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An Extension of the Axioms of Utility Theory Based on Fuzzy Rationality Measures

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Summary. We present here a (better yet, the problems involved with a) generalization of classical utility theory when basic preferences are stated by means of "rational" fuzzy preference relations. Rationality of fuzzy preference relations will be measured according to general fuzzy rationality measures. A utility function is proposed and introduced by using a "boosting" procedure on the fuzzy preference relations which may assure a linearization of the alternatives, still maintaining or improving rationality.

Mathematics is a language in which one cannot express unprecise or nebulous thoughts.
H. Poincaré

What exactly is Mathematics? Many have tried but nobody has really succeeded in defining mathematics; it is always something else.
S.M. Ulam
(Adventures of a mathematician)

1 Introduction

Fuzzy logic is nowadays a remarkably well formalized mathematical framework for dealing in a precise and formal way with concepts deriving from unprecise knowledge or thoughts. This is particularly true in the general area of decision making. Wherever a finite collection of alternatives must be analyzed, compared so to pick one, it is often the case the human beings tend to loose precision and clarity of mind. Although they are pretty able to pairwise compare the alternatives. Fuzzy preference relations represent a good formal methodology for producing algorithmic solutions to decision making problems. Utility theory has been traditionally formalized and used in such areas. Its axiomatic foundation, though, relies upon non fuzzy and non contradictory preference relations. Not very human-like in many applications today.

For instance, the amazing growth of the Internet and the WWW, the social and economic issues involved with it have caused an enormous interest in the design, testing and production of software systems which are able to help and interact with human participants in the (virtually handled) economic world. Such software systems are commonly denoted by "Intelligent Systems" and have evolved in body (implementation level), mind (inferential core) and scope from classical expert systems.

In particular, they must *do a good job of acting on their environment* (see [21]). And if we want to define them to be "rational agents" or "ideal rational agents": *for each possible sequence of inputs, an ideal rational agent does whatever action is expected to maximize its performance measure, on the basis of the evidence provided by the inputs and whatever built-in knowledge the agent has.*

Example 1. The first example of intelligent agent is given by one of the aspects of e-commerce. A company who is planning to have a marketing activity using its web server would need a software system which :

- keeps track of the customers requests
- "localizes" the requests to the customer profiles so to offer specific information to specific customers (common advertising activity done already on a simple basis using "cookies")
- "learns" customer preferences from statistical data
- "understands" from customer requests what new products could be of interest and feasible to offer given the company resources and interests;
- "redirects" the customers to other links (friendly company links) when it is necessary
- searches the web for information on competitors activities
- gives general advice and suggestions to the company managers, etc.

□

Example 2. A second example can be derived by the growing interest on distance education using the web. Many higher education institutions are already working on it and have started offering degrees which are almost entirely done on the web. In such a scenario, one could think of intelligent agents which act as tutors for specific disciplines. For instance, an "interactive mathematics tutor", would check typed formulas and results, would offer

suggestions and corrections, would understand the student preferences in terms of material and timing. The agent would have to decide which topics to stress, what exercise to assign, what suggestions to give, etc. □

In both of the examples described above, decisions must be made under probabilistic uncertainty using as well approximate reasoning techniques.

We will now describe a formal framework (see [7,8]) to solve (at least partially) such kind of decision problems. We will start from the classical axioms of utility theory and then we will describe a possible extension by introducing fuzzy preferences and rationality of preferences. To define rationality we will make use of general fuzzy rationality measures (see [4]).

2 The fuzziness of rationality

Rationality of individuals is clearly a fuzzy concept, therefore the closer an intelligent agent is to a human being the fuzzier will be its associated concept of rationality. Rationality can be seen as consistency of (degree of) preferences. Such a consistency of preferences is in fact a combination of explicit and implicit consistency. Explicit consistency is the absence of explicit contradictions, i.e. statements of type $P \wedge \neg P$, and implicit consistency is related to the judgment criteria (and their use) upon which the opinions are ultimately based.

To each individual one can assign a value of rationality between 0 (absolute irrationality) and 1 (absolute rationality). This value (degree) of rationality can be assigned in many different ways, and each of these different assignments corresponds to a different criterion for measuring the rationality of an individual. These criteria are called *fuzzy rationality measures*. Fuzzy rationality measures have been introduced and formalized in [2–5].

2.1 Fuzzy preferences

Fuzzy preference relations were introduced by Zadeh [25] in order to capture degrees of preferences (see also [10,13,14,19,26] for an extensive introduction). Two different alternatives may be considered to be better than a third one, but one preference may not be as intense as the other. In this way, given a finite set of alternatives or states X, a fuzzy binary preference relation is defined as a fuzzy set over all pairs of the cartesian product $X \times X$, so that its membership function, $\mu : X \times X \to [0,1]$, associates to each pair (x, y) the strength or intensity of preference $\mu(x, y)$ between x and y, measured in the unit interval. More intuitively, $\mu(x, y)$ gives the degree to which x is not considered to be worse than y. That is to say, μ is understood as the *weak* fuzzy preference, with a *strict* and an *indifference* part. Reflexivity ($\mu(x, x) = 1$ for all x) is therefore implied. Moreover, we shall be assuming here that such a measure of intensity of preference μ is defined on an absolute scale. This strict cardinal framework is indeed a strong assumption, but it has been justified in the past by some authors, within some particular context (see [23] for example).

