REMARKS ON A THEOREM OF TASKINEN ON SPACES OF CONTINUOUS FUNCTIONS

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Abstract. We clarify and prove in a simpler way a result of Taskinen about symmetric operators on $C(K^n)$, K an uncountable metrizable compact space. To do this we prove that, for any compact space K and any $n \in \mathbb{N}$, the symmetric injective n-tensor product of $C(K)$, $\widehat{\otimes}^n_{s,\epsilon} C(K)$, is complemented in $C(B_{C(K)^*})$, a result of independent interest. The techniques we develop allow us to extend the result in several directions. We also show that the hypothesis of metrizability and uncountability can not be removed.

1. Introduction and notation

In the remarkable paper [6], Pełczyński, guided by the purpose of explaining Milutin's theorem, developed the theory of extension operators and averaging operators between spaces of continuous functions. In [8], Taskinen applied this theory to obtain a factorization result for a certain class of polynomials defined on a $C(K)$ space, with K uncountable and metrizable. In this note we want to clarify certain aspects of Taskinen's theorem and to prove some related results. In particular, we prove that, for any compact space and any $n \in \mathbb{N}$, $\widehat{\otimes}^n_{s,\epsilon} C(K)$ is complemented in $C(B_{C(K)^*})$. This allows us to give a general factorization result valid for every compact space K (Corollary 2.2), and to obtain an improvement of [8, Theorem 2] as a corollary (Corollary 2.4). We also put bounds on the validity of Taskinen's result showing that it cannot be true neither for $C(\beta\mathbb{N}) = \ell_\infty$ nor for $c = c(\mathbb{N}^*)$, showing thus that neither the metrizability nor the uncountability of K can be dispensed with in the statement of the result.

First we explain our notation and some basic facts we use. Following [7], we say that a topological space K is compact if it is quasicompact and Hausdorff. $\mathbb{1}_K$ will stand for the function identically 1 on K. Given a Banach space X, the unit ball of its dual, B_{X^*}, is always a compact space when we endow it with the weak* topology. We call $i_X : X \hookrightarrow C(B_{X^*})$ the usual isometric injection. We write Id_X for the identity on X. Given two compact spaces K and S and a continuous function $\varphi : K \longrightarrow S$, we can always define the linear operator $\varphi^0 : C(S) \longrightarrow C(K)$ of norm one given by $\varphi^0(f)(t) = f(\varphi(t))$. If φ is injective, then φ^0 is a quotient map, and if φ is onto, then φ^0 is an injective isometry. Suppose φ is injective (resp. onto). If there exists $u : C(K) \longrightarrow C(S)$ such that $\varphi \circ u = Id_{C(K)}$ (resp. $u \circ \varphi = Id_{C(S)}$) then we say that u is an extension operator (resp. averaging operator) for φ. Moreover, if $\|u\| = 1$ and $u(\mathbb{1}_K) = \mathbb{1}_S$, then u is said to be regular. For us, n will always be a natural number. We write $\otimes^n X$ for the tensor product of X with itself n times

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and $\otimes_s^n X$ for the symmetric tensor product. We write $\widehat{\otimes}_\epsilon^n X$ for the completion of $\otimes^n X$ under the injective norm and $\widehat{\otimes}_{s,\epsilon}^n X$ for the completion of $\otimes_s^n X$ with the injective norm. We recall that there are two natural ways to define an injective norm in $\otimes_s^n X$, and that both norms are equivalent, but not equal (see [4]). If K is compact, we write K^n for the compact set $K \times \overset{(n)}{\cdots} \times K$. We recall that $\widehat{\otimes}_\epsilon^n C(K)$ is isometrically isomorphic to $C(K^n)$, the isomorphism $\vartheta : \widehat{\otimes}_\epsilon^n C(K) \longrightarrow C(K^n)$ verifying $\vartheta(f_1 \otimes \cdots \otimes f_n)(t_1, \ldots, t_n) = \prod_{m=1}^n f_m(t_m)$. Thus we identify $\widehat{\otimes}_\epsilon^n C(K)$ and $C(K^n)$, and we shift from one to another without further comment. Given two Banach spaces X and Y, we write $\mathcal{L}(X;Y)$ for the space of linear bounded operators from X into Y.

