The Team Orienteering Problem With Variable Time Windows

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Abstract

The Orienteering Problem (OP) is a well-established subject within the domain of Operational Research. Over time, numerous variants of this problem have emerged. This paper introduces the Team Orienteering Problem with Variable Time Windows (TOPVTW) as a new variant based on the Team Orienteering Problem with Time Windows. The distinguishing feature of TOPVTW lies in its adaptive time windows, which evolve dynamically as a consequence of the solution. This problem arises within the context of spread processes that must be contained. The propagation of the spread dictates the time windows, which should be observed by the teams. In turn, these teams are in charge of taking some actions which subsequently influence both the spread and, consequently, the time windows themselves. The time windows can affect both the nodes and the paths. To address this dual impact, two distinct versions of the problem are delineated, each supported by dedicated Mixed Integer Programming (MIP) models. Empirical evaluations are conducted, focusing on resource optimization for wildfire suppression. However, the scope of this paper remains limited to solving small-scale instances, leaving the exploration of decomposition techniques or heuristics for future research. This work serves as an initial stride toward further exploration and analysis within this domain.

Keywords: Combinatorial optimization, Orienteering Problem, Integer programming, Routing, Time windows

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1. Introduction

The Orienteering Problem (OP) was formally established in Golden et al. (1987), where it is defined as a problem in which a competitor has to visit a subset of control points collecting their

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rewards, so as to maximize the total score and return to the starting point within a prescribed amount of time.

From the original problem, several variants have been created as new conditions are included. In the team Orienteering Problem (TOP) (Chao et al., 1996) several teams are in charge of visiting the check points. The goal is to maximize the total collected profit, while each team abides by the time limit. Another variant is the Orienteering Problem with Time Windows (OPTW) (Kantor and Rosenwein, 1992), in which the check points have to be visited within their predefined time windows. The combination of both variants, called the Team Orienteering Problem with Time Windows (TOPTW) has also been studied in the literature (Vansteenwegen et al. (2009); Montemanni and Gambardella (2009)).

In this paper, a new variant of the classical orienteering problem is presented, which, as far as we know, has not been previously studied in the literature. It is build upon the TOPTW and will be called the Team Orienteering Problem with Variable Time Windows (TOPVTW). The novelty of the proposed variant is the dynamic definition of the time windows, which are not predefined but modified by the solution, and may affect both the nodes and the paths.

The aim of this work is to establish a formal definition of the problem, presenting some MIP formulations, and some preliminary results as well. Two alternatives of the problem will be explored. In the first one the variable time windows affect only the nodes. In the second one, variable time windows affect both nodes and paths. The contributions made here have wide applicability, since the proposed problem can be useful to model and optimize the managing of spread processes such as epidemics, wildfires, or fake news in social media.

The remaining of this paper is organized as follows: Section 2 provides an overview of the existing variants of the orienteering problem. Section 3 presents the new problem, arguing what motivates its proposal and describes two possible alternatives, bringing up real situations in which they can arise. Section 4 establishes some mathematical MIP formulations for the problems and Section 5 present results for each of them, using the TOPVTW to model wildfire suppression resources' management. In Section 6 some conclusions are drawn.

2. Literature review

The orienteering problem belongs to the family of travelling salesman problems with profits (TSSP). In this kind of problems two objectives are opposed: maximizing the collected profit and minimizing the tour length. Feillet et al. (2005) classify the problems depending on how they are addressed: the objective function combines both criteria; the objective is to maximize the collected profit, limiting the tour length with a constraint or the objective is to minimise the tour length, stating a minimum value for the reward collected.

The OP corresponds to the second set of problems described above. However, some authors have proposed multi-objective versions that deal with both criteria (Keller, 1989; Hapsari et al., 2018; Saeedvand et al., 2020). In any case, this relation with the Travelling Salesman Problem (TSP), and the fact that it is an NP-hard problem, has helped proving that the OP is also NP-hard (Golden et al., 1987; Laporte and Martello, 1990).

The OP is defined as follows: given n nodes in the Euclidean plane, each with score $s(i)$, find a route of maximum score through these nodes, beginning at 1 and ending at n, of length no greater than TMAX (Golden et al., 1987). Note that it is not necessary to visit all the nodes, and that each node should be visited at most once. The problem was originally described by Tsiligirides (1984), who called it Score Orienteering Event (SOE), or Generalized Travelling Salesman Problem (GTSP). It was Golden et al. (1987) who coined the name of Orienteering Problem (OP). It was also called the Selective Travelling Salesman problem (STSP) by Laporte and Martello (1990), who provided its first ILP formulation.

Several authors have proposed different variants of the problem including new features: the Team Orienteering Problem (TOP) (Chao et al., 1996), the Orienteering Problem with Time Windows (OPTW) (Kantor and Rosenwein, 1992), the Team Orienteering Problem with Time Windows (TOPTW) (Vansteenwegen et al., 2009; Montemanni and Gambardella, 2009), or the Generalized Orienteering Problem (GOP) (Urrutia-Zambrana et al., 2021).

In the TOP, each team follows a tour collecting the profit from a different subset of nodes, keeping to a specified time limit. In the TOPTW, the nodes also should be visited within their predefined time windows. The goal in all of them is to find the optimal combination of non-overlapping tours that maximizes the total collected profit considering a single feature. In the GOP each node has a vector of scores regarding several attributes.

In the traditional TOP, the teams are considered to be homogeneous, but they can also have different characteristics (Saeedvand et al., 2020). Furthermore, obtaining the reward of a node may rely on collaborative efforts among multiple teams. The Cooperative Orienteering Problem (COPTW) – also called the Asset Protection Problem (APP) – (van der Merwe et al., 2014) generalizes the TOPTW, requiring the synchronized cooperation among a certain number teams to collect the score from a particular node. On the contrary, the multi-visit team orienteering problem with precedence constraints (Hanafi et al., 2020) requires the ordered visit of different teams to a node to accomplish a set of assembly tasks in order to obtain its reward.

The first attempts of solving the TOPTW problem were heuristics: an Ant Colony System (ACS) by Montemanni and Gambardella (2009) and an Iterated Local Search (ILS) by Vansteenwegen et al. (2009), who also tried to solve it directly using a commercial solver, resulting in large running times. Righini and Salani (2009) used dynamic programming, improving the running times. However, the vast majority of the algorithms are based on heuristic schemes (Tricoire et al., 2010; Labadie et al., 2012; Lin and Yu, 2012; Cura, 2014; Hu and Lim, 2014; Labadie et al., 2011). Mixed schemes using branch and price with heuristics for the pricing phase can also be found (Ke et al., 2014).

