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The implicit (un)healthy life expectancy used for pricing long-term care insurance and life care annuities

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Abstract

This paper examines the implicit healthy life expectancy (HLE) used for actuarial calculations in some selected biometric data sets from Australia, China, Portugal, Spain and the US. We are interested in checking the demographic and epidemiological coherence of these data sets because this health indicator is rarely presented when authors build their biometric data sets, nor when they are used to calculate long-term care insurance (LTCI) and life care annuity (LCAs) premiums, nor when they are employed in research articles to estimate the future demand for LTC services in a particular country.

Keywords: Activity limitations, Dependence states, Healthy life expectancy, Life care annuity, Long-term care insurance.

JEL Classification G22, G5, I13, J14, J26

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14-Mar-23

Abstract (English)

Background: This paper examines the implicit healthy life expectancy (HLE) used for actuarial calculations in some selected biometric data sets from Australia, China, Portugal, Spain and the US. We are interested in checking the demographic and epidemiological coherence of these data sets because this health indicator is rarely presented when authors build their biometric data sets, nor when they are used to calculate long-term care insurance (LTCI) and life care annuity (LCAs) premiums, nor when they are employed in research articles to estimate the future demand for LTC services in a particular country.

Methods: We follow a methodology based on multistate life table methods (MLTM) that enables us to obtain a life expectancy matrix for individuals on the basis of their initial health state. Multistate models provide an easy-to-apply procedure for life and health insurance contracts, including LTC. We also present some additional indicators of longevity, mortality and morbidity, these being the median age at death, the interquartile range, the weighted modal age at death, the mortality ratio, the implicit LTC prevalence rates and the survivorship curves broken down by health state.

Results: We find several weaknesses that highlight the difficulty involved in building the biometric data sets needed to make an actuarially fair valuation of the premiums for LTCI and LCAs. We also verify the existence of the so-called "male-female health-survival paradox", i.e. the fact that women have greater longevity than men but are also likely to spend proportionally more time in some state of dependence, irrespective of the initial health state.

Conclusion: It is not surprising that insurance companies are becoming less willing to offer LTCI and LCA contracts given that they face a serious problem with the biometric aspect when it comes to calculating actuarially fair premiums with any degree of certainty. From the perspective of a potential purchaser of this type of insurance product, disclosing and explaining the summary measures of health and longevity would make it easier for them to understand the need to protect themselves against the cost of possible LTC services and also make the computation of the premiums more transparent.

Keywords: Activity limitations, Dependence states, Healthy life expectancy, Life care annuity, Long-term care insurance.

JEL: G22, G5, I13, J14, J26

Resumen (español)

Antecedentes: En este artículo se examina la esperanza implícita de vida en buena salud (EVS) que se desprende de un conjunto de datos biométricos seleccionados de Australia, China, Portugal, España y Estados Unidos. Nos interesa comprobar la coherencia demográfica y epidemiológica de estas tablas de mortalidad/morbilidad porque este indicador rara vez se presenta cuando son construidas, ni cuando se utilizan para calcular las primas de los seguros de dependencia (LTCI) y las rentas con cobertura de dependencia (Life care annuities), ni cuando se emplean en artículos de investigación para estimar la demanda futura de servicios asociados a la dependencia en un país concreto.

Métodos: Se desarrolla una metodología basada en tablas de mortalidad/morbilidad multiestado (MLTM) que nos permite obtener una matriz de esperanzas de vida de los individuos en función de su estado de salud inicial. También presentamos algunos indicadores adicionales de longevidad, mortalidad y morbilidad como son, la mediana de la edad de fallecimiento, el recorrido intercuartílico, la edad modal ponderada de fallecimiento, el ratio de mortalidad, las tasas implícitas de prevalencia de la dependencia y las curvas de supervivencia desglosadas por estado de salud.

Resultados: Encontramos varios puntos débiles en los datos analizados que ponen de manifiesto la dificultad que entraña la construcción de los datos biométricos necesarios para realizar una valoración actuarialmente justa de las primas de los seguros de dependencia y las rentas vitalicias con cobertura de dependencia. También verificamos la existencia de la llamada "paradoja salud-supervivencia hombre-mujer", es decir, el hecho de que las mujeres tienen mayor longevidad que los hombres, pero también es probable que pasen proporcionalmente más tiempo en los estados de dependencia, independientemente del estado de salud inicial.

Conclusión: No es de extrañar que las compañías de seguros estén cada vez menos dispuestas a ofrecer contratos de seguros de dependencia y rentas vitalicias de dependencia, dado que se enfrentan a un grave problema con el aspecto biométrico a la hora de calcular primas actuarialmente justas con cierto grado de certeza. Desde la perspectiva de un potencial comprador de este tipo de producto de seguro, divulgar y explicar las medidas resumidas de salud y longevidad le facilitaría comprender la necesidad de protegerse contra el coste asociado a la situación de dependencia y también haría más transparente el cálculo de las primas.

Palabras clave: Esperanza de vida libre de discapacidad, Estados de dependencia, Limitaciones de la actividad, Renta vitalicia con cobertura de dependencia, Seguro de dependencia.

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The implicit (un)healthy life expectancy used for pricing long-term care insurance and life care annuities.

1.-Introduction

The development of health measures that include detailed information on both the mortality and morbidity conditions of the population has been a focus of study since the 1960s (Molla *et al*., 2001), when the concept of health expectancy was first introduced by Sanders (1964) and Sullivan (1965). Given that life expectancy is not inextricably bound to health, researchers and policy makers were searching for a population health indicator that would combine both mortality and morbidity (Saito *et al*., 2014) and could be used for assessing and monitoring population health.

Sullivan (1971) developed a method for combining mortality and morbidity rates in a single summary measure of population health known as disability-free life expectancy (DFLE), today also referred to as healthy life expectancy (HLE) or active life expectancy (ALE). Two new concerns then grew in importance in the 1980s: the relationship between changes in mortality and morbidity, and the relatively greater burden of morbidity in older ages (Molla *et al.*, 2001). In 1993 the DFLE measure became one of the health indicators used by the Organisation for Economic Co-operation and Development (OECD) (Di Lego, 2021).

Healthy life expectancy (HLE) typically combines mortality and morbidity information to represent overall population health in a single indicator (Imai and Soneji, 2007; Majer *et al*., 2013). It measures the number of remaining life years that a person of a certain age is likely to enjoy without activity limitations, assuming current disability and mortality conditions continue to apply, and is increasingly used to complement the conventional measure of life expectancy. HLE was developed to reflect the fact that people do not generally live in perfect health during all the years of their lives, and estimates of health expectancies were very attractive judging by the results obtained by popular tools for monitoring trends in population health.

It is difficult to ignore how important and useful summary measures for population health are because, among other things, they can describe life expectancy, perform cross-national health comparisons, quantify changes in quality of life (QoL) across a country or over time, contribute to decision-making for health policies, facilitate health planning, supply a comprehensive reference for epidemiological estimates and help guide research priorities (Murray *et al*., 2002). Findings from HLE analyses are increasingly being used to shape policies and programmes, not only in the health sector but also in areas such as pension policy and sustainable development (Bogaert *et al*., 2020). Such measures are also important in the actuarial field.

Long-term care (LTC) involves a range of services including medical and nursing care, personal care services, assistance services and social services that help people live either independently or in residential settings when they can no longer carry out routine activities on their own (Barber *et al*., 2021). The costs of caring are increasing since it is labour-intensive and benefits little from technological change. The traditional providers of LTC are the state, the market (private insurance) and the family (Klimaviciute and Pestieau, 2022). This paper deals with the private insurance aspect.

From the perspective of long-term care insurance (LTCI), the key indicators are HLE and life expectancy with activity limitation. LTCI collectively refers to the range of private insurance plans and public schemes that are intended to cover the costs of care for long-term disability resulting from a person's inability to perform the everyday activities of

daily living (SAS, 2020). LTCI is not life insurance, disability or health insurance and should not be confused with them, although they may sometimes be marketed together as hybrid products.

Under an LTCI scheme, a healthy insured person (contributor) pays a premium (or contributes) up to a certain age (usually the statutory retirement age) or until disability occurs, at which point they may receive regular benefit payments to cover the costs associated with LTC. Such monthly benefits are usually payable while the individual is care-dependent and will cease upon their death or recovery from disability or when the maximum payout limit, if any, is reached. These regular payments can take the form of cash (cash-for-care benefits which might be indexed to inflation) or in-kind benefits involving the partial or total disbursement of actual recurring LTC costs. According to the American Association for Long-Term Care Insurance (AALTCI, 2022) with reference to traditional LTCI reimbursement policies in the US in 2021, 67% of LTCI benefits end because the policyholder dies, 20% end because the individual 'recovers[1]', and 13% end because the policy benefits are exhausted (i.e. all the available benefits are used up).

A life care annuity (LCA) is a lifetime annuity in which the LTC benefit is defined in terms of an uplift with respect to the basic amount. In return for the payment of a premium (either in the form of a lump sum or an amount collected over time), the LCA provides a stream of fixed-income payments for the lifetime of the annuitant. It also provides an extra stream of payments if the annuitant requires LTC. In other words, an LCA is a combined annuity and LTCI policy in a single product (Pla-Porcel *et al.*, 2016; Vidal-Meliá *et al*., 2020). In the US it is also known as a long-term care annuity.

In this paper we are very interested in investigating the biometric assumptions made by the insurers/sponsors of LTCI schemes. The inclusion of information showing the breakdown of life expectancy by health state is invaluable for illustrating the actuarial calculations used for pricing LTCI and LCAs.

