Deliberation, Leadership and Information Aggregation*

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Abstract

We analyze committees of voters who take a decision between two options as a two-stage process. In a discussion stage, voters share non-verifiable information about a private signal concerning what is the best option. In a voting stage, votes are cast and one of the options is implemented. We introduce the possibility of leadership whereby a certain voter, the leader, is more influential than the rest at the discussion stage even though she is not better informed. We study information transmission and characterize the effects of the leader on the deliberation process. We find, amongst others, that both the quality of the decision taken by the committee and how truthful voters are at the discussion stage depends non-monotonically on how influential the leader is. In particular, although a leader whose influence is weak does not disrupt the decision process of the committee in any way, a very influential leader is less disruptive than a moderately influential leader.

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1 Introduction

Committees are a common framework for making choices. In a committee, its members discuss their views on the issue at stake and then choose among the different options, usually via voting. For example, a company's chief executives meet often to decide the firm's future strategy. Ministers of a cabinet meet regularly to choose the policies the government should follow. Faculty members in a university gather during staff meetings to decide on new appointments, course programs, etc.

A frequent feature of committees is that the opinions of some of its members, leaders, are taken as more relevant by some its other members. This can happen even though leaders may not necessarily be better informed about the issue at hand than other committee members. The leaders' opinions can be persuasive to other committee members for a variety of reasons. It may be that leaders are more effective at communicating their views or that some committee members believe the leaders to be better informed. Alternatively, some committee members may want to favor the leaders' views expecting something in return.

The objective of this paper is to investigate the implications of leadership on information aggregation and choices in deliberative committees. We try to answer this question by understating how the existence of a leader affects the incentives of other committee members to manipulate information, possibly to counter the influence of the leader. Furthermore, we are also interested in understating if it is possible to mitigate the potential negative effects of a leader.

To this end, we consider a common value election setting where committee members, voters, have to decide between two options: whether to change to a certain alternative or to maintain the status quo. Voters receive a piece of private information (signal) about which option is best for the committee. Voters then meet at a discussion stage and talk about the two options. During the discussion stage voters express their support for either of the options by simultaneously reporting, truthfully or not, the value of their private signals. Afterwards, during a voting stage, each voter simultaneously casts a ballot for one of the options and the alternative beats the status quo if and only if it receives at least a given number of votes.

Within the committee, there are three types of voters. First, there is the *leader*, who is characterized by her influence on other voters. The leader's influence on other voters manifests via the second type of voters: the *followers*. Followers regard the leader's views as the truth and, thus, vote for the option that is supported by the leader during the discussion stage. The final type of voters is the *objective voters*, who want to implement the best option. Thus, followers are committee members that distort the social decision process where all other voters have an interest in implementing the best option.

In our results, we find that a situation where all voters are truthful at the discussion stage is possible if and only if there are either too few or sufficiently many followers. Too few followers means that followers are not enough as to influence the decision process of the committee in any meaningful way, because they are greatly outnumbered by objective voters. On the other hand, if there are many followers then the leader's opinion at the discussion stage is the only opinion that matters. Since the leader's only piece of information regarding what is the best option is in this case his own signal then he has incentives to truthfully report it to the committee.

When a non-majority rule is in place (more than half the votes needed for abandoning the status quo in favour of the alternative), there are situations where all voters truthfully reporting their signals cannot be an equilibrium. This happens whenever there are more than few but not too many followers: followers are enough to block the implementation of the alternative but not enough to entirely decide the outcome of the election. This is the case as if a follower's message at the discussion stage has any effect in the outcome of the election, it must be because the leader is going to support the alternative as otherwise all followers vote for the status quo and the implementation of the alternative is blocked. Thus, whenever the message revealed by a follower matters, it must be that the leader is going to support the alternative and, hence, such follower has incentives to also support the alternative regardless on her own signal, i.e. truthfully revealing her signal is not an equilibrium strategy.

Our results characterize how the existence of a leader and her followers affect information revelation in committees. Surprisingly, it is not true that more followers make truthful sharing of information more unlikely. Moreover, we also find that ceteris paribus the majority rule is the best voting rule in that it makes truthful sharing of information more likely. Thus, a way to mitigate the effects of the followers in decision making in committees is not by increasing the number of votes needed for implementing one of the options. Furthermore, in our analysis we look at what is the probability of implementing the best option as a function of the number of followers. Finally, we consider two extensions to our main model: one where the leader has a bias towards either of the two options and, thus, may take advantage of his influence on the followers, and another extension where there are two leaders.

Our contribution to the literature lies in the inclusion of leadership in deliberative voting committees. To our knowledge, ours is the first attempt at modelling this phenomenon that is much present in real life situations. Nevertheless, the papers of Coughlan (2000) and Austen-Smith and Feddersen (2006) are closely related to our work and modelling approach. These authors extend the strategic analysis of the Condorcet (1785) Jury's framework by introducing a preliminary stage where voters can share information previous to voting.[1] In

[1]Following on the work of Austen-Smith and Banks (1996), a growing literature extends the analysis of

these papers information transmitted in the deliberation stage is not verifiable and, thus, the deliberation stage is regarded as a cheap-talk game. Coughlan (2000) analyzes communication prior to a voting stage in a framework similar to ours and proves that sincere revelation of signals is obtained when voters are similar enough, independently of the voting rule employed. Austen-Smith and Feddersen (2006) study the circumstances under which the unanimity rule can support truthful information revelation when voters are uncertain about the possible preference biases of other voters. We build on the model by Austen-Smith and Feddersen (2006) by assuming that voters' preferences depend directly on the profile of signals received by all voters, but we consider the possibility that some voters (the followers) do not use all the information available to them to decide which option to vote for.

Also related to our paper are the works of Gerardi and Yariv (2007), Jackson and Tan (2013) and Dewan and Myatt (2007, 2008, 2012). Gerardi and Yariv (2007) show the equivalence of the set of equilibria for different voting rules under cheap talk when voters may use dominated strategies in the voting stage. Jackson and Tan (2013) model deliberation as the transmission of verifiable information prior to voting. In their model, a group of experts may receive privates signal or not and can choose whether to reveal the information they recive. Depending on the voting rule, experts may have incentives to hide their signals when the signal they receive goes against their ex-ante preferences. Finally, Dewan and Myatt (2007, 2008, 2012) study the role of leaders as facilitators of coordination among party activists in the process of choosing the best political platform. Party activist receive a piece of private information and want to choose the best platform, where a sense a party unity also introduces a coordination motive in their actions. By publicly communicating their information, leaders bridge differences of opinion among activist and become coordinating focal points. The focus of these papers relies on the impact of leaders' information precision and communication skills and how the costs of coordination may bias the party decisions.

The rest of the paper is organized as follows. In Section 2 we present the model and the notation. In Section 3 present our main results and a welfare analysis. In Section 4 we extend the analysis by contemplating a situation where the leaders has a biased toward either option and another extensions where there are two leaders. Finally, in Section 5 we conclude. All proofs are presented in the Appendix.

non-verifiable strategic information transmission in different voting contexts. For instance, Feddersen and Pesendorfer (1996, 1997) investigate the effects of different accuracy of voters' signals, Feddersen and Pesendorfer (1998); McLennan (1998) consider the problem under different voting rules, Martinelli (2006); Gerardi and Yariv (2008); Gerzskov and Szentes (2009) analyze the implicationsm of costly information acquisition.

2 The Model

Consider a committee formed by $N+1$ voters where $N \geq 2$ is an even number. Voters have to decide whether to implement an alternative A or to keep the status quo Q.

There are two states of nature, $S = \{A, Q\}$, and each voter receives a private signal correlated with the state of nature. The private signal received by voter i is given by $\theta_i \in \{A, Q\}$ where

$$P(\theta_i = A | S = A) = P(\theta_i = Q | S = Q) = p$$

with $p \in \left[\frac{1}{2}, 1\right]$. That is, p is the accuracy of the signal and is independent of the identity of the voter. Let $\theta = (\theta_1, \ldots, \theta_{N+1}) \in \{A, Q\}^{N+1}$ denote a profile of observed signals. For each voter i let $\theta_{-i} \in \{A, Q\}^N$ be the complementary profile of signals observed by voters other than i. Define Θ_A and Θ_Q as the set of voters who receive signal A and Q respectively. Finally, let $\Theta_A \setminus i \equiv \{j \neq i \mid \theta_j = A\}$ be the set of voters excluding i who receive signal A and let $\Theta_Q \setminus i \equiv \{j \neq i \mid \theta_j = Q\}$ be the set of voters excluding i who receive signal Q.

After each player receives a signal, a discussion stage takes place. This discussion stage takes the form of a non-binding straw poll where each voter simultaneously reveals a message $m_i \in \{A, Q\}$ to all other voters. Let $m \in \{A, Q\}^{N+1}$ denote a profile of reported messages. For each voter i let $m_{-i} \in \{A, Q\}^N$ be the complementary profile of messages reported by voters other than i. Define M_A and M_Q as the set of voters who report signal A and Q respectively. Finally, let $M_A \setminus i \equiv \{j \neq i \mid m_j = A\}$ denote the set of voters excluding i who reveal message A and let $M_Q \setminus i \equiv \{j \neq i \mid m_j = Q\}$ be the set of voters excluding i who reveal message Q.

Once the discussion stage is over voters casts their vote during the voting stage. Let $v_i \in \{A, Q\}$ be the alternative chosen by voter i and define V_A and V_Q as the set of voters who choose option A and Q respectively. The alternative A is implemented if and only if it receives at least $q \in \left\{\frac{N}{2}+1, \ldots, N+1\right\}$ votes. Note that if $q = \frac{N}{2}+1$ then the majority rule is in place whilst if $q = N+1$ then the voting rule is the unanimity rule.

Voters' preferences over the option implemented are not homogeneous. In particular, voters can be of three types: voter i is either a *leader*, $i \in L$, she is a *follower*, $i \in F$, or she is an *objective voter* $i \in O$. Initially, we assume that the set L is a singleton so there is a unique leader l, $L = \{l\}$. The preferences of each voter can be represented by a utility function $u : \{L, F, O\} \times \{A, Q\} \times \{A, Q\}^{N+1} \times \{A, Q\}^{N+1} \to [0, 1]$ where the first argument is the type of the voter, the second argument is the option implemented, the third argument is the profile of signals and the fourth argument is the profile of messages. We assume voters are expected utility maximizers.

The leader is characterized by the fact that his message at the discussion stage has a great influence in some voters. Moreover, the leader may have a biased against either option in that she prefers the alternative A if and only if there is, in her opinion, sufficient evidence in favour of A. We model this as the leader requiring at least $b \in \{0, \dots, N+1\}$ signals in favour of A in order to prefer the alternative A during the voting stage. Given that $p \geq \frac{1}{2}$, we say that the leader is *unbiased* if $b = \frac{N}{2} + 1$ as in this case he prefers the option that is more likely to match with the state of nature given the signals received by all players. We say that the leaser is *biased* if $b \neq \frac{N}{2} + 1$.[2] We represent the preferences of the leader by the following utility function:

$$u(L, A, \theta, m) = 1 - u(L, Q, \theta, m) \quad = \quad \begin{cases} 1 & \text{if } \#\Theta_A \geq b, \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

The preferences of the followers depend completely on the message revealed by the leader. Followers simply want the option revealed by the leader to win the election. Their preferences can be represented by the following utility function:

$$u(F, A, \theta, m) = 1 - u(F, Q, \theta, m) \quad = \quad \begin{cases} 1 & m_l = A, \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

Even though followers may be thought of as behavioural agents in that their target is not to choose the option that matches with the state of nature, they are fully rational agents given their utility function. Furthermore, note that the followers' aim is to have the option revealed by the leader to win the election; they are not interested in the option the leader actually votes for. This is a reasonable assumption under the interpretation of the model whereby followers want to please the leader, possibly because of career concerns or because they expect something in return for their support. Hence, followers want their backing of the leader to be noticed and messages are observable but votes are not.