Some other variants of the OP are more dynamic, considering time-dependent aspects. The Time-Dependent Orienteering problem (TDOP) (Fomin and Lingas, 2002) considers that travel times between nodes vary linearly with time, they depend on the departure time at the first location. There is a variant of the TDOP that includes time windows (TDOPTW) (Garcia et al., 2010) which can be augmented for several teams (TDTOPTW) (Garcia et al., 2013). Also, the profits can be time-dependent. In the orienteering problem with variable profits (OPVP) the obtained profits may vary with time in two ways: they can depend on the arrival time (OPATP) or on the service time (OPSTP) (Yu et al., 2019). Besides, there exist variants of the problem which combine time-dependent travel times with variable profits either depending on the arrival time (Peng et al., 2019) or the service time (Khodadadian et al., 2022).

The aforementioned papers consider time-dependent aspects, but assuming a knowledge on the change with time. Some other authors propose stochastic variants of the OP, for example, introducing uncertainties in the collected profits (OPSP) (Ilhan et al., 2008), or in the travel and service times (OPSTS) (Campbell et al., 2011). In the Stochastic OP with Time Windows

(SOPTW) waiting times are modeled as random variables (Zhang et al., 2014). Nuraiman et al. (2022) consider stochastic time windows based on fire arrival times, establishing scenarios for wind direction shifts that can modify its behaviour. Even though there is a vast number of dynamic variants of the OP with time windows, none of them consider a problem in which the decisions made have an impact on the time windows. To the best of the authors' knowledge this is the first paper describing the Team Orienteering Problem with Variable Time Windows (TOPVTW).

3. Problem description: TOPVTW

Some of the mentioned OP variants in Section 2 deal with dynamic aspects such as rewards or travel times changing over time. Some other variants of the OP include static time windows, giving rise to the OPTW. In this paper, a novel variant of the OPTW is proposed, in which the time windows are neither predefined from the beginning nor they are stochastic but depend on the tours defined by the solution itself. Since the problem arises from the OPTW it is called the Orienteering Problem with Variable Time Windows (OPVTW).

The solution to this new problem, as in the OPTW, is a route in which the nodes should be visited within their time windows. The novelty of the OPVTW is that the time windows are not known in advance but defined by the arrival time of a spread process, which can be modified by the route. Slowing down the spread may delay the closing time of several nodes.

In general, a route can go through a node several times, and only spending some service time there blocks the spread. In this case the node will be called *managed*. A node is *available* for visiting until its time window closes, so it should be left before that instant, defined as the time the spread would arrive to that node minus a safety margin. The nodes reached by the spread will be called *affected*. Note that there is no opening time for the time windows – every node is available since the beginning. A solution to the problem is a sequence of visited nodes, specifying which of them are managed, and a set of affected nodes.

The goal is to stop the spread, minimizing the value of the affected nodes. This is yet another complicating difference with the classical OPTW. In the OPVTW there is no direct reward associated to each visited node. It is not possible to ascertain how beneficial is to manage a single node, as it is the combination of several of them which determines the set of affected ones. Although a tour length can be explicitly defined, in general it is not – it depends on the instant when the spread is completely stopped or when there are no available nodes to visit.

The OPVTW arises in the context of spread processes that need to be stopped, such as wildfires, epidemics or fake news' spreading in social media.

Two alternatives to the problem are presented here. In the first one, it is assumed that the shortest path between each pair of nodes is always available. It is suitable to model epidemics or fake news that spread through a network, or the management of aerial resources for wildfire suppression, which fly between locations over the landscape. It would be equivalent to assume that there is a direct connection between each pair of nodes to create the route.

The second alternative considers that the paths between nodes are also affected by the spread process, being a path available only when every node in it can be reached before its closing time window. This can model wildfire suppression operations on the ground, since the paths followed by the brigades are continuous through the landscape and restricted by the wildfire spread.

The Orienteering Problem with Variable Time Windows can also be extended to include the cooperation among several teams. This gives rise to the Team Orienteering Problem with Variable Time Windows (TOPVTW). This problem is of special interest for modelling ground wildfire suppression operations, where usually a number of teams need to be coordinated to put the fire out. For the sake of generality, the TOPVTW will be dealt with from now on.

4. MIP models for the TOPVTW

In this section, different MIP formulations are provided for each of the TOPVTW alternatives described in Section 3. All the formulations share some features and notations, especially for the spread process, which is analogous in any of them.

Let N be a set of nodes $N = \{1, ..., n\} \cup \{0\}$ where $\{1, ..., n\}$ represent a set of locations and 0 represents a depot for the teams. For some model formulations it will be necessary to define a copy of the depot indexed by $n + 1$, so $N = \{1, ..., n\} \cup \{0, n + 1\}$.

The spread process is modelled in a graph $\mathcal{G}^s = (S, E^s)$, where S is the subset of nodes $S \subset N$ reachable by the spread and E^s the set of links through which the spread can travel. The Minimum Travel Time (MTT) methodology (Finney, 2002; Belval et al., 2015) is used to model the spread process. It starts at a known node in the graph and, if not blocked, travels to every node following the shortest path from the starting point. The MTT tracks only the first time the spread reaches a node. The depot is assumed to be out of spread's reach. The graph $\mathcal{G}^s$ is a directed graph, which is not complete in general. Recalling the examples of application from Section 3, two nodes are adjacent if they represent two individuals or communities that interact during an epidemic or in social media, or if they represent two pieces of terrain that are next to each other in a landscape during a wildfire.

On the other hand, the directed graph $\mathcal{G}^v = (V, E^v)$ is the graph through which the teams can travel, being $V \subseteq N$ the subset of nodes that can be visited by the teams and E^v the links that can be traversed by them.

Table 1 contains the definitions for the common sets, parameters and variables for the different formulations. It is assumed that the teams are homogeneous and share all their features. If this is not the case, an additional set K for the teams is created and parameters T^v_{ij}, A^v_{ij} and b_i would be T^v_{ijk}, A^v_{ijk} and b_{ik} respectively.

The basic distinction between the two TOPVTW alternatives is the availability of paths. In the following subsections both are explained in more detail and MIP formulations are provided.

4.1. Variable time windows in nodes (TOPVTW–nodes)

If the shortest path between two nodes is considered to be always available, we say that there are only variable time windows in the nodes. Every node can be reached from any other – the edges in E^v represent the shortest path between each pair of nodes in N^v. So, the problem can be modelled setting $\mathcal{G}^v$ as a complete directed graph. To simplify the problem and without loss of generality, it can be set that each visited node is also managed, so the solution for each team is a directed cycle starting and ending at the depot – just checking the availability of each managed node. Only nodes in $V \cap S$ will be considered for determining this directed cycle.

