UNCONDITIONAL BASES IN TENSOR PRODUCTS OF HILBERT SPACES

DAVID PÉREZ-GARCÍA AND IGNACIO VILLANUEVA

ABSTRACT. We prove that a tensor norm α (defined on tensor products of Hilbert spaces) is the Hilbert-Schmidt norm if and only if $\ell_2 \otimes \cdots \otimes \ell_2$, endowed with the norm α, has an unconditional basis. This extends a classical result of Kwapień and Pełczyński. The symmetric version of that statement follows, and this extends a recent result of Defant, Díaz, García and Maestre.

1. INTRODUCTION

In [16], Kwapień and Pełczyński proved that if we have a tensor norm α defined on tensor products of Hilbert spaces such that $\ell_2 \hat{\otimes}_\alpha \ell_2$ has an unconditional basis, then α has to be (equivalent to) the Hilbert-Schmidt norm. Their technique involves the Schmidt decomposition of a compact bilinear form from Hilbert spaces. The problem is that, up to now, there is no reasonable analogue of this decomposition for trilinear forms (the only version is the *normal form* given in [2], which is too complicated to be useful here). This is the reason we need new techniques to extend this result to more than two spaces. Our main result is the following (Theorem 2.5)

Theorem. *If α is a tensor norm (defined on tensor products of Hilbert spaces) such that $\hat{\otimes}^n_{\alpha,j=1}\ell_2$ has an unconditional basis, then α is equivalent to the Hilbert-Schmidt norm σ_2.*

The study of Hilbert-Schmidt operators (or bilinear forms) goes back to the work of Hilbert and Schmidt about integral equations at the beginning of the twentieth century. Since then, they have been constantly applied both in pure and applied mathematics. The generalization of this class to the multilinear setting goes back to Dwyer III [12] and was recovered independently by Pietsch [23] and Janson, Peetre and Rochberg [15], where it is shown how this class can be applied to the study of Hankel forms. Motivated by this application Cobos, Kühn and Peetre continued the study in a series of papers [2], [3], [4], [5], where they give a first solution to the study of the Schmidt decomposition of a multilinear form (the *normal* form). Very recently, Matos [17] gave a new step, relating the Hilbert-Schmidt multilinear operators to the class of multiple summing operators.

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This work was finished in [19] where it is proved that these two classes are exactly the same, giving a multilinear extension of the main theorem of [18].

In the context of the theory of polynomials in Banach spaces, the problem of finding unconditional bases in symmetric tensor products has attracted recently a lot of attention (see for instance [7] or [10] and the references therein). The symmetric case of Kwapień and Pełczyński's result appeared in [6], only for the case of two spaces. In Theorem 2.6 we extend it to the general case.

Here we will take advantage of the results in [19]. Particularly, the key result in our proof of the above Theorem will be the following result, proved in [19] (see below for the definitions):

Theorem 1.1. *Let $T : \ell_2 \times \cdots \times \ell_2 \longrightarrow \mathbb{K}$ be a multilinear form. If T is multiple 1-summing, then T is Hilbert-Schmidt and*

$$\|T\|_{S_2} \leq 2^{\frac{n}{2}} \pi_1(T).$$

The theory of multiple summing multilinear operators has been recently developed by Bombal and both authors in [1], [19], [20], [21], [22], and by Matos in [17], where it is shown how this class properly generalizes the linear behavior of p-summing operators. This paper represents then another application of this theory to the study of the structure of tensor products (other applications can be found in the above references).

We remind the reader that, for a finite sequence $(x_i)_{i=1}^m \subset X$ and $1 \leq p < \infty$, we will write $\|(x_i)_{i=1}^m\|_p^\omega$ to denote

$$\sup\left\{\left(\sum_{i=1}^m |x^*(x_i)|^p\right)^{\frac{1}{p}} : x^* \in B_{X^*}\right\}.$$

Definition 1.2. Let $1 \leq p < +\infty$. A multilinear operator $T : X_1 \times \cdots \times X_n \longrightarrow Y$ is multiple p-summing if there exists a constant $K > 0$ such that, for every choice of sequences $(x_{i_j}^j)_{i_j=1}^{m_j} \subset X_j$, the following relation holds

$$\left(\sum_{i_1,\ldots,i_n=1}^{m_1,\ldots,m_n} \|T(x_{i_1}^1, \ldots, x_{i_n}^n)\|^p\right)^{\frac{1}{p}} \leq K \prod_{j=1}^n \|(x_{i_j}^j)_{i_j=1}^{m_j}\|_p^\omega. \tag{1}$$