The final type of voters are the objective voters, who want to chose the alternative that matches with the state of nature. Thus, given that the accuracy of the signals is greater or equal to $\frac{1}{2}$ objective voters prefer the alternative A to the status quo Q if and only if there are at least $\frac{N}{2} + 1$ voters who received signal A. Objective voters' preferences can be represented

[2]Although we are assuming ad-hoc that the leader can be biased, there are different arguments that justify the existence of such bias from the optimality (and Bayesian) point of view. For instance, Coughlan (2000) considers a framework where the voters want to make the "right" choice but the cost of not implementing the right choice is different depending on the state of nature. Alternatively, in Jackson and Tan (2013) voters' preferences have a private component whereby a voter may require more (or less) than half of the signals to prefer the alternative to the status quo.

by the following utility function:

$$u(O, A, \theta, m) = 1 - u(O, Q, \theta, m) \quad = \quad \begin{cases} 1 & \text{if } \#\Theta_A \geq \frac{N}{2} + 1, \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

The messages sent during the discussion stage are given by the message strategy which is mapping from the voter's type and the signal she receives to the set of options, $\sigma^m : \{L, F, O\} \times \{A, Q\} \to \{A, Q\}$. The realization of σ^m for a certain player i is given by m_i. How voters choose between the two options during the voting stage is determined by the voting strategy. This is a map from the voters' type, the signal she receives and the set of message profiles of all voters to the set of available options, $\sigma^v : \{L, F, O\} \times \{A, Q\} \times \{A, Q\}^{N+1} \to \{A, Q\}$. The realization of σ^v for a certain player i is given by v_i.

It is assumed that all that is relevant to the game presented is common knowledge except for the realized value of the private signals: every voter only observes its own signal. Thus, players know the identity of the leader and how many followers there are in the committee. Similarly, players also know the value of p, the accuracy of the signal, and b, the bias of the leader. Once voters receive their signals and before they reveal their messages they update their beliefs on the signals received by other voters using Bayesian updating.

We are interested in studying the circumstances under which there is full information transmission. We say that voter i in group $G \in \{L, F, O\}$ truthfully reveals her signal if $\sigma^m(G, \theta_i) = \theta_i$.

Given the utility function of each voter, if all voters truthfully reveal their signals then the unique weakly dominant voting strategies for any voter i are given by:

$$\sigma^v(L, \theta_i, m) \quad = \quad \begin{cases} A & \text{if } \#M_A \geq b, \\ Q & \text{otherwise.} \end{cases} \tag{4}$$

$$\sigma^v(F, \theta_i, m) \quad = \quad \begin{cases} A & \text{if } m_l = A, \\ Q & \text{otherwise.} \end{cases} \tag{5}$$

$$\sigma^v(O, \theta_i, m) \quad = \quad \begin{cases} A & \text{if } \#M_A \geq \frac{N}{2} + 1, \\ Q & \text{otherwise.} \end{cases} \tag{6}$$

Define $\mathbf{v} : \{A, Q\}^{N+1} \to \{A, Q\}$ as the option that is implemented given a profile of messages in $\{A, Q\}^{N+1}$ when the voting strategies are given in (4), (5) and (6). In an abuse of notation, we omit the number of voters of each type in the description of $\mathbf{v}$.

Definition 1. *The message strategy σ_i^m is a **truth-telling best response for voter** i **of type** G if $\sigma^m(G, \theta_i) = \theta_i$ and for any message strategy $\bar{\sigma}^m$*

$$\mathbb{E}\left[u(G, \mathbf{v}(\theta), \theta, \theta) \mid \theta_i\right] \geq \mathbb{E}\left[u(G, \mathbf{v}(\theta_{-i}, \bar{\sigma}^m(G, \theta_i)), \theta, (\theta_{-i}, \bar{\sigma}^m(G, \theta_i))) \mid \theta_i\right],$$

where $\mathbb{E}$ *denotes the expected value operator.*

Now we are in condition to introduce the equilibrium concept we are interested in:

Definition 2. *A profile of message strategies* $(\sigma_1^m, \ldots, \sigma_{N+1}^m)$ *is a* ***fully revealing (Bayesian Nash) equilibrium (FRE)*** *if and only if for each voter* i *the message strategy* σ_i^m *is a truth-telling best response.*

A FRE is a sequential (and perfect) Bayesian Nash equilibrium of the dynamic two stage incomplete information game where voters report their true signals and vote accordingly in the voting stage. Therefore, in a FRE each voter truthfully reveals her signal and no voter has incentives to change neither his message during the discussion stage nor his vote during the voting stage.

Given that when all voters truthfully report their signals there is a unique optimal voting strategy, when looking for FRE it is sufficient to look at individual incentives to report truthfully during the discussion stage assuming all other voters report truthfully. Therefore, when checking whether reporting truthfully is a best response strategy for a voter if all other voters are truthful at the discussion stage, we only need to consider the situations where her report may influence the final outcome of the election. We refer to such situations by saying that the voter is *pivotal*.

Definition 3. *Given the truthful profile of messages* θ*, voter* i *is* ***pivotal*** *if there is a* $m_i \neq \theta_i$ *such that* $\mathbf{v}(\theta) \neq \mathbf{v}((\theta_{-i}, m_i))$*.*

There are convenient implications of the assumption of the use of undominated strategies in the voting stage. When voters use undominated voting strategies, each voter's vote does not depend on the message she reports. Moreover, since by reporting a certain option a pivotal voter can only increase the support for such option if a voter is pivotal then the voting outcome coincides with the message this voter reveals.

3 Unbiased Leader

3.1 Fully Revealing Equilibrium

We analyze first the voters' strategic incentives in the discussion stage when the leader is unbiased, $b = \frac{N}{2} + 1$. We have the following result:

Proposition 1. *Assume that the leader is unbiased.*

- *If either* $\#F \geq q$ *or* $\#F < (N+1) - (q-1)$ *then there is a FRE.*

- *If* $q > \#F \geq (N+1) - (q-1)$ *then there is no FRE.*

Given the result in Proposition 1, we have that a FRE is possible if and only if either there are either few or sufficiently many followers. When there are sufficiently many followers ($\#F \geq q$), then followers are enough as to force the implementation of any of the two options. In this case, the leader is pivotal always, and the only pivotal voter, as whatever option she reveals gets voted for by all followers and, hence, such option wins the election. As a consequence, the leader's vote always completely determines the option to be implemented. Therefore, the leader learns nothing from being pivotal and, thus, the only information she has about the signals of other voters is her own signal. In consequence, since the leader is unbiased and the accuracy of the signal is at least $\frac{1}{2}$, if she receives a certain signal she believes this signal is the one that most other voters receive. Thus, she has incentives to truthfully reveal her signal.

When there are few followers ($\#F < (N+1) - (q-1)$) followers are not enough as to influence the decision process of the committee in any meaningful way: when $\#F < (N+1) - (q-1)$ the outcome of the election is determined by what the leader and objective voters vote for and, hence, the existence the followers does not disrupt the outcome of the decision process. In this case, if a voter is pivotal then it is the case that there is a tie between the messages revealed by all other voters in favour of A and in favour of Q. Followers then are indifferent between reporting A or Q as the leader is equally likely to have received signal A or Q. Hence, reporting their true signal can be part of a FRE. Leaders and objective voters also have incentives to tell the truth: if they receive signal A then there are more A signals than Q signals in the population and similarly if they receive signal Q.

Finally, when the number of followers is not enough as too completely determine the outcome of the voting stage ($\#F < q$) but their number is enough to block the implementation of A in case all non-followers vote for the alternative ($\#F \geq N + 1 - (q-1)$) a FRE is not possible. In this situation, whenever a follower is pivotal it is the case that the leader reports A as otherwise she reports Q and all followers vote for Q, blocking the implementation of the alternative. Thus, when a follower is pivotal it is the case that the leader reports A, hence, such follower has incentives to report A regardless on her signal, i.e. she has incentives to lie.

As it can be observed from Proposition 1, there is a non-monotonic relation between the number of followers and whether or not there exists an equilibrium where all voters are truthful (FRE). Since the influence of the leader manifests itself via the followers, we have that effectively the number of followers is a measure on how influential the leader is. Thus, we can conclude from Proposition 1 that there is a non-monotonic relation between how influential the leader is and how truthful voters are at the discussion stage. As we shall see later on when the welfare analysis is presented, this non-monotonicity is also present when we calculate the probability of implementing the option that coincides with the state of nature as a function of the number of followers.

A consequence of the result in Proposition 1 is the following:

Corollary 1. *Assume that the leader is unbiased.*

i) Under the majority rule $\left(q = \frac{N}{2} + 1\right)$ *there always exists a FRE.*

ii) Under the unanimity rule $(q = N + 1)$ *if there is at least one follower then there does not exist a FRE.*

From the information revelation point of view, the majority rule outperforms any other voting rule as it makes truthful sharing of information possible for any number of followers in case the leader is unbiased. For all other super-majority rules $(q \neq \frac{N}{2} + 1)$ there exists a number of followers such that truthful sharing of information is not possible.

Another consequence of the result in Proposition 1 is that, in the degenerate case where there are no followers, a FRE is always possible regardless of the voting rule q:

Corollary 2. *Assume that the leader is unbiased. If* $\#F = 0$ *then there always exists a FRE.*

3.2 Partially Revealing Equilibrium

Proposition 1 shows that there are some circumstance where a FRE is not possible. Next we turn our attention to study situations where, although no equilibrium where all voters tell the truth exists, equilibria where most voters tell the truth are possible. In particular, whenever a FRE does not exists because a type of voter has incentives to miss-report their signal, we construct a sequential Bayesian equilibrium where a subset of voters truthfully report their signals and each voter's decision in the voting stage is based on the information reported by these truthful subset of voters.[3]

Definition 4. *We say that voter i is **uninformative** if $\sigma_i^m(\theta_i)$ is independent on θ_i. Let K be the set of uninformative voters and let K_A be the set of uninformative voters who receive signal A.*

[3] So far we have analyzed the existence of perfect Bayesian equilibria with the implicit assumption that, under the prior probability beliefs for each pair of voters i, j with $i \neq j$, $P(\theta_i = m_i \mid m_i, \theta_j) = 1$. Now instead we focus on the existence of perfect Bayesian equilibria under the assumption that there is a group of voters whose report is not informative, while the remaining voters truthfully report the signal they have received.