For this MIP formulation, a copy node for the depot is needed ($N = \{1, ..., n\} \cup \{0, n+1\}$). The adjacency relations for the teams are in the form that every node is adjacent to any other except for the depot node and its virtual copy. Any node is reachable from 0, but node 0 cannot be entered, whereas $n+1$ cannot be exited but it is reachable from any other node. Node 0 and copy node $n+1$ are not considered to be adjacent.

The specific variables of the visiting process for this MIP formulation can be found in Table 2 and the corresponding constraints are listed below.

Notation	Definition
Sets	
N	set of nodes
$S \subset N$	set of nodes that can be affected by the spread
$V \subseteq N$	set of nodes that can be visited by the teams
Parameters	
D_i	1 if node i is the starting node of the spread, 0 otherwise
C_i	Value lost if node i is affected
A^s_{ij}	1 if spread can reach j from i (adjacency matrix)
T^s_{ij}	time it takes the spread to reach j from i
M^s	a big enough constraint for the spread process
A^v_{ij}	1 if a team can reach j from i (adjacency matrix)
T^v_{ij}	time it takes a team to travel from i to j
B_i	time it takes a team to manage node i
G_{ij}	Safety margin of node i if spread comes from j
κ	Number of teams
M^v	a big enough constraint for the visiting process
Variables	
y_i	1 if node i is affected, 0 otherwise
w_{ij}	1 if spread goes from i to j, 0 otherwise
x^s_i	time instant the spread arrives to node i

Table 1: Definitions of sets, parameters, and variables common to the MIP formulations

Notation	Definition
Variables	
r_{ij}	1 if a team goes from node i to node j, 0 otherwise
z_i	1 if node i is managed, 0 otherwise
x^v_i	time instant of a team visiting node i

Table 2: Specific definitions to the TOPVTW–nodes homogeneous teams

TOPVTW–nodes. Homogeneous teams

$$f_1 = \min \sum_{i \in S} y_i C_i \tag{1}$$

$$x_i^s = 0 \quad \forall i \in S \mid D_i = 1 \tag{2}$$

$$\sum_{j \in S \mid (j,i) \in E^s} w_{ji} + D_i = y_i \quad \forall i \in S \tag{3}$$

$$w_{ij} \leqslant y_i \quad \forall (i,j) \in E^s \tag{4}$$

$$w_{ij} \leqslant y_j \quad \forall (i,j) \in E^s \tag{5}$$

$$x_i^s \leqslant x_j^s + T_{ji}^s + M^s(1 - y_i) \quad \forall (j,i) \in E^s \tag{6}$$

$$x_i^s \geqslant x_j^s + T_{ji}^s w_{ji} - M^s(1 - w_{ji}) \quad \forall (j,i) \in E^s \tag{7}$$

$$x_i^s \geqslant M^s(1 - y_i) \quad \forall i \in S \tag{8}$$

$$y_i \geqslant y_j \quad \forall (j,i) \in E^s \mid i \notin V \tag{9}$$

$$y_i + z_i \geqslant y_j \quad \forall (j,i) \in E^s \mid i \in V \tag{10}$$

$$y_i + z_i \leqslant 1 \quad \forall i \in V \cap S \tag{11}$$

$$\sum_{(0,j) \in E^v \mid j \in S} r_{0j} = \kappa \tag{12}$$

$$\sum_{(j,n+1) \in E^v \mid j \in S} r_{j,n+1} = \kappa \tag{13}$$

$$\sum_{(i,j) \in E^v \mid j \in S \cup \{n+1\}} r_{ij} = \sum_{(j,i) \in E^v \mid j \in S \cup \{0\}} r_{ji} \quad \forall i \in V \cap S \tag{14}$$

$$z_i = \sum_{(j,i) \in E^v \mid j \in S \cup \{0\}} r_{ji} \quad \forall i \in V \cap S \tag{15}$$

$$z_i \leqslant \sum_{j \in S \mid (j,i) \in E^s} y_j \quad \forall j \in V \cap S \tag{16}$$

$$x_i^v + z_i B_i + G_{ji} \leqslant x_j^s + T_{ji}^s \quad \forall (j,i) \in E^s \mid i \in V \tag{17}$$

$$x_i^v \leqslant M^v \sum_{(j,i) \in E^v} r_{ji} \quad \forall i \in V \cap S \tag{18}$$

$$x_j^v \geqslant x_{ik}^v + B_i + T_{ij}^v - M^v(1 - r_{ij}) \quad \forall (i,j) \in E^v \mid i \in S \cup \{0\}, j \in S \cup \{n+1\} \tag{19}$$

Constraint (1) is the objective function of the problem, which minimises the value of the affected nodes. Constraints (2)–(9) are related solely to the spread process. Constraints (2) set initial time of spread to 0 for the initial location. Constraints (3) determines that if node i is affected, either is the initial point of the spread or it comes from another node. Constraints (4) and (5) label nodes i and j as affected if spread travels from one to the other. Constraints (6) and (7) determine the spread binding paths and arrival time to each node. Constraints (8) establish a lower bound for the spread arrival time to a node, if it is never reached. Constraints (9) force a non-manageable node to be affected if any of its adjacent nodes is affected. Constraints (10)–(11) determine how the spread process is affected by the managing decisions. Constraints (10) state that if node j is affected, adjacent node i is either affected or managed. Constraints (11) specify that if a node is managed it cannot be affected. Constraints (12)–(16) are related to the visiting process, containing also some variables from the spread process. Constraints (12) and (13) force the teams to leave and return to the depot respectively. Constraints (14) are flow restrictions. Constraints (15) establish that a visited node should be managed and vice versa. Constraints (16) avoid the management of nodes not essential to stop the spread. They ensure that every managed node will be next to at least one affected node. These constraints reduce symmetries, keeping only those solutions for which the set of managed nodes is minimal with respect to the set of affected ones. Constraints (17) ensure that nodes are visited/managed within the time-window. Constraints (18) set visiting time to 0 for those non visited nodes. Constraints (19) define the arrival time of the teams to each node.

A secondary objective function can be added to this formulation, that minimises the latest instant any team arrives to the depot:

$$f_2 = \min\{x^{max}\} \tag{20}$$

where:

$$x^{max} = x^v_{n+1} \tag{21}$$

In case heterogeneous teams are considered, the formulation can be easily extended using a new index k for listing the teams. Constraints (1)–(9) would still apply. Constraints (10)–(19) and (21) need to be modified. A new set of constraints (22) is also required, forcing each node to be managed – and thus visited – only once by all teams.

