $u(\bar{x}_k) + v(\bar{y}_k) < U_j^{opt}$, then $\bar{G}_k$ is verified to be a suboptimal choice providing a lower utility than the reference product, G_j^{opt}.

Case (I.b): Suppose that $\bar{x}_k \geq x_j^{opt}$.

- If the second characteristic $\bar{y}_k$ of $\bar{G}_k$ is such that $u(\bar{x}_k) + v(\bar{y}_k) > U_j^{opt}$, then $\bar{G}_k$ is verified as a non-regrettable choice at Step $j+1$ of the algorithm, which implies that choosing $\bar{G}_k$ would be better that choosing G_j^{opt}.
- If the second characteristic $\bar{y}_k$ of $\bar{G}_k$ is such that $u(\bar{x}_k) + v(\bar{y}_k) < U_j^{opt}$, then rejecting G_j^{opt} to choose $\bar{G}_k$ is a regrettable choice; choosing $\bar{G}_k$ would be worse than choosing G_j^{opt}, and is a suboptimal choice prevented by the new information.

Assume that** D **proceeds with Option (II):

There are again two cases to consider, in both of which acquiring information about the first characteristic of G_{k+1} is valuable from both a verification and prevention viewpoint.

Case (II.a): Suppose that $\bar{x}_k < c_X$.

- If the first characteristic x_{k+1} of G_{k+1} is such that $u(x_{k+1}) > E_X$, then G_{k+1} would be a better choice than both *Rand* and $\bar{G}_k$ at Step $j+1$ of the algorithm. That is, G_{k+1} becomes the new partially observed reference product. It should be highlighted that even if G_{k+1} becomes an optimal choice at Step $j+1$, it could become suboptimal at Step $j+2$ if D decides to continue acquiring information on G_{k+1} and $u(x_{k+1}) + v(y_{k+1}) < U_{j+2}^{opt}$.
- If the first characteristic x_{k+1} of G_{k+1} is such that $u(x_{k+1}) < E_X$, then G_{k+1} would be a suboptimal choice at Step $j+1$ of the algorithm, confirming *Rand* as the partially observed reference product. As discussed above, G_{k+1} could become optimal at Step $j+1$ if D decides to continue acquiring information on G_{k+1} and $u(x_{k+1}) + v(y_{k+1}) > U_{j+2}^{opt}$.

Case (II.b): Suppose that $\bar{x}_k \geq c_X$.

- If the first characteristic x_{k+1} of G_{k+1} is such that $u(x_{k+1}) > u(\bar{x}_k)$, then G_{k+1} is a better choice than *Rand* and $\bar{G}_k$. That is, G_{k+1} constitutes the new partially observed reference product at Step $j+1$ of the algorithm.
- If the first characteristic x_{k+1} of G_{k+1} is such that $u(x_{k+1}) < u(\bar{x}_k)$, then $\bar{G}_k$ is confirmed as the partially observed reference product at Step $j+1$ of the algorithm.

5.2. Option (I) versus option (II)

The expected information values of Option (I) and Option (II) defined in the previous subsection can be used by D to compare the two options at each step and identify the best option **BO**.

More precisely, suppose that D is in Step j. If $V_j^I(\bar{x}_k) \geq V_j^{II}(\bar{x}_k)$, then the information corresponding to Option (I) is more valuable in expected terms than the information corresponding to Option (II). Similarly, if $V_j^I(\bar{x}_k) < V_j^{II}(\bar{x}_k)$, the information provided by Option (II) is expected to be more valuable than that provided by Option (I).

The selection criterion that D implements is described schematically below:

Selection Criterion

$$D \text{ is in Step } j \rightarrow \begin{cases} \text{if } V_j^I(\bar{x}_k) \geq V_j^{II}(\bar{x}_k) & \rightarrow \mathbf{BO} = \text{Option(I)} \\ \text{if } V_j^I(\bar{x}_k) < V_j^{II}(\bar{x}_k) & \rightarrow \mathbf{BO} = \text{Option(II)} \end{cases}$$

6. Stopping criterion

After selecting the best option, D has to decide whether or not to acquire the corresponding information. While the selection between both options is based on their expected information value, the dynamic incentives of D at Step j of the algorithm will be determined by the capacity of any additional information to provide a product better than G_j^{opt}. Based on the above discussion, we introduce a stopping criterion at Step j composed by two rules that depend on **BO**.

Remark 6. It should be emphasized that since D aims at obtaining the highest possible value from the information retrieved per step and following the standard principles of bounded rationality [31], we could define *ad hoc* a maximum number of observations that D can assimilate. However, for completeness, we provide alternative consistent rules determining the dynamic incentives of D. □

Consider first the case where D selects Option (II). In this case, we propose the following definition of dynamic incentive.

Definition 3. We define the *dynamic incentive for D at* Step j as follows:

$$\sigma_j \stackrel{def}{=} \int_{x_j^{opt}}^{x_M} \left(\int_{(x_j^{opt}+y_j^{opt}-x)}^{y_M} \mu(x)\mu(y)(u(x)+v(y))\, dy \right) dx. \quad \square \tag{13}$$

According to the definition above, D evaluates the acquisition of at least two additional observations fully describing a new product, (x, y), such that $x \geq x_j^{opt}$ and $y \geq y_j^{opt}$, before proceeding with Option (II). Thus, we assume that D applies the following rule.

- ***Control rule:***

If $V_j^I(\bar{x}_k) < V_j^{II}(\bar{x}_k)$, by the selection criterion, the information provided by Option (II) is expected to be more valuable than that corresponding to Option (I), i.e., **BO** = Option (II). In this case, D either pursues Option (II) and acquires the information when $\sigma_j >$ cost or stops acquiring any new information when $\sigma_j \leq$ cost.

The cost function can be assumed increasing in the number of previous observations, given the limited capacity of D to acquire and assimilate information. For example, it could be assumed that:

$$\text{cost} = c \cdot \frac{(j-1)}{\omega} \tag{14}$$

where ω is a predetermined maximum number of observations that can be acquired and assimilated by D and $j-1$ is the number of observations acquired until Step j. Clearly, $\omega \leq 2N$. Moreover, the value of the cost parameter c has to be such that D stops by the time ω is reached. Thus, the control rule would also work as a consistency rule.

Consider now the case where D decides to pursue Option (I). We assume that D applies the following rule.

- ***Default rule:***

If $V_j^I(\bar{x}_k) \geq V_j^{II}(\bar{x}_k)$, by the selection criterion, the information provided by Option (I) is expected to be more valuable than that corresponding to Option (II), i.e., **BO** = Option (I). In this case, D pursues Option (I) and acquires the information.

The stopping criterion that D implements is described schematically below:

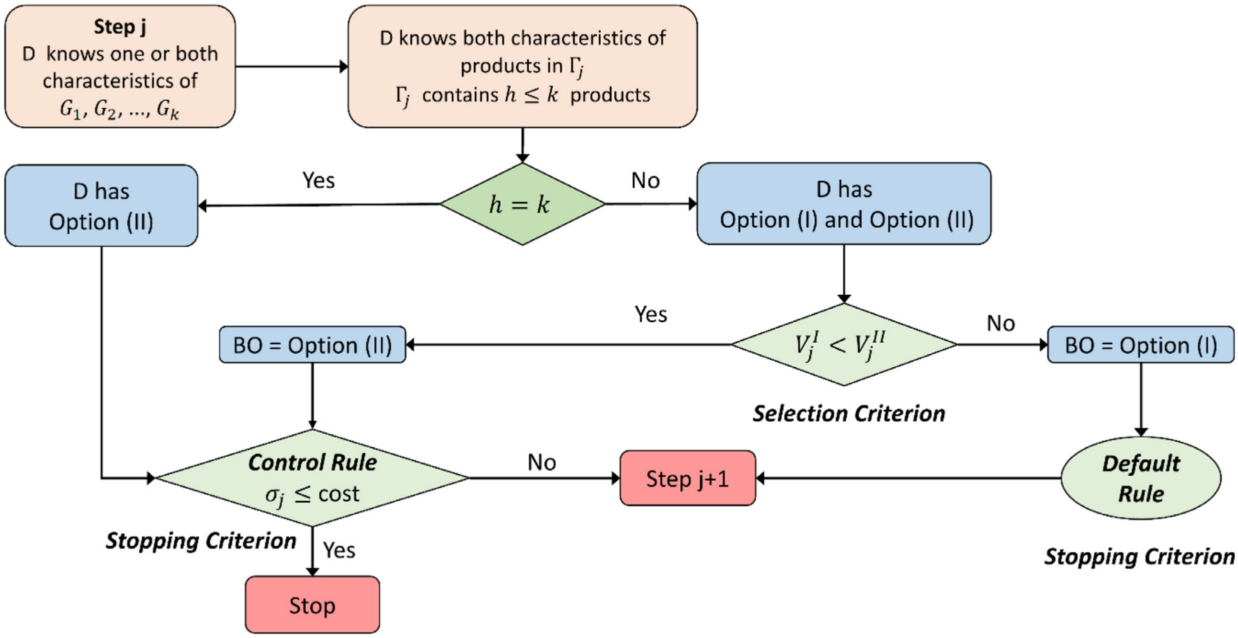


Fig. 5. Generic step of the proposed algorithm.