We examine the implicit healthy life expectancy (HLE) used for actuarial calculations in some selected biometric data sets from Australia, China, Portugal, Spain and the US. It is very difficult to find this health indicator explicitly presented in the actuarial literature on LTC, and even more difficult to find it for LCAs. It is for this reason that we check the demographic/epidemiological coherence of the biometric data sets in two ways. First, we use each data set to compute the life expectancy values for both healthy and dependent individuals and then for dependent people in each of the different states of dependence. We then verify the existence of the so-called “male-female health-survival paradox”, a phenomenon seen in developed countries where women have greater longevity but higher rates of disability and poorer health than men (Nusselder *et al*., 2019; Nielsen *et al*., 2021; Sherris and Wei, 2021). And second, we present some additional longevity, mortality and morbidity indicators, namely the median age at death, the interquartile range, the weighted modal age at death, the mortality ratio, the implicit LTC prevalence rates and the survivorship curves broken down by health state. Finally, we also want to find out how close the HLEs used for theoretically pricing these insurance products and the information used in academic research are to officially published figures and/or those used in similar investigations. In Europe, this indicator is calculated[2] annually by Eurostat

[1] They may make another claim at some time in the future.

[2] The variable used for health states in cases of disability:
Variable PH030 (limitation in the activities people usually do because of health problems for at least the last 6 months) in the EU Statistics on Income and Living Conditions (EU-SILC Survey). The EU-SILC question is: For at least the past six months, to what extent have you been limited because of a health

for EU countries and also for some European Free Trade Association countries (OECD, 2021; OECD/European Union, 2022).

To the best of our knowledge, our paper is original and makes a real contribution to the literature given that the demographic/epidemiological coherence of the selected biometric data sets has not yet been checked by looking at the life expectancy values for both healthy and dependent individuals and also for dependent people in each of the different states of dependence, and then following this up by presenting a number of additional longevity, mortality and morbidity indicators.

Our approach should enable us to answer the following research questions: What is the implicit HLE used for pricing LTCI and LCA contracts? Is this HLE in line with official published information and/or similar investigations? Does the male-female health-survival paradox hold true in the data sets analysed? Are the implicit summary measures for longevity and morbidity that are embedded in the biometric data sets coherent from a demographic/epidemiological point of view? Should it be compulsory to disclose information about the HLE and other longevity risk and morbidity indicators used in the technical bases for computing premiums for LTCI and LCA contracts?

The rest of the paper is structured as follows. Section 2 briefly describes the selected biometric data sets from Australia, China, Portugal, Spain and the US and the methodology used to compute the implicit healthy life expectancy (HLE) and check the demographic/epidemiological coherence of the data sets. Section 3 presents the main results. Section 4 discusses some issues arising from the results. We focus especially on the impact of the biometric assumptions on the pricing of LTCI and LCA contracts, the problem of increasing premiums (which is especially serious in the US LTCI market) and the advantages of LCAs for retirement planning given the uncertainty surrounding the need for LTC services and the costs involved. We also discuss whether it should be made compulsory to disclose information about the HLE and other longevity risk and morbidity indicators used in the technical bases for computing the premiums for LTCI and LCA policies. The paper ends with our concluding comments, future research possibilities and a technical appendix, which explains the methodology used to compute the life expectancy matrix based on the individual's initial health state. It also describes the mathematical expressions used to compute the additional indicators of longevity, mortality and morbidity mentioned earlier.

2.- Data and Methodology

2.1.-Data

In this section we briefly describe the biometric data sets analysed and explore their impact in the literature by examining the number of citations and the quality/ranking of the relevant journals. It is not our aim to include all the existing biometric data sets from all the countries we have selected. We could have included more, but for various reasons (difficulties in data collection, non-Markovian structure...) we decided not to.

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problem in activities people usually do? Would you say you have been: (a) severely limited? (b) limited but not severely? or (c) not limited at all?

On the basis of this variable, the proportions of population in healthy (answer code: "not limited at all") and unhealthy conditions (answer codes: "severely limited" and "limited but not severely") are calculated by sex and age. The comparability of the data on healthy life years is limited by the fact that the indicator is derived from self-reported information that can be affected by people's subjective assessment of their activity limitation (disability) and by social and cultural factors. There are also differences between countries in the formulation of the survey question on disability in the EU-SILC.

Table 1 summarizes the main features of the biometric data sets selected:

Table 1: Main features of the biometric data sets selected

Data set	Country	Data years	Levels of dependence	Gender distinction	Recovery from illness state	Citations
Artís *et al.* (2007)	Spain	1999	1	Yes	No	35
Albarrán-Lozano *et al.* (2021)	Spain	2008	4	Yes	No	0
Esquivel *et al.* (2021)	Portugal	2015	3	No	Yes	5
Robinson (1996)	US	1982-89	6	Yes	Yes	61
Friedberg *et al.* (2014)	US	1982-2010	6	Yes	Yes	56
Hariyanto *et al.* (2014a, 2014b)	Australia	1998-2003	4	Yes	Yes	40
Cui *et al.* (2022)	China	2014-17	2	Yes	Yes	0

Source: Own

2.1.1.-Spain

2.1.1.1.-Data from Artís *et al.* (2007).

To build their biometric data set the authors used information from the Spanish general population survey (1999) and prevalence rates from the Disability, Deficiency and Health Condition Survey conducted in 1999 by the Spanish National Statistics Institute (Instituto Nacional de Estadística - INE). The mortality rates for dependent people were calculated using data from the Society of Actuaries (2002). Only one level of dependence is considered and the figures are broken down by gender.

This biometric data set and/or the paper in which it was originally used has attracted the interest of numerous researchers. This can be seen from the number of citations in Google Scholar (35) and the importance of some of the journals in which they are found, e.g. the Astin Bulletin, the European Journal of Finance and Ageing International.

2.1.1.2.-Data from Albarrán-Lozano *et al.* (2021).

Using data from the EDAD 2008 survey conducted by the INE, Albarrán-Lozano *et al.* (2021) estimate transition probabilities for the dependent population. They consider five health states (four dependent and one exit state), namely:

1.- Dependent person with no entitlement to state benefits. 2.- Moderate dependence: a person who needs help to perform various basic activities of daily living (ADLs) at least once a day or periodically and/or needs limited support for their personal autonomy. 3.- Severe dependence: a person who needs help to perform various ADLs two or three times a day but does not need/want permanent help from a caregiver or extensive support services for their personal autonomy. 4.-Major dependence: a person who needs help with various ADLs several times a day and/or needs continued assistance from another person due to their total loss of physical, mental, intellectual or sensorial autonomy. And finally 5.-Death.

The authors present transition probabilities by gender and age.

At the time of writing this paper (March 2023), we were unable to find any citation in the literature for this biometric data set and/or the paper in which it was originally used.

2.1.2.-Portugal

2.1.2.1.- Data from Esquivel *et al.* (2021)

The transition probabilities of a Markovian multistate model are estimated using data from the 2015 Portuguese National Network of Continuous Care database (Esquivel *et al*, 2021). In this model the authors consider five health states: one autonomous, three dependent and one exit state, namely: 1.-autonomous, 2.-light dependence, 3.-moderate dependence, 4.-severe dependence and 5.-death. They do not provide much information about how individuals are assigned to each of these states of dependence. According to the authors: "*The choice of the five-state model is loosely justified by the widespread use of a reduced Barthel index, allowing for a general classification of elders in roughly three states of dependence, according to the performance achieved in their daily tasks*". Transition probabilities are not presented yearly by gender but for different non-uniform age intervals.

At the time of writing this paper we found five citations in the literature for this biometric data set and/or the paper in which it was originally used.

2.1.3.-USA

2.1.3.1.- Data from Robinson's (1996) care transition model (CTM)

The "life transitions" demonstrated by Chandler (2011) using mathematical software provided the basic data needed to compute the transition rates between the seven health states that make up Robinson's (1996) care transition model (CTM).

Based on data from the National Long-Term Care Survey (NLTCS) 1982-89, the seven health states contained in this CTM differ according to three variables: (a) the number of instrumental activities of daily living (IADLs) impaired, (b) the number of activities of daily living (ADLs) impaired, and (c) whether there is "cognitive impairment" (CI).

In state a, the individual has no impaired IADLs, no impaired ADLs and no cognitive impairment (i.e. they are able or healthy). In state d_1 the individual has one impaired IADL, no impaired ADLs and no CI. In state d_2, the individual has one impaired ADL and no CI. In state d_3 the individual has two impaired ADLs and no CI. In state d_4 the individual has three or more impaired ADLs and no CI. In state d_5 the individual has at most one impaired ADL but has CI. In state d_6 the individual has more than one impaired ADL and CI. In state 8 (f) the individual is dead.

Robinson's (1996) CTM and its derivatives have been used in the US insurance industry, and particularly in LTC insurance, to calculate premiums and reserves and to perform other essential computations (Chandler, 2011).

This biometric data set and/or the paper in which it was originally used has proven to be very popular in the academic literature. At the time of writing this paper, Robinson's (1996) CTM has a total of 61 Semantic Scholar citations, most of them in prestigious journals including the American Economic Review, the Journal of Public Economics, the Journal of Finance, Insurance: Mathematics and Economics, the Journal of Risk and Insurance and the Scandinavian Actuarial Journal.

2.1.3.2.-Data from Friedberg *et al*. (2014)

Friedberg *et al*. (2014) revised and updated Robinson's CTM by adding a linear time trend. They used data from the 1982-2004 National Long-Term Care Survey (NLTCS) and the 1998-2010 Health and Retirement Study (HRS). On the basis of their LTC

transition rate parameter estimate (Table 5 in their paper), a new set of health transition matrices were built.

This biometric data set and/or the paper in which it was originally used has a total of 56 Google Scholar citations, most of them in prestigious journals.

2.1.4.-Australia

2.1.4.1.-Data from Hariyanto *et al*. (2014a, 2014b)

Hariyanto *et al*. (2014a, 2014b) estimate transition probabilities between levels of disability as defined by the Australian Survey of Disability, Ageing and Carers, produced by the Australian Bureau of Statistics for the period 1998-2003. They assume that an individual in any state involving core activity limitation can only improve by one category over a one-year interval, if and only if they survive the year and do not deteriorate to a more severe state of dependence.

Their model provides five health states: one healthy and four dependent (with core activity limitation in their terminology). In state r, the individual is able or healthy with no activity limitation.

The four levels of core activity limitation are determined according to whether a person needs help with, has difficulty with or uses aids or equipment for any of the core activities (communication, mobility and self-care). A person's overall level of core activity limitation is determined by their highest level of limitation in these activities.