TOPVTW–nodes. Heterogeneous teams

(1)–(9)

$$y_i + \sum_{k \in K} z_{ik} \geqslant y_j \quad \forall (j,i) \in E^s \mid i \in V \tag{10'}$$

$$y_i + \sum_{k \in K} z_{ik} \leqslant 1 \quad \forall i \in V \cap S \tag{11'}$$

$$\sum_{j \in V \cap S} r_{0jk} = 1 \quad \forall k \in K \tag{12'}$$

$$\sum_{j \in V \cap S} r_{j,n+1,k} = 1 \quad \forall k \in K \tag{13'}$$

$$\sum_{j \in V \cap S \cup \{n+1\}} r_{ijk} = \sum_{j \in V \cap S \cup \{0\}} r_{jik} \quad \forall i \in V \cap S, k \in K \tag{14'}$$

$$z_{ik} = \sum_{(j,i) \in E^v} r_{jik} \quad \forall i \in V \cap S, k \in K \tag{15'}$$

$$z_{ik} \leqslant \sum_{j \in S \mid (j,i) \in E^s} y_j \quad \forall i \in V \cap S, k \in K \tag{16'}$$

$$x^v_{ik} + z_{ik} B_{ik} + G_{ji} \leqslant x^s_j + T_{ji} \quad \forall k \in K, (j,i) \in E^s \mid i \in V \tag{17'}$$

$$x^v_{ik} \leqslant M^v \sum_{(j,i) \in E^v} r_{jik} \quad \forall i \in V \cap S, k \in K \tag{18'}$$

$$x^v_{jk} \geqslant x^v_{ik} + B_{ik} + T^v_{ijk} - M^v(1 - r_{ijk}) \quad \forall k \in K, (i,j) \in E^v \mid i \in S \cup \{0\}, j \in S \cup \{n+1\} \tag{19'}$$

$$x^{max} \geqslant x^v_{n+1} \quad \forall k \in K \tag{21'}$$

$$\sum_{k \in K} z_{ik} \leqslant 1 \quad \forall i \in V \cap S \tag{22}$$

Note that, although this formulation would also be valid for the case of homogeneous teams, it would yield many symmetries than can be avoided using the constraints (10)–(19). This will be shown in Section 5.

4.2. Variable time windows in nodes and paths (TOPVTW–paths)

As well as in the TOPVTW–nodes, the goal is to determine the nodes to be managed, in order to minimise the value of the affected ones. However, in this alternative, the shortest path between two consecutive managed nodes might not be available when it is required. Thus, secondary paths need to be considered. It is said then that there are variable time windows affecting both the nodes and the paths. If the time window of any of the nodes to be visited within a path is already closed, the path becomes unavailable. The solution for each team is a closed walk of visited nodes, in which only some of them are managed. It is a directed closed walk because although each node can only be managed once, it can be visited several times.

In this paper, two ways of modelling the TOPVTW–paths are explored. The first one considering the original incomplete graph and letting the model determine the shortest paths, ensuring their availability. It will be *called implicit shortest paths formulation.* And the second one, assuming a complete multigraph build upon the original one, where ℓ links between each pair of nodes represent secondary pre-calculated shortest paths between them. In this case the model chooses the shortest one among the ℓ paths, based on their availability. This one will be called *explicit shortest paths formulation.*

4.2.1. Implicit shortest paths formulation

Given a set V of nodes and the set of edges E^v for the teams, an incomplete graph is considered. The goal is to find the closed walk that each of the teams follows, starting and ending at the depot, that minimises the value of the affected nodes. Not all the nodes in the walk must be managed. The walk of a team can overlap with itself and with the walks of other teams, as long as a node is not managed more than once. A time window constraint must be observed in each of the visited nodes regardless of them being managed or not.

In this TOPVTW alternative, a node can be visited several times, thus, the classical formulation of the TOPTW is not suitable to track the time correctly. Instead of using flow constraints, a walk is created for each of the teams using a new set L, that defines the position in the walk of a team. Each of the teams needs to be addressed independently, so the index k is necessary to differentiate them, either dealing with homogeneous or heterogeneous teams. In this case a node can be visited several times, so no copy for the depot is needed ($N = \{1, ..., n\} \cup \{0\}$). Specific sets and variables for this formulation can be found in Table 3.

Notation	Definition
Sets	
L	positions in the walk of a team
Variables	
r_{lik}	1 if node i is visited by team k in position l, 0 otherwise
z_{lik}	1 if node i in position l is managed by team k, 0 otherwise
x^v_{lk}	time instant node in position l is visited by team k
$u^v_{ll'k}$	travelling time for team k from node in position l to node in position l'

Table 3: Specific definitions to the TOPVTW–paths implicit shortest paths formulation

The spread process is modelled using constraints (2)–(9), which are not explicitly written again for the sake of simplicity. The main objective function is also the same as in Section 4.1, as expressed in constraint (1). The constraints modelling the visiting process are (23)–(36).

TOPVTW–paths implicit shortest paths formulation

$$(1)\text{–}(9)$$

$$y_i + \sum_{k\in K, l\in L | l>1} z_{lik} \geqslant y_j \quad \forall (j,i) \in E^s \mid i \in V \tag{23}$$

$$y_i \leqslant 1 - \sum_{k\in K, l\in L | l>1} z_{lik} \quad \forall i \in V \cap S \tag{24}$$

$$r_{10k} = 1, \quad \forall k \in K \tag{25}$$

$$\sum_{l>1} r_{l0k} = 1, \quad \forall k \in K \tag{26}$$

$$\sum_{i\in V} r_{lik} \leqslant 1, \quad \forall l \in L, k \in K \tag{27}$$

$$\sum_{i\in V} r_{lik} \leqslant \sum_{i\in V} r_{l-1,ik}, \quad \forall k \in K, l \in L \mid l > 1 \tag{28}$$

$$\sum_{i\in V} \sum_{l'>l} r_{l'ik} \leqslant (1 - r_{l0k}) \sum_{l'>l} 1, \quad \forall k \in K, l \in L \mid l > 1 \tag{29}$$

$$\sum_{j\in V | (i,j)\notin E^v} r_{ljk} \leqslant 1 - r_{l-1,ik}, \quad \forall i \in V, k \in K, l \in L \mid l > 1 \tag{30}$$

$$z_{lik} \leqslant r_{lik}, \quad \forall i \in V \cap S, k \in K, l \in L \mid l > 1 \tag{31}$$