Stopping Criterion

$$D \text{ is in Step } j \;\rightarrow\; \begin{cases} \text{if } \mathbf{BO} = \text{Option(I)} \;\underset{\text{default rule}}{\rightarrow}\; D \text{ pursues Option(I)} \\ \text{if } \mathbf{BO} = \text{Option(II)} \;\underset{\text{control rule}}{\rightarrow}\; \begin{cases} \text{if } \sigma_j \le \text{cost} \rightarrow D \text{ stops} \\ \text{if } \sigma_j > \text{cost} \rightarrow D \text{ pursues Option(II)} \end{cases} \end{cases}$$

Remark 7. In the proposed stopping criterion, the dynamic incentive of Definition 3 plays a role only in the case where **BO**= Option (II), that is, D controls whether or not to stop acquiring information at Step j only when he should start with a new product. At the same time, D acquires the next piece of information by default if such information is the second characteristic of the partially observed reference product $\bar{G}_k$, that is, when **BO**= Option (I). However, the default rule can be replaced by another control rule relative to only one characteristic, that is, to the second characteristic of $\bar{G}_k$. To do so, we could define the dynamic incentive for D at Step j when **BO**= Option (I) as follows:

$$\delta_j \overset{def}{=} \int_{(x_j^{opt}+y_j^{opt}-x)}^{y_M} \mu(y)(u(x)+v(y))\,dy \tag{15}$$

Hence, replace the default rule with the following one: if $V_j^I(\bar{x}_k) \ge V_j^{II}(\bar{x}_k)$, i.e. **BO**= Option (I), then:
$\delta_j > \frac{\text{cost}}{2}$ $\rightarrow$ D pursues Option (I) and acquires the information;
$\delta_j \le \frac{\text{cost}}{2}$ $\rightarrow$ D stops. □

Fig. 5 represents the general step of the proposed algorithm explicitly, including the selection and the stopping criteria.

7. Simulations

The numerical simulations introduced through this section illustrate how the relative width of the domains on which the characteristics of the products are defined determines the information acquisition behavior of D.

At the generic Step j, letting the value $\bar{x}_k$ vary in X, $P_j^+(\bar{x}_k)$, $P_j^-(\bar{x}_k)$, $Q_j^+(\bar{x}_k)$ and $Q_j^-(\bar{x}_k)$ can be interpreted as set-valued functions of the observed value $\bar{x}_k$. Similarly, $V_j^I(\bar{x}_k)$ and $V_j^{II}(\bar{x}_k)$ define two functions that we will refer to as continuation and starting functions. Finally, $\sigma_j(x, y)$ is a function of (x, y) varying in $X \times Y$ that will be called incentive function.

We start by describing the information acquisition incentives of D for given values of x_j^{opt}, y_j^{opt} and $\bar{x}_k$, and analyzing how modifications of these reference values affect the behavior of D. We will assume uniform probability densities and linear utility functions, $u(x) = x$ and $v(y) = y$, on both product characteristics throughout the numerical simulations presented in this section and the next one. Also, a value of $\alpha = 0.5$ will be exogenously assigned so as to focus solely on the effects derived from modifications of the reference values. This latter assumption will be relaxed in the following section, where the effects that the attitude toward regret of D has on the dynamic structure of the algorithm are explicitly analyzed.

Fig. 6 presents the $X = [0,\ 10]$ and $Y = [0,\ 5]$ scenario. Note that the spread of the information value exhibited by $V_j^I(\bar{x}_k)$ equals 7.5, since $U(x_M, y_M) - U_{rand} = x_M + y_M - c_X - c_Y =\ 15 - 7.5 = 7.5$ and $U_{rand} - U(x_m, y_m) = c_X + c_Y - x_m - y_m =\ 7.5 - 0 = 7.5$. Similarly, the spread of the information value exhibited by $V_j^{II}(\bar{x}_k)$ equals 5, since $u(x_M) - E_X = x_M - c_X = 10 - 5 = 5$ and $E_X - u(x_m) = c_X - x_m = 5 - 0 = 5$. Thus, the spread of the information value exhibited by the continuation option, $V_j^I(\bar{x}_k)$, is 1.5 times that of the starting one, $V_j^{II}(\bar{x}_k)$.

Fig. 6(a) assumes that the fully observed reference product, G_j^{opt}, delivers a lower utility than *Rand*, i.e. $U_j^{opt} < E_X + E_Y$. That is, the reference product to be improved upon by the new observations is given by $c_X + c_Y$. It should be noted that the starting function $V_j^{II}(\bar{x}_k)$ remains unchanged throughout the numerical simulations of Fig. 6, since it does not depend on the values of x_j^{opt}, while the continuation function $V_j^I(\bar{x}_k)$ is determined by them.

Figs. 6(b) and (c) have been introduced to illustrate the shifts in the value of the continuation functions as the reference values defining G_j^{opt} are modified.

Fig. 6(b) describes two cases where $U_j^{opt} > E_X + E_Y = 7.5$, namely, $(x_j^{opt} = 6, y_j^{opt} = 2.5)$ and $(x_j^{opt} = 7, y_j^{opt} = 2.5)$. Note that $V_j^I(\bar{x}_k)$ shifts rightward as the value of x_j^{opt} increases, which motivates D to continue acquiring information on a given partially observed product, $\bar{x}$. An identical effect can be observed when y_j^{opt} increases, leading to the $(x_j^{opt} = 6, y_j^{opt} = 3.5)$ reference product.

That is, the unit increase in y_j^{opt} has the same effect on the continuation incentives of D as the unit increase in x_j^{opt}. Intuitively, both modifications lead to the same increment in the reference product that determines the relative value of the information acquired. Moreover, the uniformity of the probability densities and linearity of the utilities reinforce the symmetry of the effect, which would not prevail if D were averse to risk or his beliefs determined by the value of $\bar{x}$.

Fig. 6(c) illustrates the additional rightward shift of $V_j^I(\bar{x}_k)$ that takes place as G_j^{opt} increases further, i.e. $(x_j^{opt} = 6, y_j^{opt} = 4.5)$. Note that the formal structure of the information acquisition model implies that the minimum of $V_j^I(\bar{x}_k)$ is attained at the value of $\bar{x}$ such that $\bar{x} + c_Y = x_j^{opt} + y_j^{opt}$.

It should be highlighted that the simulations presented in Fig. 6 imply that D will be unable to observe a *complete* product whose characteristics provide a utility higher than that of *Rand*. This will be the case since the continuation and starting functions cross at $\bar{x} = 2.5$ in Fig. 6(a), which implies that the highest potential realization of the second characteristic, $y_M = 5$, would only suffice to fully observe a product with a utility as high as *Rand*. We analyze the strategic implications from such an evaluation constraint in the following section.

However, this information acquisition pattern can be reversed, since an increase in the potential variability exhibited by the Y characteristic can be defined so as to lead continuation to dominate the behavior of D. At the same time, intermediate situations justifying the uncertain random behavior of D can also be designed. Fig. 7 has been introduced to illustrate both these potential situations as well as to provide intuition regarding the dynamic structure of the algorithm and its reference products.

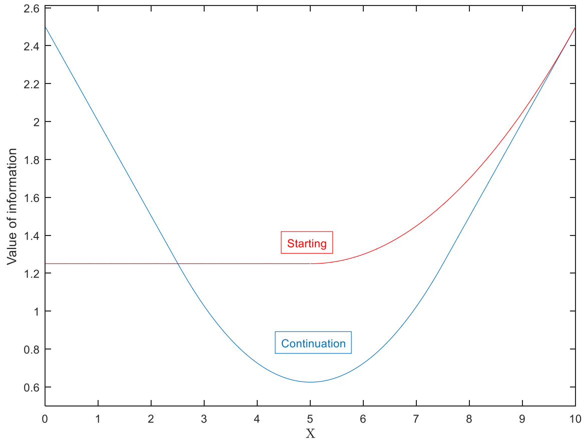
Fig. 7(a) describes the $X = [15,\ 20]$ and $Y = [10,\ 20]$ scenario. In this case, the spread of the information value exhibited by $V_j^I(\bar{x}_k)$ equals 7.5, since $U(x_M, y_M) - U_{rand} = x_M + y_M - c_X - c_Y =\ 40 - 32.5 = 7.5$ and $U_{rand} - U(x_m, y_m) = c_X + c_Y - x_m - y_m =, 32.5 - 25 = 7.5$.

Similarly, the spread of the information value exhibited by $V_j^{II}(\bar{x}_k)$ equals 2.5, since $u(x_M) - E_X = x_M - c_X = 20 - 17.5 = 2.5$ and $E_X - u(x_m) = c_X - x_m = 17.5 - 15 = 2.5$. Thus, the spread of the information value exhibited by the continuation option is 3 times that of the starting one. We will also assume that the fully observed reference product is given by $x_j^{opt} = 18$ and $y_j^{opt} = 16$.

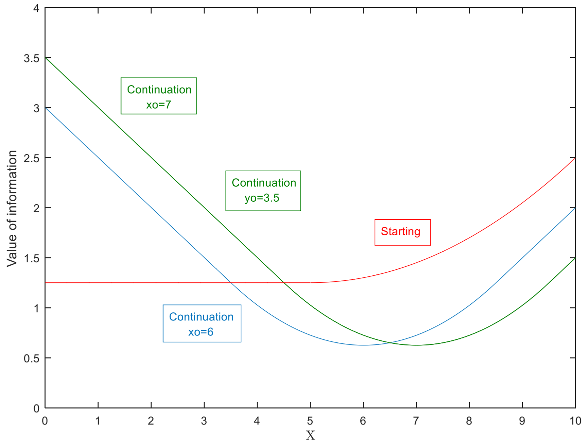
Note that $x_j^{opt} = 18$ determines the shape of the starting payoff described in Fig. 7(a), since D must improve upon the initial characteristic of the reference product when acquiring information on a new product. On the other hand, the entire reference product is used to define the integral limits of the continuation function $V_j^I(\bar{x}_k)$. More importantly, Fig. 7(a) illustrates how an increment in the spread of the information value exhibited by the continuation alternative can be defined so that it prevails over the starting one through the entire information acquisition process.