In state d_1, the individual has mild limitations. They need no help and have no difficulty with any of the core activity tasks, but they use aids and equipment and cannot easily walk 200 meters, cannot walk up and down stairs without a handrail, cannot easily bend to pick up an object from the floor, cannot use public transport, can use public transport but need help or supervision, or need no help or supervision but have difficulty using public transport. In state d_2, the individual has moderate limitations. They need no help but have difficulty with a core activity task.

In state d_3, the individual has severe limitations. They sometimes need help with a core activity task, have difficulty understanding or being understood by family or friends or can communicate more easily using sign language or other non-spoken forms of communication, have impaired ADLs and no cognitive impairment. In state d_4, the individual has profound limitations. They are unable to do, or always need help with, a core activity task. In state 5 (f) the individual is dead.

This biometric data set and/or the paper in which it was originally used has a total of 40 Google Scholar citations, most of them in prestigious journals including the Journal of Population Ageing, the Journal of Pension Economics and Finance, the Multiple Sclerosis Journal, Accounting & Finance and Population Studies.

2.1.5.-China

2.1.5.1.-Data from Cui *et al*. (2022)

Two types of data are used in this study. The first comes from the sixth and seventh waves of the Chinese Longitudinal Healthy Longevity Survey (CLHLS) in 2014 and 2017. The CLHLS follows the principle of strict random sampling and covers 23 of the 31 provinces in China. The second comes from the China Life Insurance Mortality Table, which is used to calculate the survival rate for all age groups.

To classify dependent people the authors use three indexes – ADLs, IADLs and CI – to define four health states corresponding to four levels: health (state 1), health impairment (state 2), disability (state 3) and death (state 4). State 1 means no health obstacle in any of the three indexes, state 3 is defined as having three or more obstacles in daily activities or scoring less than 16 in cognitive function, and the complementary state is defined as state 2.

Using a Markov model with piecewise constant transition probabilities, they provide one-year state transition matrices by gender and health state for various open-ended age intervals. These transition probabilities are a function of the age interval and unrelated to the initial age.

As far as we know at the time of writing this paper, this biometric data set and/or the paper in which it was originally used has not been cited in the literature.

2.2.-Methodology

2.2.1.- The life expectancy matrix based on the individual's initial health state

Healthy life expectancy (HLE) can be estimated using a variety of health attributes, and we should bear in mind that its values vary by definition, by measures of health and by methods of calculation (Saito *et al*., 2014). Health expectancy is defined as the "*general term referring to the entire class of indicators expressed in terms of life expectancy in a given state of health (however defined). Health expectancies are hypothetical measures and indicators of current health and mortality conditions. Health expectancies include both 'positive' and 'negative' health states, which may be defined in terms of impairment, disability, handicap, self-rated health, or other concepts. The sum of health expectancies in a complete set of health states should always equal total life expectancy*" (Robine, 2002).

Health expectancy is thus an analysis of both healthy and unhealthy years of life in which health can be defined across various dimensions. Life expectancy can also be divided into more than two health states, such as healthy years, mildly disabled and severely disabled years, as long as the states involved are mutually exclusive. Commonly used terms for healthy years are disability-free life expectancy, active life expectancy, healthy life years and healthy life expectancy (Saito *et al*., 2014).

The most widely used measure is disability, from which we get disability-free life expectancy (DFLE), the most classic of all healthy life expectancy indicators. The measurement of disability can vary, however, and indeed does vary considerably from one study to another. This means that comparability between some estimates and others, even when dealing with the same indicator, cannot be assured (Gutiérrez-Fisac *et al*. 2010).

Three commonly used methods for estimating DFLE (or "healthy life-years") are the Sullivan method, the multistate life table method (MLTM) and the double decrement method (DDM) (Imai and Soneji, 2007; Majer *et al*., 2013; Di Lego, 2021). However, they require different kinds of data and can yield different results (Barendregt *et al*. 1994).

The simpler Sullivan method estimates health expectancy by combining mortality data with external information on cross-sectional prevalence for each health state (Sullivan 1971). The key idea is to combine the period life table, which is the main way of calculating life expectancy, with the age-specific disability prevalence estimated from cross-sectional survey data. The Sullivan method simply breaks down the total number of person-years lived, which is obtained from the period life table, into disability years

and DFLE based on the proportions of time spent with and without disability, which is in turn measured from the cross-sectional disability survey (Imai and Soneji, 2007).

The actuarial approach used to compute the life expectancy of active people broken down into healthy and unhealthy life years is clearly linked to the multistate life table method (MLTM). Multistate models provide an easy-to-apply procedure for life and health insurance contracts, including LTC. Each state represents a particular state for the policy holder. The benefits considered in an LCA are associated with sojourns in or transitions between states (Denuit *et al*., 2019). The usual approach to the representation of LTC covers is multistate modelling, which is especially valuable for pricing LCAs.

The double decrement method (DDM) is a special type of MLTM where the only possible transition is from disability to death, and thus the probability of remission from any given health state is zero. It is used when the disability state is considered irreversible or the probabilities of recovery are negligible (Di Lego, 2021). In actuarial terminology this method is also known as the "inception and annuity model" (SAS, 2020).

A multiple state transition model applied to compute annuity rates for LTCI contracts describes a subject's movements within a set of various states: active (healthy) (a), disabled ($j \in \{1,2,\dots,n\}$-level dependent) ($d_j, j \in \{1,2,\dots,n\}$) and deceased ($f$).

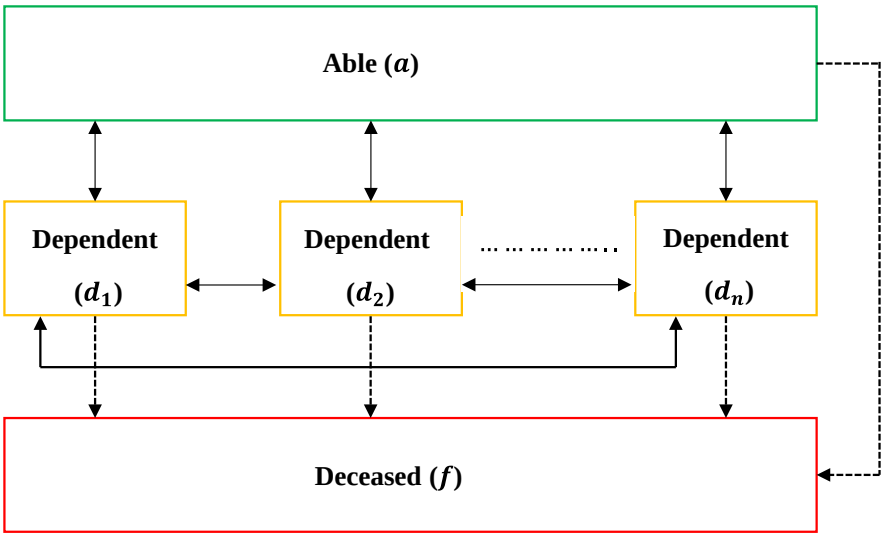
This discrete-time Markov multistate model takes into account n levels of dependence where no more than one transition in a year is assumed, with level 1 being the least severe and level n the most. The individual's health dynamics depend on the reversible illness-death (RID) model, in which transitions are modelled from the initial healthy state to the absorbing death state. The RID model allows recovery from the illness state.

Considering time to be discrete has the advantage of simplifying the inferential procedures for processes with time-dependent transition probabilities because it is easier to deal with matrix multiplication than with differential or difference equations with non-homogeneous coefficients (Lièvre *et al*., 2003).

Although the Markov model (like any other) might be accused of oversimplification, for models that use variations in state space and intensities, the Markov set-up can deal with extremely complex phenomena (Norberg, 2002).

Figure 1 shows the transitions into and out of the various states that will be considered in the model for computing life expectancies based on a multistate model for valuing LCAs/LTCI with n levels of disability (Haberman and Pitacco, 1999; Pitacco, 2014; Pla-Porcel *et al.*, 2017). The relationships denoted by the dotted lines in the diagram represent the different paths to the absorbent state.

Figure 1: A multistate model for valuing LCAs/LTCI with n levels of disability



Source: Own

From an actuarial point of view, the model for calculating annuity factors can be seen as a multistate non-homogeneous discrete Markov process, $\{S(t), t \in \mathbb{Z}^+\}$, where $S(t)$ is the random variable that represents the current state of the process at time t with values in a finite state space, $\mathcal{S} = \{a, d_1, \dots \dots, d_n, f\}$, with just one state at any time along with a set of direct possible unidirectional ordered pair transitions:

$$\mathcal{T} = \left\{ \begin{array}{c} (a, d_1), .. (a, d_n), (a, f), \\ (d_1, a), \dots, (d_1, d_n), (d_1, f), \\ (d_2, a), \dots (d_2, d_n), (d_2, f), \\ \dots \dots \dots \dots \dots \dots \dots \\ (d_{n-1}, a), (d_{n-1}, d_1), \dots, (d_{n-1}, f), \\ (d_n, a), (d_n, d_1) \dots (d_n, f) \end{array} \right\}$$

The process is non-homogeneous if we take into account the age of the individuals.

In this type of model the transition probabilities depend only on the current state of the process. Therefore pair $(\mathcal{S}, \mathcal{T})$ is the multiple state model used. It can be said that this framework modelled using Markov assumptions makes it easy to compute relevant probabilities and expected values (Norberg, 2002).

In short, this methodology (see the Technical Appendix) enables us to obtain the following life expectancy matrix based on the individual's initial health state (Table 2).

The diagonal line of shaded cells in Table 2 shows the life expectancy likely to be lived in that same (initial) health state. The last column shows the individual's total life expectancy linked to the initial health state, i.e. each cell in this column is the sum of the cells in its corresponding row.