$$\sum_{l \in L | l > 1} \sum_{k \in K} z_{lik} \leqslant 1, \quad \forall i \in V \cap S \tag{32}$$

$$z_{lik} \leqslant \sum_{(j,i) \in E^s} y_j, \quad \forall i \in V \cap S, k \in K, l \in L \mid l > 1 \tag{33}$$

$$x^v_{lk} = x^v_{l-1,k} + u^v_{l-1,lk} + \sum_{i \in V \cap S} z_{lik} B_{ik}, \quad \forall k \in K, l \in L \mid l > 1 \tag{34}$$

$$u^v_{l-1,lk} \geqslant T^v_{jik} \left(r_{lik} + r_{l-1,jk} - 1 \right), \quad \forall (j,i) \in E^v, k \in K, l \in L \mid l > 1 \tag{35}$$

$$x^v_{lk} + z_{lik} B_{ik} + G_{ji} \leqslant x^s_j + T^s_{ji} + M^v (1 - r_{lik}), \quad \forall (j,i) \in E^s \mid i \in V, k \in K, l \in L \mid l > 1 \tag{36}$$

Constraints (23) and (24) determine the behaviour of the spread, modified by the management. Constraints (25) and (26) force the teams to start and return to the depot respectively. Each team can only visit one node in position l (27) and node in position l can only be visited if node in position $l-1$ has been visited (28). Observe that not all the positions L need to be used. Last visited node should be the depot (29) and teams can only move from i to j if they are adjacent (30). Only visited cells can be managed (31) and each node can only be managed once (32). The constraints that reduce symmetries in the solutions are also included (33). The arrival time of each team to each node is governed by (34); where the teams travel time is calculated by (35). Constraints (36) ensure that the teams leave any node before its time window closes.

As well as in the formulation for the TOPVTW–nodes in Section 4.1 a secondary objective may be added, using constraint (20). But in this case constraint (21) needs to be modified to define x^{max} in the following way:

$$x^{max} \geq x^v_{lk} \quad \forall k \in K, l \in L \tag{37}$$

4.2.2. Explicit shortest paths

Even though in the TOPVTW–paths a node can be visited several times, it can be managed only once. Based on this feature, a different formulation recovering the flow constraints is now proposed, as a way of modelling the movement of the teams between managed nodes. However, since the shortest path between them might not be available, alternative paths must be defined.

Given an incomplete graph for the teams as in the previous section, the first step is to transform it into a complete multigraph. Each of the ℓ links between any pair of nodes represents one of the ℓ-shortest paths between them, arranged in ascending order. To calculate the first ℓ-shortest paths between every pair of nodes a procedure based on the algorithm by Yen (1971) is used. The output is summarized in two parameters: $P_{\ell ijq}$ that specifies whether or not node q is on the ℓ-shortest path between node i and j and $SP_{\ell ijq}$ that specifies the time it takes to arrive to node q on the ℓ-shortest path from i to j. An example of this transformation can be found in Figure 1. The first shortest path between A and E is the dashed line, with length 2: $P_{1AEA} = P_{1AEB} = P_{1AEE} = 1$, $SP_{1AEA} = 0$, $SP_{1AEB} = 1$, $SP_{1AEE} = 2$. The second shortest path is the solid line, with length 3: $P_{1AEA} = P_{1AEC} = P_{1AED} = P_{1AEE} = 1$, $SP_{1AEA} = 0$, $SP_{1AEC} = 1$, $SP_{1AED} = 2$, $SP_{1AEE} = 3$.

(a) Original incomplete graph. All edges have weight equal to one.

(b) Connections between nodes A and E in the completed multigraph.

Figure 1: Example of transformation from the original incomplete graph to a complete graph, updating the connections between A and E.

Once this information is calculated and considering the multigraph complete for the teams, the goal is to determine the directed cycle of managed nodes that each team should follow, starting and ending at the depot, to minimise the value of the affected nodes. The combination of this directed cycle of managed nodes with the selected shortest path between each pair of them will give the complete closed walk for each team. To ensure the selected shortest path is available, a time window constraint must be observed at all the nodes included in the selected shortest paths.

Same as in the MIP formulation in Section 4.1, a copy node for the depot is needed $(n+1)$. Also, only nodes in $V \cap S$ are to be managed. The specific sets, variables and parameters for this formulation can be found in Table 4. This formulation is based on flow constraints like the one for the TOPVTW–nodes, so the constraints of the model are essentially the same, except for those new ones needed for selecting alternative available paths between managed locations. This formulation takes constraints (1)–(18) and constraints (19) are replaced by (38). Constraints (39)–(41) are introduced to describe how the shortest paths between managed locations are selected.

Notation	Definition
Sets	
$\mathcal{L}$	index of shortest paths arranged in ascending length
Parameters	
$P_{\ell ijq}$	If node q is traversed on ℓ-shortest path from i to j
$SP_{\ell ijq}$	Travel time from i to q on the way to j using the ℓ-shortest path
Variables	
$h_{\ell ij}$	1 if ℓ-shortest path from i to j is selected, 0 otherwise
r_{ij}	1 if any team travels from nodes i to node j, 0 otherwise
z_i	1 if node i is managed
x_i^v	time instant node i is visited
u_{ij}^v	travel time from i to j (depending on shortest path selected)

Table 4: Specific definitions to the TOPVTW–paths explicit shortest paths formulation homogeneous teams

TOPVTW–paths explicit shortest paths formulation. Homogeneous teams.

$$(1)\text{–}(18)$$

$$x_j^v \geqslant x_i^v + B_i + u_{ij}^v - M^v(1 - r_{ij}) \quad \forall (i,j) \in E^v \mid i \in S \cup \{0\}, j \in S \cup \{n+1\} \tag{38}$$

$$u_{ij}^v = \sum_{\ell \in \mathcal{L}} SP_{\ell ijj} h_{\ell ij} \quad \forall (i,j) \in E^v \mid i \in S \cup \{0\} \wedge j \in S \cup \{n+1\} \tag{39}$$

$$r_{ij} = \sum_{\ell \in \mathcal{L}} h_{\ell ij} \quad \forall (i,j) \in E^v \mid i \in S \cup \{0\} \wedge j \in S \cup \{n+1\} \tag{40}$$

$$\begin{aligned} & x_i^v + SP_{\ell ijq} + B_i \leqslant x_{q'}^s + M^v(1 - h_{\ell ij}) \\ & \qquad \forall (i,j) \in E^v \mid i \in S \cup \{0\} \wedge j \in S \cup \{n+1\}, P_{\ell ijq} = 1, \\ & \qquad\qquad \forall (q', q) \in E^s \mid q \in V \wedge q \neq i \wedge q' \notin \{i,j\}, \ell \in \mathcal{L} \end{aligned} \tag{41}$$

Constraints (38) replace constraints (19) to include the travel time between i and j as a variable, instead of as a parameter. This travel time is calculated in constraints (39). Constraints (40) relate travel variables r_{ij} with shortest path selection variables $h_{\ell ij}$. Constraints (41) ensure the availability of paths.