Intuitively, the dominance of the continuation payoff can be eliminated by decreasing the relatively larger information value spread of the function $V_j^I(\bar{x}_k)$. Thus, we change the domains of the variables to $X = [0,\ 10]$ and $Y = [0,\ 10]$ and assume that $x_j^{opt} = 5$ and $y_j^{opt} = 5$, coinciding with *Rand*. In this case, the spread of the information value exhibited by $V_j^I(\bar{x}_k)$ equals 10, since $U(x_M, y_M) - U_{rand} = x_M + y_M - c_X - c_Y = 20 - 10 = 10$ and $U_{rand} - U(x_m, y_m) = c_X + c_Y - x_m - y_m =\ 10 - 0 = 10$. The spread of the information value exhibited by $V_j^{II}(\bar{x}_k)$ equals 5, since $u(x_M) - E_X = x_M - c_X = 10 - 5 = 5$ and $E_X - u(x_m) = c_X - x_m = 5 - 0 = 5$. That is, the spread of the information value of the continuation option is 2 times that of the starting one.

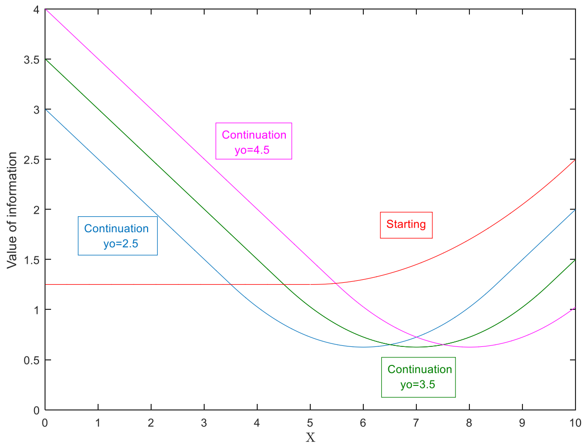
Fig. 7(b) illustrates how continuation dominates starting for all realizations located below the reference value $c_X = 5$, while both functions overlap for realizations above $c_X = 5$. This overlap is caused by the way functions $V_j^I(\bar{x}_k)$ and $V_j^{II}(\bar{x}_k)$ have been defined and the fact that $x_j^{opt} = 5$ together with $y_j^{opt} = 5$ deliver the same utility as *Rand*.



(a). $x_j^{opt} + y_j^{opt} < c_X + c_Y$

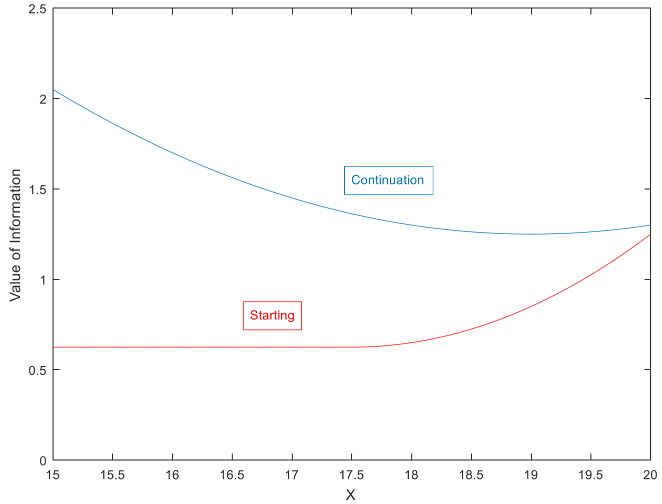


(b). $x_j^{opt} = 6, y_j^{opt} = 2.5$ and $x_j^{opt} = 7, y_j^{opt} = 2.5$

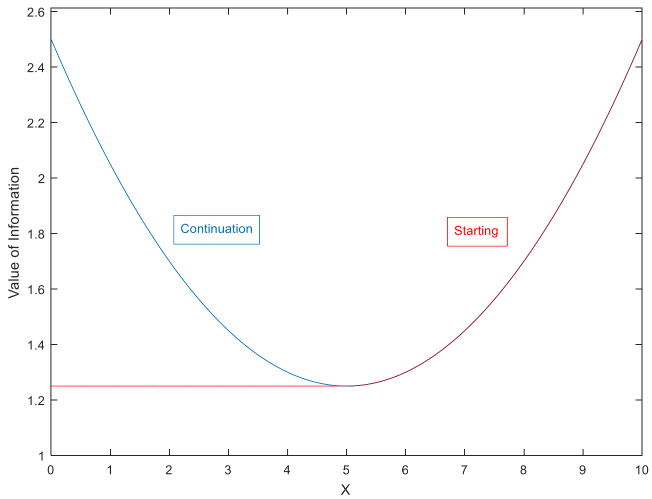


(c). Adding $x_j^{opt} = 6, y_j^{opt} = 4.5$ to 4(b)

Fig. 6. $V_j^I(\bar{x}_k)$ and $V_j^{II}(\bar{x}_k)$ functions in the $X = [0,\ 10]$ and $Y = [0,\ 5]$ scenario.



(a). $X = [15, 20]$ **and** $Y = [10, 20]$ **with** $x_j^{opt} = 18$ **and** $y_j^{opt} = 16$



(b). $X = [0, 10]$ **and** $Y = [0, 10]$ **with** $x_j^{opt} = 5$**,** $y_j^{opt} = 5$ **and**
$x_j^{opt} = 2$**,** $y_j^{opt} = 8$

Fig. 7. $V_j^I(\bar{x}_k)$ and $V_j^{II}(\bar{x}_k)$ functions as G_j^{opt} varies when information is acquired.

As illustrated in Fig. 6(b), the same incentive structure should be obtained if we modify the values of the reference product while providing a utility equal to *Rand*. Fig. 7(b) describes also the scenario with $x_j^{opt} = 2$ and $y_j^{opt} = 8$, where, despite the relative differences in the values of x_j^{opt} and y_j^{opt}, $V_j^I(\bar{x}_k)$ and $V_j^{II}(\bar{x}_k)$ remain unchanged since they are still determined by a reference product providing a utility identical to *Rand*.

The numerical results described through this section yield the following proposition.

Proposition 4. The information acquisition behavior of D is determined by the spread of the information value exhibited by $V_j^I(\bar{x}_k)$ relative to $V_j^{II}(\bar{x}_k)$ as follows.

a. A relatively lower variability of the Y characteristic of the products favors the starting behavior of D, who will observe partially as many products as possible.
b. A relatively higher variability of the Y characteristic of the products favors the continuation behavior of D, who will observe fully as many products as possible.

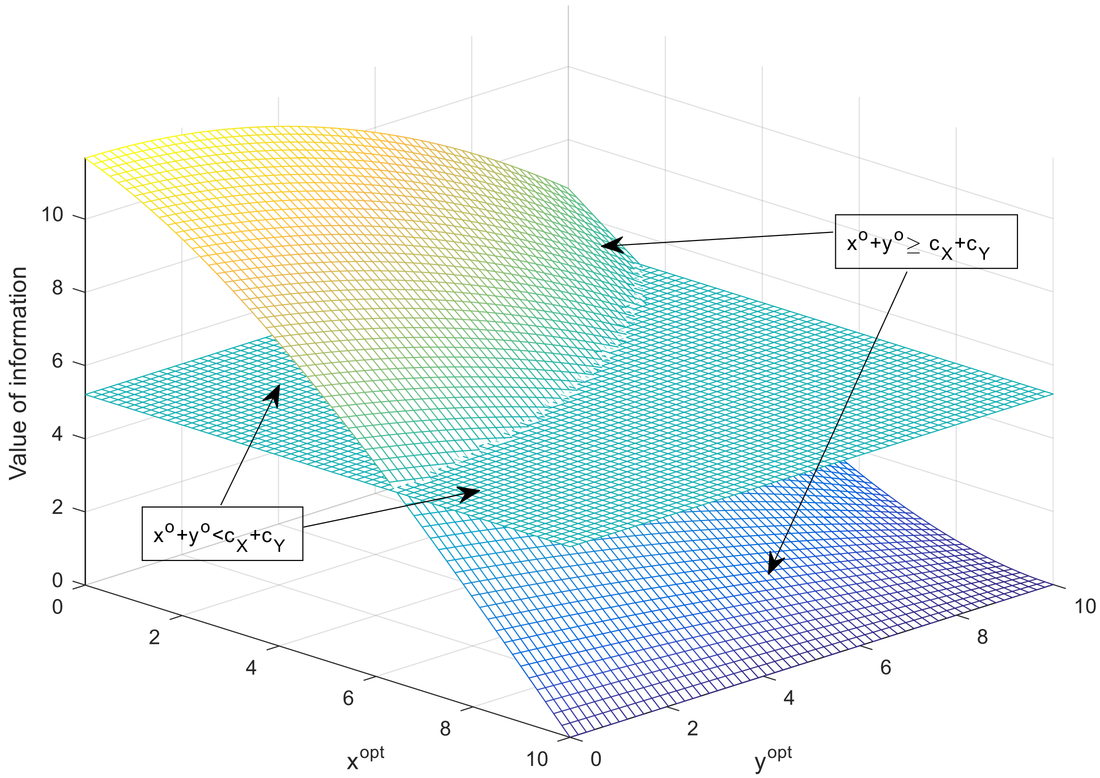


Fig. 8. Incentive function $\sigma_j(x, y)$ in the $X = [0,\ 10]$ and $Y = [0,\ 10]$ scenario.