Without such an assumption, many operations on preferences may not be meaningful at all. Addition and subtraction on intensities of preference need [illegible] scale at least and multiplication requires that intensities are measured at least on a ratio scale (see [18,24] for an extensive discussion on measurement theoretic aspects of fuzzy sets).

2.2 Complete fuzzy preference relations

In some cases we can also assume that the fuzzy binary preference relations μ are complete, where completeness is defined as

$$\mu(x,y)+\mu(y,x)\geq 1 \quad \forall x,y\in X.$$

Completeness is required in order to assure that the set of states is feasible and comprehensive (see [16], for example, for an axiomatic discussion). Then, the values [illegible]

$$\mu_I(x,y)=\mu(x,y)+\mu(y,x)-1$$
$$\mu_B(x,y)=\mu(x,y)-\mu_I(x,y)$$
$$\mu_W(x,y)=\mu(y,x)-\mu_I(x,y)$$

can be understood, respectively, as the degree to which the two alternatives are indifferent (xIy), the degree of strict preference of x over y (xBy, *x is better than y*) and the degree of strict preference of y over x (xWy, *x is worse than y*). In this way, preference between two alternatives x and y will be explained by means of these three intensity values (xBy, xIy, xWy), each one associated to one possible crisp relation between both alternatives in such a way that [illegible]

$$\mu_B(x,y)+\mu_I(x,y)+\mu_W(x,y)=1 \quad \forall x,y\in X.$$

2.3 Incomplete fuzzy preference relations

If $\mu(x,y)+\mu(y,x)\leq 1$, the value $1-\mu(x,y)-\mu(y,x)$ can be understood either as the degree to which the comparison has not been accepted or the two alternatives are both rejected (see [1]).

Therefore, given an arbitrary fuzzy preference relation μ for any fixed pair of alternatives x,y we define

$$\mu_{PI}(x,y)=\max(\mu(x,y)+\mu(y,x)-1,0)$$
$$\mu_{NI}(x,y)=\max(1-\mu(x,y)-\mu(y,x),0)$$

in order to capture the degrees of possitive indifference and negative indifference (incomparability). Notice, in particular that $\mu_{PI}(x,y)$ is the *Lukasiewicz* T-norm representing in this case $x\geq y$ and $y\geq x$ (see [16,13]). It is then obvious that the meaning of the expressions

$$\mu_B(x,y)=\mu(x,y)-\mu_{PI}(x,y)$$
$$\mu_W(x,y)=\mu(y,x)-\mu_{PI}(x,y)$$

is kept. Obviously, this model is based on the assumption that both positive and negative indifference are basically indifferences, so that standardized (complete) preferences can be defined

$$\mu^*(x,y)=\mu(x,y)+\mu_{NI}(x,y)=1-\mu_W(y,x)$$

The μ^* will be called the *completion* of μ.

Moreover, each value

$$\sigma(x,y)=1-\mu_{NI}(x,y)=\min\{\mu(x,y)+\mu(y,x),1\} \qquad (1)$$

can be associated to the degree to which the comparison between the pair of alternatives x,y is being supported.

2.4 Fuzzy rationality measures

Given now a complete (or the completion of a non complete one) fuzzy preference relation μ, we must assign to μ a degree or rationality. In the context of fuzzy binary preference relations, a standard assumption to characterize consistency or rationality is max-min transitivity (see [25] and [10,26]). Under this hypothesis if x is better than y with intensity $\mu(x,y)$ and y is better than z with intensity $\mu(y,z)$ then the degree to which x is better than z, i.e. $\mu(x,z)$, can never be lower than both values $\mu(x,y)$ and $\mu(y,z)$. The intuitive meaning of this property which extends transitivity is clear. As pointed out in [10], the strength of the link between two elements must be greater than or equal to the strength of any chain involving other elements. However, the property of being max-min transitive is crisp, that is, each relation either is or is not max-min transitive. Therefore, assuming max-min transitivity as key property for consistency and therefore rationality, does not allow a fuzzy classification.

General fuzzy rationality measures, that is to say functions ρ which map fuzzy preference relations into the interval $[0,1]$ and which allow a fuzzy classification have been introduced in [4]. Let us recall the axiomatic definition of any fuzzy rationality measure ρ.

(R1) Foundation: *If X is a finite set of alternatives, then $\rho(\mu)=1$ for any crisp binary preference μ being a strict chain on X.*

(R2) Name Invariance: *Given $\mu : X \times X \rightarrow [0,1]$ and a permutation $\pi : X \rightarrow X$, then*

$$\rho(\mu^{\pi}) = \rho(\mu)$$

where $\mu^{\pi}(x,y) = \mu(\pi(x), \pi(y))$ for all $x, y \in X$.

(R3) Symmetry:

$$\rho(\neg\mu) = \rho(\mu)$$

where

$$\neg\mu(x,y) = \mu(y,x).$$

for all $x, y \in X$.