A secondary objective can also be added in this formulation as well as in the TOPVTW–nodes, using constraints (20) and (21) from previous section.

When considering heterogeneous teams, the formulation can be extended using a new index k, as previously done in Section 4.1. Constraints (38)–(41) need to be replaced by (38′)–(41′).

TOPVTW–paths explicit shortest paths formulation. Heterogeneous teams.

$$(1)\text{–}(9),\ (10')\text{–}(19'),\ (22)$$

$$x^v_{jk} \geqslant x^v_{ik} + B_i + u^v_{ijk} - M^v(1 - r_{ijk}) \quad \forall k \in K, (i,j) \in E^v \mid i \in S \cup \{0\}, j \in S \cup \{n+1\} \tag{38$'$}$$

$$u^v_{ijk} = \sum_{\ell \in \mathcal{L}} SP_{\ell ijj} h_{\ell ijk} \quad \forall k \in K (i,j) \in E^v \mid i \in S \cup \{0\} \wedge j \in S \cup \{n+1\} \tag{39$'$}$$

$$r_{ijk} = \sum_{\ell \in \mathcal{L}} h_{\ell ijk} \quad \forall k \in K, (i,j) \in E^v \mid i \in S \cup \{0\} \wedge j \in S \cup \{n+1\} \tag{40$'$}$$

$$x^v_{ik} + SP_{\ell ijq} + B_i \leqslant x^s_{q'} + M^v(1 - h_{\ell ijk}) \tag{41$'$}$$

$$\forall k \in K, (i,j) \in E^v \mid i \in S \cup \{0\} \wedge j \in S \cup \{n+1\}, P_{\ell ijq} = 1,$$

$$\forall (q', q) \in E^s \mid q \in V \wedge q \neq i \wedge q' \notin \{i,j\}, \ell \in \mathcal{L}$$

5. An application of the TOPVTW: Wildfire Suppression

In this study, the MIP models described in Section 4 are used to optimize resource allocation for wildfire suppression.

The term *wildfire suppression* comprises a set of actions undertaken to put a wildfire out. They belong to the preparedness and response phase of the management cycle of a natural disaster (Tomasini and van Wassenhove, 2009). In general, once a wildfire starts, all the available resources should be organized to address it as soon as possible. For this purpose, a powerful tool is needed. In particular, the TOPVTW can be useful to model this problem.

To test the proposed MIP formulations a set of realistic instances has been created. The instances describe landscapes of different sizes and features, in which a wildfire can spread, which must be stopped by two teams. There are 25 instances with different terrain and weather features per landscape size. The terrain is divided in parcels, which are represented as the nodes of a graph, as it can be seen in Figure 2. The spread parameters such as the spread travel times between adjacent nodes have been calculated according to the methodology from Belval et al. (2015) and Wei et al. (2011). The necessary parameters have been taken from FlamMap (Finney et al., 2006), which considers factors such as the slope, the wind or the type of vegetation. Weather conditions

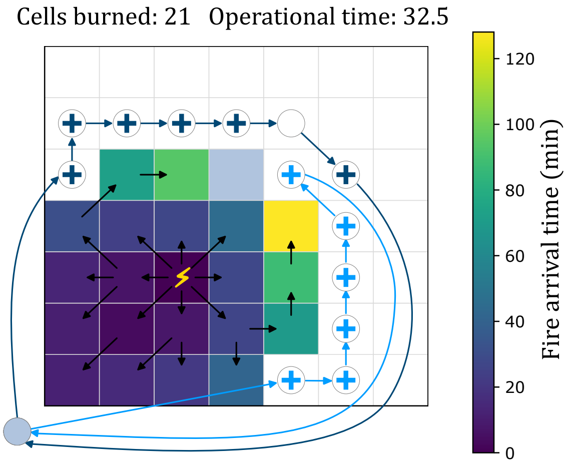

Figure 2: Example of a landscape with a solution: two teams containing the wildfire

are assumed to be constant. The teams are assumed to be homogeneous, having all common features, and their travel times are calculated based on their velocity and the slope and aspect of the terrain. In the TOPVTW-paths implicit formulation the cardinality of L is set to 15. In the case of the TOPVTW-paths explicit formulation 10 shortest paths are pre-calculated between each pair of nodes.

There are two objectives in the problem: f_1 (Constraint (1)) is the value of the affected nodes – which is not a linear function of simple rewards – and f_2 (Constraint (20)) which represents the last moment any team returns to the depot. To handle both objectives this paper proposes a lexicographic approach in two steps. In the first step, the main objective f_1 is minimised. This yields a set of managed and affected nodes. This information is stored and used to run the model again. The second step distributes the managed nodes among the teams, determining the tours that minimise f_2.

5.1. Computational results

In this section some performance measures for each of the TOPVTW models are presented, in order to compare them. The value of the objective functions f_1 and f_2 as well as the final GAP and running times serve this purpose. A running time limit of 30 minutes have been set for each step.

Figure 3, present a summary of results for the MIP formulations of the TOPVTW-nodes: homogeneous teams with one and two teams and heterogeneous teams with two teams. Regardless of the formulation or the number of teams, the value of the objective function f_1 increases with landscape size, because a larger piece of landscape needs to be protected with the same amount of resources. The final MIP gap also increases as the landscape enlarges. Focusing on running times it is clear that the bottleneck is the first step, in which most of the running time is spent, increasing rapidly with landscape size.