c. If μ and η are uniform probability densities and D is risk-neutral (i.e. u and v are linear utility functions), then an identical variability of the X and Y characteristics of the products favors the random behavior of D for a given subset of $\bar{x}_k$ values. □

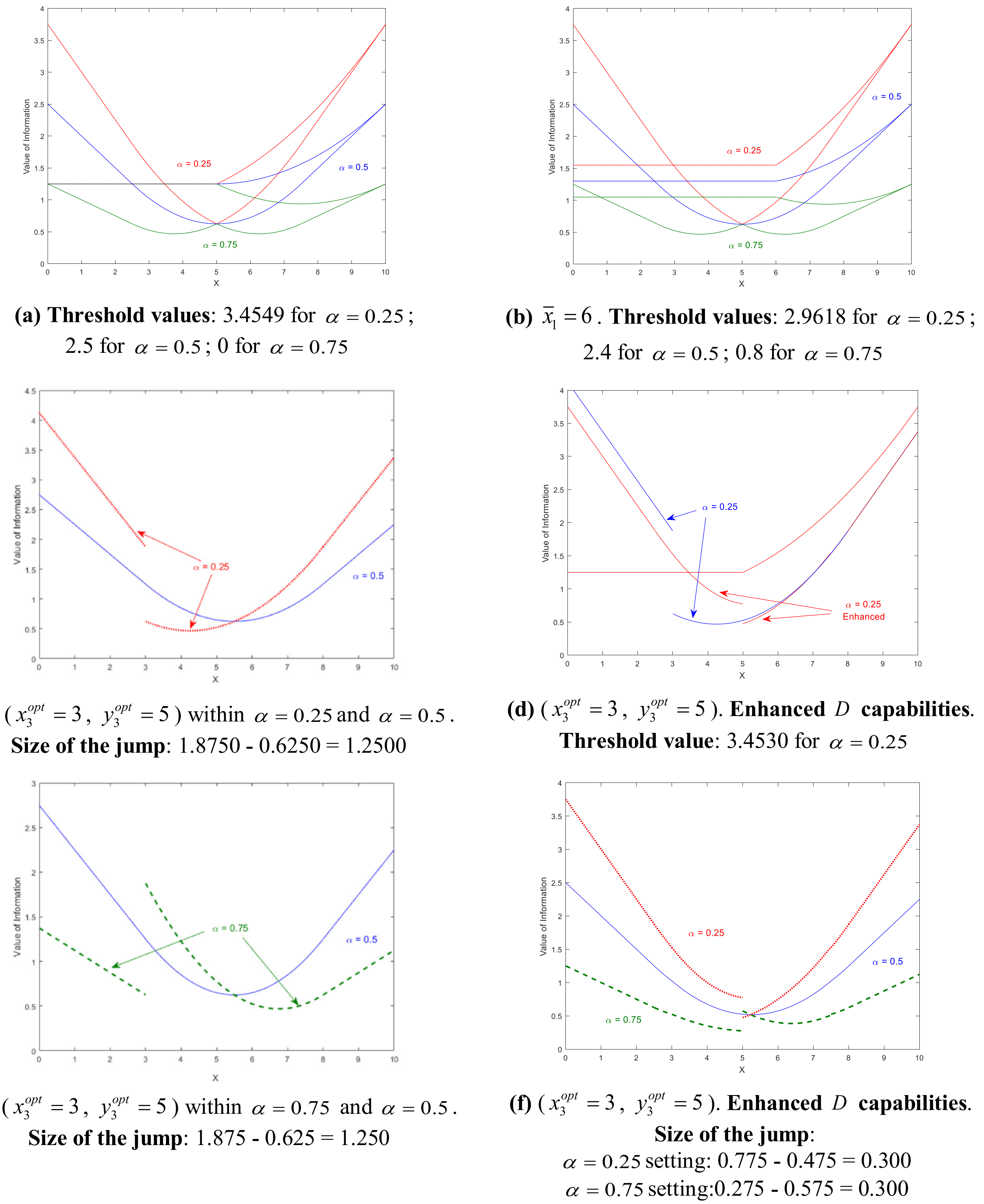
Finally, Fig. 8 plots the incentive function $\sigma_j(x, y)$ determining the dynamic behavior of D within the $X = [0,\ 10]$ and $Y = [0,\ 10]$ scenario. The graph of $\sigma_j(x, y)$ is composed by two sheets separated by the diagonal plane of equation $x^{opt} + y^{opt} = 10$, where 10 is the utility derived from *Rand*. The sheet on the left of the separating plane is given by a horizontal plane ($z = 5$) while the sheet on the right is a decreasing function. Note that the plane defines the value of information when $x^{opt} + y^{opt} < c_X + c_Y$ while the decreasing function refers to the case $x^{opt} + y^{opt} \geq c_X + c_Y$. In this regard, it may also be assumed that D does not exhibit the discontinuity triggered by the *Rand* product in his information acquisition incentives.

That is, the incentive function defined for the case $x^{opt} + y^{opt} \geq c_X + c_Y$ may be used as a reference through the whole domain on which both product characteristics are defined, since it describes asymmetric improvements relative to any potential combination that may be observed within the $X = [0,\ 10]$ and $Y = [0,\ 10]$ scenario. The constant search incentives determined by *Rand* preserve the consistency of the analysis presented throughout the paper, which would be modified if we were to eliminate the *Rand* reference product.

8. Sequential search incentives and value of information

Given the analysis regarding the basic behavior of the $V_j^I(\bar{x}_k)$ and $V_j^{II}(\bar{x}_k)$ functions provided in the previous section, the following simulations describe explicitly the information acquisition process of D and the search incentives that follow from the different products observed, both partially and completely. In this regard, the set of simulations presented in Fig. 9 correspond to the $X = [0,\ 10]$ and $Y = [0,\ 5]$ scenario.

Fig. 9(a) describes the initial $V_2^I(\bar{x}_1)$ and $V_2^{II}(\bar{x}_1)$ functions for three values of the α parameter, i.e. $\alpha = 0.25,\ 0.5,\ 0.75$, reflecting different attitudes of D toward regret. Note how the continuation alternative becomes increasingly plausible as the value of α decreases. However, as emphasized in the previous section, D may continue acquiring information on a partially observed product within the $\alpha = 0.5$ and $\alpha = 0.75$ settings never to observe a complete product whose utility is higher than that of *Rand*. Indeed, in order to prevent continuation from delivering a complete product better than *Rand*, we just need an initial observation such that $V_3^I(\bar{x}_1) = V_3^{II}(\bar{x}_1)$ for $x_2 = 2.5$ in the $\alpha = 0.25$ setting. That is, any initial observation above $\bar{x}_1 = 6.3189$ would prevent continuation from delivering a completely observed product with a utility higher than *Rand* in all three α settings.



(a) Threshold values: 3.4549 for $\alpha = 0.25$; 2.5 for $\alpha = 0.5$; 0 for $\alpha = 0.75$

(b) $\bar{x}_1 = 6$. **Threshold values**: 2.9618 for $\alpha = 0.25$; 2.4 for $\alpha = 0.5$; 0.8 for $\alpha = 0.75$

(c) ($x_3^{opt} = 3$, $y_3^{opt} = 5$) within $\alpha = 0.25$ and $\alpha = 0.5$. **Size of the jump**: 1.8750 - 0.6250 = 1.2500

(d) ($x_3^{opt} = 3$, $y_3^{opt} = 5$). **Enhanced** D **capabilities. Threshold value**: 3.4530 for $\alpha = 0.25$

(e) ($x_3^{opt} = 3$, $y_3^{opt} = 5$) within $\alpha = 0.75$ and $\alpha = 0.5$. **Size of the jump**: 1.875 - 0.625 = 1.250

(f) ($x_3^{opt} = 3$, $y_3^{opt} = 5$). **Enhanced** D **capabilities. Size of the jump**: $\alpha = 0.25$ setting: 0.775 - 0.475 = 0.300 $\alpha = 0.75$ setting: 0.275 - 0.575 = 0.300

Fig. 9. $V_j^I(\bar{x}_k)$ and $V_j^{II}(\bar{x}_k)$ in the sequential $X = [0, 10]$ and $Y = [0, 5]$ scenario.

The intuition validating this result follows from the different $V_3^I(\bar{x}_1)$ and $V_3^{II}(\bar{x}_1)$ functions presented in Fig. 9(b), which have been generated after assuming that D observes $\bar{x}_1 = 6$. Given this initial realization, D starts acquiring information on a new product in all three settings. Note that the only function affected by this decision is $V_2^{II}(\bar{x}_1)$, which shifts its kink to $x_2 = 6$ (together with the threshold values defined within the different α settings). Given the new threshold values, continuing under $\alpha = 0.5$ and $\alpha = 0.75$ leads once again to products whose utility is below that of *Rand*. Only the $\alpha = 0.25$

setting allows for continuing and completely observing a product with a higher utility than *Rand*, but for a lower range of x_2 values relative to that of x_1.

It therefore follows that products with mediocre X characteristics placed in the initial positions on the search process may be considered as viable choices by D while other potentially better products with relatively high Y characteristics may be ignored.