We invite the reader to take a look at [8, Theorem 2] and we make the following remarks: If X, Y are Banach spaces, we say that an n-linear map $T : Y \times \overset{(n)}{\cdots} \times Y \longrightarrow X$ is *ϵ-continuous* if its linearization $\hat{T} : \otimes^n Y \longrightarrow X$ is a continuous operator when we endow $\otimes^n Y$ with the injective topology. It follows immediately from the closed graph theorem that $T : Y \times \overset{(n)}{\cdots} \times Y \longrightarrow X$ is ϵ-continuous if and only if, given a norming set $D \subset X^*$, for every $x^* \in D$, $x^* \circ T : Y \times \overset{(n)}{\cdots} \times Y \longrightarrow \mathbb{K}$ is ϵ-continuous. Notice also that, if $T : Y \times \overset{(n)}{\cdots} \times Y \longrightarrow X$ is ϵ-continuous and we call $\hat{T}_\epsilon : \widehat{\otimes}_\epsilon^n Y \longrightarrow X$ the associated continuous linear operator, then the mapping $\hat{T}_\epsilon^* : X^* \longrightarrow (\widehat{\otimes}_\epsilon^n Y)^*$ is weak*-weak* continuous. Note also that the mapping $\delta : S \longrightarrow C(S)^*$, $t \mapsto \delta_t$ is continuous when we consider the weak* topology in $C(S)^*$.

Using these remarks, we get that [8, Theorem 2] can actually be reformulated as **Theorem (T').** *a) Let K and S be compact spaces, K metric uncountable, let $n \in \mathbb{N}$, $n > 1$ and let P be a continuous symmetric n-linear map $P : C(K) \times \overset{(n)}{\cdots} \times C(K) \longrightarrow C(S)$. If P is ϵ-continuous, then there exist linear operators $A \in \mathcal{L}(C(K);C(K))$ and $B \in \mathcal{L}(C(K);C(S))$ such that, for all $f_1, \ldots, f_n \in C(K)$,*

$$P(f_1, \ldots, f_n) = B\left(\prod_{m=1}^n A(f_m)\right).$$

Conversely, if P can be represented in the above form, then it is ϵ-continuous.

b) Let K and n be as above, let X be a Banach space and let P be a continuous symmetric n-linear map $P : C(K) \times \overset{(n)}{\cdots} \times C(K) \longrightarrow X$. If P is ϵ-continuous then there exist linear operators $A \in \mathcal{L}(C(K);C(K))$ and $B \in \mathcal{L}(C(K);X^{**})$ such that, for all $f_1, \ldots, f_n \in C(K)$,

$$P(f_1, \ldots, f_n) = B\left(\prod_{m=1}^n A(f_m)\right).$$

Conversely, if X is reflexive and P can be represented in the above form, then P is ϵ-continuous.

In the rest of this note, together with other results of independent interest, we give a much simpler proof of Theorem (T') (Corollary 2.4), showing among other things that there is no need to distinguish the cases (a) and (b), that there is no need for B to finish in X^{**} and that the operator A can be chosen to be an isometric injection independent of P.

2. The results

Our main result is the following

Theorem 2.1. *For every compact space K and every $n \in \mathbb{N}$, $\widehat{\otimes}^n_{s,\epsilon} C(K)$ is complemented in $C(B_{C(K)^*})$.*

Proof. It is known that, for every Banach space X and every $n \in \mathbb{N}$, $\widehat{\otimes}^n_{s,\epsilon} X$ is isometrically contained in $C(B_{X^*})$, the inclusion mapping taking $x \otimes \overset{(n)}{\cdots} \otimes x \mapsto i_X(x)^n$. We see now that, for the case of $C(K)$ spaces, this copy is complemented.

All we need to prove is the existence of a quotient map $Q : C(B_{C(K)^*}) \longrightarrow \widehat{\otimes}^n_{s,\epsilon} C(K)$ such that, for every $f \in C(K)$, $Q(i_{C(K)}(f)^n) = f \otimes \overset{(n)}{\cdots} \otimes f$.