As expected, better results are obtained when two teams are considered instead of only one – their collaboration allows for smaller values of f_1 and f_2. It is noteworthy that also the final MIP gaps and running times are smaller when two teams are considered. Comparing the homogeneous and heterogeneous formulations similar values of f_1 and f_2 are obtained. Nevertheless, MIP gaps and running times are smaller when using the homogeneous formulation, denoting that it might be tighter than the heterogeneous one. Thus, when homogeneous teams are considered, the specific homogeneous formulation should be preferred. Figure 4 shows the summary of results for the three MIP models dedicated to the TOPVTW-pahts: the implicit formulation and the explicit formulation for both homogeneous and heterogeneous teams. Regarding the differences between the explicit formulations, the homogeneous one achieves better results for f_1 and f_2 than the heterogeneous one. Besides, it obtains smaller values for the MIP gaps and running times. When comparing the explicit homogeneous formulation to the implicit one they show similar objective values f_1 and f_2, however, the explicit homogeneous formulation performs better in terms of MIP gaps and running times, especially in the second step. It must be noted that the explicit formulation requires a pre-processing phase involving the calculation of the ℓ-shortest paths connecting every pair of nodes. Figure 5 shows total running times, which include reading and model construction times, solver durations for both steps, alongside the pre-processing time specific to the explicit models. It demonstrates that even considering the pre-processing times the explicit formulation for homogeneous teams outperforms the implicit one in terms of running time. All this being considered indicates that, when homogeneous teams are considered, the explicit formulation for homogeneous teams is the most adequate.

Comparing the implicit formulation to the explicit heterogeneous formulation, the first one performs better in terms of values of f_1. Values of f_2 for biggest landscapes are not comparable because the explicit heterogeneous formulation achieved worse results in the first step. The explicit heterogeneous formulation performs better in terms of MIP gaps and running time for small

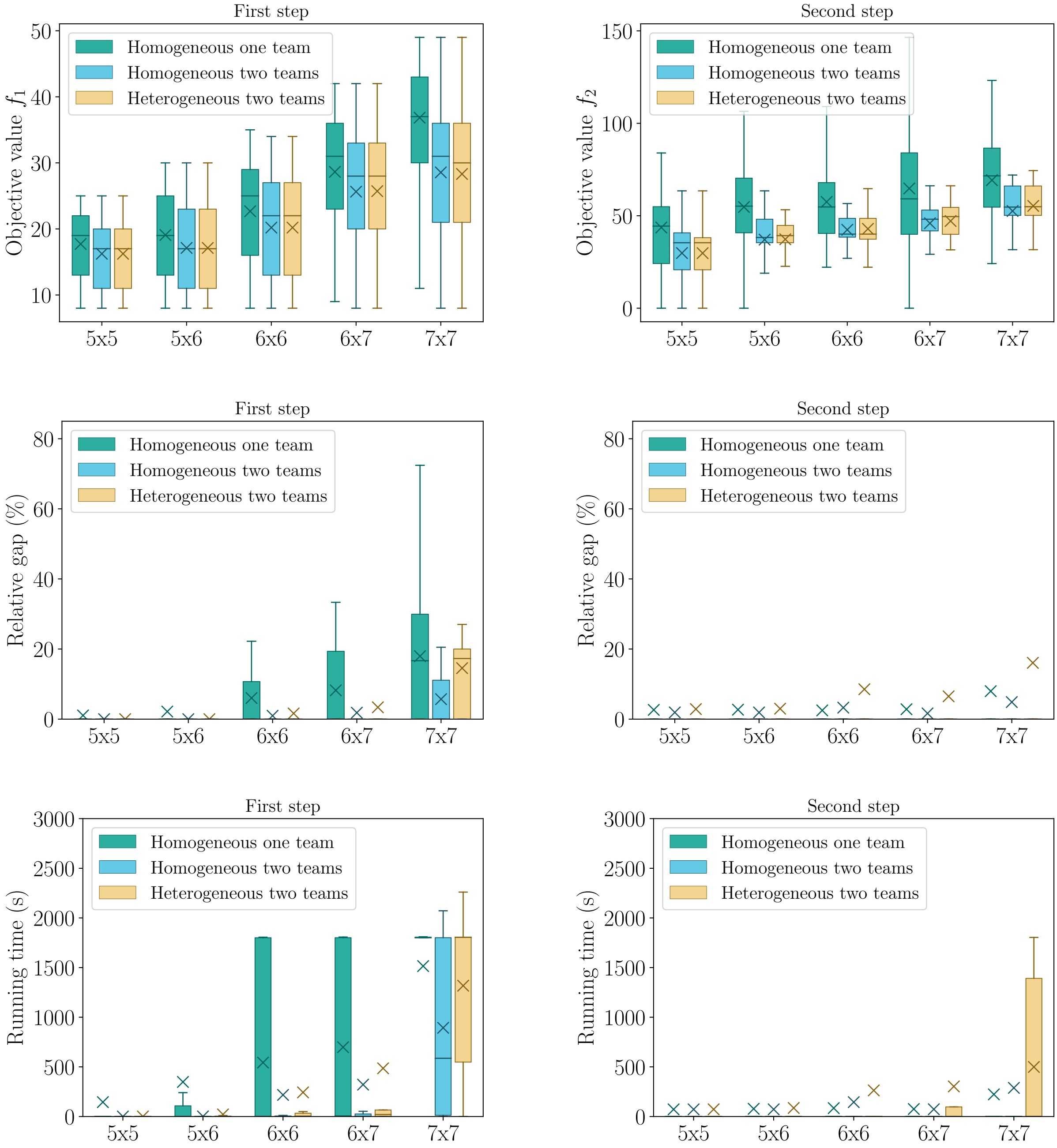

Figure 3: Performance measures TOPVTW–nodes homogeneous for one and two teams and heterogeneous formulation for two teams. Outliers have not been plotted and ×-like symbols represent average values.