We analyze first the optimal voting strategies for each type of voter under the existence of a a group of voters whose messages are not informative. For this section we assume that if the number of truthful signals supporting each option is the same then voters choose the conservative option of maintaining the status quo.[4]

Lemma 1. *Assume the leader is unbiased. Assume further that $K \neq \emptyset$ and all $j \notin K$ report truthfully, and the set K is common knowledge. For each voter i the unique weakly dominant voting strategies is given by:*

$$\sigma^v(L,\theta_i,m) = \begin{cases} A & \text{if } i \notin K \text{ and } \#(M_A \setminus K) > \frac{N+1-\#K}{2} \text{ or} \\ & \text{if } i \in K \text{ and } \#(M_A \setminus K) + \mathbb{1}_{\theta_i = A} > \frac{N+1-(\#K-1)}{2}, \\ Q & \text{otherwise.} \end{cases} \tag{7}$$

$$\sigma^v(F,\theta_i,m) = m_l. \tag{8}$$

$$\sigma^v(O,\theta_i,m) = \begin{cases} A & \text{if } i \notin K \text{ and } \#(M_A \setminus K) > \frac{N+1-\#K}{2} \text{ or} \\ & \text{if } i \in K \text{ and } \#(M_A \setminus K) + \mathbb{1}_{\theta_i = A} > \frac{N+1-(\#K-1)}{2}, \\ Q & \text{otherwise.} \end{cases} \tag{9}$$

where $\mathbb{1}_{\theta_i = A}$ is the indicator function that takes the value 1 if $\theta_i = A$ and 0 otherwise.

Lemma 1 states that an unbiased leader and objective voters vote for A if and only if more than half of the truthful evidence presented at the discussion stage points at A as the option that matches with the state of nature. This is a consequence of the utility functions of each voter and that voters update their beliefs in a Bayesian way.

Define $\mathbf{v}' : \{A,Q\}^{N+1} \to \{A,Q\}$ as the option that is implemented given a profile of messages in $\{A,Q\}^{N+1}$ when the voting strategies are given in (7), (8) and (9). Notice that in an abuse of notation we are omitting the number of voters of each type in the description of $\mathbf{v}'$.

The next two definitions introduce the equilibrium concept under the prior belief that there is a group of uninformative voters and the remaining voters truthfully report the signal they receive.

Definition 5. *Given the set of uninformative voters , K, the message strategy σ^m is a* ***truth-telling best response for voter*** *$i \notin K$* ***of type*** *G if and only if $\sigma^m(G,\theta_i) = \theta_i$ and for any message strategy $\bar{\sigma}^m$*

$$E\left[u(G,\mathbf{v}'(m),\theta,m) \mid \theta_i\right] \geq E\left[u(G,\mathbf{v}'(m_{-i},\bar{\sigma}^m(G,\theta_i)),\theta,(m_{-i},\bar{\sigma}^m(G,\theta_i))) \mid \theta_i\right]$$

where m is the message profile such that $m_j = \theta_j$ if $j \notin K$ and m_j is independent on θ_j if $j \in K$.

[4] We make this assumption to reduce the number of cases that need to be considered. Note that this assumption is not needed when all voters are truthful as the committee has an odd number of voters.

Definition 6. *A profile of message strategies* $(\sigma_1^m, \ldots, \sigma_{N+1}^m)$ *is a* ***partially revealing (Bayesian Nash) equilibrium*** *(PRE) if and only if there exists a set of voters* $K \in \{0, \ldots, N+1\}$ *such that for all voter* $i \notin K$ *the message strategy* σ_i^m *is a truth-telling best response given* K *and for all voter* $j \in K$ *the message strategy* σ_j^m *does not depend on* θ_j*.*

Note that we are not specifying how uninformative voters reveal their messages because this is not needed. The only requirement is that their message strategies does not use the value of the signal they receive. An example of such strategy for voter $i \in K$ could be m_i equals A with probability $\frac{1}{2}$ and Q with probability $\frac{1}{2}$.

A PRE is a perfect (sequential) Bayesian equilibrium under the prior belief that the messages reported by uninformative voters do not provide relevant information, and the remaining voters truthfully report their signal. Notice that the voting strategies in (4), (5) and (6) are equivalent to those in (7), (8) and (9) if $b = \frac{N}{2} + 1$ and $K = \emptyset$. Similarly, a FRE is a PRE where $K = \emptyset$.

In our next result we investigate the existence of a PRE in situations where a FRE is not possible. Thus, we restrict our attention to situations where $q > \#F > N + 1 - (q - 1)$. Moreover, if a PRE exists then we focus on equilibria where the set of uninformative voters is minimal.

Since followers' only concern is the message reported by the leader and, given that $q > \#F > N + 1 - (q - 1)$, they can always force the voting outcome to be Q, we have that followers are the most obvious candidates to be uninformative voters. We show that a PRE where objective voters are not uninformative can always be constructed. However, only if the accuracy of the signal, p, is low enough a PRE where the leader is also not uninformative exists.

Proposition 2. *Assume that the leader is unbiased and* $q > \#F \geq N + 1 - (q - 1)$ *and let* $K' = F \cup \{l\}$*,*

- *If* $q \neq N + 1$ *and*

$$P\left(\#(\Theta_A \smallsetminus K') > \#(\Theta_Q \smallsetminus K') + 1 | \#(\Theta_A \smallsetminus K') \geq \#(\Theta_Q \smallsetminus K')\right) \leq \frac{1}{2}$$

then there exists a PRE with followers as the set of uninformative voters: $K = F$*.*

- *If* $q = N + 1$ *and*

$$P\left(\#(\Theta_A \smallsetminus K') > \#(\Theta_Q \smallsetminus K') + 1 | \#(\Theta_A \smallsetminus K') > \#(\Theta_Q \smallsetminus K')\right) \leq \frac{1}{2}$$

then there exists a PRE with followers as the set of uninformative voters: $K = F$*.*

- *Otherwise, there is no PRE with with the followers as the set of uninformative voters, but there exists a PRE with followers and the leader as the set of uninformative voters:* $K = F \cup \{l\}$.

The result in Proposition 2 states that we can always find a PRE where objective voters reveal their signals truthfully. The reason is that an objective voter is pivotal only if there are as many messages in favour of A as messages in favour of Q in the population of truthful voters (excluding her own message). Thus, such objective voter has incentives to be truthful at the voting stage.

On top of that, Proposition 2 implies that we can construct a PRE where the leader is truthful if only if the accuracy of the signal is low enough. The reason is that, as the leader is pivotal if only if there are at least as many A signals as Q signals in the population of truthful voters, if the accuracy of the signal is high then there are significantly more A signals than Q signals and the leader has incentives to report A ignoring her own signal. In particular, for the case where $q \neq N+1$, if the leader is pivotal then necessarily $\#(\Theta_A \setminus K') \geq \#(\Theta_Q \setminus K')$. That is, amongst those who tell the truth excluding the leader there are at least as many voters with A signals than with Q signals. This is true as otherwise objective voters and the leader vote for Q and the status quo is maintained regardless on the message revealed by the leader. However, if the accuracy of the signal, p, is high then $\#(\Theta_A \setminus K') \geq \#(\Theta_Q \setminus K')$ implies that $\#(\Theta_A \setminus K') > \#(\Theta_Q \setminus K') + 1$ is a more likely event than $\#(\Theta_A \setminus K') \in \{\#(\Theta_Q \setminus K'), \#(\Theta_Q \setminus K') + 1\}$. Thus, if the former happens then the leader has incentives to reveal A regardless on the signal she receives while if the latter occurs then the leader has incentives to report truthfully.

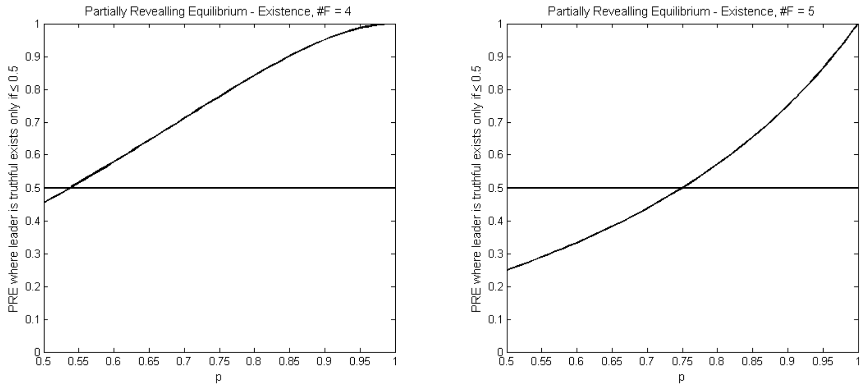
A consequence of Proposition 2 is that the voting rule $q = N+1$ reduces the possible values of the signal p for which a PRE where the leader is truthful exists when compared to any other voting rule $q \neq N+1$. Moreover, Proposition 2 implies that if $q = N+1$ and $\#F$ is odd then there is no PRE with $K = F$. This is because if $\#F$ odd then since $N+1$ is odd we have that $\#(\Theta_A \setminus K') + \#(\Theta_Q \setminus K')$ is even and, hence, $\#(\Theta_A \setminus K') > \#(\Theta_Q \setminus K')$ implies $\#(\Theta_A \setminus K') > \#(\Theta_Q \setminus K') + 1$.

If $q \neq N+1$ then the condition for the existence of a PRE with $K = F$ can be rewritten as

$$\frac{P\left(\#(\Theta_A \setminus K') > \frac{N+1-\#K'}{2} + \frac{1}{2}\right)}{P\left(\#(\Theta_A \setminus K') \geq \frac{N+1-\#K'}{2}\right)} \quad \leq \quad \frac{1}{2}.$$

In Figure 1 we plot the inequality above to illustrate the values of the accuracy of the signal p for which a PRE where the leader is truthful is possible.

Figure 1: $N+1=9$, $q=7$.



Note that in general it is not true that more followers make the existence of a PRE where the leader is truthful less likely. For example, assume that $N+1=9$ and $q=7$ and consider two possibilities: $\#F=4$ and $\#F=5$. In this case, $q>\#F\geq N+1-(q-1)$ and, thus, a FRE is not possible. Moreover, if $\#F=4$ then $\frac{N+1-K'}{2}=2$ and, thus, the condition for the existence of a PRE where the leader is truthful can be rewritten as

$$\frac{P\left(\#(\Theta_A\smallsetminus K')\geq 3\right)}{P\left(\#(\Theta_A\smallsetminus K')\geq 2\right)} \leq \frac{1}{2}. \tag{10}$$

If, on the other hand, $\#F=5$ then $\frac{N+1-K'}{2}=\frac{3}{2}$ and, thus, the condition for the existence of a PRE where the leader is truthful can be rewritten as

$$\frac{P\left(\#(\Theta_A\smallsetminus K')\geq 3\right)}{P\left(\#(\Theta_A\smallsetminus K')\geq 2\right)} \leq \frac{1}{2}. \tag{11}$$

It can be checked that, since $\#F=4$ implies $\#O=4$ and $\#F=5$ implies $\#O=3$, equation (10) is satisfied for a smaller set of parameter values for p than equation (11). Thus, in this case, an extra follower makes the existence of a PRE where the leader is truthful more likely.