Table 2: Life expectancies at age x broken down into health states						
Initial health state	**States**					**Total**
	$\boldsymbol{a}$	$\boldsymbol{d_1}$	$\boldsymbol{d_2}$	… … ….	$\boldsymbol{d_n}$	
$\boldsymbol{a}$	e_x^{aa}	$e_x^{ad_1}$	$e_x^{ad_2}$	… … ….	$e_x^{ad_n}$	e_x^{a}
$\boldsymbol{d_1}$	$e_x^{d_1a}$	$e_x^{d_1d_1}$	$e_x^{d_1d_2}$	…	$e_x^{d_1d_n}$	$e_x^{d_1}$
$\boldsymbol{d_2}$	$e_x^{d_2a}$	$e_x^{d_2d_1}$	$e_{x+k}^{d_2d_2}$	… … ….	$e_x^{d_2d_n}$	$e_x^{d_2}$
… …	… ..	… ..	… ..	… … ….	… ..	… ..
$\boldsymbol{d_n}$	$e_x^{d_na}$	$e_x^{d_nd_1}$	$e_x^{d_nd_2}$	… … ….	$e_x^{d_nd_n}$	$e_x^{d_n}$
Source: Own						

where:

e_x^a is the life expectancy for active people aged (x). This can be broken down into the health states (active or dependent) they can expect to experience. It should be stressed that this relationship is only true at the initial age.

e_x^{aa} is the dependence-free life expectancy (or "healthy life years"). This indicates how many years of their total remaining life an active person aged (x) can expect to live free of activity limitation.

$e_x^{ad_j}$ is the life expectancy for a $j \in \{1,2,..n\}$-level dependent person. This can be defined as the number of years an active person aged (x) can expect to spend with $(j) \in \{1,2,..n\}$-level activity limitation.

$e_x^{d_j}$ is the life expectancy for a $j \in \{1,2,..n\}$-level dependent person aged (x). This can be broken down into the health states (active or dependent) that they can expect to experience. It should be stressed that this relationship is only true at the initial age.

$e_x^{d_id_i}$ indicates how many years of their total remaining life expectancy a person aged (x) in state (i) of dependence can expect to live in the same state of dependence.

$e_x^{d_ia}$ is the life expectancy expected to be spent free of activity limitation for a person aged (x) in state (i) of dependence.

2.2.2.- Other lifespan variation indexes: median age at death, the interquartile range (IQR) and the (weighted) adult modal age at death.

There are several indexes for calculating lifespan variation and each has different underlying properties (Van Raalte and Caswell, 2013). Median and modal ages at death are seldom proposed as measures for studying longevity. The mean age at death, or life expectancy, is generally preferred. However, all three measures show central tendencies and are therefore important. They complement one another with information on the "centre" of the distribution of deaths. (Canudas-Romo, 2010).

The median age at death, Md, is the age when half of the hypothetical cohort members have died, i.e. when the number of people surviving to the exact age of x (l_x) in any health state is equal to half the initial cohort aged x_e, $l_{Md} = \frac{l_{x_e}}{2}$. In our case the cohort is aged 65 or older and the health state is able (or with j-level activity limitation).

The interquartile range (IQR), also known as the middle 50% (Wilmoth and Horiuchi, 1999), is a measure that equals the distance between the lower and upper quartiles of the

age distribution of death in a life table. The measure decreases as age at death becomes less variable. The IQR has a twofold appeal as a single measure of variability in the life table. Firstly, it is very simple to calculate because it equals the difference between the ages where the survivorship curve, S(x), crosses 0.25 and 0.75, and secondly, it is easy to interpret as it is the length of the span of ages containing the middle 50% of deaths.

The adult modal age at death, *M*, i.e. the age (beyond infancy) at which the largest single number of deaths occur, is used as an indicator to analyse mortality disparities at older ages. The mode is both a natural measure of the length of life and a good basis for measuring its dispersion (Kannisto, 2001). Under a given mortality regime, *M* represents the most common or "typical" length of life among adults (Diaconu *et al*, 2022).

When the individuals that form part of a group can exit for reasons other than death (i.e. move from one group to another) as in our case, an improvement or worsening of their state of health is considered, and therefore the previous indicator needs to be modified. Given a starting age and an initial health state (able or disabled), there is a mode for each state ($M_{x_e}^{ij}$). By taking into account the percentages of deaths in the various states, we get the weighted adult modal age at death ($\bar{M}_{x_e}^{i}$). In our setting this new indicator is more consistent and informative than the unweighted adult modal age at death, especially in relation to life expectancy and the median.

Life expectancy statistics are hardly ever presented as interval estimates (i.e. with confidence bounds). For national-level populations, life expectancy is very accurately estimated. Apart from the smallest countries, population size and deaths are large enough to ensure that the confidence intervals are so narrow that they become unhelpful and can therefore be omitted (Deville *et al*., 2015). Given that the biometric data we are going to use later in this paper do not provide information about deaths and exposures to risk, we cannot properly calculate the confidence intervals of life expectancy (Hanley, 2022).

2.2.3.- Further indicators of mortality and morbidity.

There are a number of other mortality and morbidity indicators that are very interesting from an LTC perspective when analysing biometric data sets. They can give us additional information about how realistic the data set is.

We show the implicit LTC prevalence rate[3], λ_{x+k}^{j} – which is the ratio between the number of dependent people in dependence level j and the number of individuals aged $x+k$ – and the average LTC prevalence rate, $\bar{\lambda}_{x}^{j}$. The total prevalence rate is a key element in the public field, where there is a need to assess how the demand and cost burden of LTC will evolve over the coming years under a given set of biometric assumptions.

We also explore the mortality ratio, δ_{x+k}^{j}, for dependent people aged $x+k$ in dependence level j. This is the ratio between the mortality rates for dependent people and active people and the average mortality ratio ($\bar{\delta}_{x}^{j}$). Generally speaking, disabled people have a lower life expectancy than active people, but the difference in longevity tends to decrease notably as individuals get older. However, the real situation is much more complex given that the mortality of disabled people basically depends on the cause and severity of their disability (Pitacco, 2014; Biessy, 2017).

Finally, we also show the family of survivorship curves (Mathers, 2002; Cheung *et al*., 2005; Ebeling *et al*., 2018) so as to provide a visual representation of the multistate life

[3] From an epidemiological point of view, this can be defined as the ratio between the number of cases of a disease divided by the number of exposures to risk in a specific population over a particular period.

tables. A survivorship curve indicates, for each age along the x-axis, the proportion of an initial cohort that will still be alive at that age. The area below the curve represents life expectancy. In our case we will show several survivorship curves indicating life expectancy at a given age broken down by health state.

3.-Results

The procedure followed in this section is:

1.-The results are presented by country/authors and summarized in tables or graphs.

2.-The consistency of the results is studied via a series of questions: Is life expectancy in line with what would be expected according to official data and/or other similar studies? Is the percentage of time expected to be spent in a dependent health state reasonable? Is the mortality of dependents realistic? Is the life expectancy according to health state consistent? Is the male-female health-survival paradox fulfilled? Are the resulting age-specific prevalence rates consistent with the empirical evidence?

3.-The main weaknesses are highlighted.

3.2.1.-Spain

The biometric data from Artís *et al*. (2007) (Tables 3 and 4 and Figures 2, 3 and 4) have a standard structure relating to life expectancy broken down by gender for both the active and the dependent population. Life expectancy in both cases is higher for women than for men, and the amount of time that an initially active individual might expect to spend in a dependent state is also higher for women than for men. Life expectancy in 1999 for active individuals at age 65 was higher than that for the general population by a margin of 1.85 and 1.48 years for men and women respectively. The amount of time likely to be spent in a dependent state (22.93% and 26.89% for men and women respectively) is much lower than that calculated using the data in Gutiérrez-Fisac *et al*. (2010) (29.94% and 38.86% for men and women respectively for 1999).

Table 3: Life expectancy matrix in years at age 65 and percentage of life expectancy likely to be spent in each health state

Gender	**Males**			**Females**		
States	$\boldsymbol{a}$	$\boldsymbol{d_1}$	**Total**	$\boldsymbol{a}$	$\boldsymbol{d_1}$	**Total**
$\boldsymbol{a}$	13.89	4.13	18.02	15.85	5.83	21.68
	77.07%	22.93%	100%	73.11%	26.89%	100%
$\boldsymbol{d_1}$	---	8.03	8.03	---	13.74	13.74
	---	100%	100%	---	100%	100%

Source: Own based on data from Artis *et al.* (2007)

In this case the structure of longevity for men and women is notably different for dependent people. For men, the mode ($\bar{M}^i_{65}$) value is surprisingly low. The mortality ratio (between 7 and 9) is in line with that reported by the SOA (2002).

Table 4: Further indicators of mortality and morbidity

Gender	**Males**		**Females**	
Indicators/states	$\boldsymbol{a}$	$\boldsymbol{d_1}$	$\boldsymbol{a}$	$\boldsymbol{d_1}$
$\boldsymbol{A^i}$ $\boldsymbol{(= 65 + e^i_{65})}$[4] (years)	83.02	73.03	86.68	78.74
$\boldsymbol{Md^i}$ (years)	83.27	71.66	87.28	78.34
$\boldsymbol{Q^i_3}$ (years)	88.35	76.76	91.66	84.80
$\boldsymbol{Q^i_1}$ (years)	77.74	67.97	82.32	72.01
$\boldsymbol{IQR^i_{65}}$ (years)	10.61	8.79	9.34	12.79
$\boldsymbol{\bar{M}^i_{65}}$ (years)	84.20	65.95	88.35	77.83

[4] The average age at death for people still alive at age 65.

Table 4: Further indicators of mortality and morbidity				
Gender	**Males**		**Females**	
Indicators/states	$\boldsymbol{a}$	$\boldsymbol{d_1}$	$\boldsymbol{a}$	$\boldsymbol{d_1}$
$\bar{\lambda}^{i}_{65}\%$	77.7	22.3	73.74	26.26
$\bar{\delta}^{d}_{65}\%$	---	8.75	---	7.28
Source: Own based on data from Artis *et al.* (2007)				

The result of the interquartile range ($\boldsymbol{IQR^{i}_{65}}$) seems to indicate that the female group of dependent individuals has greater heterogeneity because, despite the fact that life expectancy at age 65 falls from 21.68 to 13.74 years when moving from active to dependent, the interquartile range value increases from 9.34 to 12.79 years. This is not the same as the pattern observed for men, where the transition from active to dependent brings a decrease in both life expectancy and the interquartile range.

The following figures will help us to verify the demographic coherence and different behaviours in mortality and morbidity for both men and women.