To see this, let ε_m take one of the values $+1$ or -1 ($1 \leq m \leq n$) and let $J_{\varepsilon_1,\ldots,\varepsilon_n} : K \times \overset{(n)}{\cdots} \times K \longrightarrow B_{C(K)^*}$ be the continuous (when we consider the weak* topology in $B_{C(K)^*}$) mapping defined by

$$J_{\varepsilon_1,\ldots,\varepsilon_n}(t_1, \ldots, t_n) = \frac{1}{n} \sum_{m=1}^{n} \varepsilon_m \delta_{t_m}.$$

Define now $Q' : C(B_{C(K)^*}) \longrightarrow \widehat{\otimes}^n_{\epsilon} C(K)$ by

$$Q' = \frac{n^n}{n! 2^n} \sum_{\varepsilon_m = \pm 1} \varepsilon_1 \cdots \varepsilon_n J^0_{\varepsilon_1,\ldots,\varepsilon_n}$$

and let $Q = \pi \circ Q'$, where $\pi : \widehat{\otimes}^n_{\epsilon} C(K) \longrightarrow \widehat{\otimes}^n_{s,\epsilon} C(K)$ is the usual projection. To see that $Q(i_{C(K)}(f)^n) = f \otimes \overset{(n)}{\cdots} \otimes f$, it suffices to check that, for every $f \in C(K)$ and $(t_1, \ldots, t_n) \in K^n$,

$$\prod_{m=1}^{n} f(t_m) = Q'(i_{C(K)}(f)^n)(t_1, \ldots, t_n).$$

Let us notice that

$$f(t_1) \cdots f(t_n) = \langle f, \delta_{t_1} \rangle \cdots \langle f, \delta_{t_n} \rangle = \langle f \otimes \overset{(n)}{\cdots} \otimes f, \delta_{t_1} \otimes \cdots \otimes \delta_{t_n} \rangle$$

considering $f \otimes \overset{(n)}{\cdots} \otimes f$ as a symmetric operator acting on $\delta_{t_1} \otimes \cdots \otimes \delta_{t_n} \in \otimes^n C(K)^*$. So, we can apply the polarization formula to get that

$$f(t_1) \cdots f(t_n) = \frac{1}{n! 2^n} \sum_{\varepsilon_m = \pm 1} \varepsilon_1 \cdots \varepsilon_n \langle f, \sum_{m=1}^{n} \varepsilon_m \delta_{t_m} \rangle^n =$$

$$= \frac{n^n}{n! 2^n} \sum_{\varepsilon_m = \pm 1} \varepsilon_1 \cdots \varepsilon_n \langle f, \frac{1}{n} \sum_{m=1}^{n} \varepsilon_m \delta_{t_m} \rangle^n =$$

$$= \frac{n^n}{n! 2^n} \sum_{\varepsilon_m = \pm 1} \varepsilon_1 \cdots \varepsilon_n i_{C(K)}(f)^n \left(\frac{1}{n} \sum_{m=1}^{n} \varepsilon_m \delta_{t_m}\right) =$$

$$= \frac{n^n}{n! 2^n} \sum_{\varepsilon_m = \pm 1} \varepsilon_1 \cdots \varepsilon_n J^0_{\varepsilon_1,\ldots,\varepsilon_n}(i_{C(K)}(f)^n)(t_1, \ldots, t_n) = Q'(i_{C(K)}(f))^n(t_1, \ldots, t_n).$$

□

The next corollary is somehow a general version of Theorem (T'), valid for every compact K.

Corollary 2.2. *Let K be a compact Hausdorff space, $n \in \mathbb{N}$ and let $T : \widehat{\otimes}_\epsilon^n C(K) \to X$ be a symmetric operator. Then there exists a linear operator $B' : C(B_{C(K)^*}) \longrightarrow X$ such that*

$$T(f_1 \otimes \cdots \otimes f_n) = B' \left(\prod_{m=1}^{n} i_{C(K)}(f_m) \right)$$

Proof. We follow the notation of Theorem 2.1. It suffices to consider $B' = T \circ Q'$. □

It is now very easy to prove the announced improvement of Theorem (T'). First we need a technical lemma, which is implicit in [8, Proof of Theorem 2] and is a standard consequence of an extension theorem of Borsuk-Dugundji and a famous result of Milutin. We include a sketch of the proof for completeness.