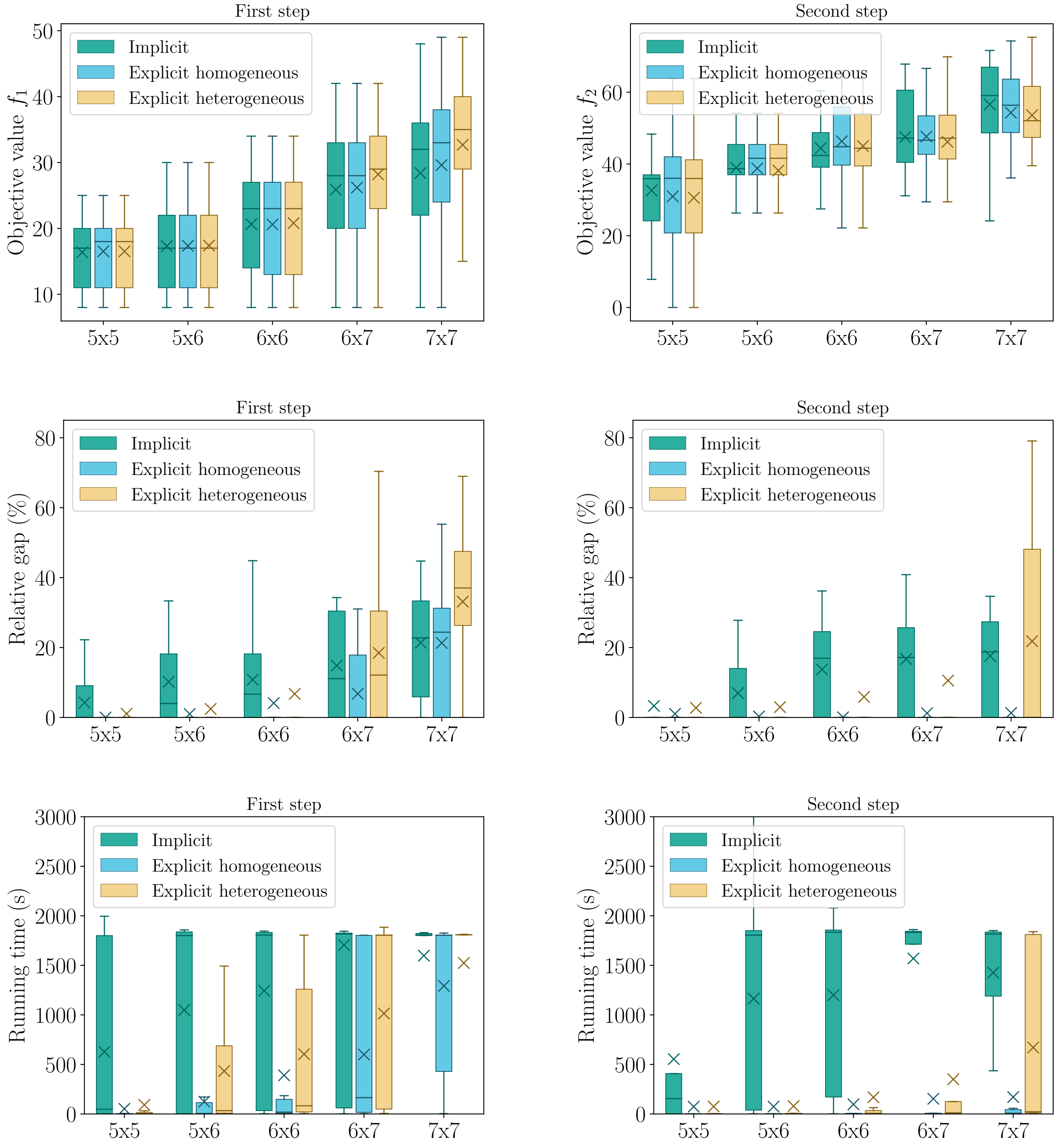

Figure 4: Performance measures of the TOPVTW–paths implicit and explicit homogeneous and explicit heterogeneous formulations. Outliers have not been plotted and ×-like symbols represent average values.

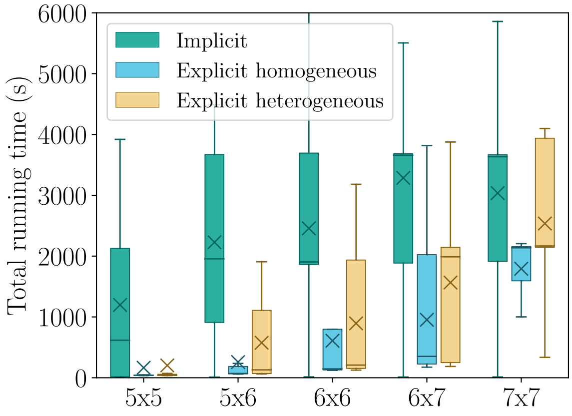

Figure 5: Total running times for the three MIP TOPVTW-paths models. They include reading, model construction and solver times, as well as pre-processing times specific to explicit models.

landscapes. However, these measures worsen rapidly as the landscape size increases, exceeding the ones for the implicit formulation in the biggest landscapes. This analysis might indicate that the implicit formulation is more adequate when heterogeneous teams are considered. Even so, these results must be taken cautiously, because in this study only homogeneous teams are considered. It might be the case that considering teams that are indeed heterogeneous can eliminate symmetries, yielding better results for the explicit heterogeneous formulation. In any case, there is a limitation for both formulations. In the explicit one it is not clear how many paths should be calculated in advance – they must be enough to avoid infeasibilities, while in the implicit one the index l limits the number of nodes that can be visited by each of the teams. Thus, L should be big enough to not impose a boundary, but small enough to avoid having more variables than necessary. Considering the applicability of the models, running times are too large to be useful in real situations. The response phase of a wildfire is a very stressful context in which decisions need to be made quickly. Decomposition or heuristic/matheuristic methods should be explored, leaving them as open problems for future research on this topic.

6. Conclusions and future work

This study introduces a novel problem within the realm of orienteering problems, enlarging this family. It describes a new variant of the Team Orienteering Problem with Time Windows in which the time windows are variable, changing with the solution. This problem, called the Team

Orienteering Problem with Variable Time Windows (TOPVTW), arises in the context of spread processes that must be stopped. If no interaction were applied, the time windows would be known in advance, given by the behaviour of the spread. However, interventions aimed at hindering its advance result in modifications to the closing time of these windows.

Two alternatives of the TOPVTW are introduced, depending on whether the time windows affect the nodes (TOPVTW–nodes) or both the nodes and the paths (TOPVTW–paths). These two alternatives have been described, and MIP formulations have been provided for each of them in Section 4. A MIP formulation for the TOPVTW–nodes with homogeneous teams is shown, which can be easily modified to describe heterogeneous teams. Also, two MIP formulations for the TOPVTW–paths are described. An implicit formulation lets the model define the shortest path between two consecutively managed nodes, whereas an explicit one includes a pre-processing phase that calculates these shortest paths – the model just selects one of them from the available ones. Only one implicit formulation is presented, no matter whether the teams are homogeneous or heterogeneous. The explicit formulation is provided for homogeneous teams, which can also be modified for the case of heterogeneous teams being involved.

The TOPVTW arises in the context of wildfire suppression, where the available resources need to be arranged to minimise the value of the area burned. Some experiments have been conducted to test the MIP models in landscapes of different sizes and features, where wildfires with various conditions have been simulated. The results of these experiments can be found in Section 5.

To the best of the authors' knowledge, this is the first paper describing the Team Orienteering Problem with Variable Time Windows which also includes some applications and alternatives, as well as different MIP formulations.

Given the large running times, the development of decomposition techniques or heuristics seem required, but they are out of the scope of this paper and left for future research. Also, the features of fire spreading are very particular, and further experiments might explore the models' behaviour when other spread processes are considered. Despite its limitations, this work establishes a starting point to continue exploring the topic.

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