(R4) Principle of persistent degree of rationality *Let Y be a non-empty finite set of alternatives and let x be an extra alternative not belonging to Y. Let us consider a fuzzy preference $\mu : Y \times Y \rightarrow [0,1]$ such that $\mu(y,z) = 1, \mu(z,y) = 0, \forall y \in Y_1, \forall z \in Y_2$ for some Y_1, Y_2 partition of Y, and an extension μ' such that*

$$\mu'(y,z) = \mu(y,z), \forall y,z \in Y$$

$$\mu'(y,x) = 1, \mu'(x,y) = 0, \forall y \in Y_1$$

$$\mu'(x,z) = 1, \mu'(z,x) = 0, \forall z \in Y_2$$

$$\mu'(x,x) = 1$$

Then it must be

$$\rho(\mu') \geq \rho(\mu).$$

The last axiom of fuzzy rationality measures partitions the set of rationality measures into three sets: normal, pessimistic and optimistic.

(R5) Regularity: Given $\mu : X \times X \rightarrow [0,1]$ defined over X, $(\bar{x}, \bar{y})$ arbitrary pair of alternatives, let $P_{\mu}(\bar{x}, \bar{y})$ be the collection of pair of real numbers (a,b) such that

(i) $0 \leq \mu(\bar{x}, \bar{y}) + a \leq 1$

(ii) $0 \leq \mu(\bar{y}, \bar{x}) + b \leq 1$

(iii) $\mu(\bar{x}, \bar{y}) + \mu(\bar{y}, \bar{x}) + a + b \geq 1$

For every pair $(a,b) \in P_{\mu}(\bar{x}, \bar{y})$ we will denote by

$$\Delta_{\mu}((\bar{x}, \bar{y}), (a,b))$$

the fuzzy preference relation defined as follows

$$\Delta_{\mu}((\bar{x}, \bar{y}), (a,b))(x,y) = \begin{cases} \mu(\bar{x}, \bar{y}) + a \text{ if } (x,y) = (\bar{x}, \bar{y}) \\ \mu(\bar{y}, \bar{x}) + b \text{ if } (x,y) = (\bar{y}, \bar{x}) \\ \mu(x,y) \quad \text{otherwise} \end{cases}.$$

Let ρ be a fuzzy rationality measure, then

(1) ρ is normal if

$$\rho(\Delta_{\mu}((\bar{x}, \bar{y}), (a,0))), \quad \rho(\Delta_{\mu}((\bar{x}, \bar{y}), (a,-a))), \quad \rho(\Delta_{\mu}((\bar{x}, \bar{y}), (a,a)))$$

are monotone functions of a.

(2) ρ is pessimistic if there exists a value $a \in [0,1]$ such that

(2.1) $\rho(\Delta_{\mu}((\bar{x}, \bar{y}), (b,0))) \geq \rho(\Delta_{\mu}((\bar{x}, \bar{y}), (c,0)))$

(2.2) $\rho(\Delta_{\mu}((\bar{x}, \bar{y}), (b,-b))) \geq \rho(\Delta_{\mu}((\bar{x}, \bar{y}), (c,-c)))$

(2.3) $\rho(\Delta_{\mu}((\bar{x}, \bar{y}), (b,b))) \geq \rho(\Delta_{\mu}((\bar{x}, \bar{y}), (c,c)))$

for all b, c such that either $a > b > c$ or $a < b < c$.

(3) ρ is optimistic if there exists a value $a \in [0,1]$ such that

(3.1) $\rho(\Delta_{\mu}((\bar{x}, \bar{y}), (b,0))) \leq \rho(\Delta_{\mu}((\bar{x}, \bar{y}), (c,0)))$

(3.2) $\rho(\Delta_{\mu}((\bar{x}, \bar{y}), (b,-b))) \leq \rho(\Delta_{\mu}((\bar{x}, \bar{y}), (c,-c)))$

(3.3) $\rho(\Delta_{\mu}((\bar{x}, \bar{y}), (b,b))) \leq \rho(\Delta_{\mu}((\bar{x}, \bar{y}), (c,c)))$

for all b, c such that either $a > b > c$ or $a < b < c$.

2.5 Some examples

We will now give some examples of fuzzy rationality measures. The proofs that they satisfy the appropriate axioms can be found in [4].

- A fuzzy preference relation μ is *max-min transitive* whenever

$$\mu(x,y) \geq \min\{\mu(x,z), \mu(z,y)\}$$

for all z and for any fixed pair of alternatives x, y.
Let us define

$$\rho_{maxmin}(\mu) = \begin{cases} 1 \text{ if } \mu \text{ is max-min transitive} \\ 0 \text{ otherwise} \end{cases}$$

The above is a pessimistic fuzzy rationality measure.

- Next example is based upon Orlovsky's choice set of unfuzzy nondominated alternatives. Let us define for any arbitrary non-empty subset $\mathbf{Y}$ of alternatives

$$Y^{\mu}_{UND} = \{x \in Y | \mu(x,y) \geq \mu(y,x), \forall y \in Y\}$$

for any given fuzzy preference $\mu : X \times X \rightarrow [0,1]$. Then the following map is an optimistic fuzzy rationality measure:

$$\rho_N(\mu) = \begin{cases} \min\{\frac{1}{|Y^{\mu}_{UND}|} | Y^{\mu}_{UND} \neq \emptyset\} \text{ if } Y^{\mu}_{UND} \neq \emptyset, \forall Y \subseteq X, Y \neq \emptyset \\ 0 \quad \text{otherwise} \end{cases}$$

Note that ρ_N will take value 1 if and only if there is just one unfuzzy nondominated alternative for each non-empty set of alternatives. Moreover, value 0 is reached if and only if there exists some subset with no unfuzzy nondominated alternatives. In between these two cases, the larger the choice sets are, the lower is the degree of rationality. Therefore, this rationality measure captures the fact that usually a unique final decision must be chosen, and the bigger is the choice the more complex will be the procedure required to select an alternative.