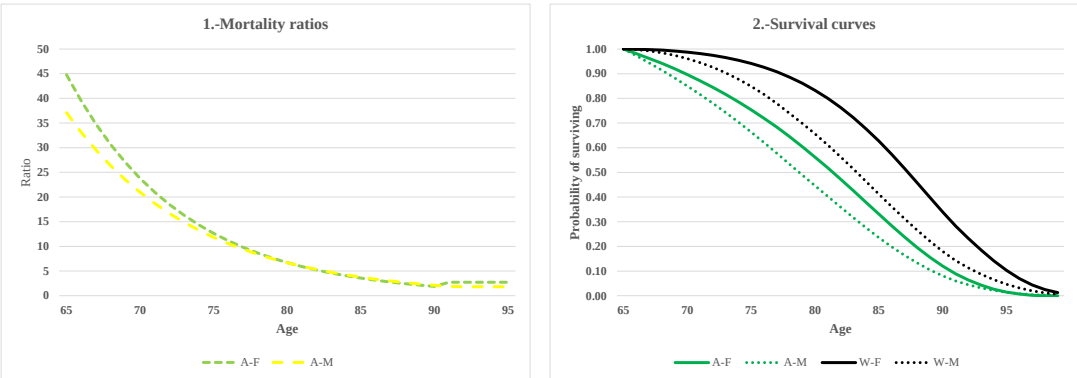


Figure 2: Mortality ratios and survivorship curves by health state
Source: Own based on data from Artís *et al.* (2007)

Graph 1 in Figure 2 shows mortality ratios by age and gender (green curve for females and yellow for males). The mortality ratio (dependent/healthy population) shows that dependent people have a higher (much higher) yearly probability of dying than healthy people, but the difference tends to decrease notably as people get older.

Graph 2 in Figure 2 shows survivorship curves by age, gender and initial health state. Active females are represented by the solid green line and males by the dotted green line. Black curves represent the total population (active and dependent together). As mentioned above, the area below the survivorship curve represents life expectancy. The difference between the two curves (for the same gender) expresses the number of years that an active individual of a given age will spend in a dependent state.

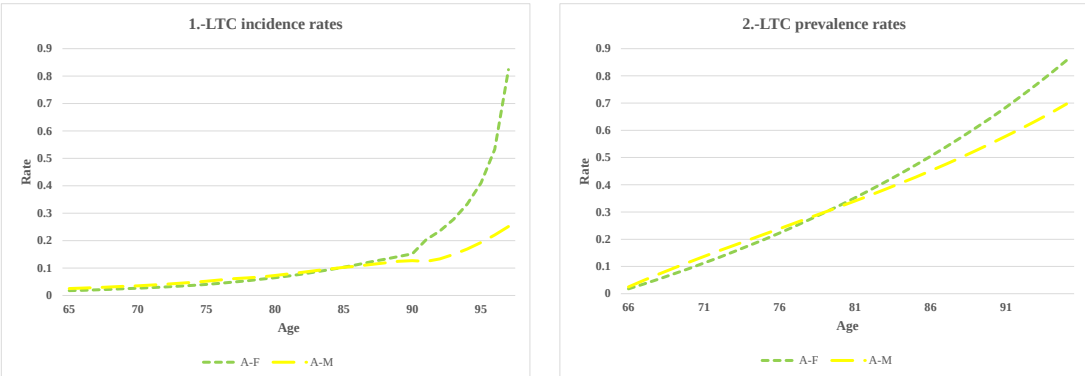


Figure 3: LTC morbidity rates.
Source: Own based on data from Artís *et al.* (2007)

Graph 1 in Figure 3 shows the LTC incidence rates[5] for active people by age and gender (green for females, yellow for males). Both rates increase with age, but the patterns are different. At lower ages the incidence rates are higher for men than for women, but after a certain age – in this case 85 – the trend reverses and the rates become much higher for women than for men.

Graph 2 in Figure 3 shows the specific LTC prevalence rates by age that would result from combining the mortality and incidence rates. As expected, and given that there is no possibility of recovery, the rates are particularly high for the very elderly. Up to age 79 the prevalence rates would be higher for men than for women.

As Table 4 indicates, between 22.3% (males) and 26.3% (females) of the population aged 65 and over would be receiving LTC benefits according to the hypothetical (public) pension scheme.

Figure 4 shows the evolution of the synthetic population by age and health states, with Graph 1 for women and Graph 2 for men. The proportion of dependent females (dotted red lines) by age would reach a peak of nearly 30% (21% for males) of the initial group of active people for those aged 84 (80 in the case of males). The number of dependent people would exceed the number of active individuals for age groups 86 and 89 onwards for females and males respectively.

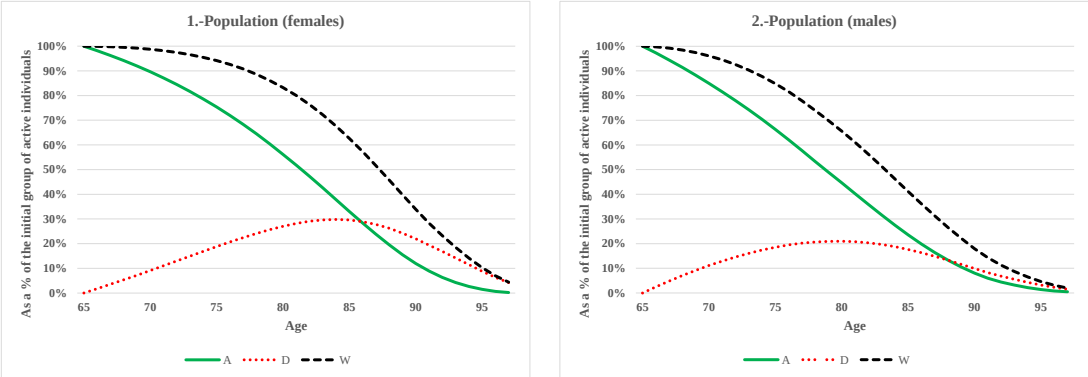


Figure 4: Comparison of population by health state and gender.
Source: Own based on data from Artís *et al.* (2007)

Next we analyse the biometric data from Albarrán-Lozano *et al*. (2021) (Tables 5, 6 and 7).

Table 5: Life expectancy matrix in years at age 65 and percentage of life expectancy likely to be spent in each health state

States	Males					Females				
	d_1	d_2	d_3	d_4	T	d_1	d_2	d_3	d_4	T
d_1	7.16	0.53	0.36	1.06	9.11	6.26	0.53	0.34	0.87	8
	78.59%	5.84%	3.93%	11.63%	100%	78.28%	6.62%	4.19%	10.91%	100%
d_2	---	4.51	0.53	2.06	7.1	---	6.36	0.85	3.54	10.75
	---	63.51%	7.46%	29.02%	100%	---	59.16%	7.9%	32.94%	100%
d_3	---	---	3.71	2.38	6.09	---	---	5.4	2.92	8.32
	---	---	60.87%	39.13%	100%	---	---	64.88%	35.12%	100%
d_4	---	---	---	5.39	5.39	---	---	---	6.99	6.99
	---	---	---	100%	100%	---	---	---	100%	100%

Source: Own based on data from Albarrán-Lozano *et al.* (2021)

[5] The ratio of new cases of a disease divided by the number of exposures to risk in a specific population over a particular period.

At first glance, the demographic parameters that result from this data set appear to be fairly coherent given the amount of time dependent people will spend in the same state. However, there is one piece of information that seems to be clearly inconsistent, insofar as women with a d_1 activity level limitation have a lower life expectancy at age 65 (8.00 years) than those with a d_2 activity level limitation (10.75 years). This is not so in the case of men.

It is also striking that for men with a d_1 activity level limitation, life expectancy at age 65 is greater (9.11 years) than for women (8 years) with the same level of dependence.

The data in Table 6 confirm this inconsistency.

Table 6: Further indicators of mortality and morbidity

Indicators/states	Males				Females			
	d_1	d_2	d_3	d_4	d_1	d_2	d_3	d_4
A^i (years)	74.11	72.1	71.09	70.39	73	75.75	73.32	71.99
Md^i (years)	73.1	70.91	69.79	69.04	72.28	75.6	71.76	70.38
Q_3^i (years)	78.29	75.28	73.78	72.74	75.78	81	77.03	74.81
Q_1^i (years)	68.93	67.73	67.11	66.73	69.14	69.18	68.06	67.40
IQR_{65}^i (years)	9.36	7.55	6.67	6.01	6.64	11.82	8.97	7.41
$\bar{M}_{65}^i$ (years)	70.53	68.92	68.35	65.88	72.38	72.08	68.81	65.93
$\bar{\lambda}_{65}^i$%	79.7	5.54	3.73	11.03	79.46	6.23	3.94	10.27
$\bar{\delta}_{65}^d$%	3.8	3.74	4.69	4.66	12.27	2.81	6.91	6.87

Source: Own based on data from Albarrán-Lozano *et al.* (2021)

Finally, the two graphs presented in Figure 5 below will help us to better understand the situation.