In that case, we define the *multiple p-summing norm* of T by $\pi_p(T) = \min\{K : K \text{ verifies } (1)\}$

We will use the following straightforward characterization of multiple 1-summing operators

Lemma 1.3. *A multilinear operator $T : X_1 \times \cdots \times X_n \longrightarrow Y$ is multiple 1-summing if and only if there exists a constant K such that for every $1 \leq j \leq n$, for every $m_j \in \mathbb{N}$ and for every $u_j : \ell_\infty^{m_j} \longrightarrow X_j$ with $\|u_j\| \leq 1$, we have that*

$$\sum_{i_1,\ldots,i_m=1}^{m_1,\ldots,m_n} |T(u_1(e_{i_1}), \ldots, u_n(e_{i_n}))| \leq K$$

where $(e_{i_j})_{i_j=1}^{m_j}$ is the canonical basis in $\ell_\infty^{m_j}$. In that case we have that $\pi_1(T)$ is just the infimum of such constants K.

The notation will be the standard in the context and [9], [11] will be our basic references for unexplained notation.

All along this paper all the operators are supposed to be continuous. Given X, Y Banach spaces, $\mathcal{L}(X,Y)$ will denote the Banach space of linear (and continuous) operators. B_X will be the closed unit ball of the Banach space X and $\mathbb{K}$ will denote the scalar field, either $\mathbb{R}$ or $\mathbb{C}$. If $\lambda \in \mathbb{K}^k$, we write $d_\lambda : \mathbb{K}^k \longrightarrow \mathbb{K}^k$ for the diagonal operator $d_\lambda(x) = (\lambda_i x_i)_{i=1}^k$.

Let X be a Banach space and let I be either $\mathbb{N}$ or $\{1, \dots, m\}$. A sequence $(x_i)_{i\in I}$ of non-zero vectors is an unconditional basic sequence in X if there is a constant K such that

$$\left\| \sum_{i\in I} \epsilon_i \mu_i x_i \right\| \leq K \left\| \sum_{i\in I} \mu_i x_i \right\|$$

for every $\epsilon_i, \mu_i \in \mathbb{K}$ with $|\epsilon_i| \leq 1$. The best of such constants K is called the unconditional constant of $(x_i)_{i\in I}$. If a Banach space X has an unconditional basis, we define $\mathrm{ub}(X)$ as the infimum of the unconditional constants of all the unconditional basis of X.

In this paper, we will deal with tensor norms of order n defined on tensor products of Hilbert spaces. We will call them simply *tensor norms;* that is, for us, a tensor norm α will be a method of ascribing to each choice $H_1, H_2, \dots, H_n$ of n Hilbert spaces, a norm (that we will also call α) in the tensor product $H_1 \otimes \cdots \otimes H_n$ such that

i) $\epsilon \leq \alpha \leq \pi$, where ϵ and π are, respectively, the injective and projective norms.
ii) For every $u_j \in \mathcal{L}(H_j, K_j)$ $(1 \leq j \leq n)$, where H_j, K_j are Hilbert spaces for $1 \leq j \leq n$, we have that $u_1 \otimes \cdots \otimes u_n$ is $\alpha - \alpha$ continuous with norm less or equal than $\prod_{j=1}^n \|u_j\|$.

We will write $\otimes_{\alpha,j=1}^n H_j$ for the tensor product endowed with the norm α, and $\hat{\otimes}_{\alpha,j=1}^n H_j$ for its completion.

We do not know of any general reference for tensor norms of order n, though one can find the definition and some properties in [14]. However, everything we are going to use is a straightforward generalization of the case $n = 2$, for which we refer to [9] (specially to [9, Chapter 26], where the case of tensor norms defined on Hilbert spaces is treated). If α is a tensor norm, α' will be its dual tensor norm.

We will need the following result that appeared in [7, Remark 1] (see [24] and [25] for the case of two spaces). We write it in the case we are going to use.

Theorem 1.4. *If α is a tensor norm, we have that $\hat{\otimes}_{\alpha,j=1}^n \ell_2$ has an unconditional basis if and only if $f_{i_1}^1 \otimes \cdots \otimes f_{i_n}^n$ is an unconditional basis in $\hat{\otimes}_{\alpha,j=1}^n \ell_2$ for any choice of orthonormal basis $(f_{i_j}^j)_{i_j=1}^\infty \subset \ell_2$. In fact, in that*

case, the unconditional constant of $f_{i_1}^1 \otimes \cdots \otimes f_{i_n}^n$ *is bounded by*

$$2^{n+1}\,\mathrm{ub}(\hat{\otimes}_{\alpha,j=1}^{n}\ell_2).$$

Finally, we recall the definition of the Hilbert-Schmidt norm σ_2.