Lemma 2.3. *Let K, S be two metrizable uncountable compact sets. Then there exists an isometric injection $j : C(S) \hookrightarrow C(K)$ and a norm one projection $p : C(K) \longrightarrow C(S)$ such that $p \circ j = Id_{C(S)}$ and such that, for every $f_1, \ldots, f_n \in C(S)$,*

$$\prod_{m=1}^{n} f_m = p \left(\prod_{m=1}^{n} j(f_m) \right).$$

Proof. Let $\Delta \subset K$ be a compact subset homeomorphic to the Cantor set. Let $R : C(K) \longrightarrow C(\Delta)$ be the restriction operator associated to the inclusion $\Delta \subset K$ and $E : C(\Delta) \to C(K)$ the Borsuk-Dugundji extension operator. On the other hand, according to [6, Theorem 5.6], there exists a continuous and onto application $\varphi : \Delta \longrightarrow S$ admitting a regular averaging operator. Call u this averaging operator. Then it can be checked that $j = E \circ \varphi^0$ and $p = u \circ R$ satisfy the required conditions. □

Corollary 2.4 (Taskinen [8])**.** *Let K be a metrizable uncountable compact set. Then there exists an isometric injection $A : C(K) \hookrightarrow C(K)$ so that, for every $n \in \mathbb{N}$ and every symmetric operator $T : \widehat{\otimes}_\epsilon^n C(K) \longrightarrow X$ there exists $B \in \mathcal{L}(C(K); X)$ such that*

$$T(f_1 \otimes \cdots \otimes f_n) = B \left(\prod_{m=1}^{n} A(f_m) \right).$$

Conversely, if $T : C(K) \times \overset{(n)}{\cdots} \times C(K) \longrightarrow X$ is a multilinear operator which admits such a representation, then T is ϵ-continuous and symmetric.

Proof. Since K is metrizable, $C(K)$ is separable, hence $B_{C(K)^*}$ is metrizable (and uncountable). We consider the operators $j : C(B_{C(K)^*}) \hookrightarrow C(K)$ and $p : C(K) \longrightarrow C(B_{C(K)^*})$ given by Lemma 2.3, and it suffices to define $A = j \circ i_{C(K)}$, $B = B' \circ p$, where B' is as in Corollary 2.2. The converse is clear, since the multiplication operator $\prod : C(K) \times \overset{(n)}{\cdots} \times C(K) \longrightarrow C(K)$ is ϵ-continuous. Symmetry is obvious. □

Note that, for any uncountable metrizable compact space S, the same proof shows that we could have chosen an isometric injection $A : C(K) \hookrightarrow C(S)$ and an operator $B \in \mathcal{L}(C(S); X)$.

It is also possible to give a multilinear version of this result. First we need a lemma which is a simple formality.

Lemma 2.5. *Let K be a compact space, $n \in \mathbb{N}$. Then there exist isometric injections $i_m \in \mathcal{L}(C(K); \widehat{\otimes}_\epsilon^n C(K))$ $(1 \leq m \leq n)$ such that, for every operator $T : \widehat{\otimes}_\epsilon^n C(K) \longrightarrow X$*

$$T(f_1 \otimes \cdots \otimes f_n) = T\left(\prod_{m=1}^{n} i_m(f_m)\right).$$

Proof. It suffices to define $i_m : C(K) \hookrightarrow \widehat{\otimes}_\epsilon^n C(K)$ by $i_m(f) = \mathbb{1}_K \otimes \cdots \otimes \mathbb{1}_K \otimes \overset{(m)}{f} \otimes \mathbb{1}_K \otimes \cdots \otimes \mathbb{1}_K$ □