3.3 Welfare Analysis

In this section we turn our attention to the welfare analysis. When talking about welfare, we assume that the objective of the committee is to choose the best option, meaning the option that coincides with the state of nature.[5] The question we raise in this section is: how does the existence of the followers affect the likelihood of implementing the best option?

[5] An alternative would be to study the aggregate utility of the committee but this goes against the interpretation of the model whereby followers are a distortion to the decision process.

Given the result in Corollary 2, if there were no followers then a FRE would always exists for all voting rules q. Moreover, if there were no followers then in a FRE the best option is implemented if and only if at least half the committee, $\frac{N}{2}+1$, receives the signal that matches with the sate of nature. Thus, the probability of implementing the best option in this case is given by $\sum_{i=\frac{N}{2}+1}^{N+1}\binom{N+1}{i}p^i(1-p)^{N+1-i}$. Hence, when exploring how the existence of followers affects the likelihood of implement the best option we take this value as the benchmark.

Note that, even if all voters are truthful, the existence of followers may introduce distortions: even if all voters report true information at the discussion stage, followers do not use this information when voting, as they vote for the option the leader reveals at the discussion stage. Moreover, even if followers do not use all the information revealed at the discussion stage, their existence may not reduce the probably of implementing the right option if there are only a few of them. If there are only a few followers then the option implemented coincides with what objective voters vote for, which is based on all the messages revealed at the discussion stage.

To simplify the exposition we reduce significantly the number of cases to be considered by focusing on situations where if either $\#F \geq q$ or $\#F < N+1-(q-1)$ then voters strategies constitute a FRE while if $q > \#F \geq N+1-(q-1)$ then voters strategies constitute a PRE where the set of uninformative voters is $K = F$. That is, for studying the welfare implications of followers we focus on the equilibria where information revelation at the discussion stage is maximal. Define $P(S)$ as the probability by which the committee selects the best option, i.e. implements the option that matches with the state of nature.

Proposition 3. *Assume that if either $\#F \geq q$ or $\#F < N+1-(q-1)$ then voters strategies constitute a FRE. Moreover if $q > \#F \geq N+1-(q-1)$ then assume voters strategies constitute a PRE where the set of uninformative voters is $K = F$. The probability of implementing the option that matches with the state of nature, $P(S)$, is given by:*

$$P(S) = \begin{cases} p & \text{if } \#F \geq q, \\ w(N, \#F, p) & \text{if } q > \#F \geq N+1-(q-1), \\ \sum_{i=\frac{N}{2}+1}^{N+1}\binom{N+1}{i}p^i(1-p)^{N+1-i} & \text{otherwise.} \end{cases}$$

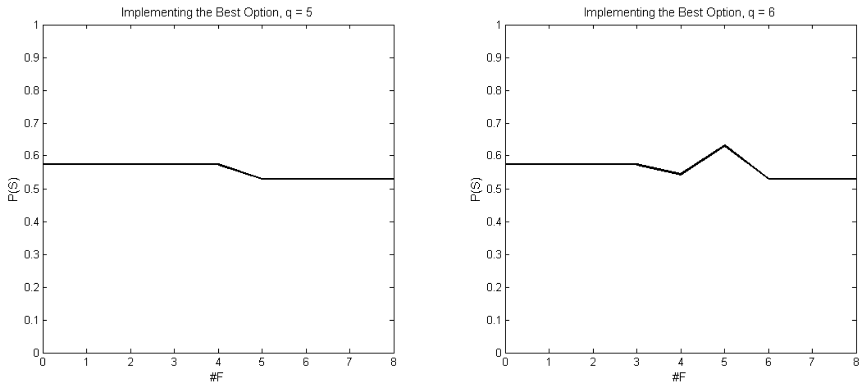
with

$$w(N, \#F, p) = \frac{1}{2}\left(p + \sum_{i=\lceil\frac{N+1-F}{2}\rceil}^{N+1-F}\binom{N+1-F}{i}p^i(1-p)^{N+1-F-i}\right).$$

In words, Proposition 3 means that if there are few followers, $\#F < N+1-(q-1)$, then the committee implements the best option if and only if most most members receive the correct private signal. If there are many followers, $\#F \geq q$, then the committee implements the best option if and only if the leader receives the correct signal. If there are sufficiently few but not too many followers, $q > \#F \geq N+1-(q-1)$, then the committee implements the best option if and only if this option is A, the leader receives A and most truthful voters also receive A, or if the state of nature is Q and either the leader receives Q or most truthful voters receive Q.

In order to get a better understanding of Proposition 3 we present Figure 2, were we plot the probability of implementing the best option as a function on the number of followers for different voting rules q given the accuracy of the signal p. In Figure 2 the committee has a size of 9 voters. We have chosen a low value for the accuracy of the signal, $p = 0.53$, so that a PRE where the leader is truthful exists in this case (see Proposition 2).[6]

Figure 2: $N+1=9$, $p=0.53$.



Proposition 3 has several implications. First, followers do not represent a problem for the implementation of the best option if and only if there are only a few of them ($\#F < N+1-(q-1)$). In this case the option implemented coincides with what objective voters vote for, which is based on all the messages revealed at the discussion stage. Hence, there is no loss of information due to the existence of followers and, hence, the committee makes the best decision possible.

Second, again we observe a non-monotonicity in the effects of the number of followers: it is not true that for a given voting rule q more followers decrease the probability of implementing the right option. In Figure 2 this can be seen in the case where $q = 6$ (half plus one votes are needed to implement the alternative). In this situation, the probability of implementing the right option when there are 4 followers is smaller than where there are 5 of them: with

[6]For higher values of p such that no PRE where the leader is truthful exists, we have that the probability of implementing the option that coincides with the state of nature, $P(S)$, when $q > \#F \geq N+1-(q-1)$ is lower as the leader is no longer telling the truth.

4 followers, 2 out of 4 objective voters need to receive the right signal in order for the right option to be implemented, on the other hand if there are 5 followers then only 2 our of 3 objective voters need to receive the right signal.

Third, the majority rule is not necessarily the one that maximizes the probability of implementing the right option for any number of followers. Take the example in Figure 2 and assume there are 5 followers. If $q = \frac{N}{2} + 1$ then 5 out of 9 members of the committee need to receive the signal that matches with the state of nature in order to implement the right decision. However, if q is such that $q > \#F \geq N + 1 - (q - 1)$ (take $q = 6$ for instance) and the state is Q then either the leader or at least 2 out of 3 objective voters need to receive the signal that matches with the state of nature for the committee to take the right decision. Hence, as it is the case in the right plot in Figure 2, with 5 followers the committee is more likely to make the right decision when $q > \#F \geq N + 1 - (q - 1)$ than when $q = \frac{N}{2} + 1$.

4 Extensions

4.1 Biased Leader

Consider now the case where the leader is biased, i.e. $b \neq \frac{N}{2} + 1$. The existence of a biased leader affects the strategic incentives of all voters. Effectively, this fact precludes the possibility of full information aggregation unless the leader has enough followers and the accuracy of the signals is high enough.

Proposition 4. *Assume that the leader has a bias $b \neq \frac{N}{2} + 1$. A FRE exists if and only if $\#F \geq q$ and p is high enough relative to b.*

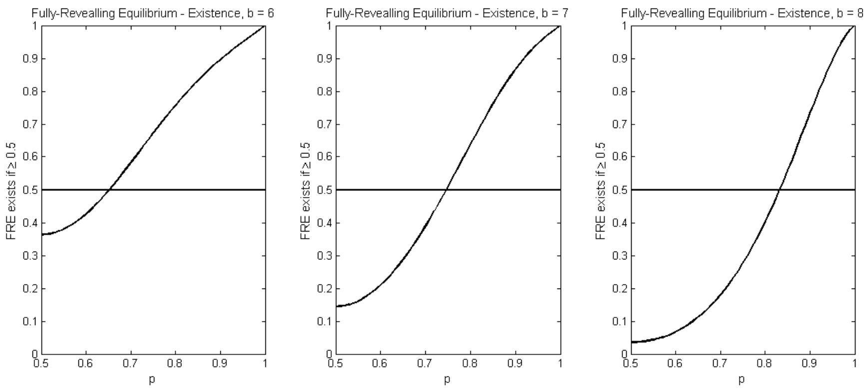
Proposition 4 states that with a biased leader truthful information transmission occurs only if there are enough followers ($\#F \geq q$). Thus, in comparison with the situation where the leader is unbiased, the case where there are few followers ($\#F < N + 1 - (q - 1)$) is not compatible with a FRE any more. The reason for this is that when $\#F < N + 1 - (q - 1)$ the leader is pivotal if and only if her message can influence objective voters. This is possible only if the number of signals in favor of both alternatives, excluding the leader's own signal, is the same. Since the leader is biased, this means that independently on the signal she receives she has incentives to report according to her bias: reveal A if $b < \frac{N}{2} + 1$ and reveal Q if $b > \frac{N}{2} + 1$.

The reason why according to Proposition 4 a FRE is possible only if p is high enough relative to b is the following. If $\#F \geq q$ then the leader is the only pivotal voter and in deciding whether or not to report truthful she has to anticipate how many voters receive the same signal as he does. If the accuracy of the signal, p, is high enough then most voters receive the same signal as she does (in particular, more voters than her own bias). Thus, in this case the leader has incentives to be truthful. On the other hand, if p is not high enough then even though the leader expects more than half other voters to receive the same signal as she does, the leader does not expect the number of voters receiving her same signal to be higher than her bias. Thus, in this case, the leader ignores her signal and reports according to her bias.

A consequence of Proposition 4 is that, once again, the majority rule is the voting rule that makes truthful sharing of information more likely, ceteris paribus. Moreover, the chances of implementing the best option are significantly reduced in the presence of a biased leader: either the leader is not truthful and all followers' vote is unrelated with what is the best option, or the leader is truthful ($\#F \geq q$) but her opinion is the only one that matters.

In Figure 3 we explore a situation where $N+1=9$ and $\#F \geq q$ and show the values of p for which a FRE is possible given different bias levels. The higher the bias of the leader the more accurate is the precision of the signal needed for a FRE to exist.

Figure 3: $N+1=9$, $\#F \geq q$.



Proposition 4 deals with the case where the leader is the only voter who is biased. Although a FRE does not exits in the situations when objective voters suffice to determine the voting outcome, if objective voters had the same bias as the leader then a FRE exists. In this case, whenever the leader (or any objective voter) is pivotal her report determines the votes of all the voters that share her same preferences. Thus, whether the leader prefers A or Q depends on the signal she receives and consequently she has incentives to reveal truthfully.[7] Conversely, if the leader is unbiased and objective voters are biased (all with the same bias) then the converse arguments show that a FRE is not possible. Finally, it is clear that if the

[7]This result is parallel to the main result of Coughlan (2000).

leader is biased and $\#F < N + 1 - q$ ($\#O \geq q$) then a PRE exist where $F \cup \{l\}$ is the set of uninformative voters.