- If we just assign value 1 whenever all choice sets Y^{μ}_{UND} are not empty and value 0 otherwise, we will get the following *normal* fuzzy rationality measure (see [15] for a necessary and sufficient condition for the existence of Orlovsky's choice set):

$$\rho_{UND}(\mu) = \begin{cases} 1 \text{ if } Y^{\mu}_{UND} \neq \emptyset, \forall Y \subseteq X, Y \neq \emptyset \\ 0 \text{ otherwise} \end{cases}$$

If we consider the dual set of the set of unfuzzy nondominated alternatives, i.e., the set of unfuzzy dominated alternatives, defined as follows:

$$Y^{\mu}_{UD} = \{x \in Y | \mu(x, y) \leq \mu(y, x), \forall y \in Y\}$$

for any non empty set of alternatives $Y \subseteq X$. Then the following mapping defines another normal fuzzy rationality:

$$\rho_D(\mu) = \begin{cases} 1 \text{ if } X^{\mu}_{UND} \cup X^{\mu}_{UD} \neq \emptyset \\ 0 \text{ otherwise} \end{cases}$$

This last fuzzy rationality measure captures the fact that from a decision point of view, a fuzzy binary preference relation will become a decision aid procedure whenever either a set of alternatives which is equivalent from a decision point of view has been given, or we are able to choose a smaller subset (best or worst alternatives), in such a way that the number of alternatives to be considered in the next step is reduced.

2.6 Some more comments on fuzzy rationality measures

There is a significant computational problem related with fuzzy rationality measures and the feasibility of their use in real life applications. Indeed, some of the rationality measures proposed, though intuitively (and axiomatically) sound, appear to be quite complex from a computational point of view (see [6]).

Finally we must point out the importance of the completeness assumption. Rationality measures, as considered in this paper, do not apply to non complete fuzzy preference relation, although incomparable alternatives indeed appear in practice (see [11–13,16] for a general approach to valued preference binary relations, where weak and strict intensity preferences, together with indifference and incomparability, are simultaneously modeled). Rationality under incomparability (as modeled in [12]) appears then to be a very interesting subject for future research.

3 The axioms of utility theory

First of all, we will introduce a reminder about the axioms of utility theory, following [21].

Let us assume a "rational" agent and a set of basic alternatives or states, $A_1, \ldots, A_n$. Such states can be combined to obtain new ones which are basically lotteries. In details, if we have the states $A_1, A_2, \ldots, A_l$ then for any probability value $p_1, p_2, \ldots, p_l$ such that $\sum_{i=1}^{l} p_i = 1$ we obtain the lottery $\begin{pmatrix} p_1 & \cdots & p_l \\ A_1 & \cdots & A_l \end{pmatrix}$, that is to say, the state where with probability p_i we have A_i. The axioms of utility theory which would follow, will not be concerned with utility but with preferences. Preferences can be modeled as a binary relation $\mu(A_i, A_j) \in \{0, 1\}$ defined over the set of states. In particular,

- if $\mu(A_i, A_j) > \mu(A_j, A_i)$ we say that the agent strictly prefers A_i to A_j, i.e. $A_i \succ A_j$;
- if $\mu(A_i, A_j) = \mu(A_j, A_i)$ we say that the agent is indifferent to the two states, i.e. $A_i \sim A_j$; (and such an indifference could be either "positive" or "negative");
- if $\mu(A_i, A_j) \geq \mu(A_j, A_i)$ we say that the agent does not prefer A_j over A_i, i.e. $A_i \succeq A_j$. Such a preference is denoted as "weak" preference.

The agent rationality can be characterized, for example, with the absence of (either strict or weak) preference cycles (see [20]). If the binary relation is complete (i.e. $\mu(A_i, A_j) = 1$ or $\mu(A_j, A_i) = 1$ holds for every pair $A_i, A_j \in X$) weak *acyclity* condition is equivalent to classical crisp transitivity of weak binary relations. Classical crisp binary order preference relations, that is, those relations verifying reflexivity and transitivity, represent ideal examples of consistent crisp preference relations. An alternative weaker proposal of consistency for crisp binary relations based on the idea of quasi-transitivity has been given in [22], by assuming transitivity just for the associated strict preferences. This alternative assumption can be justified from a theoretical point of view because it is well known that transitivity does not in practice hold for indifferences (see also [20]).

The above considerations are formally translated into the first two axioms below. The remaining four axioms are, instead, related to the way the preference relation evolves when we move onto lotteries built from the given states.