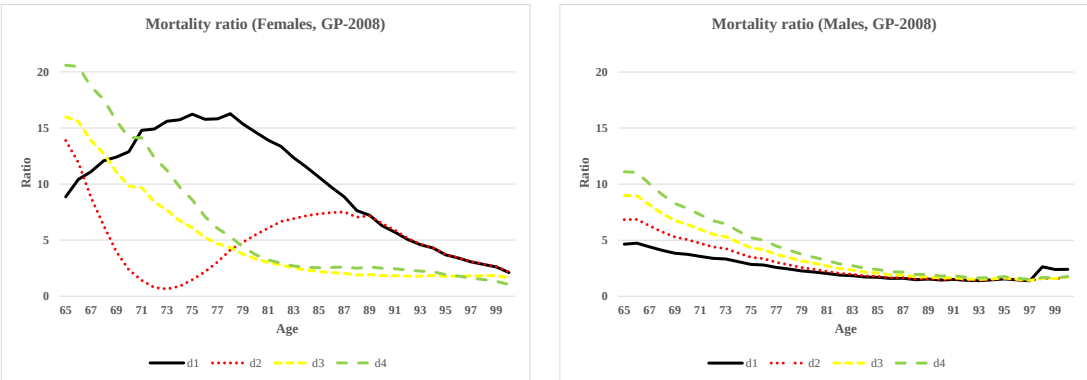


Figure 5: Comparison of mortality ratios: dependent people/general population.
Source: Own based on Albarrán-Lozano *et al.* (2021)

Figure 5 shows the mortality ratios (Graph 1 for females and Graph 2 for males). In this case the ratio between the dependent and the general population shows a pattern for females – especially for d_1 and d_2 – that reveals anomalies that are difficult to explain or justify. For dependence level 1 (d_1), the mortality ratio presents irregularities and increases until approximately age 79, then it practically coincides with level d_2 from age 88. The behaviour of the ratio for d_2 could be labelled as erratic, with a very pronounced decrease during the early years, reaching values of less than one for the age range 72-74. Then there comes an inexplicable growth phase until age 87, followed by another decrease in the ratio. As can be seen in Graph 2, the profile of the mortality ratio for men is fairly standardized except in the later ages, and the ratio decreases according to the age reached.

Figure 6, which partially reproduces the mortality data shown in Table 6, helps us to understand the different behaviour observed in both the female group (Graph 1) and the

male group (Graph 2). It enables us to clearly visualize the information provided in Table 6 relating to the interquartile range (IQR), also known as the middle 50%. While in Graph 2 the representation might be considered to fall within the limits of what could be expected, i.e. that an increase in the level of dependence reduces both life expectancy and the IQR, in Graph 1 this is not true for level d_2, where both life expectancy and the IQR increase, indicating more variability and uncertainty.

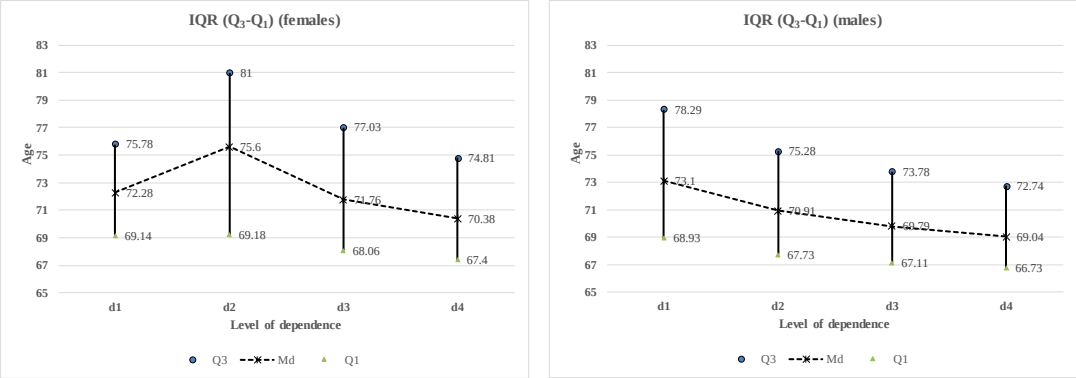


Figure 6: IQR_{65}^{d}.

Source: Own based on data from Albarrán-Lozano *et al*. (2021)

3.2.2.-Portugal

The biometric data from Esquivel *et al*. (2021) (Tables 7 and 8) show both a lower life expectancy for the active population (12.78 years) and a relatively small percentage of time that they can expect to spend in a state of dependence (36.37%) compared to the data provided by Eurostat (2022) for the general population. For 2015, which is the data reference year for compiling the homogeneous probability transition matrices, Eurostat (2022) estimates a life expectancy of 20 years with 69.50% of total life expectancy spent in poor health[6].

Table 7: Life expectancy matrix in years at age 65 and percentage of life expectancy likely to be spent in each health state

States	a	d_1	d_2	d_3	Total
a	7.8	3.45	0.81	0.72	12.78
	61.03%	26.99%	6.31%	5.67%	100%
d_1	4.89	5.05	0.92	0.92	11.78
	41.52%	42.91%	7.78%	7.79%	100%
d_2	3.19	3.02	2.1	1.58	9.89
	32.31%	30.51%	21.19%	15.99%	100%
d_3	1.91	1.98	0.99	3.22	8.1
	23.56%	24.4%	12.29%	39.75%	100%

Source: Own based on data from Esquivel *et al*. (2021)

Table 8: Further indicators of mortality and morbidity

Indicators/states	a	d_1	d_2	d_3
A^i (years)	77.78	76.78	74.89	73.09
Md^i (years)	77.8	76.22	73.26	70.63
Q_3^i (years)	83.53	82.83	80.94	78.16
Q_1^i (years)	71.7	70.17	68.07	66.95

[6] The comparisons made throughout the Eurostat study should be considered with great caution, since poor health is not necessarily equivalent to being dependent or having activity limitations.

Table 8: Further indicators of mortality and morbidity				
Indicators/states	$\boldsymbol{a}$	$\boldsymbol{d_1}$	$\boldsymbol{d_2}$	$\boldsymbol{d_3}$
$\boldsymbol{IQR^i_{65}}$ (years)	11.83	12.66	12.87	11.21
$\boldsymbol{\bar{M}^i_{65}}$ (years)	78.33	74.69	69.02	68.45
$\boldsymbol{\bar{\lambda}^i_{65}}$%	59.45	28.09	6.56	5.9
$\boldsymbol{\bar{\delta}^d_{65}}$%	---	1.42	2.91	4.52
Source: Own based on data from Esquivel *et al.* (2021)				

Esquivel *et al*. (2021) calculate through simulation that the life expectancy at 65 of active individuals is 15.21 years and compare this figure to that of the general population in Portugal for 2015. They admit that it is much lower but justify it by saying that "*The population in the RNCCI system is mostly a population with comorbidities, which induces higher mortality and, as a consequence, reduces the life expectancy. We remark that we are considering a sub-population of Portuguese individuals that, during 2015, used the National Network of Continuing Care*." Certainly the justification would be valid for dependent individuals but not for active individuals, and our figure is almost 2.5 years lower than that calculated by the authors.

As far as the life expectancy of people in states of dependence is concerned, we do not have the necessary data to estimate whether the figure given would fall within what might be considered acceptable, but we can at least say that it has a certain coherence since the higher the level of activity limitation the lower the life expectancy. However, the very low percentage of time that dependent people with activity limitation d_3 would be expected to spend in that state (39.75%) is surprising, when the rest of the time would be spent in states with less limitation. Indeed, for 23.56% of the time they would be considered active. This is clearly not in line with what has been observed for other European countries, where the possibility of recovery is generally disregarded given the prevailing chronic character of LTC disability in most of the severe states of dependence (see for example Van der Gaag *et al*., 2013; Pitacco, 2014 and 2016; Albarrán-Lozano *et al*., 2017). More recently, using Swiss data, Fuino and Wagner (2018) reported very low probabilities (less than 0.05%) of recovery transition.

A similar observation could be made considering populations with levels of activity limitation d_2 and d_1.

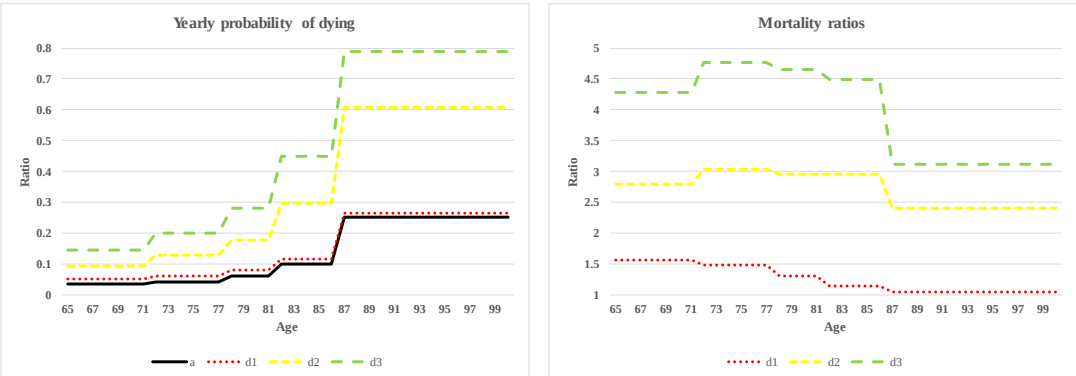


Figure 7: Yearly probability of dying and mortality ratios
Source: Own based on data from Esquivel *et al.* (2021)

Figure 7 shows the yearly probability of dying by age and health state (Graph 1) and the mortality ratios (Graph 2). The yearly probability of dying is increasing by age for all the health states considered, and the higher the level of dependence, the higher the probability of dying by age.

In general it can be said that the mortality ratios are relatively low (between 1.42 and 4.52) and irregular and that the ratios corresponding to dependence levels d_2 and d_3 present a pattern that is difficult to explain, as can be seen in Figure 7, Graph 2.

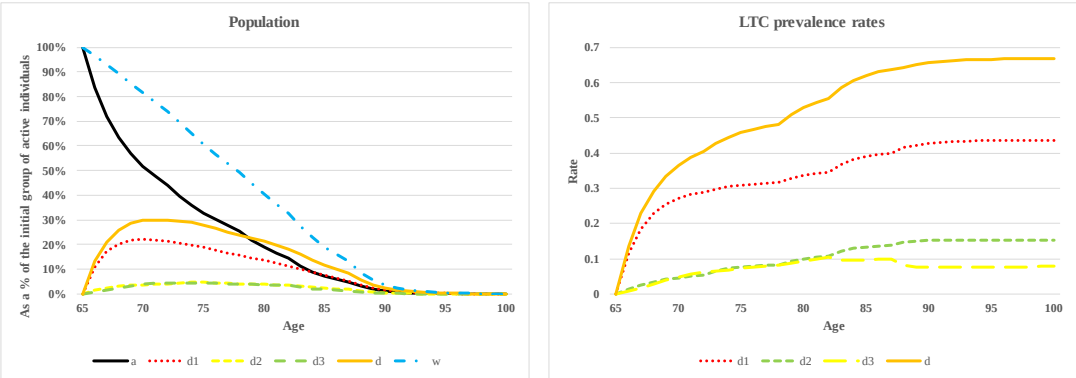


Figure 8: Population and LTC prevalence rates by health state
Source: Own based on data from Esquivel *et al.* (2021)

Figure 8 shows the evolution of the synthetic population (Graph 1) and the LTC prevalence rates (Graph 2), both of them by age and health states. According to Graph 1, the proportion of dependent people (solid orange line) by age would reach a peak at around 30% of the original group of active individuals (at age 65) once they reach age 79, when dependent individuals would outnumber active. For the entire range of ages considered, the group with the most individuals is always the one with the lowest level of dependence d_1. As can be seen in Graph 2, the prevalence rates for the most advanced ages would be relatively high (solid orange line), more than 60% from age 84, but still above 40% for age 88 and upwards (dotted red line) in the case of those with dependence level d_1. All this is clearly related to the comments made earlier regarding the high recovery rates observed.