4.2 Multiple Leaders

In this section we extend our results to situations where the set of leaders contains two voters $L = \{l, l'\}$. While the preferences and voting behavior of leaders and objective voters are not affected by the existence of more than one leader, followers' behavior changes. We assume that followers act as a type of objective voters that only care about the signals both leaders send: followers strictly prefer a certain option if and only if both leaders send the same signal and are indifferent between the two options if and only if the leaders send different signals. Formally, the utility function that represents the preferences of the followers in the scenario with two leaders l and l' is given by

$$u(F, A, \theta, m) = 1 - u(F, Q, \theta, m) \quad = \quad \begin{cases} 1 & \text{if } m_l = m_{l'} = A, \\ \frac{1}{2} & \text{if } m_l \neq m_{l'}, \\ 0 & \text{otherwise.} \end{cases}$$

A weakly dominant voting strategy for the followers when all other voters are truthful is given by

$$\sigma^v(F, \theta_i, m) \quad = \quad \begin{cases} A & \text{if } m_l = m_{l'} = A, \\ Q & \text{otherwise.} \end{cases} \tag{12}$$

In order to reduce the number of cases that need to be considered we assume that in case followers are indifferent between the alternative A and the status quo Q then they vote for the status quo.

Proposition 5. *Let $L = \{l, l'\}$ and assume that both leaders are unbiased.*

- *If either $\#F \geq q$ or $\#F < (N + 1) - (q - 1)$ then there is a FRE.*
- *If $q > \#F \geq (N + 1) - (q - 1)$ then there is no FRE.*

Comparing the results in Propositions 1 and 5 we can see that the existence of a second leader does not change the incentives to reveal information truthfully during the discussion stage. Next we extend this finding to the case where leaders are biased:

Proposition 6. *Let $L = \{l, l'\}$ with respective biases b, b' and assume that at least one of the leaders is biased, i.e. either $b \neq \frac{N}{2} + 1$ or $b' \neq \frac{N}{2} + 1$. There exists a FRE if and only if $\#F \geq q$, $b, b' \geq \frac{N}{2} + 1$ and p is high enough relative to b and b'.*

We can conclude from Propositions 5 and 6 that our model is robust to the addition of an extra leader.

5 Conclusions

In this paper we have analyzed information aggregation in deliberative committees under the presence of leadership. Deliberation is modeled as a cheap talk game where voters share non-verifiable information about what is the best choice. We have shown that the presence of a leader and voters who follow the information revealed by the leader (followers) may introduce distortions in the decision process. Specifically, followers may have incentives to misreport their private information to obtain additional support for the option that matches with the leader's revealed information. Conversely, the leader may have incentives to compensate the followers' effect whenever her private information does not coincide with the best choice. This issues are more prevalent when the leader may have an "a priori" bias for one of the options. Surprisingly, the problem is alleviated if there are many followers; the leader knows that her report determines the final outcome, thus, if the quality of private information is sufficient then it is more likely that she supports the best choice (even if her preferences were ex-ante biased). Similar results and intuitions apply to the case in which the leadership in the committee is shared between two voters.

We want to conclude highlighting some possible extensions and directions of further research. The key concept of this paper is that the discussion stage defines the preferences of the members of the committee about the final outcome. We have chosen the most simple framework and modeled deliberation as cheap talk revelation games. There is a growing literature on debate, strategic argumentation and persuasion that could inspire new lines of research incorporating additional realism in the definition of the preferences of the members of the committee.[8] In this paper, we have also made abstraction of reputation issues and the dynamic component of the leadership. Since we have dealt with static elections where only two options are available, our focus on pure strategies and FRE and PRE becomes natural. In a dynamic setting, reputation effects could compensate the direct incentives of biased leaders to support the option towards which they have a biased, such analysis would need to consider more elaborated (mixed) strategies and beliefs.

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[8]See, for instance, Glazer and Rubinstein (2001); Spiegler (2006) and references therein.

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A Appendix: Proofs

Proof of Proposition 1. Assume first that $\#F \geq q$ or $\#F < (N+1)-(q-1)$. We consider the strategic incentives in the message stage for the different types of voters.

Leader Consider the leader l and assume that the remaining voters truthfully report their signal. That is, for each $i \neq l$, $m_i = \theta_i$. First, we characterize the circumstances under which l is pivotal.

Assume that $\#F \geq q$. Since for each $f \in F$ and each $m_l' \in \{A, Q\}$, $v_f(m_l', m_{-l}) = m_l'$, then $\mathbf{v}(m_l', m_{-l}) = m_l'$ and the leader is pivotal.

Assume that $\#F < (N+1)-(q-1)$. If $\#\Theta_A \setminus l \neq \#\Theta_Q \setminus l$ then all objective voters and the leader vote for a certain option that is independent on the leader's report. Since $\#(O+1) \geq q$, for each $m_l, m_l' \in \{A, Q\}$, $\mathbf{v}(m_l, m_{-l}) = \mathbf{v}(m_l', m_{-l})$ and the leader is not pivotal. If $\#\Theta_A \setminus l = \#\Theta_Q \setminus l$ and $q \neq N+1$ then all objective voters and followers vote according to the leader's report and since $q \neq N+1$ the leader is pivotal. Finally, if $\#\Theta_A \setminus l = \#\Theta_Q \setminus l$ and $q = N+1$, then the alternative A is the voting outcome when the leader receives $\theta_l = A$ and reports $m_l = A$, and Q is the voting outcome otherwise. Thus, the leader is pivotal only when $\theta_l = A$.

Summing up, provided that either $\#F \geq q$ or $\#F < (N+1)-(q-1)$, the leader is pivotal if and only if either:

- $\#F \geq q$ or,
- $(N+1)-(q-1) > \#F$ ($\#O \geq q-1$) and either
  - $\#\Theta_A \setminus l = \#\Theta_Q \setminus l$ and $q \neq N+1$ or,
  - $\#\Theta_A \setminus l = \#\Theta_Q \setminus l$, $q = N+1$ and $\theta_l = A$,

Next, we explore whether there exists a truth-telling best response for the leader whenever she is pivotal and all other voters are truthful.

If $\#F \geq q$ then the leader learns nothing about the signals of other players from being pivotal. Thus, since since $p \geq \frac{1}{2}$ implies $P\left(\#\Theta_A > \#\Theta_Q \middle| \theta_l = A\right) = P\left(\#\Theta_A < \#\Theta_Q \middle| \theta_l = Q\right) \geq \frac{1}{2}$, $m_l = \theta_l$ is a best response for l.

If $\#F < (N+1) - (q-1)$ and $\#\Theta_A \setminus l = \#\Theta_Q \setminus l$, then whenever $\theta_l = A$ we have $\#\Theta_A > \#\Theta_Q$ and whenever $\theta_l = Q$ we have $\#\Theta_A < \#\Theta_Q$. Therefore, $u(L, A, \theta, m) > u(L, Q, \theta, m)$ if and only if $\theta_l = A$ and $m_l = \theta_l$ is a best response for l.

Followers Consider an arbitrary follower f. If $\#F \geq q$ then f is not pivotal as in this case the outcome of the voting stage depends entirely on the message revealed by the leader. If $\#\Theta_A \setminus f \neq \#\Theta_Q \setminus f$ then f is not pivotal as her message does not influence any vote. Finally, if $\#F < (N+1) - (q-1)$ ($\#O + 1 \geq q$) and $\#\Theta_A \setminus f = \#\Theta_Q \setminus f$, then m_f determines the vote of the leader and all the objective voters. Since $\#O + 1 \geq q$, f is pivotal. Moreover, since $P(\theta_l = A | \#\Theta_A \setminus f = \#\Theta_Q \setminus f) = P(\theta_l = Q | \#\Theta_A \setminus f = \#\Theta_Q \setminus f) = \frac{1}{2}$, $m_f = \theta_f$ is is a best response for f.

Objective Voters Consider an arbitrary objective voter o. A necessary condition for o to be pivotal is that $\#\Theta_A \setminus o = \#\Theta_Q \setminus o$ as otherwise her message does not influence any vote. If $\theta_o = A$ then $\#\Theta_A \setminus o = \#\Theta_Q \setminus o$ implies $\#\Theta_A > \#\Theta_Q$ and, similarly, $\theta_o = Q$ implies $\#\Theta_A < \#\Theta_Q$. Thus, with the same arguments we use for the leader, $m_o = \theta_o$ is the best response for o.

In conclusion, whenever $\#F \geq q$ or $\#F < (N+1) - (q-1)$ there exists a truth-telling best response for every voter. Hence, if either $\#F \geq q$ or $\#F < (N+1) - (q-1)$ there is a FRE.

To conclude the proof assume now that $q > \#F \geq (N+1) - (q-1)$ and consider an arbitrary follower f. If $\#\Theta_A \setminus f \neq \#\Theta_Q \setminus f$, then f is not pivotal as her message does not influence any vote. Moreover, if $q > \#F \geq (N+1) - (q-1)$ and $\theta_l = Q$ then all followers vote for Q and the number of votes f can influence are at most $q-1$. Thus, f is not pivotal. However, if $q > \#F \geq (N+1) - (q-1)$, $\#\Theta_A \setminus f = \#\Theta_Q \setminus f$ and $\theta_l = A$, m_f determines the vote of the leader and all objective voters. If $m_f = A$ then all voters vote for A and A is the voting outcome. If $m_f = Q$ then only the followers vote for A and since $\#O + 1 \geq q$ then Q is the voting outcome and f is pivotal. Note that since $m_l = \theta_l = A$, necessarily $u(F, A, \theta, m) > u(F, Q, \theta, m)$ and $m_f = A$ is the best-response for f independently of the signal she receives. Thus, there is no FRE with $q > \#F \geq (N+1) - (q-1)$. □

Proof of Lemma 1. We proceed by consider each type of voter individually:

Leader Assume first that $l \notin K$. Since the leader is unbiased she derives one unit of utility from voting the alternative A if and only if the number of signals in the population support the alternative A is greater or equal than $\frac{N}{2}+1$. Given that the report of the voters in K is not informative, the leader only considers $N+1-\#K$ truthful signals. Let $x \equiv \{j \notin K \mid m_j = A\} \in \{0,1,\dots,N+1-\#K\}$ Thus, the leader votes for A if and only if $P\left(\#\Theta_A \geq \frac{N}{2}+1 | \#(\Theta_A \setminus K) = x\right) > P\left(\#\Theta_A < \frac{N}{2}+1 | \#(\Theta_A \setminus K) = x\right)$.[9] This can be rewritten as

$$P\left(\#K_A \geq \frac{N}{2}+1-x | \#(\Theta_A \setminus K) = x\right) \quad > \quad P\left(\#K_A < \frac{N}{2}+1-x | \#(\Theta_A \setminus K) = x\right)$$

which holds if and only if

$$\sum_{i=\frac{N}{2}+1-x}^{\#K} P\left(\#K_A = i | \#(\Theta_A \setminus K) = x\right) \quad > \quad \sum_{i=0}^{\frac{N}{2}-x} P\left(\#K_A = i | \#(\Theta_A \setminus K) = x\right),$$

$$\sum_{i=\frac{N}{2}+1-x}^{\#K} P\left(\#K_A = i \cap \#(\Theta_A \setminus K) = x\right) \quad > \quad \sum_{i=0}^{\frac{N}{2}-x} P\left(\#K_A = i \cap \#(\Theta_A \setminus K) = x\right).$$

The inequality above can be rewritten as

$$\sum_{i=\frac{N}{2}+1-x}^{\#K} \binom{\#K}{i} \left(p^{i+x}(1-p)^{N+1-i-x} + (1-p)^{i+x}p^{N+1-i-x}\right) \quad >$$
$$\sum_{i=0}^{\frac{N}{2}-x} \binom{\#K}{i} \left(p^{i+x}(1-p)^{N+1-i-x} + (1-p)^{i+x}p^{N+1-i-x}\right).$$

Comparing term by term the components of both sums at either side of the inequality above leads to the conclusion that the inequality holds if and only if $\#K + x > N+1-x$. Thus, the leader votes for A if and only if $x > \frac{N+1-\#K}{2}$. The proof for the case where $i \in K$ follows easily from the arguments above and, hence, its omitted.

