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ABSTRACT

Learning Theoretical Computing from the Mathematical Impossibility Results of the CS Curriculum

RAFAEL DEL VADO-VÍRSEDA, Complutense University of Madrid, Madrid, Madrid, Spain

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Learning Theoretical Computing from the Mathematical Impossibility Results of the CS Curriculum

Rafael del Vado Vírveda
Complutense University of Madrid
rdelvado@mat.ucm.es

ABSTRACT

In this paper, we present a novel technique to systematically introduce questions of computability and algorithmic complexity in the curricula of the early years of Computer Science degrees. We propose to start from those results of impossibility that are already included in the curriculum of CS-gearred mathematics courses, and around these classical impossibility theorems, we identify motivating and interesting theoretical computing questions. We provide a helpful list of impossibility results and motivating problems, and we analyze how and where they could be introduced in the CS mathematics curriculum. We also provide a preliminary evaluation of the effectiveness of the proposed technique to demonstrate that the introduction of theoretical computing questions from impossibility results could increase students' academic results in theoretical computing subjects and mathematics subjects of the CS curriculum.

CCS CONCEPTS

• **Mathematics of computing** → *Discrete mathematics*; Probability and statistics; *Calculus*; • **Theory of computation** → **Computability**; **Complexity theory and logic**.

KEYWORDS

Computability; Complexity; Logic; Impossibility; Mathematics

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1 INTRODUCTION AND MOTIVATION

Theoretical Computer Science (TCS) topics are critical for any student who wants to acquire a solid grasp of computing, but the amount of mathematics required to understand these topics often scares away students. On the other hand, students who are exposed to the basic TCS concepts in their early college years have a higher chance of succeeding and probably enjoying TCS topics more than students who have not been exposed. In order to help to improve this situation, it is necessary to establish a more stimulating relationship between the curriculum of the CS mathematics subjects of the

first university years and the theoretical content of the TCS subjects. In this work we offer a novel technique to systematically introduce TCS questions in CS subjects with mathematical content of the first university years (*Discrete Mathematics*, *Mathematical Logic*, *Calculus*, *Linear Algebra*, *Probability*, *Statistics*, etc.) in order to better motivate students to study TCS subjects. Our technique consists of starting from those results of impossibility that are already included in the curricula of these mathematical subjects of CS degrees, in an intuitive but a sufficient formal way, and the interesting and motivating stories connected to these problems. For example, a mathematical impossibility that generates great curiosity among students, especially because of the number of historical anecdotes, is the impossibility of finding a general formula to solve the fifth degree equation, or the impossibility to solve diophantic equations such as $x^n + y^n = z^n$ in Fermat's last theorem. Around these classic results of impossibility, conveniently adapted and presented to students, we have located related TCS questions that allow students to intuitively reason about their computability and algorithmic complexity, without the need to introduce complicated concepts about Turing machines or any other abstract computing model. For example, interesting computability questions, such as the impossibility of finding an algorithm to determine whether any diophantic equation has integer solutions, or algorithmic complexity questions, such as the $\mathcal{NP}$ -completeness of the *Boolean Satisfiability Problem* (SAT), which is easily obtained by considering diophantic equations whose solutions and operations are interpreted in the modulo 2 arithmetic, are presented to CS students of *Discrete Mathematics*. Students find it very motivating that these theoretical questions are linked to problems and situations related to their daily lives, such as the undecidability of whether a team has a forced win in the *Super Smash Bros* video game, or that the problem of determining whether the end of a video game screen such as *Super Mario Bros* can be reached from the beginning is $\mathcal{NP}$ -complete.

2 IMPOSSIBILITY RESULTS

The impossibility results together with their computability and complexity questions have been meticulously classified and distributed throughout the curriculum of CS math courses. **Table 1** describes a selected list of impossibility results and motivating problems about computability and complexity to help instructors better motivate TCS topics. All these TCS questions have been progressively presented to students within the context of each didactic unit in which their curriculum is structured and the level of knowledge required by the students for its adequate assimilation. Although the technique is applied in the curriculum of CS-gearred mathematics courses for lower-level undergraduate students, it is also possible to partially apply the list of problems in **Table 1** in the last years of pre-university education, so that students can acquire an intuitive

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Table 1: The CS mathematics curriculum with impossibility results and theoretical computing questions