Definition 1.5. A multilinear form $T : H_1 \times \cdots \times H_n \longrightarrow \mathbb{K}$ defined on Hilbert spaces is Hilbert-Schmidt if

$$\left(\sum_{i_1\in I_1,\ldots,i_n\in I_n} \left|T(e_{i_1}^1,\ldots,e_{i_n}^n)\right|^2\right)^{\frac{1}{2}} < \infty, \tag{2}$$

where $(e_{i_j}^j)_{i_j\in I_j} \subset H_j$ is an orthonormal basis $(1 \leq j \leq n)$. It is easy to see that the expression (2) does not depend on the choice of the orthonormal basis [17, Proposición 5.1]. In that case, (2) is the Hilbert-Schmidt norm of T and we will denote it by $\|T\|_{S_2}$. The class of Hilbert-Schmidt forms is denoted by $S_2(H_1,\ldots,H_n)$.

Now (see [17]) one can see the Hilbert-Schmidt norm σ_2 as the tensor norm that verifies

$$\left(\hat{\otimes}_{\sigma_2,j=1}^{n}H_j\right)^* = S_2(H_1,\ldots,H_n).$$

2. The Result

We need first some lemmas.

Lemma 2.1. *If* $k \leq m$*, each* u *in the unit ball of* $\mathcal{L}(\ell_2^k,\ell_2^m)$ *can be written as a convex combination of isometries. As a consequence, if* $k \in \mathbb{N}$*, each* u *in the unit ball of* $\mathcal{L}(\ell_2^k,\ell_2)$ *can also be written as a convex combination of isometries.*

Proof. By Krein-Milman's Theorem, it is enough to show that every extreme point of the unit ball of $\mathcal{L}(\ell_2^k,\ell_2^m)$ is an isometry.

Let $u : \ell_2^k \longrightarrow \ell_2^m$ be with $\|u\| = 1$. We consider its Schmidt decomposition [11, Theorem 4.1]

$$u(\cdot) = \sum_{i=1}^{k} \lambda_i(\cdot|e_i)f_i,$$

where $(e_i)_{i=1}^k \subset \ell_2^k, (f_i)_{i=1}^k \subset \ell_2^m$ are orthonormal sequences and

$$\|u\| = \lambda_1 \geq \cdots \geq \lambda_k.$$

If u is not an isometry we have that $\lambda_k < 1$ and then u can be written as a convex combination of

$$\sum_{i=1}^{k-1} \lambda_i(\cdot|e_i)f_i + (\cdot|e_k)f_k \quad \text{and} \quad \sum_{i=1}^{k-1} \lambda_i(\cdot|e_i)f_i - (\cdot|e_k)f_k,$$

which are also in the unit ball of $\mathcal{L}(\ell_2^k,\ell_2^m)$. □

Lemma 2.2. *If $u : \ell_\infty^k \longrightarrow \ell_2$ verifies $\|u\| \leq 1$, then we can find an element $\lambda \in B_{\ell_2^k}$, and a convex combination of isometries $v_r : \ell_2^k \longrightarrow \ell_2$, say $\sum_{r=1}^R \mu_r v_r$, such that*

$$u = \pi_2(u) \sum_{r=1}^R \mu_r (v_r \circ d_\lambda),$$

where $\pi_2(u)$ is the 2-summing norm of u.

Proof. By Pietsch' Factorization Theorem, there exists a regular probability measure ν on $\{1, \ldots, k\}$ and an operator $v : L_2(\nu) \longrightarrow \ell_2$, with $\|v\| = \pi_2(u)$, such that the following diagram commute

$$\begin{array}{ccc} \ell_\infty^k & \xrightarrow{u} & \ell_2 \\ {\scriptstyle \mathrm{id}}\downarrow & \nearrow_{v} & \\ L_2(\nu) & & \end{array}$$

Now, the operator $w : L_2(\nu) \longrightarrow \ell_2^k$ given by $w(x) = \left(x_i \sqrt{\nu(i)}\right)_{i=1}^k$ is an isometry and so we can extend v to ℓ_2^k with the same norm (we still denote the extension by v). Then, if we call $\lambda = \left(\sqrt{\nu(i)}\right)_{i=1}^k \in B_{\ell_2^k}$, we obtain the following diagram

$$\begin{array}{ccc} \ell_\infty^k & \xrightarrow{u} & \ell_2 \\ {\scriptstyle d_\lambda}\downarrow & \nearrow_{v} & \\ \ell_2^k & & \end{array}$$