Followers Since the followers always prefer the alternative that matches the report of the leader, they always vote according to the leader's report.

Objective Voters The result follows directly from the arguments in the analysis of the leader's optimal voting strategy. □

[9] Recall that we assume that if the number of truthful signals supporting each option is the same then the status quo is preferred.

Proof of Proposition 2. Assume that the leader is unbiased and $q > F \geq N + 1 - (q - 1)$. Assume there is a group of uninformative voters K. Note that if voter j belongs to K the she is not pivotal as in a PRE uninformative voter's messages are ignored. We proceed by studying the individual incentives of at the discussion stage of each of the different types of voters to check whether their messages can be informative.

Leader Let $K' = K \cup \{l\}$. Given that $q > \#F \geq (N+1) - (q-1)$, there are three possible scenarios where the leader is pivotal:

- $\#(\Theta_A \setminus K') > \#(\Theta_Q \setminus K')$. In this case if the leader reveals A then all voters choose A and the alternative is implemented. On the other hand, if the leader reveals Q then followers choose Q and since $\#F \geq (N + 1) - (q - 1)$ implies $\#O + 1 \leq q - 1$ we have that the status quo is maintained. Hence, the leader is pivotal.

- $\#(\Theta_A \setminus K') = \#(\Theta_Q \setminus K')$. In this situation followers and objective voters vote according to the leader's message, while the leader herself votes according to θ_l. If $q \neq N + 1$, then the leader's report determines the voting outcome and l is pivotal. If $q = N + 1$, then the voting outcome is A only if the leader receives $\theta_l = A$. Therefore, the leader is pivotal if and only if either $q \neq N + 1$ or $q = N + 1$ and $\theta_l = A$.

- $\#(\Theta_A \setminus K') < \#(\Theta_Q \setminus K')$. In this scenario the leader and objective voters vote for Q and, thus, the voting outcome is Q. Therefore, the leader is not pivotal.

Assume that $q \neq N + 1$. The leader is pivotal if and only if $\#(\Theta_A \setminus K') \geq \#(\Theta_Q \setminus K')$. If $\#(\Theta_A \setminus K') > \#(\Theta_Q \setminus K') + 1$ then the leader prefers alternative A to Q and, thus, she reports $m_l = A$ independently of the signal she receives. If $\#(\Theta_A \setminus K') = \#(\Theta_Q \setminus K') + 1$ then the leader prefers alternative A to Q if and only if $m_l = A$ and, thus, has incentives to report truthfully. Finally, if $\#(\Theta_A \setminus K') = \#(\Theta_Q \setminus K')$, then the leader prefers the voting outcome to coincide with the signal she has received and, therefore, reports truthfully. Therefore, if

$$P\left(\#(\Theta_A \setminus K') > \#(\Theta_Q \setminus K') + 1 | \#(\Theta_A \setminus K') \geq \#(\Theta_Q \setminus K')\right) > \frac{1}{2}$$

the leader reports A independently of the signal she has received. Otherwise, the leader's best response is to report truthfully, $m_l = \theta_l$.

Consider now the case where $q = N + 1$. If $\theta_l = A$ then the leader is pivotal if and only if $\#(\Theta_A \setminus K') \geq \#(\Theta_Q \setminus K')$. Thus, given that $\#(\Theta_A \setminus K') \geq \#(\Theta_Q \setminus K')$ and $\theta_l = A$ imply $\#(\Theta_A \setminus K) > \#(\Theta_Q \setminus K)$ the leader has incentives to report truthfully.

Finally, if $q = N+1$ and $\theta_l = Q$ then the leader is pivotal if and only if $\#(\Theta_A \setminus K') > \#(\Theta_Q \setminus K')$. Thus, since $\#(\Theta_A \setminus K') > \#(\Theta_Q \setminus K')$ implies $\#(\Theta_A \setminus K) \geq \#(\Theta_Q \setminus K)$ if it is true that $\#(\Theta_A \setminus K) = \#(\Theta_Q \setminus K)$ then the leader has incentives to report Q, thus, she has incentives to report truthfully. If, on the other hand, $\#(\Theta_A \setminus K) > \#(\Theta_Q \setminus K)$ then the leader has incentives to report A even though she received signal Q, i.e. she has incentives to miss-report. Hence, if

$$P\left(\#(\Theta_A \setminus K') > \#(\Theta_Q \setminus K') + 1 | \#(\Theta_A \setminus K') > \#(\Theta_Q \setminus K')\right) > \frac{1}{2}$$

the leader reports A independently of the signal she has received. Otherwise, the leader's best response is to report truthfully.

Followers Consider an arbitrary follower f. Given the voting strategies in (7), (8) and (9) if $q > \#F \geq (N+1)-(q-1)$ then the alternative A is implemented if and only if all followers (including f) vote for A, which can happen only if $m_l = A$. Thus, a necessary condition for a follower to be pivotal is that $m_l = A$. Hence, such follower has incentives to report A regardless of her signal.

Objective Voters Consider an arbitrary objective voter o and let $K'' = K \setminus \{o\}$. A necessary condition for objective voter o to be pivotal is that $\#(\Theta_A \setminus K'') = (\Theta_Q \setminus K'')$ or $\#(\Theta_A \setminus K'') - 1 = (\Theta_Q \setminus K'')$ as otherwise her message does not influence any vote. Thus, o prefers the voting outcome to coincide with the signal she has received and she sends the message $m_o = \theta_o$.

To conclude the proof, note that the followers are always uninformative: $F \subseteq K$. Moreover, as we have just shown for objective voters it is always a best response to truthfully report their signal whilst the leader is truthful if and only if $q \neq N+1$ and

$$P\left(\#(\Theta_A \setminus K') > \#(\Theta_Q \setminus K') + 1 | \#(\Theta_A \setminus K') \geq \#(\Theta_Q \setminus K')\right) \leq \frac{1}{2}$$

or $q = N+1$ and

$$P\left(\#(\Theta_A \setminus K') > \#(\Theta_Q \setminus K') + 1 | \#(\Theta_A \setminus K') > \#(\Theta_Q \setminus K')\right) \leq \frac{1}{2}.$$

□

Proof of Proposition 3. If $\#F \geq q$ then in a FRE the option implemented coincides with the message revealed by the leader. Thus, the probability that the option implemented matches with the state of nature given that the leader is truthful equals the probability that the signal of the leader coincides with the state of nature, i.e. p.

Assume that $q > \#F \geq N+1-(q-1)$ and the conditions in Proposition 2 for the leader not to be uninformative are satisfied. Since $q > \#F \geq N+1-(q-1)$ implies $\#O < q-1$ then the option implemented is A only if the leader's message is A and either the leader or one objective voter votes for A. Given that the leader and all the objective voters vote for the same option then the alternative A is implemented if and only if all voters vote for it. Thus, in a PRE where the leader truthfully reveals her signal A is implemented if and only if $\theta_l = A$ and $\#(\Theta_A \setminus F) > \frac{N+1-\#F}{2}$ (see equation (9)). Thus, as both states are equally likely the probability of implementing the option that matches with the state of nature is given by a function $w(N, \#F, p)$ where

$$\begin{aligned} w(N,\#F,p) &= \frac{1}{2}P\left(\theta_l = A \cap \#(\Theta_A \setminus F) > \frac{N+1-\#F}{2} | S = A\right) + \\ & \frac{1}{2}P\left(\theta_l = Q \cup \#(\Theta_Q \setminus F) \geq \frac{N+1-\#F}{2} | S = Q\right), \end{aligned}$$

which can be rewritten as

$$\begin{aligned} w(N,\#F,p) &= \frac{1}{2}\left(p \sum_{i=\lfloor \frac{N-F-1}{2} \rfloor +1}^{N-F} \binom{N-F}{i} p^i (1-p)^{N-F-i}\right) + \\ & \frac{1}{2}\left(p + \sum_{i=\lceil \frac{N+1-F}{2} \rceil}^{N+1-F} \binom{N+1-F}{i} p^i (1-p)^{N+1-F-i} - \right. \\ & \left. p \sum_{i=\lfloor \frac{N-F-1}{2} \rfloor +1}^{N-F} \binom{N-F}{i} p^i (1-p)^{N-F-i}\right) \\ &= \frac{1}{2}\left(p + \sum_{i=\lceil \frac{N+1-F}{2} \rceil}^{N+1-F} \binom{N+1-F}{i} p^i (1-p)^{N+1-F-i}\right). \end{aligned}$$

If $\#F < (N+1)-(q-1)$ then $\#O \geq q-1$ and since the leader is unbiased in a FRE the option implemented coincides with the option objective voters and the leader vote for. The option objective voters and the leader vote for is A if and only if $\#\Theta_A \geq \frac{N}{2}+1$. Thus, the probability of implementing the option that matches with the state of nature is given by

$$\begin{aligned} P\left(\#\Theta_A \geq \frac{N}{2}+1 \middle| S = A\right) &= P\left(\#\Theta_Q \geq \frac{N}{2}+1 \middle| S = Q\right) \\ &= \sum_{i=\frac{N}{2}+1}^{N+1} \binom{N+1}{i} p^i (1-p)^{N+1-i}. \end{aligned}$$

□

Proof of Proposition 4. Assume first that $\#F \geq q$. From the arguments in Proposition 1, if $\#F \geq q$ then the leader is always pivotal and she is the only pivotal voter. Thus, to check the existence of a FRE it suffices to explore the incentives of the leader to truthfully reveal her signal. If $\#F \geq q$ the leader learns nothing about the signals of other players from the fact that she is pivotal. Thus, if $b > \frac{N}{2}+1$ and $\theta_l = Q$ then $P\left(\#\Theta_A < b|\theta_l = Q\right) > \frac{1}{2}$ and the leader has incentives to truthfully report his signal. If $b > \frac{N}{2}+1$ and $\theta_l = A$ then whether the leader wants to report truthfully or not depends on whether or not $\#\Theta_A \geq b$ given $\theta_l = A$, which depends on the accuracy of the signal p and the value of b. If $b < \frac{N}{2}+1$ and $\theta_l = Q$ then again whether the leader wants to report truthfully or not depends on de accuracy of the signal p and the value of b as in the previous case. Finally, if $b < \frac{N}{2}+1$ and $\theta_l = A$ then $P\left(\#\Theta_A \geq b|\theta_l = A\right) > \frac{1}{2}$ and the leader wants to truthfully report his signal.