Consider now the case where D observes a product completely within the $\alpha = 0.25$ setting and that it equals ($x_3^{opt} = 3$, $y_3^{opt} = 5$). That is, the initial observation is located within the interval [2.5, 3.4549] defined by the corresponding $V_2^I(\bar{x}_1)$ and $V_2^{II}(\bar{x}_1)$ functions. In the $\alpha = 0.25$ setting, D continues with the partially observed product defined by $x_1 = 3$, leading to the situation described in Figs. 9(c) and (d). The threshold value illustrated in Fig. 9(d) is given by $x_2 = 3.4530$, which preserves the continuation capacity of the $\alpha = 0.25$ setting as in the initial stage described in Fig. 9(a). On the other hand, given $x_1 = 3$, D would start acquiring information on a new product within the $\alpha = 0.5$ and $\alpha = 0.75$ settings, leading to the partially observed product ($\bar{x}_1 = 3$, $y_1 = c_Y$) and the search incentives defined by the corresponding functions described in Fig. 9(a).

8.1. Symmetry and continuity

Figs. 9(c) and (e) illustrate the discontinuities arising within the $\alpha = 0.25$ and $\alpha = 0.75$ settings after D completely observes ($x_3^{opt} = 3$, $y_3^{opt} = 5$). These figures also show that $V_j^I(\bar{x}_k)$ does not experience a jump within the $\alpha = 0.5$ setting, given the symmetry inherent to the α and $(1-\alpha)$ expressions within the continuation functions. A second source of discontinuity is given by the asymmetries in the realizations of Y relative to the domain within which the characteristic is defined. In the current section, we illustrate how the width of the resulting gap can be reduced to a considerable extent by enhancing the computational capacities of D within all settings.

A simple way of preventing the potential non-existence of a threshold value determined by $V_j^I(\bar{x}_k)$ and $V_j^{II}(\bar{x}_k)$ is to introduce a rule of thumb defining the behavior of D based on the relative distance to the extremes of the jump of the corresponding $V_j^I(\bar{x}_k)$ function. For example, in the $\alpha = 0.25$ case described in Fig. 9(c), it could be assumed that values of $V_3^{II}(\bar{x}_2)$ above 1.250 lead D to start acquiring information on a new product. A similar rule could be applied to the $\alpha = 0.75$ counterpart setting described in Fig. 9(e), with values of $V_3^{II}(\bar{x}_2)$ below 1.2500 leading D to continue acquiring information on the product partially observed.

The *ad hoc* quality of the previous rule implies that the spread of the jump should be reduced so as to preserve behavioral consistency within the search environment being analyzed. This can be done by enhancing the computational capacity of D. For instance, applying the $V_j^I(\bar{x}_k)$ continuation function defined in Eq. (11) to the product ($x_3^{opt} = 3$, $y_3^{opt} = 5$) we obtain

$$V_3^I(\bar{x}_2) = \begin{cases} \underbrace{(1-\alpha)\int_{P_3^-(\bar{x}_2)} \eta(y)\big(U_3^{opt} - u(\bar{x}_2) - v(y)\big)\,dy}_{\text{Verification}}, & \text{if } \bar{x}_2 < x_3^{opt} + y_3^{opt} - y_M = 3 \\ \underbrace{\alpha\int_{P_3^-(\bar{x}_2)} \eta(y)\big(U_3^{opt} - u(\bar{x}_2) - v(y)\big)\,dy}_{\text{Prevention}} + & \\ \underbrace{(1-\alpha)\int_{P_3^+(\bar{x}_2)} \eta(y)\big(u(\bar{x}_2) + v(y) - U_3^{opt}\big)\,dy}_{\text{Verification}}, & \text{if } x_3^{opt} + y_3^{opt} - y_M \le \bar{x}_2 < x_3^{opt} + y_3^{opt} = 8 \\ \underbrace{(1-\alpha)\int_{P_3^+(\bar{x}_2)} \eta(y)\big(u(\bar{x}_2) + v(y) - U_3^{opt}\big)\,dy}_{\text{Verification}}, & \text{if } \bar{x}_2 \ge x_3^{opt} + y_3^{opt} = 8 \end{cases} \quad (11.1)$$

The first term of Eq. (11.1) delimits the realizations of the X characteristic that do not suffice to provide a product with a utility higher than $x_j^{opt} + y_j^{opt}$ even when paired with y_M. Similarly, the last term of the equation delimits the realizations of the X characteristic that suffice on their own to provide a product with a utility higher than $x_j^{opt} + y_j^{opt}$. The middle term corresponds to the expression introduced in Eq. (11) for $\bar{x}_k < x_j^{opt}$. The lack of symmetry implied by the $y_1 = 5$ realization leads to the discontinuities observed in Fig. 9(c) and (e). In other words, a realization of $y_1 = 2.5$ would have preserved the continuity of the $V_3^I(\bar{x}_2)$ functions within the $\alpha = 0.25$ and $\alpha = 0.75$ settings.

The computational capacity of D can be enhanced to account for the *Rand* reference product, which implies that the new product being observed may not deliver a utility higher than G_j^{opt} but be better than a random choice, U_{rand}, and therefore useful from an informational viewpoint as part of the set $\hat{\Gamma}_j$. This additional computational requirement reduces the gap generated within the $V_j^I(\bar{x}_k)$ functions since it considers additional product comparisons that smooth the effects derived from the asymmetries in the realizations of Y and α. The resulting expression is provided in Eq. (11.2) below

$$V_3^I(\bar{x}_2)=\begin{cases} \underbrace{(1-\alpha)\int_{P_3^-(\bar{x}_2)}\eta(y)(U_{rand}-u(\bar{x}_2)-v(y))\,dy}_{\text{Verification}}, & \text{if } \bar{x}_2 < Rand - y_M = 2.5 \\ \underbrace{\alpha\int_{P_3^+(\bar{x}_2)}\eta(y)(u(\bar{x}_2)+v(y)-U_{rand})\,dy}_{\text{Prevention}}+ & \\ \underbrace{(1-\alpha)\int_{P_3^-(\bar{x}_2)}\eta(y)(U_{rand}-u(\bar{x}_2)-v(y))\,dy}_{\text{Verification}}, & \text{if } Rand - y_M \le \bar{x}_2 < x_3^{opt}+y_3^{opt}-y_M = 3 \\ \underbrace{(1-\alpha)\int_{P_3^+(\bar{x}_2)}\eta(y)\left(u(\bar{x}_2)+v(y)-U_3^{opt}\right)dy}_{\text{Verification}}+\underbrace{\alpha\int_{P_3^+(\bar{x}_2)}\eta(y)(u(\bar{x}_2)+v(y)-U_{rand})\,dy}_{\text{Prevention}}+ & \\ \underbrace{(1-\alpha)\int_{P_3^-(\bar{x}_2)}\eta(y)(U_{rand}-u(\bar{x}_2)-v(y))\,dy}_{\text{Verification}}, & \text{if } x_3^{opt}+y_3^{opt}-y_M \le \bar{x}_2 < c_X = 5 \\ \underbrace{(1-\alpha)\int_{P_3^+(\bar{x}_2)}\eta(y)\left(u(\bar{x}_2)+v(y)-U_3^{opt}\right)dy}_{\text{Verification}}+\underbrace{(1-\alpha)\int_{P_3^+(\bar{x}_2)}\eta(y)(u(\bar{x}_2)+v(y)-U_{rand})\,dy}_{\text{Verification}}+ & \\ \underbrace{\alpha\int_{P_3^-(\bar{x}_2)}\eta(y)(U_{rand}-u(\bar{x}_2)-v(y))\,dy}_{\text{Prevention}}, \text{ if } c_X \le \bar{x}_2 < Rand = 7.5 & \\ \underbrace{(1-\alpha)\int_{P_3^+(\bar{x}_2)}\eta(y)\left(u(\bar{x}_2)+v(y)-U_3^{opt}\right)dy}_{\text{Verification}}+ & \\ \underbrace{\alpha\int_{P_3^-(\bar{x}_2)}\eta(y)\left(U_3^{opt}-u(\bar{x}_2)-v(y)\right)dy}_{\text{Prevention}}, & \text{if } Rand \le \bar{x}_2 < x_3^{opt}+y_3^{opt} = 8 \\ \underbrace{(1-\alpha)\int_{P_3^+(\bar{x}_2)}\eta(y)\left(u(\bar{x}_2)+v(y)-U_3^{opt}\right)dy}_{\text{Verification}}, & \text{if } \bar{x}_2 \ge x_3^{opt}+y_3^{opt} = 8 \end{cases} \tag{11.2}$$

Eq. (11.2) introduces the reference product *Rand* so that *D* considers the potential value derived from information not only when improving upon G_j^{opt} at Step *j*, but also when observing a product that provides a utility higher than the random default one, U_{rand}. In this regard, *D* may account for potential improvements and worsenings within the verification and prevention areas that are defined with respect to different sets of products used as relative references. That is, together with G_j^{opt}, *D* may be assumed to consider additional products located above *Rand* when defining the function $V_j^I(\bar{x}_k)$, which would increase further the requirements imposed on his computational capacities.

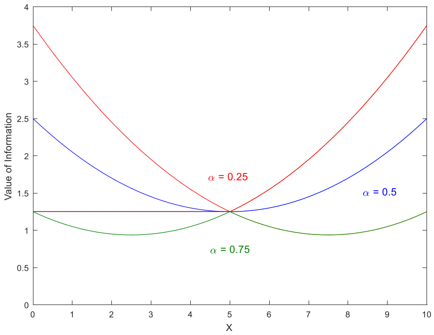
Note how *Rand* is the sole main reference within the first two terms of Eq. (11.2), within which *D* cannot yet improve upon the G_j^{opt} product. In the third expression, after being able to improve upon G_j^{opt}, *D* still considers *Rand* as the lower bound for any potential improvement. The same is true for the fourth term, where, after surpassing the value of the first random characteristic, c_X, *D* adjusts the verification and prevention intervals resulting from U_{rand}.