Finally, if we apply Lemma 2.1, we obtain that $\frac{v}{\pi_2(u)}$ can be written as a convex combination of isometries $\frac{v}{\pi_2(u)} = \sum_{r=1}^R \mu_r v_r$ and we are done. □

Lemma 2.3. *If $T : \ell_2 \times \overset{n}{\cdots} \times \ell_2 \longrightarrow \mathbb{K}$ is a multilinear form, we have that*

$$\pi_1(T) \leq K_G^n \sup \left\{ \sum_{i_1, \ldots, i_n = 1}^{m_1, \ldots, m_n} \left| \lambda_{i_1}^1 \cdots \lambda_{i_n}^n T\left(f_{i_1}^1, \ldots, f_{i_n}^n\right) \right| \right\} \tag{3}$$

where K_G is Grothendieck's constant and the supremum is taken among all the orthonormal sequences $(f_{i_j}^j)_{i_j=1}^{m_j}$ of ℓ_2 and all the elements $\lambda^j \in B_{\ell_2^{m_j}}$.

Proof. Following Lemma 1.3 we consider an arbitrary $\epsilon > 0$ and operators $u_j : \ell_\infty^{m_j} \longrightarrow \ell_2$ with $\|u\| \leq 1$ such that

$$\pi_1(T) - \epsilon \leq \sum_{i_1, \ldots, i_n = 1}^{m_1, \ldots, m_n} \left| T\left(u_1(e_{i_1}), \ldots, u_n(e_{i_n})\right) \right|$$

where $\left(e_{i_j}\right)$ denotes the canonical basis of $\ell_\infty^{m_j}$. Now we use Lemma 2.2 to obtain, for each $1 \leq j \leq n$, an element $\lambda^j \in B_{\ell_2^{m_j}}$ and a convex combination of isometries $\sum_{r_j=1}^{R_j} \mu_{r_j}^j v_{r_j}^j$ such that

$$u_j = \pi_2(u_j) \sum_{r_j=1}^{R_j} \mu_{r_j}^j (v_{r_j}^j \circ d_{\lambda^j}).$$

By Grothendieck's Theorem, $\pi_2(u_j) \leq K_G$ for every j and therefore,

$$\pi_1(T) - \epsilon \leq \sum_{r_1,\ldots,r_n=1}^{R_1,\ldots,R_n} \mu_{r_1}^1 \cdots \mu_{r_n}^n K_G^n \sum_{i_1,\ldots,i_n=1}^{m_1,\ldots,m_n} \left|\lambda_{i_1}^1 \cdots \lambda_{i_n}^n T\left(v_{r_1}^1(e_{i_1}), \ldots, v_{r_n}^n(e_{i_n})\right)\right|.$$

Now, for each $1 \leq j \leq n$ and each r_j, we have that $\left(v_{r_j}^j(e_{i_j}^j)\right)_{i_j=1}^{m_j}$ is an orthonormal sequence in ℓ_2. Then

$$K_G^n \sum_{i_1,\ldots,i_n=1}^{m_1,\ldots,m_n} \left|\lambda_{i_1}^1 \cdots \lambda_{i_n}^n T\left(v_{r_1}^1(e_{i_1}), \ldots, v_{r_n}^n(e_{i_n})\right)\right|$$

is bounded by the right hand side of (3) and we are done. □

Lemma 2.4. *If α is a tensor norm such that $\hat{\otimes}_{\alpha,j=1}^n \ell_2$ has an unconditional basis, then we have that*

$$\alpha \leq K \operatorname{ub}\left(\hat{\otimes}_{\alpha,j=1}^n \ell_2\right) \sigma_2$$

in $\otimes_{i=1}^n \ell_2$, where $K = 2^{\frac{3}{2}n+1} K_G^n$.

Proof. By Theorem 1.4, for any orthonormal sequence $(f_{i_j}^j)_{i_j=1}^{m_j}$ in ℓ_2 ($1 \leq j \leq n$), the unconditional constant of $\left(f_{i_1}^1 \otimes \cdots \otimes f_{i_n}^n\right)_{i_1,\ldots,i_n=1}^{m_1,\ldots,m_n}$ in $\hat{\otimes}_{\alpha,j=1}^n \ell_2$ is bounded by

$$2^{n+1} \operatorname{ub}\left(\hat{\otimes}_{\alpha,j=1}^n \ell_2\right).$$

Therefore, if $T : \ell_2 \times \cdots \times \ell_2 \longrightarrow \mathbb{K}$ belongs to the unit ball of $\left(\hat{\otimes}_{\alpha,j=1}^n \ell_2\right)^*$, we have that