We can implicitly compute the value of p relative to b for the leader to truthfully reveal his signal if either $b > \frac{N}{2}+1$ and $\theta_l = A$ or $b < \frac{N}{2}+1$ and $\theta_l = Q$. If $b > \frac{N}{2}+1$ and $\theta_l = A$ then the leader wants to truthfully reveal his signal if

$$\begin{aligned} P\left(\#\Theta_A \geq b|\theta_l = A\right) &= \sum_{i=b-1}^{N} \binom{N}{i} \left[p^{i+1}(1-p)^{N-i} + p^{N-i}(1-p)^{i+1}\right] \\ &\geq \frac{1}{2}. \end{aligned}$$

On the other hand, if $b < \frac{N}{2}+1$ and $\theta_l = Q$ then the leader wants to truthfully reveal his signal if

$$\begin{aligned} P\left(\#\Theta_A < b|\theta_l = Q\right) &= P\left(\#\Theta_Q \geq N+1-b|\theta_l = Q\right) \\ &= \sum_{i=N-b}^{N} \binom{N}{i} \left[p^{i+1}(1-p)^{N-i} + p^{N-i}(1-p)^{i+1}\right] \\ &\geq \frac{1}{2}. \end{aligned}$$

In conclusion, if $\#F \geq q$ then the leader follows a truth-telling best response if and only if p is high enough relative to b. Moreover, since followers and objective voters are never pivotal, truth-telling is always a best response for them. Thus, if $\#F \geq q$ then there exists a FRE if and only if p is high enough relative to b.

Assume now that $\#F < q$. We show that in this situation for at least one type of voter it is not a best-response strategy to report their true signal.

Firstly, consider the case where $\#F > N+1-q$ ($\#O < q-1$). Let f be an arbitrary follower. Note that whenever $\theta_l = Q$, then every follower votes for Q and Q is the voting outcome independently of the signal reported by f. However, if $\theta_l = A$, then all the followers vote for A but they do not suffice to determine the voting outcome. Therefore, whenever f

is pivotal, $\theta_l = A$. Since in this situation f wants the voting outcome to be the alternative A independently of the signal she receives, her best response is always to report $m_f = A$, which precludes the existence of a FRE.

Next consider consider the case where $\#F < N + 1 - q$ ($\#O \geq q$). In this situation objective voters completely determine the outcome of the voting stage. With the arguments in the proof of Proposition 1, the leader is pivotal if and only if $\#\Theta_A \setminus l = \#\Theta_Q \setminus l$. Assume first that $b > \frac{N}{2} + 1$. If l is pivotal, then $\#\Theta_A < b$ and l prefers Q over A independently of θ_l. Hence, the leader's best response is always to report $m_l = Q$. Assume now that $b < \frac{N}{2} + 1$, then if the leader is pivotal $\#\Theta_A \geq b$ and l prefers A to be implemented independently of θ_l. Hence, the leader's best response is always to to report $m_l = A$. Thus, the leader's best response if $\#F < N + 1 - q$ is to report the signal that matches her bias, which precludes the existence of a FRE.

Finally, consider the case where $\#F = N + 1 - q$ ($\#O = q - 1$). We have two possible situations: $q \neq N + 1$ and $q = N + 1$. We analyze first the case where $q \neq N + 1$. Assume that $b > \frac{N}{2} + 1$ and consider the leader's incentives in the following three possible scenarios:

- $\#\Theta_A \setminus l > \#\Theta_Q \setminus l$. In this case every objective voter votes for A regardless of the leader's report. Note that the vote of the leader, v_l, does not depend on m_l. If $\#\Theta_A \geq b$, then $v_l = A$ and the voting outcome is A independently of m_l. Thus, l is not pivotal. If $\#\Theta_A < b$ then $v_l = Q$. If $m_l = A$, then every follower and every objective voter voter for A and the voting outcome is A. If $m_l = Q$ then the leader and the followers vote for Q and the voting outcome is Q. Hence l is pivotal only if $\#\Theta_A < b$.

- $\#\Theta_A \setminus l = \#\Theta_Q \setminus l$. In this situation followers and objective voters always vote according to the leader message and are enough to determine the voting outcome. That is, l is pivotal. Notice that $\#\Theta_A \setminus l = \#\Theta_Q \setminus l$ and $b > \frac{N}{2} + 1$ implies $\#\Theta_A < b$.

- $\#\Theta_A \setminus l < \#\Theta_Q \setminus l$. In this case every objective voter votes for Q regardless of the leader's report. Since $\#F < q$ the voting outcome is Q regardless of the message sent by the leader and, thus, l is not pivotal.

Summing up, if $b > \frac{N}{2} + 1$ and l is pivotal then $\#\Theta_A < b$ and the leader's best response is always to report $m_l = Q$, following her bias. A parallel argument applies to prove that if $b < \frac{N}{2} + 1$ and l is pivotal then $\Theta_A \geq b$ and l's best response is to report $m_l = A$ regardless of her signal.

To conclude, consider the case $\#F = N + 1 - q$ and $q = N + 1$. Clearly, this implies that $\#F = \emptyset$. Assume that $b > \frac{N}{2} + 1$. Since there is no follower and the leader's vote does not depend on her signal, the only possibility for the leader to be pivotal is that her message affects the vote of the objective voters. This fact implies that whenever l is pivotal $\#(\Theta_A \setminus l) = \#(\Theta_Q \setminus l)$ and, thus, $\#\Theta_A < b$. Therefore, the leader's best response is always to report $m_l = Q$ independently of the signal she receives. If $b < \frac{N}{2} + 1$ similar arguments apply and we have that whenever l is pivotal then $\#\Theta_A > b$ and the leader's best response is to report $m_l = A$ regardless of her signal. Hence, a biased leader's best response if all other voters report truthfully is to report the signal that favors her bias. That is, a FRE is not possible. □

Proof of Proposition 5. Assume first that $\#F \geq q$ or $\#F < (N+1)-(q-1)$. We consider the individual incentives to deviate from the truth-telling best response for each of the different types of voters.

Leader Consider the leader l and assume that for each $i \neq l$, $m_i = \theta_i$. We consider different cases.

Consider first the case $\#F \geq q$. If $\theta_{l'} = A$ then for each $f \in F$ we have that $v_f = m_l$ and since $\#F \geq q$ it is true that l is pivotal. If $\theta_{l'} = Q$ then for each $v_f = Q$ for all $f \in F$ and, therefore, l is not pivotal.

If $\#F \geq q$ and l is pivotal then l learns that $\theta_{l'} = A$. If $\theta_l = A$, then

$$P\left(\Theta_A \geq \frac{N}{2} + 1 | \theta_l = \theta_{l'} = A\right) > \frac{1}{2}.$$

Therefore, l's best response is to report truthfully. Moreover, if $\theta_l = Q$, then since

$$P\left(\Theta_A \geq \frac{N}{2} + 1 | \theta_l = Q, \theta_{l'} = A\right) \quad = \quad P\left(\Theta_A < \frac{N}{2} + 1 | \theta_l = Q, \theta_{l'} = A\right)$$

l is indifferent between reporting A and reporting Q. In particular, reporting $m_l = Q$ is a best response. Therefore, if $\#F \geq q$ then truth-telling is a best response for l.

Consider next the case where $\#F < N + 1 - q$, which in turn implies $\#O \geq q - 1$. In this situation the other leader, l', and objective voters' vote determine the outcome of the election. Since l' is unbiased and for each $o \in O$ and each $m_l \in \{A, Q\}$ we have that objective voters and l' vote for the same option. Thus, if $\#(\Theta_A \setminus l) \neq \#(\Theta_Q \setminus l)$ then l is not pivotal as her message can influence $\#F$ votes only. On the other hand, if $\#(\Theta_A \setminus l) = \#(\Theta_A \setminus l')$ then l's message determines the vote of all voters (except her own vote) and, thus, she is pivotal. Therefore, if $\#F < N+1-(q-1)$ then whenever l is pivotal $\#(\Theta_A \setminus l) \neq \#(\Theta_Q \setminus l)$ and the option she prefers coincides with θ_l. In conclusion, $m_l = \theta_l$ and l has incentives to truthfully report her signal.

Consider now the situation where $\#F = N+1-q$, which in turn implies $\#O = q-2$. In this case followers together with l are enough to block the implementation of A at the voting stage if they all vote for Q. If $\#(\Theta_A \setminus l) \neq \#(\Theta_Q \setminus l)$ then for each $m_l, m_{l'} \in \{A, Q\}$ and each $o \in O$, $v_l(m_l, \theta_{-l}) = v_l(m'_l, \theta_{-l}) = v_{l'}(m_l, \theta_{-l}) = v_{l'}(m'_l, \theta_{-l}) = v_o(m_l, \theta_{-l}) = v_o(m'_l, \theta_{-l})$ and l is not pivotal. Assume thus that $\#(\Theta_A \setminus l) = \#(\Theta_Q \setminus l)$. There are two cases to be considered:

- If $\theta_{l'} = A$, then for each $i \neq l$ and each $m_l \in \{A, Q\}$, $v_i(m_l, \theta_{-l}) = m_l$ and $v_l(m_l, \theta_{-l}) = \theta_l$. Thus, $\mathbf{v}(m_l, \theta_{-l}) = m_l$ and l is pivotal.

- If $\theta_{l'} = Q$ then for each $f \in F$ and each $m_l \in \{A, Q\}$, $v_f(m_l, \theta_{-l}) = Q$. Moreover, for each $o \in O$, $v_o(m_l, \theta_{-l}) = v_{l'}(m_l, \theta_{-l}) = m_l$ and $v_l(m_l, \theta_{-l}) = \theta_l$. Assume that $\theta_l = A$. If $m_l = A$ then $\mathbf{v}(m_l, \theta_{-l}) = A$, while if $m_l = Q$ then only l votes for A and $\mathbf{v}(m'_l, \theta_{-l}) = Q$. Thus, l is pivotal. Finally assume that $\theta_l = Q$, then for each $f \in F$ and each $m_l, m'_l \in \{A, Q\}$ $v_l(m_l, \theta_{-l}) = v_l(m'_l, \theta_{-l}) = v_f(m_l, \theta_{-l}) = v_f(m'_l, \theta_{-l}) = Q$ and $\mathbf{v}(m_l, \theta_{-l}) = \mathbf{v}(m'_l, \theta_{-l}) = Q$. Therefore, l is not pivotal.

Thus, if $\#F < N+1-(q-1)$ whenever l is pivotal $\#(\Theta_A \setminus l) = \#(\Theta_Q \setminus l)$ and l's best response is to send the message that coincides with her signal. Thus, $m_l = \theta_l$ and l reports truthfully.

Followers The analysis of the incentives of the followers is parallel to the case where there is a unique leader. If $\#F \geq q$ then the followers are never pivotal. If $\#F < N+1-(q-1)$ (i.e. $\#O \geq q-2$) then a follower f is pivotal if and only if $\#(\Theta_A \setminus f) = \#(\Theta_Q \setminus f)$. Since

$$P\left(\theta_l = \theta_{l'} | \#(\Theta_A \setminus f) = \#(\Theta_Q \setminus f)\right) = \frac{1}{2}$$

then followers are indifferent between sending the message $m_f = A$ or $m_f = Q$. Thus, it is a weakly dominant strategy to send the message that coincides with their signal. Hence, $m_f = \theta_f$ and followers report truthfully.