(A1) Orderability: given any two states A_i, A_j it must be true

$$(A_i \succ A_j) \vee (A_i \sim A_j) \vee (A_i \prec A_j)$$

(A2) Transitivity: given any three states A_i, A_j, A_h it must be true

$$(A_i \succ A_j) \wedge (A_j \succ A_h) \rightarrow (A_i \succ A_h)$$

(A3) Continuity: this axiom expresses the fact that if $(A_i \succ A_j \succ A_h)$ then there exists a probability value p such that the agent is indifferent between A_j and the lottery where that yields A_i with probability p and A_h with probability $1-p$, that is to say

$$A_i \succ A_j \succ A_h \rightarrow \exists p | \begin{pmatrix} p & 1-p \\ A_i & A_h \end{pmatrix} \sim A_j$$

(A4) Substitutability:

$$A_i \sim A_j \rightarrow \begin{pmatrix} p & 1-p \\ A_i & A_h \end{pmatrix} \sim \begin{pmatrix} p & 1-p \\ A_j & A_h \end{pmatrix}$$

(A5) Monotonicity:

$$A_i \succ A_j \rightarrow \{p \geq q \leftrightarrow \begin{pmatrix} p & 1-p \\ A_i & A_j \end{pmatrix} \succeq \begin{pmatrix} q & 1-q \\ A_i & A_j \end{pmatrix}\}$$

(A6) Decomposability:

$$\begin{pmatrix} p & 1-p \\ A_i & \begin{pmatrix} q & 1-q \\ A_j & A_h \end{pmatrix} \end{pmatrix} \sim \begin{pmatrix} p & (1-p)q & (1-p)(1-q) \\ A_i & A_j & A_h \end{pmatrix}$$

3.1 The utility principle

Given the above axioms, there exists a real valued function U which verifies

$$U(A_i) > U(A_j) \leftrightarrow A_i \succ A_j$$

$$U(A_i) = U(A_j) \leftrightarrow A_i \sim A_j$$

Such a function, defined over the set of states and lotteries, can be supposed to be normalized, that is to say, to have values between 0 (minimal utility) and 1 (maximal utility). A possible way to define a utility function is the following:

- let A be a state of maximal utility and A' be a state of minimal utility;
- define $U(A) = 1$ and $U(A') = 0$;
- for any other state A_i, in view of axiom (A3), there exists a probability value p such that

$$A_i \sim \begin{pmatrix} p & 1-p \\ A & A' \end{pmatrix}$$

Put then $U(A_i) = p$.

4 The axioms of fuzzy utility theory

Let $A_1, \ldots, A_n$ be a set of basic alternatives or states. Since our intelligent agent is supposed to be rational, we suppose that its preference relation μ defined over the set of basic alternatives must verify $\rho(\mu) > 0$, for some fuzzy rationality measure ρ. Such a property is obviously the extension to the fuzzy case of the axioms (A1) and (A2) of utility theory, above described.

(FA1) Rationality:

$$\rho(\mu) > 0$$

4.1 Extending the set of alternatives

Next question we need to address is the following: how do we extend μ when a lottery $\begin{pmatrix} p & 1-p \\ A_i & A_j \end{pmatrix}$ is added to the set of alternatives, in such a way that the rationality value does not decrease ? Let A_h be a basic state, then

$$\mu(A_h, \begin{pmatrix} p & 1-p \\ A_i & A_j \end{pmatrix})$$

must clearly be equal to

- $\mu(A_h, A_i)$ if $p = 1$
- $\mu(A_h, A_j)$ if $p = 0$

In general, we will assume

(FA2) Extension: for any probability value p, and basic alternatives A_i, A_j, and A_h

$$\min(\mu(A_h, A_i), \mu(A_h, A_j)) \leq \mu(A_h, \begin{pmatrix} p & 1-p \\ A_i & A_j \end{pmatrix})$$

and

$$\mu(A_h, \begin{pmatrix} p & 1-p \\ A_i & A_j \end{pmatrix}) \leq \max(\mu(A_h, A_i), \mu(A_h, A_j))$$

Analogously,

$$\min(\mu(A_i, A_h), \mu(A_j, A_h)) \leq \mu(\begin{pmatrix} p & 1-p \\ A_i & A_j \end{pmatrix}, A_h)$$

and

$$\mu(\begin{pmatrix} p & 1-p \\ A_i & A_j \end{pmatrix}, A_h) \leq \max(\mu(A_i, A_h), \mu(A_j, A_h))$$

In particular, we have

- for $\mu(A_h, A_i) = 1$ and for any p,

$$\mu(A_h, A_j) \le \mu(A_h, \begin{pmatrix} p & 1-p \\ A_i & A_j \end{pmatrix}) \le 1$$

- for $\mu(A_i, A_h) = 1$ and for any p,

$$\mu(A_j, A_h) \le \mu(\begin{pmatrix} p & 1-p \\ A_i & A_j \end{pmatrix}, A_h) \le 1$$

As a consequence, if $\mu(A_h, A_i) = \mu(A_i, A_h) = 1$

$$\mu_f(A_h, A_j) \le \mu_f(A_h, \begin{pmatrix} p & 1-p \\ A_i & A_j \end{pmatrix}).$$

Axiom (FA2) is justified by the axiom of persistent rationality. Indeed, if we can partition the set of basic alternatives into Y_1 and Y_2 such that

$$\mu(A, A') = 1, \mu(A', A) = 0, \forall A \in Y_1, \forall A' \in Y_2$$

the above assumption will guarantee us that the rationality of the extended preference relation will not decrease.