3.2.3.-USA

Tables 10 and 11 give detailed information about life expectancy broken down into the various health states, while Tables 12 and 13 add further mortality and morbidity indicators. The diagonal of the matrices (shaded cells) embedded in the tables shows life expectancy in years and how much of it is likely to be spent in that same health state. The expected number of years allocated to the possible range of health states depends on the starting state by age. The data shown for the life expectancy of active individuals (15.35 years and 19.02 years respectively for men and women) almost coincide with those calculated by Crimmins *et al*. (2016). However, there is a sizeable divergence in the time that active individuals are expected to spend in a state of dependence. In Robinson's model (1996), active life expectancy is 79.14% and 72.22% of total life expectancy for males and females respectively, whereas Crimmins *et al*. (2016) estimate 49.06% and 52.10% for males and females respectively.

Table 10: Life expectancy matrix in years at age 65 and percentage of life expectancy likely to be spent in each state (males)

States	a	d_1	d_2	d_3	d_4	d_5	d_6	Total
a	12.15	1.85	0.30	0.12	0.33	0.36	0.24	15.35
	79.14%	12.06%	1.98%	0.75%	2.12%	2.37%	1.57%	100%
d_1	4.72	5.87	0.72	0.30	0.93	0.64	0.61	9.88
	37.35%	45.02%	4.62%	1.79%	4.40%	4.16%	2.66%	100%
d_2	3.13	3.50	1.12	0.21	0.51	0.39	0.26	9.12
	34.26%	38.42%	12.28%	2.26%	5.64%	4.32%	2.82%	100%
	2.11	2.62	0.64	1.12	0.83	0.33	0.32	7.95

Table 10: Life expectancy matrix in years at age 65 and percentage of life expectancy likely to be spent in each state (males)

States	a	d_1	d_2	d_3	d_4	d_5	d_6	Total
d_3	26.48%	32.89%	7.99%	14.05%	10.38%	4.17%	4.03%	100%
d_4	0.81	1.12	0.30	0.36	2.34	0.17	0.33	5.44
	14.94%	20.64%	5.60%	6.61%	42.95%	3.14%	6.12%	100%
d_5	2.15	2.63	0.55	0.28	0.80	1.53	0.44	7.95
	25.65%	31.45%	6.54%	3.36%	9.52%	18.26%	5.25%	100%
d_6	0.68	0.94	0.25	0.26	1.46	0.24	1.30	5.13
	13.29%	18.33%	4.93%	5.02%	28.43%	4.69%	25.30%	100%

Source: Own based on data from Robinson's (1996) care transition model (CTM)

Table 11: Life expectancy matrix in years at age 65 and percentage of life expectancy likely to be spent in each state (females)

States	a	d_1	d_2	d_3	d_4	d_5	d_6	Total
a	13.76	2.69	0.51	0.21	0.72	0.58	0.59	19.02
	72.22%	14.14%	2.67%	1.08%	3.79%	3.02%	3.09%	100%
d_1	4.72	5.87	0.72	0.30	0.93	0.64	0.61	13.79
	34.22%	42.57%	5.19%	2.18%	6.77%	4.67%	4.40%	100%
d_2	4.12	4.90	1.41	0.34	1.05	0.63	0.60	13.04
	31.55%	37.58%	10.84%	2.59%	8.05%	4.79%	4.59%	100%
d_3	2.91	3.84	0.93	1.33	1.50	0.55	0.68	11.72
	24.80%	32.79%	7.90%	11.30%	12.78%	4.66%	5.77%	100%
d_4	1.30	1.96	0.53	0.56	3.44	0.33	0.69	8.83
	14.73%	22.25%	6.03%	6.39%	38.97%	3.78%	7.86%	100%
d_5	2.90	3.80	0.81	0.43	1.48	1.82	0.82	12.06
	24.01%	31.49%	6.76%	3.60%	12.23%	15.08%	6.83%	100%
d_6	1.12	1.71	0.47	0.44	2.50	0.41	1.79	8.44
	13.28%	20.29%	5.51%	5.25%	29.61%	4.90%	21.16%	100%

Source: Own based on data from Robinson's (1996) care transition model (CTM)

Table 12: Further indicators of mortality and morbidity (females)

Indicators/states	a	d_1	d_2	d_3	d_4	d_5	d_6
A^i (years)	84.05	78.79	78.04	76.72	73.83	77.06	73.44
Md^i (years)	84.67	77.76	76.75	74.77	70.99	74.97	70.83
Q_3^i (years)	90.80	85.93	85.54	83.56	78.62	83.65	77.76
Q_1^i (years)	77.78	70.81	69.90	68.94	67.68	69.74	67.61
IQR_{65}^i (years)	13.02	15.12	15.64	14.62	10.94	13.91	10.15
$\bar{M}_{65}^i$ (years)	84.73	72.76	72.05	68.48	67.70	68.78	67.91
$\bar{\lambda}_{65}^i$%	71.55	14.51	2.73	1.11	3.87	3.09	3.13
$\bar{\delta}_{65}^d$%	---	3.32	4.53	4.39	7.22	2.77	7.12

Source: Own based on data from Robinson's (1996) care transition model (CTM)

Table 13: Further indicators of mortality and morbidity (males)

Indicators/states	a	d_1	d_2	d_3	d_4	d_5	d_6
A^i (years)	80.35	74.88	74.12	72.95	70.44	73.38	70.13
Md^i (years)	80.60	72.90	71.92	70.77	68.67	71.56	68.57
Q_3^i (years)	86.62	80.53	78.97	76.87	72.67	77.60	71.91
Q_1^i (years)	74.55	68.65	67.83	67.64	66.69	67.88	66.66
IQR_{65}^i (years)	12.07	11.88	11.14	9.23	5.98	9.72	5.25
$\bar{M}_{65}^i$ (years)	78.71	68.61	68.19	67.95	67.05	68.25	67.19
$\bar{\lambda}_{65}^i$%	78.46	12.47	2.05	0.77	2.19	2.45	1.62
$\bar{\delta}_{65}^d$%	---	3.38	4.56	4.48	7.32	2.87	7.20

Source: Own based on data from Robinson's (1996) care transition model (CTM)

The discrepancies are even greater but in the opposite direction when we look at the values provided by Manton and Land (2000), who use the same database and a similar period of time (the National Long-Term Care Surveys of older Americans for the years 1982 to 1996). These authors estimate 94.49% and 94.87% active life expectancy as a proportion of total life expectancy for males and females respectively.

In both tables our attention is drawn to state d_5 (those unable to perform one ADL and with cognitive impairment) because its associated total life expectancy is higher than that calculated for states d_2, d_3, d_4 and, logically, for state d_6 too. At first sight and from an epidemiological point of view, it might be said that this observation shows some kind of data anomaly because it is known that life expectancy decreases (or should decrease) when an individual's disability state worsens. However, there may not be an anomaly (Biessy, 2017) if we consider the French experience, where two types of mortality were found in disabled people: the first due to severe illness causing entry into dependence with a very short life expectancy, and the second caused by capacity erosion due to age and conditions such as dementia, which has a much longer mean duration.

The mortality ratio (dependent/healthy population) (Figure 10) shows that dependent people have a higher (much higher) yearly probability of dying than healthy people, but the difference tends to decrease notably as they get older. The extra mortality for dependent individuals is very noticeable and the "anomaly" involving the group of people labelled d_5 is clear to see.

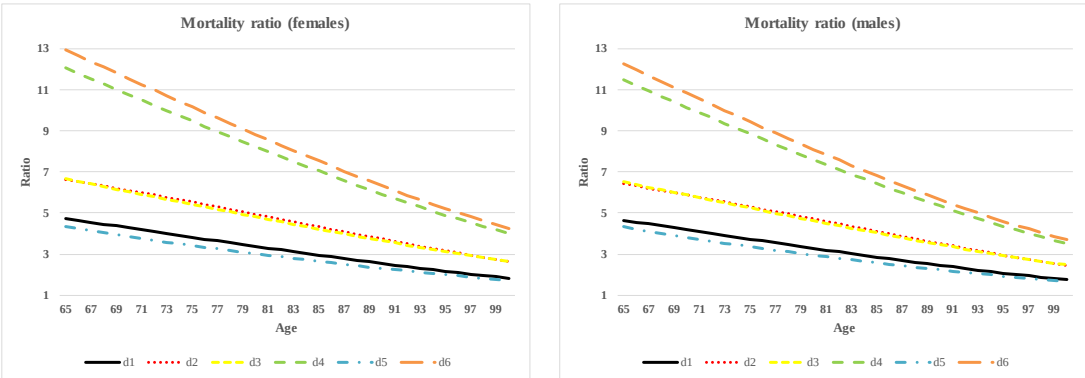


Figure 10: Comparison of mortality ratios: dependent/active people
Source: Own based on data from Robinson (1996)

The synthetic LTC prevalence rates by age and gender that would result from combining mortality and incidence rates are shown in Figure 11. The rates for the most severe states of dependence – d_5 and d_6 – would be particularly high for the very elderly (especially females). The highest average prevalence rate ($\bar{\lambda}_{65}^{d_1}$) (see Tables 10 and 11) would be for the least severe state of dependence, 12.47% for males and 14.51% for females, whereas the lowest would be for dependent people in state d_3 ($\bar{\lambda}_{65}^{d_3}$), barely reaching 0.77% for males and 1.11% for females. The total average LTC prevalence rate ($\bar{\lambda}_{65}^{d_j}$) would be 21.54% and 28.45% for males and females respectively.