After improving upon the utility provided by *Rand*, the last two terms focus on U_j^{opt}. The expressions within these terms coincide with those of the original model described in Eq. (11.1), since we focus exclusively on the G_j^{opt} product. However, the domains on which they are defined differ, since *Rand* constitutes the reference product in (11.2) while Eq. (11.1) considers the capacity of the new product to improve upon G_j^{opt}. Fig. 9(c), (e) and (f) illustrate how the enhanced computational capacity of *D* reduces the gap exhibited by the different $V_3^I(\bar{x}_2)$ functions from a value of 1.250 to 0.300. It should be highlighted that the symmetry inherent to the $\alpha = 0.5$ setting preserves the continuity of the corresponding $V_3^I(\bar{x}_2)$ function though its image values are modified, as can be observed from a direct comparison between Fig. 9(e) and (f).

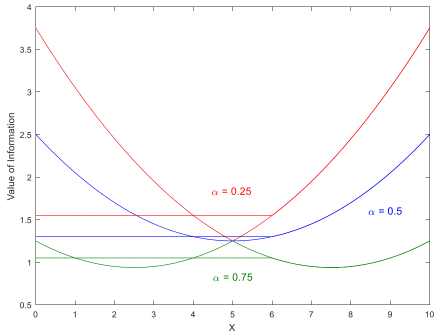
A similar modification will be applied to reduce the gaps arising in the next set of simulations, where the domains of characteristics *X* and *Y* are equated, i.e. the relative importance of the *Y* characteristic is increased so as to reflect the identical [0, 10] evaluation intervals considered for all product characteristics in online evaluation environments.

8.2. Online evaluation environments

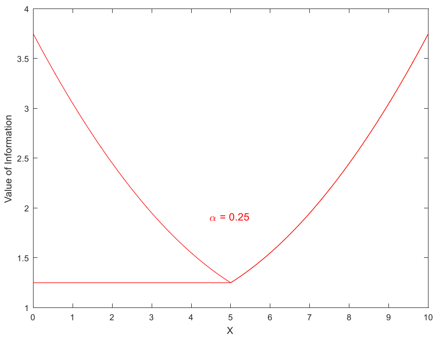
The set of simulations presented in Fig. 10 describes the information acquisition behavior of *D* within the $X = [0,\ 10]$ and $Y = [0,\ 10]$ scenario, where the relative importance assigned to the second characteristic of the products has been increased in order to equate the spread of both variables, as is generally the case in online evaluation environments.



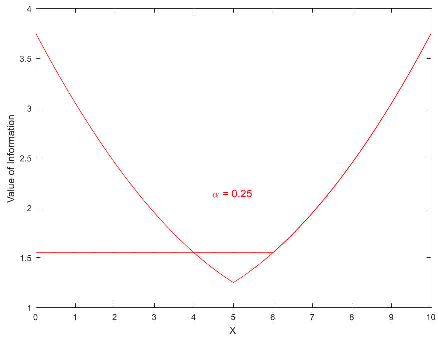
(a0) Threshold values: [5, 10] for $\alpha = 0.25$; [5, 10] for $\alpha = 0.5$; 1 and [5, 10] for $\alpha = 0.75$



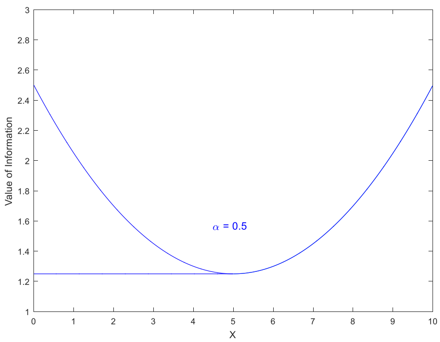
(b0) $\bar{x}_1 = 6$. Starting in all α settings



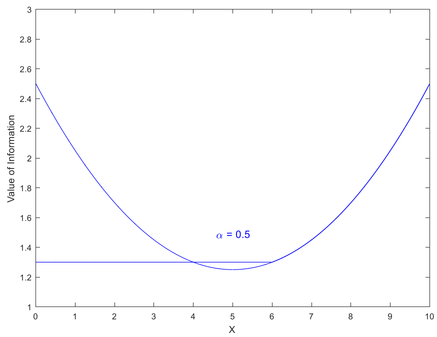
(a1) The $\alpha = 0.25$ setting



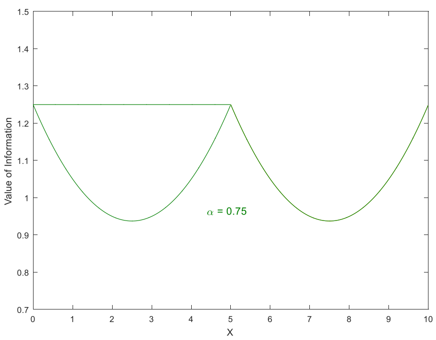
(b1) Threshold values: 4 and [6, 10] for $\alpha = 0.25$



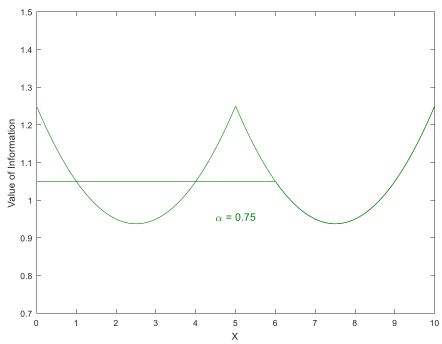
(a2) The $\alpha = 0.5$ setting



(b2) Threshold values: 4 and [6, 10] for $\alpha = 0.5$

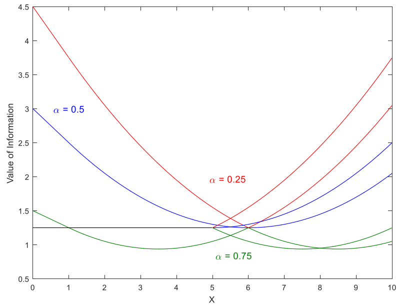


(a3) The $\alpha = 0.75$ setting

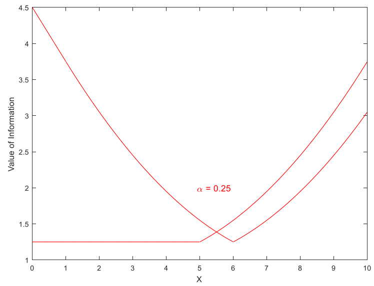


(b3) Threshold values: 1, 4 and [6, 10] for $\alpha = 0.75$

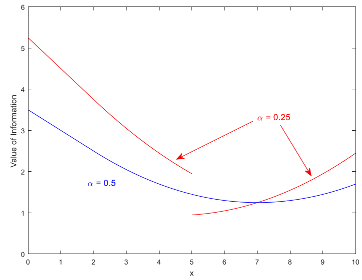
Fig. 10. $V_j^I(\bar{x}_k)$ and $V_j^{II}(\bar{x}_k)$ in the sequential $X = [0,\ 10]$ and $Y = [0,\ 10]$ scenario.



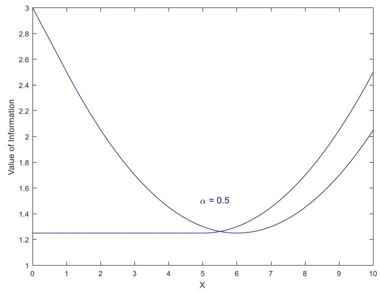
(c0) ($x_3^{opt} = 6$, $y_3^{opt} = 5$). $\bar{x}_1 = 6$. Continuing in all α settings



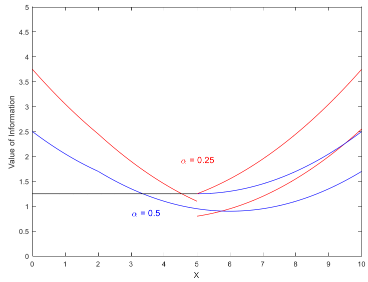
(c1) ($x_3^{opt} = 6$, $y_3^{opt} = 5$).
Threshold value: 5.5 for $\alpha = 0.25$



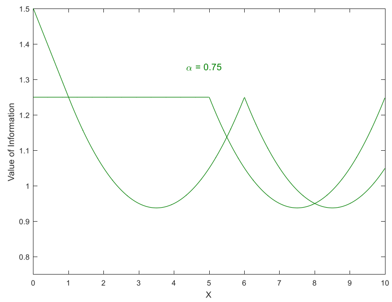
(d1) ($x_6^{opt} = 5$, $y_6^{opt} = 7$) with $\alpha = 0.25$ and $\alpha = 0.5$.
Size of the jump: 1.9500 - 0.9500= 1



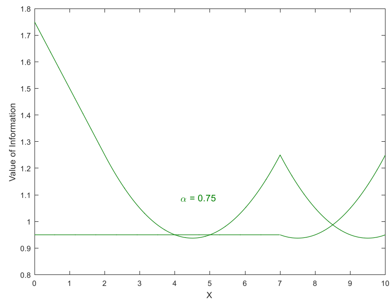
(c2) ($x_3^{opt} = 6$, $y_3^{opt} = 5$).
Threshold value: 5.5 for $\alpha = 0.5$



(d2) ($x_6^{opt} = 5$, $y_6^{opt} = 7$). **Threshold values**:
4.5359 for $\alpha = 0.25$; 3.3542 for $\alpha = 0.5$
Size of the jump with $\alpha = 0.25$: 1.10 - 0.80 = 0.3



(c3) ($x_3^{opt} = 6$, $y_3^{opt} = 5$)
Threshold values: 1, 5.5 and 8 for $\alpha = 0.75$



(d3) ($x_7^{opt} = 7$, $y_7^{opt} = 5$).
Threshold values: 4, 5 and 8.5 for $\alpha = 0.75$

Fig. 10. Continued

Similarly to the previous scenario, the different Fig. 10(a) describe the initial $V_2^I(\bar{x}_1)$ and $V_2^{II}(\bar{x}_1)$ functions for the $\alpha = 0.25$, 0.5 and 0.75 parameter values. Note how the continuation and starting incentives are identical in all settings, as illustrated by Fig. 10(a1) to (a3), and that both functions overlap for realizations of the X characteristic above c_X. For comparison purposes, assume that the first observation is given by $x_1 = 6$. In this case, D is indifferent between continuing and starting in all α settings.