In view of axiom (FA2) we can propose an analytical definition of the fuzzy preference relation, to be axiomatically justified.

Definition (F-def). For any probability value p, and alternatives A_i, A_j, and A_h

$$\mu(A_h, \begin{pmatrix} p & 1-p \\ A_i & A_j \end{pmatrix}) = p\mu(A_h, A_i) + (1-p)\mu(A_h, A_j)$$

and

$$\mu(\begin{pmatrix} p & 1-p \\ A_i & A_j \end{pmatrix}, A_h) = p\mu(A_i, A_h) + (1-p)\mu(A_j, A_h).$$

Moreover, given any $k \ge 2$, $X_1, \ldots, X_k$ alternatives and $p_1, \ldots, p_k$ such that $\sum_{i=1}^{k} p_i = 1$ we extend μ as follows

- let $B = \begin{pmatrix} p_1 & \cdots & p_k \\ X_1 & \cdots & X_k \end{pmatrix}$,
- given any alternative A and probability value q let $C = \begin{pmatrix} q & (1-q)p_1 & \cdots & (1-q)p_k \\ A & X_1 & \cdots & X_k \end{pmatrix}$

then

$$\mu(\begin{pmatrix} q & 1-q \\ A & B \end{pmatrix}, C) = \mu(C, \begin{pmatrix} q & 1-q \\ A & B \end{pmatrix}) = 1.$$

From (F-def) it follows that

$$\mu_f(A_h, \begin{pmatrix} p & 1-p \\ A_i & A_j \end{pmatrix}) = p\mu_f(A_i, A_h) + (1-p)\mu_f(A_j, A_h).$$

It also follows that the decomposability axiom is properly extended.

Notice that in case μ is crisp, and $A_i \succ A_h \succ A_j$, then we may suppose that $\mu(A_h, \begin{pmatrix} p & 1-p \\ A_i & A_j \end{pmatrix}) = 1$ if $p \le 1/2$ and 0 otherwise. Analogously, we may suppose that $\mu(\begin{pmatrix} p & 1-p \\ A_i & A_j \end{pmatrix}, A_h) = 1$ if $p \ge 1/2$ and 0 otherwise. That is to say, we put $1/2$ as a threshold and we increase μ to 1 if it is at least $1/2$, whereas we decrease to 0 if it less than $1/2$. In this case, for $p = 1/2$, we obtain $A_h \sim \begin{pmatrix} p & 1-p \\ A_i & A_j \end{pmatrix}$, which, without differences in intensity of preferences, seems to be the only appropriate understanding of axiom (A3) of continuity. We can conclude then that axiom (FA2) as extended by (F-def) is also a fuzzy extension of axiom (A3).

4.2 The problem of indifference

Let now $X = \begin{pmatrix} p & 1-p \\ A_i & A_h \end{pmatrix}$ and $Y = \begin{pmatrix} p & 1-p \\ A_j & A_h \end{pmatrix}$. If $\mu_f(A_i, A_j) = 1$ can we automatically claim that $\mu_f(X, Y) = 1$? The answer is no! Indeed, we must take into account the preference values which relate A_i and A_j to A_h. Following (F-def) we have

- for $p = 1$, $\mu_f(X, Y) = \mu_f(A_i, A_j)$
- for $p = 0$, $\mu_f(X, Y) = 1$

In general, we have

$$\mu_f(X, Y) = p^2\mu_f(A_i, A_j) + p(1-p)(\mu_f(A_i, A_h) + \mu_f(A_j, A_h)) + (1-p)^2$$

4.3 Monotonicity

The monotonicity condition is implied by (F-def). Specifically, suppose that $\mu(A_i, A_j) > \mu(A_j, A_i)$ and let $X = \begin{pmatrix} p & 1-p \\ A_i & A_j \end{pmatrix}$ and $Y = \begin{pmatrix} q & 1-q \\ A_i & A_j \end{pmatrix}$ then for all probability values p, q we have

$$\mu(X, Y) = pq + (1-p)q\mu(A_j, A_i) + p(1-q)\mu(A_i, A_j) + (1-p)(1-q)$$

and

$$\mu(Y, X) = pq + (1-q)p\mu(A_j, A_i) + q(1-p)\mu(A_i, A_j) + (1-p)(1-q)$$

Thus

$$\mu(X, Y) - \mu(Y, X) = \alpha(\mu(A_i, A_j) - \mu(A_j, A_i))$$

where $\alpha = p(1-q) - (1-p)q$ and it is clear that $\alpha \ge 0$ if and only if $p \ge q$.

4.4 How do we define a utility function?

We use the 5-th and last axiom of fuzzy rationality measures

We have the following

Theorem 1 (Boosting Theorem). *If ρ is either a normal or an optimistic fuzzy rationality measure, by using the regularity axiom we can obtain from μ a fuzzy preference relation μ^* such that*

- $\rho(\mu^*) \geq \rho(\mu)$
- *for every pair of alternatives (x,y) such that $\mu(x,y) \leq \mu(y,x)$ we have*

$$\mu^*(x,y) \leq \mu^*(y,x)$$

- *for every pair of alternatives (x,y) either*
  - $\mu^*(x,y) = \mu^*(y,x)$, *or*
  - $\max(\mu^*(x,y), \mu^*(y,x)) = 1$, *or*
  - $\min(\mu^*(x,y), \mu^*(y,x)) = 0$.