Proposition 2.6. *Let K be a metrizable uncountable compact space, $n \in \mathbb{N}$. Then there exist isometric injections $A_m \in \mathcal{L}(C(K); C(K))$ $(1 \leq m \leq n)$ so that, for every operator $T : \widehat{\otimes}_\epsilon^n C(K) \longrightarrow X$ there exists an operator $B \in \mathcal{L}(C(K); X)$ such that,*

$$T(f_1 \otimes \cdots \otimes f_n) = B\left(\prod_{m=1}^{n} A_m(f_m)\right).$$

Proof. Since K is metrizable uncountable, so is K^n. We consider the operators $j : C(K^n) \hookrightarrow C(K)$ and $p : C(K) \longrightarrow C(K^n)$ given by Lemma 2.3, and we define $B = T \circ p$, $A_m = j \circ i_m$, where i_m are the operators defined in Lemma 2.5. □

Remark 2.7. In Corollaries 2.2, 2.4 and Proposition 2.6 the operator B (or B') can always be written as $B = T \circ \pi$, where π is a quotient map. Hence, for any surjective operator ideal $\mathcal{U}$, $B \in \mathcal{U}$ if and only if $T \in \mathcal{U}$. In particular B is (weakly) compact if and only if T is (weakly) compact.

We presently put some bounds on the validity of Corollary 2.4 and Proposition 2.6.

Example 2.8. Proposition 2.6 and Corollary 2.4 need not hold if K is not metrizable.

To see this, let K be a space such that $C(K)$ is a Grothendieck space. It is known that $C(K)\hat{\otimes}_\epsilon C(K)$ contains a complemented copy of c_0 (e.g. see [1]), so let $T : C(K)\hat{\otimes}_\epsilon C(K) \longrightarrow c_0$ be a quotient map. If Proposition 2.6 applied to K, then the corresponding operator $B : C(K) \longrightarrow c_0$ would also be onto, which is not possible.

For the symmetric case, let K be a completely disconnected compact space (in that case $C(K)$ is Grothendieck). Then $C(K)$ contains a copy of ℓ_∞, which in turn contains a copy of ℓ_1. Hence, there is a quotient map $q : C(K) \longrightarrow \ell_2$ ([3, Corollary 4.16]). In [2, Section 4.4] it is shown that the ϵ topology on tensor products respects quotients from $C(K)$ spaces (in general from $\mathcal{L}_\infty$ spaces). So, $q \otimes q : C(K)\hat{\otimes}_\epsilon C(K) \longrightarrow \ell_2\hat{\otimes}_\epsilon \ell_2$ is a quotient map. In [5] it is proved that $\ell_2\widehat{\otimes}_{s,\epsilon}\ell_2$ has an isomorphic copy of c_0, and this copy is complemented because $\ell_2\widehat{\otimes}_{s,\epsilon}\ell_2$ is separable. So, let $Q : \ell_2\hat{\otimes}_\epsilon \ell_2 \longrightarrow c_0$ be a symmetric quotient map and let $T = Q \circ (q \otimes q)$. This is a symmetric quotient map from $C(K)\hat{\otimes}_\epsilon C(K)$ onto c_0, so, if Corollary 2.4 applied to K, then the operator B provided by that corollary would be a quotient map from $C(K)$ onto c_0, a contradiction.

Example 2.9. Proposition 2.6 and Corollary 2.4 need not hold if K is metrizable and countable.

To see this, let $c = C(\mathbb{N}^*)$, where $\mathbb{N}^*$ is the Alexandroff compactification of $\mathbb{N}$. Let $\varphi : c \longrightarrow c_0$ be an onto isometric isomorphism and let $p : \mathbb{N} \longrightarrow \mathbb{N}$ be the function assigning n to the n-th prime number (so $p(1) = 2$, $p(2) = 3$, $p(3) = 5$ etc.). Finally, let

$$T : c(\mathbb{N}^* \times \mathbb{N}^*) = c\hat{\otimes}_\epsilon c \longrightarrow c_0$$

be the operator given by

$$T(h) = \sum_{n,m=1}^{\infty} \widetilde{\varphi}(h)(n,m)e_{p(n)^m},$$

where $\widetilde{\varphi} = \varphi \otimes \varphi : c\hat{\otimes}_\epsilon c \longrightarrow c_0\hat{\otimes}_\epsilon c_0$.