If D decides to start, the first partially observed product would be given by $(x_1 = 6,\ y_1 = c_Y)$ in all α settings, whose resulting value functions are described in Fig. 10(b0). Note that the continuation functions, $V_3^I(\bar{x}_1)$, remain unchanged and that modifications of $V_3^{II}(\bar{x}_1)$ due to partially observed products do not generate discontinuities. Moreover, starting does not preclude continuation from delivering a product with a utility higher than $Rand$, as was the case within the $\alpha = 0.5$ and $\alpha = 0.75$ settings in the $Y = [0,\ 5]$ scenario.

Assume now that D observes $x_1 = 6$, but this time he decides to continue in all α settings. That is, consider the case where D completely observes a product, which is assumed to be given by $(x_3^{opt} = 6,\ y_3^{opt} = 5)$ in order to emphasize the differences with the scenario described in Fig. 10(b) for a partially observed product delivering the same (expected) utility. Fig. 10(c0) represents the different search incentives faced by D. Note that the symmetry of the $y_1 = 5$ realization within its [0, 10] domain preserves the continuity of the continuation functions in all α settings.

Assume that after fully observing $(x_3^{opt} = 6,\ y_3^{opt} = 5)$, D starts acquiring information on a new product and observes $x_2 = 5$. The functions in Fig. 10(c1) and (c2) illustrate the continuation incentives of D within the $\alpha = 0.25$ and $\alpha = 0.5$ settings, while Fig. 10(c3) shows that D prefers to start within the $\alpha = 0.75$ setting.

Two assumptions can be made regarding the $\alpha = 0.25$ and $\alpha = 0.5$ cases. The next product observed could be assumed to be equal to $(x_2 = 5,\ y_2 = 3)$, which brings D back to the scenario described in Fig. 10(c0), since D must start observing a new product using $(x_6^{opt} = 6,\ y_6^{opt} = 5)$ as the reference one. An alternative scenario would consist of D observing $(x_6^{opt} = 5,\ y_6^{opt} = 7)$ as the second product, which is the case described in Fig. 10(d1) and (d2). Similarly, an observation of $x_2 = 5$ leads in the $\alpha = 0.75$ setting to the scenario described in Fig. 10(c3), where the partially observed product $(\bar{x}_2 = 5,\ y_2 = c_Y)$ is used as a starting reference.

Consider the discontinuity caused by the different α values in Fig. 10(d1) and (d2). As already stated, the $\alpha = 0.5$ setting is symmetric and its continuation function continuous. On the other hand, the $\alpha = 0.25$ and $\alpha = 0.75$ settings are asymmetric and their continuation functions discontinuous unless, as illustrated in Fig. 10(c) and (d3), the second characteristic equals the c_Y value, which is at the same distance from both extremes of the domain.

Fig. 10(d1) illustrates the discontinuity arising in the $\alpha = 0.25$ setting. Once again, we describe below how the existing gap can be substantially reduced by enhancing the computational capacities of D. Note that the function $V_6^I(\bar{x}_3)$ remains continuous within the $\alpha = 0.5$ setting though the image values of its enhanced version are different. Fig. 10(d2) describes the resulting threshold points determining the search incentives of D, with the $\alpha = 0.25$ setting exhibiting higher continuation incentives than the $\alpha = 0.5$ one.

The enhanced continuation equation illustrated in Fig. 10(d2) for the $(x_6^{opt} = 5,\ y_6^{opt} = 7)$ product is given by

$$V_6^I(\bar{x}_3) = \begin{cases} \underbrace{\alpha \int_{P_6^+(\bar{x}_3)} \eta(y)(u(\bar{x}_3) + v(y) - U_{rand})\, dy}_{\text{Prevention}} + \\ \underbrace{(1-\alpha) \int_{P_6^-(\bar{x}_3)} \eta(y)(U_{rand} - u(\bar{x}_3) - v(y))\, dy}_{\text{Verification}}, & \text{if } \bar{x}_3 < x_6^{opt} + y_6^{opt} - y_M = 2 \\ \underbrace{\alpha \int_{P_6^+(\bar{x}_3)} \eta(y)\left(u(\bar{x}_3) + v(y) - U_6^{opt}\right) dy}_{\text{Prevention}} + \underbrace{\alpha \int_{P_6^+(\bar{x}_3)} \eta(y)(u(\bar{x}_3) + v(y) - U_{rand})\, dy}_{\text{Prevention}} + \\ \underbrace{(1-\alpha) \int_{P_6^-(\bar{x}_3)} \eta(y)(U_{rand} - u(\bar{x}_3) - v(y))\, dy}_{\text{Verification}}, & \text{if } x_6^{opt} + y_6^{opt} - y_M \le \bar{x}_3 < x_{rand} = 5 \\ \underbrace{(1-\alpha) \int_{P_6^+(\bar{x}_3)} \eta(y)\left(u(\bar{x}_3) + v(y) - U_6^{opt}\right) dy}_{\text{Verification}} + \underbrace{(1-\alpha) \int_{P_6^+(\bar{x}_3)} \eta(y)(u(\bar{x}_3) + v(y) - U_{rand})\, dy}_{\text{Verification}} + \\ \underbrace{\alpha \int_{P_6^-(\bar{x}_3)} \eta(y)(U_{rand} - u(\bar{x}_3) - v(y))\, dy}_{\text{Prevention}}, & \text{if } x_{rand} \le \bar{x}_3 \le Rand = 10 \end{cases} \tag{11.3}$$

The intuition defining the terms that compose Eq. (11.3) is identical to the one used to describe Eq. (11.2), but accounting for a different optimal completely observed product.

We conclude by assuming that $x_3 = 7$ in all the settings described in Figs. 10(d2) and 8(d3). As a result, D would start acquiring information within the $\alpha = 0.25$ and $\alpha = 0.5$ settings, while continuing with the product partially observed in the $\alpha = 0.75$ one. Assume, for completeness, that this final product completely observed by D is $(x_7^{opt} = 7,\ y_7^{opt} = 5)$. In this regard, the value of the incentive functions determined by the ($x_4^{opt} = 6$, $y_4^{opt} = 5$) and ($x_6^{opt} = 5$, $y_6^{opt} = 7$) products equals σ_4 =4.0867 and $\sigma_6 = 4.1083$, respectively. Thus, for example, a cost $c = 3$ for a given parameter $\omega = 20$ would allow D to proceed acquiring information on several additional fully observed products.

We provide below a summary of the products observed by D and emphasize the fact that the same set of products has been displayed through the search process in each α setting.

- If $\alpha = 0.25$: D observes (6, 5), (5, 7), and (7, -). This setting presents a behavior similar to the $\alpha = 0.5$ one while incentivizing the continuation option for a wider range of X values.
- If $\alpha = 0.5$: D observes (6, 5), (5, 7), and (7, -). An increase in the relative importance of regret leads D to restrict his continuation incentives. Thus, if a firm wants its products to be completely observed by D but they display low X characteristics, a setting where D exhibits relatively low α levels constitutes a preferred search environment. Moreover, this setting constitutes a valuable option if the X characteristic is relatively high and the firm has its product located at the beginning of the search process, decreasing the incentives of D to completely observe further products and increasing its selection probability.
- If $\alpha = 0.75$: D observes (6, 5), (5, -), and (7, 5). Increasing the relative importance assigned to regret further leads to threshold multiplicity as the search process evolves, which, at the same time, allows D to acquire information on potentially better completely observed products after fully observing the first one. Thus, firms whose products display relatively high X characteristics but are not located at the beginning of the search process would prefer this setting to any of the previous ones.

Remark 8. We conclude by emphasizing the importance of recall within the current information acquisition framework. Assume that D initially observes $x_1 = 6$. Then, within the $\alpha = 0.75$ setting illustrated in Fig. 10(a3), he would start acquiring information on a new product. After proceeding through the information acquisition process described in Fig. 10, he would end up in Fig. 10(d3). At this point, given his capacity to recall previous products, D would continue acquiring information on the initial partially observed product corresponding to $x_1 = 6$ instead of leaving it aside as a suboptimal alternative. ■

9. Conclusion

We have defined an information acquisition model using the value of information as a verification and regret-prevention mechanism that accounts for the main features of the sequential search environments described by different branches of the academic literature.