$$\sum_{i_1,\ldots,i_n=1}^{m_1,\ldots,m_n} \left|\lambda_{i_1}^1 \cdots \lambda_{i_n}^n T\left(f_{i_1}^1, \ldots, f_{i_n}^n\right)\right| \leq 2^{n+1} \operatorname{ub}\left(\hat{\otimes}_{\alpha,j=1}^n \ell_2\right) \prod_{j=1}^n \left\| \sum_{i_j=1}^{m_j} \lambda_{i_j}^j f_{i_j}^j \right\|$$

for every choice of scalars $\lambda_{i_j}^j$. Therefore, Lemma 2.3 tells us that T is multiple 1-summing with

$$\pi_1(T) \leq 2^{n+1} K_G^n \operatorname{ub}\left(\hat{\otimes}_{\alpha,j=1}^n \ell_2\right).$$

Theorem 1.1 gives us then that T is Hilbert-Schmidt with

$$\|T\|_{S_2} \leq 2^{\frac{n}{2}} 2^{n+1} K_G^n \operatorname{ub}\left(\hat{\otimes}_{\alpha,j=1}^n \ell_2\right)$$

and we are done.

□

We can prove now our main result. We restate it as follows

Theorem 2.5. *There exists positive constants $K, K' > 0$ that depends only on n such that, if α is a tensor norm such that $\hat{\otimes}^n_{\alpha,j=1}\ell_2$ has an unconditional basis, then*

$$\frac{1}{K \operatorname{ub}\left(\hat{\otimes}^n_{\alpha,j=1}\ell_2\right)}\sigma_2 \leq \alpha \leq K' \operatorname{ub}\left(\hat{\otimes}^n_{\alpha,j=1}\ell_2\right)\sigma_2. \tag{4}$$

Proof. The existence of K' is given by Lemma 2.4. To obtain the other inequality, we consider the dual tensor norm α'. By Theorem 1.4, we have that the canonical tensor basis $(e_{i_1}\otimes\cdots\otimes e_{i_n})_{i_1,\ldots,i_n=1}^{\infty}$ is unconditional in $\hat{\otimes}^n_{\alpha,j=1}\ell_2$, with unconditional constant bounded by $2^{n+1}\operatorname{ub}\left(\hat{\otimes}^n_{\alpha,j=1}\ell_2\right)$. Therefore, for each $N\in\mathbb{N}$, as

$$\left(\otimes^n_{\alpha',j=1}\ell_2^N\right) = \left(\otimes^n_{\alpha,j=1}\ell_2^N\right)^*,$$

the unconditional constant of $(e_{i_1}\otimes\cdots\otimes e_{i_n})_{i_1,\ldots,i_n=1}^{N}$ in $\otimes^n_{\alpha',j=1}\ell_2^N$ is uniformly bounded by

$$2^{n+1}\operatorname{ub}\left(\hat{\otimes}^n_{\alpha,j=1}\ell_2\right)$$

and, therefore, $(e_{i_1}\otimes\cdots\otimes e_{i_n})_{i_1,\ldots,i_n=1}^{\infty}$ is an unconditional basis also in $\otimes^n_{\alpha',j=1}\ell_2$, with constant bounded by $2^{n+1}\operatorname{ub}\left(\hat{\otimes}^n_{\alpha,j=1}\ell_2\right)$.

We can use now Lemma 2.4 to obtain that

$$\alpha' \leq K' \operatorname{ub}\left(\hat{\otimes}^n_{\alpha',j=1}\ell_2\right)\sigma_2 \leq K'2^{n+1}\operatorname{ub}\left(\hat{\otimes}^n_{\alpha,j=1}\ell_2\right)\sigma_2.$$

As $\sigma_2' = \sigma_2$, we obtain the left inequality of (4) with $K = K'2^{n+1}$.

□

We can also give a symmetric version of Theorem 2.5. We just have to reproduce the proof given in [6, Corollary 4] for the case of two spaces. For definitions and notation on symmetric tensor products see [13].

Theorem 2.6. *If α is a symmetric tensor norm such that $\otimes^{n,s}_{\alpha}\ell_2$ has an unconditional basis, then α is equivalent to the Hilbert-Schmidt symmetric tensor norm.*

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E-mail address: dperezg@escet.urjc.es

Área de Matemática Aplicada, ESCET, Universidad Rey Juan Carlos, Departamental II, 28933 Móstoles (Madrid)

E-mail address: ignaciov@mat.ucm.es

Departamento de Análisis Matemático, Facultad de Matemáticas, Universidad Complutense de Madrid, Madrid 28040, Spain