Objective Voters The analysis of objective voters incentives is parallel to the case with a unique leader. If $\#F \geq q$ then the objective voters are never pivotal. If $\#F < N+1-(q-1)$ (i.e. $\#O \geq q-2$) then a objective voter o is pivotal if and only if $\#(\Theta_A \setminus o) = \#(\Theta_Q \setminus o)$. Thus, with the same arguments as the ones used in the proof of Proposition 1 $m_o = \theta_o$ is the best response for o.

To conclude the proof, assume that $q > \#F \geq (N+1)-(q-1)$, which in turn implies $\#O \leq q-3$. In this case followers suffice to block the implementation of alternative A. For an arbitrary follower f to be pivotal it is necessary that $\theta_l = \theta_{l'} = A$ and $\#(\Theta_A \setminus f) = \#(\Theta_Q \setminus f)$. In this case, f prefers A to Q independently of the signal she receives and, hence, she has incentives to miss-report. $\square$

Proof of Proposition 6. Assume first that $\#F \geq q$. In this case followers and objective voters are never pivotal, thus, for each $i \in \{F, O\}$ we have that $m_i = \theta_i$ is a best response and both followers and objective voters report truthfully.

Consider leader l and assume that $b \geq \frac{N}{2} + 1$. In this case leader l is pivotal only if $\theta_{l'} = A$. Thus, if $\theta_l = A$ then since l has no information about the signals of neither followers nor objective voters she prefers A to Q if and only if

$$P\left(\#\Theta_A \geq b | \theta_l = \theta_{l'} = A\right) \geq \frac{1}{2}.$$

Note that the inequality above holds is true for $p = 1$ and since $P\left(\#\Theta_A \geq b | \theta_l = \theta_{l'} = A\right)$ is an increasing polynomial in p, for each b there is an interior value of p such that the inequality is satisfied: l prefers A to Q and $m_l = A$ is a best response for l. On the other hand, if $\theta_l = Q$ then $P\left(\#\Theta_A \leq \frac{N}{2} | \theta_l = A, \theta_{l'} = Q\right) \geq \frac{1}{2}$. Therefore, $P\left(\#\Theta_A \geq b | \theta_l = A, \theta_{l'} = Q\right) \leq \frac{1}{2}$ and l reports $m_l = Q$. Hence, if p is high enough relative to b we have that $m_l = \theta_l$ is the best-response for l. The same logic applies to l' and, thus, a FRE exists if $\#F \geq q$, $b, b' \geq \frac{N}{2} + 1$ and p is high enough relative to b and b'.

Consider now that $b < \frac{N}{2} + 1$. By similar arguments as the ones used above, whenever l is pivotal it is necessarily the case that $\theta_{l'} = A$. Thus, if $\theta_l = A$, then

$$P\left(\#\Theta_A \geq b | \theta_l = \theta_{l'} = A\right) \geq P\left(\#\Theta_A \geq \frac{N}{2} + 1 | \theta_l = \theta_{l'} = A\right)$$

implies

$$P\left(\#\Theta_A \geq b | \theta_l = \theta_{l'} = A\right) \geq \frac{1}{2},$$

Therefore, l prefers A to Q and truth-telling is a best response. On the other hand, if $\theta_l = Q$ then

$$\begin{aligned} P\left(\#\Theta_A \geq \frac{N}{2} + 1 | \theta_l = Q, \theta_{l'} = A\right) &= P\left(\#\Theta_Q \geq \frac{N}{2} + 1 | \theta_l = Q, \theta_{l'} = A\right) \\ &= \frac{1}{2}. \end{aligned}$$

Since $P\left(\#\Theta_A \geq b | \theta_l = Q, \theta_{l'} = A\right) \geq P\left(\#\Theta_A \geq \frac{N}{2} + 1 | \theta_l = Q, \theta_{l'} = A\right)$ then l prefers A to Q and $m_l = A$ is the best response for l. Thus, l has incentives to miss-report and a FRE does not exist.

Assume now that $\#F < q$, we need to show that there is always a voter whose best response whenever all other voters truthfully report their signals is to miss-report. We proceed by considering three possible scenarios: $q > \#F \geq (N+1) - (q-1)$, $\#F \leq N - q - 1$ and $\#F \in \{N - q, N - (q-1)\}$.

Consider first the case in which followers are enough to veto the alternative A: $q > \#F \geq (N+1) - (q-1)$. Consider a follower f, whenever f is pivotal necessarily $\theta_l = \theta_{l'} = A$. Thus, provided f is pivotal she prefers A to Q and $m_f = A$ is f's best response strategy independently on her signal. That is, she miss-reports.

Consider now the case where objective voters are enough to determine the voting outcome: $\#O \geq q$ ($\#F \leq N - q - 1$). A biased leader l is pivotal if and only if $\#(\Theta_A \setminus l) = \#(\Theta_Q \setminus l)$. If $b > \frac{N}{2} + 1$ then $\#\Theta_A < b$ and l prefers Q to A. Thus, $m_l = Q$ is l's best response independently of θ_l. On the other hand, if $b < \frac{N}{2} + 1$ then $\#\Theta_A \geq b$ and l prefers A to Q. Thus, $m_l = A$ is l's best response independently of θ_l. Hence, the best response for a biased leader is to report according to her bias and a FRE is not possible.

It only remains to study the case where $\#O \in \{q-2, q-1\}$ ($\#F \in \{N-q, N-(q-1)\}$). Consider first a situation where $b > \frac{N}{2} + 1$ and $b > b'$. If $\theta_{l'} = Q$ then all followers vote for Q. Moreover, since l's voting behavior does not depend on her message, for l to be pivotal her report has to change the voting behavior of either objective voters: $\#(\Theta_A \setminus l) = \#(\Theta_Q \setminus l) = \frac{N}{2}$, or the other leader l': $(\#\Theta_A \setminus l) = b' - 1$. Thus, since $b > \frac{N}{2} + 1$ and $b > b'$ if l is pivotal and $\theta_{l'} = Q$ then l prefers Q to A independently of θ_l and thus miss-reports. If, on the other hand, $\theta_{l'} = A$ then l's report affects the vote of all followers. If the message sent by l affects the vote of either objective voters or the other leader l' , then either $\#(\Theta_A \setminus l) = \#(\Theta_Q \setminus l) = \frac{N}{2}$ or $(\#\Theta_A \setminus l) = b' - 1$ and again l has incentives to missreport. Suppose then that m_l does not affect the vote of neither objective voters nor the other leader l'.

If both the objective voters and the other leader l' vote for Q independently of m_l then $\#\Theta_A < b$ and l prefers Q to A. Since in this situations both leaders and objective voters vote for Q and $\#O \in \{q-2, q-1\}$ the outcome is Q regardless of l's report and l is not pivotal. Assume now that objective voters and the other leader l' vote for A independently of m_l. If in this situation l prefers A to Q at the voting stage once all signals are revealed (she knows her own signal and all other voters are truthful at the discussion stage) then the voting outcome is A and again l is not pivotal. If instead l prefers Q to A at the voting stage once all signals are revealed then as objective voters and the other leader l' vote for A independently of m_l if $m_l = A$ we have that l is the only voter that votes for Q (thus A is implemented) whilst if $m_l = Q$ then followers and l vote for Q and the objective voters and l' vote for A. In this case if $\#O = q - 1$ then the outcome is always A and l is not pivotal. On the other hand if $\#O = q - 2$ then the followers and l are enough to veto A and the voting outcome is Q. Hence, l is pivotal if $\#O = q-2$, $\theta_{l'} = A$ and l prefers Q to A at the voting stage once all signals are revealed. Finally, if objective voters and the remaining leader vote for different alternatives it is true that $\#\Theta_A < b$ and l prefers Q to A. Hence, whenever l is pivotal and she prefers Q to A at the voting stage once all signals are revealed (independently on whether she received signal A or Q). Hence, if l is pivotal she has incentives to miss-report by sending the message Q at the discussion stage regardless on the signal she receives. A symmetric argument applies to the case where $b < \frac{N}{2}+1$ and $b < b'$ to prove that l's best response is to report A regardless on her signal.

Consider now the case where $b > \frac{N}{2}+1$ but $b = b'$. Note that l is pivotal only if l's report changes the voting decision of either followers, objective voters or the decision of the other leader l'. If l's message determines the voting decision of the followers but leaves unchanged the decision of objective voters and the remaining leader then as in the previous paragraph l prefers Q to A regardless on her signal. On the other hand, since $b' \neq \frac{N}{2}+1$ if l is pivotal then her message either determines the voting decision of objective voters ($\#(\Theta_A \setminus l) = \#(\Theta_Q \setminus l)$) or the voting decision of l' ($\#(\Theta_A \setminus l) = b-1$). If l is pivotal because her message determines the vote of objective voters then as $\#(\Theta_A \setminus l) = \#(\Theta_Q \setminus l) = \frac{N}{2}$ implies $\#\Theta_A < b$ we have that l prefers Q to A regardless on her signal. Conversely, if l is pivotal because her message determines the vote of the other leader l' then $\#\Theta_A = b-1$ then whether she prefers A or Q is determined by her own signal. If $P\left(\#(\Theta_A \setminus l) = \frac{N}{2}|\theta_l\right) \geq P\left(\#(\Theta_A \setminus l) = b-1|\theta_l\right)$ then l is at least as likely to be pivotal because $\#(\Theta_A \setminus l) = \frac{N}{2}$ as she is because $(\Theta_A \setminus l) = b-1$. Hence, l has incentive to report Q regardless on her signal and, thus, she miss-reports. A symmetric argument applies to the case where $b < \frac{N}{2}+1$ and $b = b'$ to prove that l's best response is to report A regardless on her signal.

In order to conclude the proof assume that $b > \frac{N}{2}+1$, $b = b'$ and $P\left(\#(\Theta_A \setminus l) = \frac{N}{2}|\theta_l\right) < P\left(\#(\Theta_A \setminus l) = b-1|\theta_l\right)$. For any arbitrary objective voter $o \in O$ this last inequality implies $P\left(\#(\Theta_A \setminus l) = \frac{N}{2}|\theta_o\right) < P\left(\#(\Theta_A \setminus l) = b-1|\theta_o\right)$. Since followers' voting decision is only influenced by both leaders' reports, o is pivotal only if either $\#(\Theta_A \setminus o) = \#(\Theta_Q \setminus o)$ (her message determines the voting decision of the remaining objective voters) or $\#(\Theta_A \setminus o) = b-1$ (her message determines the voting decision of the leaders). If o is pivotal because she changes the decision of the leaders then $\#\Theta_A \geq \frac{N}{2}+1$ and l prefers A to Q. Conversely, if l is pivotal because she changes the objective voters then whether se prefers A or Q depends on her own signal. Since $P\left(\#(\Theta_A \setminus l) = \frac{N}{2}|\theta_o\right) < P\left(\#(\Theta_A \setminus l) = b-1|\theta_o\right)$ it is more likely that o is pivotal because her message determines the voting behavior of the leaders ($\#(\Theta_A \setminus l) = b-1$) than because $\#(\Theta_A \setminus l) = \frac{N}{2}$. Hence, o's best response is to report A regardless on the signal she receives. A symmetric argument applies to the case where $b < \frac{N}{2}+1$ and $b = b'$ to prove that o's best response is to report Q regardless on her signal. $\square$