Given the adaptive characterization of our algorithm, D has been assumed to adapt his reference values as he acquires information and observes the characteristics of different products. At each step of the algorithm, the best fully observed alternative determines the reference product when deciding whether or not to continue acquiring information on a partially observed one. On the other hand, the reference product is given by the best partially observed one when D decides to start acquiring information on a new product. Moreover, D has been allowed to subjectively assign different importance to the prevention and verification properties of the information acquired.

We have shown that the incentives of D to continue acquiring information on a given product are determined by the relative spread exhibited by the domains on which the product characteristics are defined. In this regard, a decrease in the spread of the secondary characteristic relative to that of the primary one increases the incentives of D to acquire partial information about new products. On the other hand, the relative spread of the secondary characteristic can be increased so that the value derived from fully observing the products dominates. At the same time, intermediate spread scenarios where indifference arises, leading D to behave randomly, can also be defined.

A real-life environment where our model can be immediately applied is given by the sequential information acquisition processes taking place in online environments, where large amounts of information about different product characteristics are available at low acquisition costs [3], providing a suitable framework for regret to arise among DMs [29]. In this regard, the numerical example presented in Section 8.2 has assumed D to observe a total of five characteristics from three different products through a process that can be directly applied to online search and evaluation frameworks such as those defined by standard recommender engines. As illustrated, the decisions made by D will be determined, among others, by the relative importance assigned to the characteristics being considered and the attitude of D toward regret. Note that the current evaluation framework allows also for modifications in the subjective beliefs and risk attitudes of D when defining his information acquisition incentives.

D has been endowed with basic memory capacities throughout the analysis, while noting that absent recall he would have based his search behavior on the initial scenarios defined for all settings in Sections 7 and 8. Moreover, additional capabilities have been required from D in order to preserve computational consistency through his search process, implying that the emergence of indecision observed in real-life environments constitutes a plausible phenomenon [13].

From a computational standpoint, the current evaluation framework could be adapted and implemented as a decision rule within more complex algorithm structures to simulate the behavior of different groups of DMs based on their subjective capabilities. The generality and flexibility of the model allows to easily develop further extensions and applications to decision theory, psychology, economics and operational research. In particular, the dynamic characterization of the algorithm allows for intertemporal comparisons of incentives determined by the potential realizations of the characteristics.

This type of comparisons constitutes a particularly useful framework when considering the forward-looking behavior of DMs within strategic environments, allowing for a wide array of additional applications to management research areas.

Acknowledgement

The authors would like to thank the anonymous reviewers and the editor for their insightful comments and suggestions.

References

[1] R.P. Bagozzi, D. Belanche, L.V. Casaló, C. Flavián, The role of anticipated emotions in purchase intentions, Psychol. Mark. 33 (8) (2016) 629–645.
[2] N.O. Bakir, G.A. Klutke, Information and preference reversals in lotteries, Eur. J. Oper. Res. 210 (3) (2011) 752–756.
[3] N. Carr, The Shallows: What the Internet is Doing to Our Brains, W.W. Norton & Company, 2011.
[4] J.-G. Cegarra-Navarro, P. Soto-Acosta, A.K.P. Wensley, Structured knowledge processes and firm performance: the role of organizational agility, J. Bus. Res. 69 (5) (2016) 1544–1549.
[5] Y.C. Chen, R.A. Shang, C.Y. Kao, The effects of information overload on consumers' subjective state towards buying decision in the internet shopping environment, Electron. Commer. Res. Appl. 8 (1) (2009) 48–58.
[6] L. Denant-Boèmont, R. Petiot, Information value and sequential decision-making in a transport setting: an experimental study, Transp. Res. Part B: Methodol. 37 (4) (2003) 365–386.
[7] D. Di Caprio, F.J. Santos-Arteaga, M. Tavana, An optimal sequential information acquisition model subject to a heuristic assimilation constraint, Benchmarking: An Int. J. 23 (4) (2016) 937–982.
[8] D. Di Caprio, F.J. Santos-Arteaga, M. Tavana, The optimal sequential information acquisition structure: a rational utility-maximizing perspective, Appl. Math. Model. 38 (2014) 3419–3435.
[9] M. Fels, On the value of information: why people reject medical tests, J. Behav. Exp. Econ. 56 (2015) 1–12.
[10] F. Fouss, Y. Achbany, M. Saerens, A probabilistic reputation model based on transaction ratings, Inf. Sci. 180 (11) (2010) 2095–2123.
[11] P. Goodwin, G. Wright, Decision Analysis For Management Judgment, third ed., Wiley, Chichester, UK, 2004.
[12] B.L. Hallen, E.C. Pahnke, When do entrepreneurs accurately evaluate venture capital firms' track records? A bounded rationality perspective, Acad. Manag. J. 59 (5) (2016) 1535–1560.
[13] H.G. Jeong, A. Drolet, Variety-seeking as an emotional coping strategy for chronically indecisive consumers, Mark. Lett. 27 (1) (2016) 55–62.
[14] J.D. Johnson, Health-related information seeking: is it worth it? Inf. Process. Manag. 50 (5) (2014) 708–717.
[15] D. Kahneman, A. Tversky, Choices, Values, and Frames, Cambridge University Press, 2000.
[16] J.S. Lerner, Y. Li, P. Valdesolo, K.S. Kassam, Emotion and decision making, Ann. Rev. Psychol. 66 (2015) 799–823.
[17] G. Loomes, R. Sugden, Regret theory: An alternative theory of rational choice under uncertainty, Econ. J. 92 (1982) 805–824.
[18] A. Mas-Colell, M.D. Whinston, J.R. Green, Microeconomic Theory, Oxford University Press, New York, NY, 1995.
[19] C. Massey, D. Robinson, R. Kaniel, Can't wait to look in the mirror: The impact of experience on better-than-average effect, in: Proceedings of the Annual INFORM Meeting, Pittsburgh, PA, 2006.
[20] B. McCall, J. McCall, The Economics of search: v. 1. (Routledge Advances in Experimental and Computable Economics), Routledge, 2007.
[21] J. Medhurst, I.M. Stanton, H. Bird, A. Berry, The value of information to decision makers: an experimental approach using card-based decision gaming, J. Oper. Res. Soc. 60 (6) (2009) 747–757.
[22] M. Mintzberg, Patterns in strategy formulation, Manag. Sci. 24 (9) (1978) 934–948.
[23] C.A. O'Reilly, Variations in decision makers' use of information sources: the impact of quality and accessibility of information, Acad Manag. J. 25 (4) (1982) 756–771.
[24] J. Park, W.T Hill, J. Bonds-Raacke, Exploring the relationship between cognitive effort exertion and regret in online vs. offline shopping, Comput. Human Behav. 49 (2015) 444–450.
[25] J.P. Ponssard, On the concept of the value of information in competitive situations, Manag. Sci. 22 (7) (1976) 739–747.
[26] F.J. Santos-Arteaga, D. Di Caprio, M. Tavana, A self-regulating information acquisition algorithm for preventing choice regret in multi-perspective decision making, Bus. Inf. Syst. Eng. 6 (3) (2014) 165–175.
[27] K. Sarangee, J.B. Schmidt, J.P. Wallman, Clinging to slim chances: the dynamics of anticipating regret when developing new products, J. Prod. Innov. Manag. (2013), doi:10.1111/jpim.12041.
[28] A. Schepanski, W.C. Uecker, The value of information in decision making, J. Econ. Psychol. 5 (2) (1984) 177–194.
[29] B. Schwartz, The Paradox of choice: Why more is Less, Ecco Press, New York, 2004.
[30] B. Schwartz, What does it mean to be a rational decision maker? J. Market. Behav. 1 (2) (2015) 113–145.
[31] H.A. Simon, Administrative Behavior, The Free Press, New York, 1997.
[32] J.E. Smith, C. Ulu, Technology adoption with uncertain future costs and quality, Oper. Res. 60 (2) (2012) 262–274.
[33] M. Tavana, D. Di Caprio, F.J. Santos-Arteaga, A multi-criteria perception-based strict-ordering algorithm for identifying the most-preferred choice among equally-evaluated alternatives, Inf. Sci. 381 (2017) 322–340.
[34] M. Tavana, D. Di Caprio, F.J. Santos-Arteaga, Modeling sequential information acquisition behavior in rational decision making, Decis. Sci. 47 (4) (2016) 720–761.
[35] M. Tavana, F.J. Santos-Arteaga, D. Di Caprio, K. Tierney, Modeling signal-based decisions in online search environments: a non-recursive forward-looking approach, Inf. Manag. 53 (2) (2016) 207–226.
[36] C. Ulu, J.E. Smith, Uncertainty, information acquisition, and technology adoption, Oper. Res. 57 (3) (2009) 740–752.
[37] L. Wang, Y.-M. Wang, L. Martínez, A group decision method based on prospect theory for emergency situations, Inf. Sci. 418-419 (2017) 119–135.
[38] L.L. Wilde, On the formal theory of inspection and evaluation in product markets, Econometrica 48 (5) (1980) 1265–1279.
[39] T.M. Williams, K. Samset, K.J. Sunnevåg, Making Essential Choices With Scant Information, Palgrave Macmillan, 2009.
[40] D. Xianzhong, M. Xu, G.R. Kaye, Knowledge workers for information support: executives' perceptions and problems, Inf. Syst. Manag. 19 (1) (2002) 81–88.
[41] M. Zeelenberg, Robust satisficing via regret minimization, J. Mark. Behav. 1 (2015) 157–166.