To see that T is well defined suppose that $f \in B_c$, $g \in B_c$. Then $T(f \otimes g) = \sum_{n,m=1}^{\infty} \varphi(f)(n)\varphi(g)(m)e_{p(n)^m}$. Note that, for any $\epsilon > 0$, there exists $N_1 \in \mathbb{N}$ such that, for any $n > N_1$, $\varphi(f)(n) < \epsilon$ and there exists $N_2 \in \mathbb{N}$ such that, for any $n > N_2$, $\varphi(g)(n) < \epsilon$. Then, for any $n > p(N_1)^{N_2}$, $T(f \otimes g)(n) < \epsilon$. The continuity of T follows easily from the fact that, if $i \neq j$, the sets $\{p(i)^m : m \in \mathbb{N}\}$ and $\{p(j)^m : m \in \mathbb{N}\}$ are disjoint.

Suppose that Proposition 2.6 applied to c, let $n = 2$, let $i_1, i_2 : c \longrightarrow c$ be the operators given by the proposition and let $B : c \longrightarrow c_0$ be the operator associated to T. For every $n \in \mathbb{N}$, let $A_n = \{t \in \mathbb{N}^* \text{ such that } i_1(e_n)(t) \geq \frac{1}{10\|B\|\cdot\|i_2\|}\} = i_1(e_n)^{-1}([\frac{1}{10\|B\|\cdot\|i_2\|}, \infty])$.

Claim: For every $n \in \mathbb{N}$, $\infty \in A_n$.

Proof of the claim: Since A_n is closed, if A_n is infinite then it must have an accumulation point which is necessarily ∞, since it is the only accumulation point of $\mathbb{N}^*$. Suppose then that A_n is finite and $\infty \notin A_n$. Then $A_n = \{t_1, \ldots, t_m\} \subset \mathbb{N}$. Then, for any $f \in B_c$,

$$i_1(e_n)i_2(f) = \sum_{j=1}^{m} i_1(e_n)(t_j)i_2(f)(t_j)e_{t_j} + i_1(e_n)i_2(f)\chi_{\mathbb{N}^*\setminus A_n}.$$

Call $g = i_1(e_n)i_2(f)\chi_{\mathbb{N}^*\setminus A_n} \in c$ and note that $\|g\| \leq \frac{1}{10\|B\|}$. So,

$$B(i_1(e_n)i_2(f)) = \sum_{j=1}^{m} i_1(e_n)(t_j)i_2(f)(t_j)B(e_{t_j}) + B(g).$$

So, note that the set $\{B(i_1(e_n)i_2(f)) : f \in B_c\}$ is contained in a finite dimensional subspace plus $\frac{1}{10}$ of the unit ball, and this clearly can not be as large as $\{T(e_n \otimes f) : f \in B_c\}$, which by construction of T contains the unit ball of an infinite dimensional subspace of c_0 isometric to c_0. So, we see that, for every $n \in \mathbb{N}$, $\infty \in A_n$ and the claim is proved.

There exists an infinite set $N_1 = \{n_1 < n_2 < \cdots\} \subset \mathbb{N}$ such that, for every $n \in N_1$, $i_1(e_n)(\infty)$ has the same sign (this happens if the scalar field is $\mathbb{R}$; if the scalar field is $\mathbb{C}$ the reasonings are similar).

So, let $s_m = \sum_{j=n_1}^{n_m} e_{n_j}$. Then $\|s_m\| = 1$ but

$$\lim_{m\to\infty} \|i_1(s_m)\| \geq \lim_{m\to\infty} |i_1(s_m)(\infty)| = +\infty,$$

a contradiction to the continuity of i_1.

To prove that Corollary 2.4 does also not apply to c we could reason similarly with the symmetric operator $S : c\hat{\otimes}_\epsilon c \longrightarrow c_0$ given by $S(f\otimes g) = T(f\otimes g)+T(g\otimes f)$.

At this moment we are not sure whether a similar counterexample can be used for every countable metrizable compact space.

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