Proof. Let $(\bar{x},\bar{y})$ be an arbitrary pair of alternatives. Suppose, w.l.o.g. that $\mu(\bar{x},\bar{y}) \geq \mu(\bar{y},\bar{x})$.

Suppose first that ρ is normal. Since from the regularity condition we have that $\rho(\Delta_\mu((\bar{x},\bar{y}),(a,-a)))$ is a monotone function of a then

- if for $a > 0$ we have that $\rho(\Delta_\mu((\bar{x},\bar{y}),(a,-a)))$ does not decrease we can reach either the value $\Delta_\mu((\bar{x},\bar{y}),(a,-a))(\bar{x},\bar{y}) = 1$ or the value $\Delta_\mu((\bar{x},\bar{y}),(a,-a))(\bar{y},\bar{x}) = 0$.
- if instead for $a > 0$ we have that $\rho(\Delta_\mu((\bar{x},\bar{y}),(a,-a)))$ decreases then by choosing $a = -(\mu(\bar{x},\bar{y}) - \mu(\bar{y},\bar{x}))/2 < 0$ we have that
  - $\Delta_\mu((\bar{x},\bar{y}),(a,-a))(\bar{x},\bar{y}) = \Delta_\mu((\bar{x},\bar{y}),(a,-a))(\bar{y},\bar{x})$.
  - $\rho(\Delta_\mu((\bar{x},\bar{y}),(a,-a)))$ increased.

In both cases the theorem is proven when ρ is normal.

Suppose now that ρ is optimistic. We know that there exists a value $a \in [0,1]$ such that

$$\rho(\Delta_\mu((\bar{x},\bar{y}),(b,0))) \leq \rho(\Delta_\mu((\bar{x},\bar{y}),(c,0)))$$

for all b, c such that either $a > b > c$ or $a < b < c$.

- If $\mu(\bar{x},\bar{y}) + a < 1$ and $\mu(\bar{y},\bar{x}) - a > 0$ then $\rho(\Delta_\mu((\bar{x},\bar{y}),(b,-b)))$ is a non decreasing function of b for $b > a$ and so we can reach either the value $\Delta_\mu((\bar{x},\bar{y}),(b,-b))(\bar{x},\bar{y}) = 1$ or the value $\Delta_\mu((\bar{x},\bar{y}),(b,-b))(\bar{y},\bar{x}) = 0$.
- if $a = 1 - \mu(\bar{x},\bar{y})$ then $\rho(\Delta_\mu((\bar{x},\bar{y}),(b,-b)))$ increases when b decreases from $1 - \mu(\bar{x},\bar{y})$ to $\mu(\bar{y},\bar{x}) - \mu(\bar{x},\bar{y})$. So we reach a situation where $\Delta_\mu((\bar{x},\bar{y}),(b,0))(\bar{x},\bar{y}) = \mu(\bar{y},\bar{x})$.
- Analogously if $\mu(\bar{y},\bar{x}) - a = 0$

Remark 1. The proof of the theorem is quite easy although technically tedious. It can be done locally by taking two alternatives at a time x, y and either decreasing (alt. increasing) $\mu(x,y)$ and/or increasing (alt. decreasing) $\mu(y,x)$. For pessimistic f.r.m. our intuition suggests that it is possible to take two alternatives, increase on one hand and then readjust things all over so to have a higher degree of rationality. It can be easily seen by considering some examples with max-min transitivity which is a pessimistic fuzzy rationality measure. Therefore, it can be also somehow generalized to the pessimistic case, although we have no formal and definite proof yet.

We can now pick the best and worst alternatives by respectively maximizing and minimizing the function

$$u(x) = \sum_{y \neq x} \mu_B(x,y) - \sum_{y \neq x} \mu_W(x,y)$$

Whenever, the boosting theorem allows us to partition X in several subsets $X_1, \ldots, X_k$ such that for every $x \in X_i$ and $y \in X_j$ if $i < j$ then either $\mu(x,y) = 1$ and $\mu(y,x) < 1$, or $\mu(x,y) > 0$ and $\mu(y,x) = 0$, the above choices are in some sense the result of a *linearization* of the alternatives. Note also that the above definition is consistent with the famous Debreu's theorem ([9]) that for mutual preferential independent preference values (i.e. for each pair (x,y) the value $\mu(x,y)$ does not depend on the other alternatives) the utility function can be seen as a linear combination of utility values dependent only on the single alternative.

Finally, once we find the alternatives of maximal and minimal utility we can act as in the crisp case.

5 A simple example

Consider the following fuzzy preference values over a set of four alternatives.

1	0.2	0.8	0.6
0.9	1	0.5	0.4
0.5	0.6	1	0.3
0.6	0.7	0.8	1

Suppose the entry (i,j) of the array gives us the value $\mu(x_i, x_j)$. It can be easily seen that $\rho_N(\mu) = 0$ since $Y^\mu_{UND} = \emptyset$ for $Y = \{x_1, x_2, x_3\}$.