	CS Math Curriculum	Impossibility Results	Computability Questions	Complexity Questions
Calculus	Irrational numbers.	$\sqrt{2}$, π , e are not rational numbers.	Representation in a computer.	<i>Monte Carlo estimation of π.</i>
	Real numbers.	Real numbers are uncountable.	The <i>continuum hypothesis</i> is undecidable (or independent).	Efficient algorithms for computing with real numbers.
	Complex numbers.	Equations with no solution in the real numbers.	Computability of transcendental numbers.	Fractals generated by complex numbers.
	Exponential functions and series.	Exponential growth in the <i>chessboard legend</i> .	Calculating exponential growth.	<i>Towers of Hanoi</i> , Chess, Checkers, Go (board games).
	Differentiation and Integration.	Impossibility <i>Liouville's theorem</i> for elementary integration.	Decision of the integrability problem (<i>Richardson's theorem</i>).	Numerical integration and differentiation methods.
Linear Algebra Geometry	Linear algebra and matrices.	Matrix multiplication is not commutative.	Undecidability of the <i>matrix mortality problem</i> .	What is the fastest algorithm for matrix multiplication?
	Optimization problems.	Solving the <i>knapsack problem</i> .	<i>Conway's Game of Life</i> .	<i>Travelling Salesman Problem</i> .
	Trigonometry, triangles, polygons.	The three impossible problems of Greek antiquity.	Computing regular polygons with ruler and compass.	<i>Mathekreis</i> and <i>Mathe Quadrat</i> polygonal puzzles.
	2D Geometric transformations.	Tiling a mutilated chessboard.	Undecidability of tiling problems.	<i>Jean Claude Constantin</i> puzzles.
	Foundations of 3D Geometry.	Independence of <i>Parallel Postulate</i> .	Axioms for constructive geometry.	3D Tiling puzzles.
Prob. Stat.	Probability.	Can we predict throwing a dice?	Chaos theory and fractals.	The <i>Three-Body Problem</i> .
	Statistics.	Integrating the normal probability density function.	Undecidability of the improper integral convergence.	The <i>Galton board</i> and the normal approximation.
Logic Disc. Math.	Algebraic equations.	Resolution of the equation of the fifth degree, <i>Fermat's theorem</i> .	Undecidability of diophantine equations (<i>Matiyasevich's theorem</i>).	<i>Boolean Satisfiability Problem</i> (SAT).
	Graph theory.	<i>Königsberg bridge problem</i> .	<i>Kruskal's tree theorem</i> .	<i>Graph coloring problem</i> .
	Logic.	<i>Bertrand Russell's paradox</i> .	<i>Gödel's incompleteness theorems</i> .	<i>Presburger arithmetic</i> .
		Undecidability of first-order logic.	<i>The halting problem</i> .	Automated theorem proving.

and progressive knowledge of these basic TCS concepts and their associated abilities even before they finish high school, beyond traditional programming courses [1].

3 THE EXPERIMENTAL SETUP

To estimate the impact of our curricular technique, we have evaluated the introduction of TCS questions based on impossibility results in the common mathematical subjects of the first years of the CS curriculum (*Logic and Discrete Mathematics*, *Calculus and Linear Algebra*, *Probability and Statistics*). The experience has been carried out during the 2018/2019 academic year in the Mathematics and Computer Science Degrees of several university centres located in Spain, whose students, between 18 and 23 years old, have an average socioeconomic status. For each of these subjects, the educational experience has been applied simultaneously in two random groups of between 35 and 40 students, under the same initial conditions, and taught by a same instructor. In one of the groups the TCS questions have been introduced during the development of the subject's curriculum, while in the other group TCS questions have not been introduced. The content of the curriculum taught in each group has been the same, but in the group in which the experience has been developed, a weekly class has been reserved to present the corresponding TCS questions in **Table 1**. In the other group, this class has been dedicated to carrying out more exercises or to solve doubts about them. To compensate this class, in the group in which the educational experience has been carried out, resolution of exercises have been made through a Virtual Campus. In this way, both groups have done the same exercises and have seen the same curricular content.

4 EVALUATION AND PRELIMINARY RESULTS

At the end of each four-month period, both groups have taken the same tests on the content of the subject. The final grades obtained by each group reflect an increase from 10% to 15% in the results obtained by the students subjected to the new curricular technique. In all these tests, at least one question has been included to measure the specific development of skills related to TCS thinking. Since the educational experience has been carried out during 2018/2019, there are groups of students who have already undergone it and who have also taken TCS subjects (*Computability and Complexity*, *Formal Languages and Automata Theory*). We have been able to preliminarily measure the results of the students in these TCS subjects and compare them with students who did not undergo our curricular technique. The student feedback also shows a remarkable increase in the results and motivations of these TCS subjects.

5 CONCLUSIONS AND FUTURE WORK

Nowadays, CS departments are discussing with where to put TCS topics in the beginning CS courses to achieve better academic results in TCS subjects. The good preliminary results show that the introduction of TCS topics from impossibility results seems to affect the motivation of students and improve the students' grades in CS-gear math courses and TCS subjects. Practical implications for future work are to collect more data on this curricular technique.

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