We now try to boost the preference values hoping to linearize and improving the rationality.

For instance, if we put $\mu(x_2, x_3) = 0.6$ then $\{x_1, x_2, x_3\}^\mu_{UND} = \{x_2\}$ and $\rho_N(\mu) = 1/2$

Now, we remember that

$$\mu_I(x,y) = \mu(x,y) + \mu(y,x) - 1$$
$$\mu_B(x,y) = \mu(x,y) - \mu_I(x,y)$$
$$\mu_W(x,y) = \mu(y,x) - \mu_I(x,y)$$

and we have the following table for the values (μ_I, μ_S, μ_W)

	0.1, 0.1, 0.8	0.3, 0.5, 0.2	0.2, 0.4, 0.4
0.1, 0.8, 0.1		0.2, 0.4, 0.4	0.1, 0.3, 0.6
0.3, 0.2, 0.5	0.2, 0.4, 0.4		0.1, 0.2, 0.7
0.2, 0.4, 0.4	0.1, 0.6, 0.3	0.1, 0.7, 0.2	

It follows that

$$u(x_1) = 1 - 1.4 = -0.4$$
$$u(x_2) = 1.5 - 1.1 = 0.4$$
$$u(x_3) = 0.8 - 1.6 = -0.8$$
$$u(x_4) = 1.7 - 0.9 = 0.8$$

Therefore x_4 is the alternative of maximal utility whereas x_3 is the alternative of minimal utility.

6 Final comments

We described a possible fuzzy extension of the classical axioms of utility theory. Our goal is to provide a framework for decision making under uncertainty and approximate knowledge to be a part of any "intelligent system" inferential core. We would like to stress the fact that our formalization is certainly not the only possible one (for another formalization see for instance [17]). The novelty of this approach rests in the use of fuzzy rationality measures to formalize "reasonable orderings" of alternatives or lotteries and in the mathematical possibility to partially readjust the fuzzy preferences so to make the decisional problem easier. Many questions about the proposed formalization remain unanswered and seem to us to be good topics for future investigations. Let us mention two which are both related to the boosting theorem.

1. Clarify what happens when we are dealing with pessimistic fuzzy rationality measures.
2. Even when dealing with normal or optimistic fuzzy rationality measures, the proposed theorem is not an algorithm. How do we algorithmically proceed and when do we stop ?

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Hybrid Probabilistic-Possibilistic Mixtures and Utility Functions *

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Abstract. A basic building block in the standard mathematics of decision under uncertainty is the notion of probabilistic mixture. In order to generalize decision theory to non probabilistic uncertainty, one approach is to generalize mixture sets. In the recent past it has been proved that generalized mixtures can be non trivially defined, and they have been instrumental in the development of possibilistic utility theory. This paper characterizes the families of operations involved in generalized mixtures, due to a previous result on the characterization of the pairs of continuous t-norm and t-conorm such that the former is conditionally distributive over the latter. What is obtained is a family of mixtures that combine probabilistic and possibilistic mixtures via a threshold. It is based on a restricted family of t-conorm/t-norm pairs which are very special ordinal sums. Any practically useful theory of pseudo-additive measures must use such special pairs of operations in order to extend the additivity property, and the notion of probabilistic independence.

1 Introduction

Utility theory is based on the notion of mathematical expectation. Its axiomatic foundations, following von Neumann and Morgenstern [19] rely on the notion of probabilistic mixtures, see [15]. It has been recently shown by Dubois et al. in [6] that the notion of mixtures can be extended to pseudo-additive measures (sometimes called decomposable measures, see [9,16,20]), including the case of possibility theory (see [12]). Possibilistic mixtures strikingly differ from probabilistic measures, because they account for a non-convex structure. However, changing sum into maximum, and the product into a triangular norm, possibilistic mixtures have properties that parallel those of probabilistic mixtures. In particular, possibilistic mixtures form the underpinnings of possibilistic utility theory, as proposed by Dubois and Prade in [11] and more recently systematized by Dubois, Godo et al. in [7].

The aim of this paper is to address the following question: what else remains possible beyond possibilistic and probabilistic mixtures? This question is addressed, from a mathematical point of view, by taking advantage of a

* The paper was written during the stay of the second author as visiting professor at University Paul Sabatier, in Toulouse, in June, 1999.

Studies in Fuzziness and Soft Computing

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ISSN 1434-9922
ISBN 3-7908-1303-6 Physica-Verlag Heidelberg New York

Cataloging-in-Publication Data applied for
Die Deutsche Bibliothek – CIP-Einheitsaufnahme
Fodor, János; De Baets, Bernard; Perny, Patrice: Preferences and decisions under incomplete knowledge: with 15 tables / János Fodor; Bernard De Baets; Patrice Perny (eds.). – Heidelberg; New York: Physica-Verl., 2000
(Studies in fuzziness and soft computing; Vol. 51)
ISBN 3-7908-1303-6

Physica-Verlag is a company in the BertelsmannSpringer publishing group.

Printed in Germany

Hardcover Design: Erich Kirchner, Heidelberg
SPIN 10767311 88/2202-5 4 3 2 1 0 – Printed on acid-free paper

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