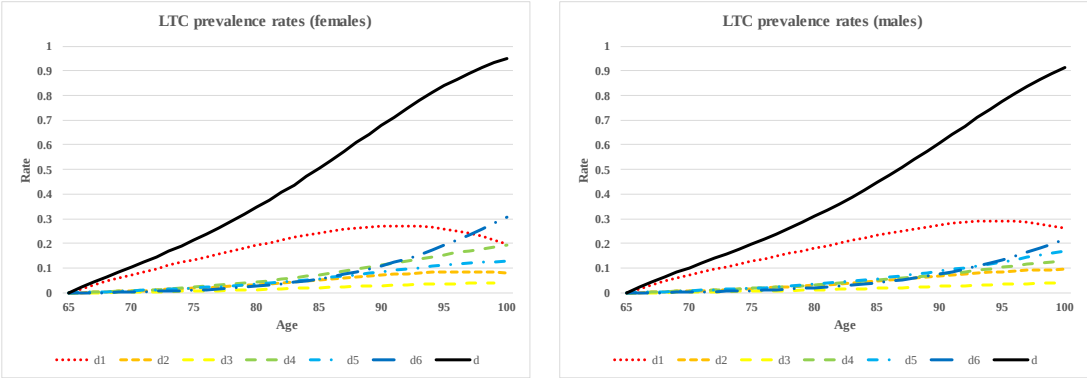


Figure 11: Comparison of LTC prevalence rates.
Source: Own based on data from Robinson (1996)

According to the above prevalence rates by age, the total number of dependent people would exceed that of the active population from age 87 in males and age 85 in females. For males, the least severe level of dependence d_1 would predominate over the entire range of ages considered, while for females it would be the most important up to age 97, when level d_6 would become most predominant.

Another aspect that stands out in these data is the different range of values observed for $IQR_{65}^{a/d}$ for males and females. In the case of males, the increase in the level of activity limitation – apart from that mentioned in the case of d_5 – implies a decrease in the average age of death for people still alive at 65. The same applies to the median, mode and IQR_{65}^{i}values. For females, however, this is not the case, despite the fact that the increase in the level of activity limitation also implies a decrease in the average age of death for people still alive at 65, the median, the mode, and the interquartile range value increases, which would seem to indicate greater heterogeneity within each group.

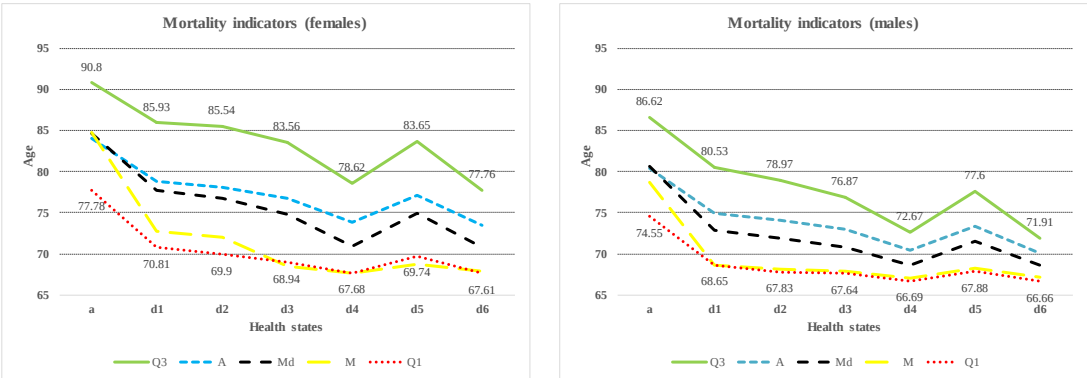


Figure 12: Mortality indicators.
Source: Own based on data from Robinson (1996)

Finally, Figure 12 shows the mortality indicators for females (Graph 1) and males (Graph 2). It can be seen that for both groups in the able state, the average age of death for people still alive at 65 and the median and the mode have very close values. However, when states of dependence are taken into account, the weighted average mode is very close to Q_1 (and well below the mean and median values) and the mean begins to differ from the median, which is higher in all cases.

Friedberg *et al*. (2014) revised and updated Robinson's CTM by adding a linear time trend. On the basis of their LTC transition rate parameter estimate (Table 5 in their paper), we have built a new set of health transition matrices.

Tables 14, 15, 16 and 17 respectively replicate Tables 10, 11, 12 and 13 above, and to a certain extent the results are unexpected because life expectancy is shorter, which is not in line with the trend observed in the US. Life expectancy there has improved steadily and substantially over the period from 1996 to 2014, but at a slower rate than in other high-income countries, particularly for women. The data obtained from the Friedberg *et al*. (2014) model also shows a greater amount of time likely to be spent in the same health state.

Table 14: Life expectancy matrix in years at age 65 and percentage of life expectancy likely to be spent in each state (males)

States	a	d_1	d_2	d_3	d_4	d_5	d_6	Total
a	10.94	1.39	0.23	0.09	0.30	0.23	0.19	13.69
	81.81%	10.42%	1.74%	0.67%	2.23%	1.74%	1.39%	100%
d_1	4.10	5.59	0.69	0.29	0.93	0.58	0.59	9.13
	35.66%	46.19%	4.80%	1.86%	4.67%	4.07%	2.75%	100%
d_2	2.73	3.26	1.11	0.20	0.51	0.36	0.25	8.43
	32.42%	38.68%	13.22%	2.39%	6.08%	4.27%	2.94%	100%
d_3	1.80	2.37	0.62	1.12	0.83	0.30	0.32	7.38
	24.46%	32.19%	8.36%	15.25%	11.28%	4.11%	4.35%	100%
d_4	0.68	0.98	0.28	0.34	2.21	0.15	0.34	5.08
	13.29%	19.23%	5.55%	6.76%	45.54%	3.01%	6.62%	100%
d_5	1.86	2.40	0.53	0.27	0.79	1.53	0.95	8.33
	22.29%	28.83%	6.34%	3.27%	9.48%	18.38%	11.41%	100%
d_6	0.56	0.81	0.23	0.24	1.41	0.22	1.32	4.80
	11.76%	16.89%	4.82%	5.01%	29.39%	4.68%	27.44%	100%

Source: Own based on data from Friedberg *et al.* (2014)

In short, the data obtained from the Friedberg *et al*. (2014) model show that life expectancy for active (and dependent) people would have decreased and that there would have been a clear compression of morbidity, which is not in line with the work carried out by Freedman *et al*. (2016), Freedman and Spillman (2016) or Jia and Lubetkin (2020).

Table 15: Life expectancy matrix in years at age 65 and percentage of life expectancy likely to be spent in each state (females)

States	a	d_1	d_2	d_3	d_4	d_5	d_6	Total
a	12.28	2.23	0.43	0.18	0.71	0.41	0.50	16.71
	73.41%	13.32%	2.55%	1.08%	4.24%	2.44%	2.97%	100%
d_1	4.10	5.59	0.69	0.29	0.93	0.58	0.59	12.77
	32.10%	43.75%	5.40%	2.28%	7.21%	4.57%	4.58%	100%
d_2	3.54	4.60	1.40	0.33	1.06	0.57	0.58	12.08
	29.34%	38.03%	11.61%	2.75%	8.75%	4.73%	4.81%	100%
d_3	2.45	3.52	0.90	1.33	1.52	0.50	0.67	10.90
	22.51%	32.30%	8.24%	12.24%	13.94%	4.58%	6.19%	100%
d_4	1.07	1.73	0.50	0.54	3.43	0.30	0.70	8.27
	12.90%	20.97%	6.01%	6.57%	41.44%	3.64%	8.47%	100%
d_5	2.46	3.49	0.78	0.42	1.48	1.81	1.33	11.77
	20.88%	29.64%	6.67%	3.60%	12.53%	15.34%	11.34%	100%
d_6	0.92	1.50	0.43	0.42	2.45	0.38	1.82	7.92
	11.57%	18.93%	5.43%	5.30%	30.93%	4.86%	17.79%	100%

Source: Own based on data from Friedberg *et al.* (2014)

Table 16: Further indicators of mortality and morbidity (males)

Indicators/states	a	d_1	d_2	d_3	d_4	d_5	d_6
A^i (years)	78.37	74.13	73.43	72.38	70.08	73.33	69.80
Md^i (years)	77.81	72.55	71.65	70.59	68.60	70.86	67.95
Q_3^i (years)	84.70	78.75	77.75	75.81	71.91	76.55	71.73
Q_1^i (years)	71.74	68.55	67.76	67.60	66.67	67.84	66.64

Table 16: Further indicators of mortality and morbidity (males)							
Indicators/states	a	d_1	d_2	d_3	d_4	d_5	d_6
IQR^i_{65} (years)	12.96	10.2	9.99	8.21	5.24	8.71	5.09
$\bar{M}^i_{65}$ (years)	73.59	68.37	68.01	67.84	66.94	68.16	67.09
$\bar{\lambda}^i_{65}$%	81.12	10.82	1.81	0.70	2.31	1.80	1.44
$\bar{\delta}^d_{65}$%	---	2.61	3.71	3.66	5.96	2.35	6.14
Source: Own based on data from Friedberg *et al.* (2014)							

Table 17: Further indicators of mortality and morbidity (females)							
Indicators/states	a	d_1	d_2	d_3	d_4	d_5	d_6
A^i (years)	81.72	77.77	77.08	75.90	73.27	76.77	72.92
Md^i (years)	81.81	76.63	75.67	73.87	70.85	74.61	70.69
Q^i_3 (years)	88.80	84.57	83.67	81.76	77.62	81.82	76.80
Q^i_1 (years)	74.70	70.59	69.73	68.84	67.63	69.63	67.56
IQR^i_{65} (years)	14.1	13.98	13.94	12.92	9.99	12.19	9.24
$\bar{M}^i_{65}$ (years)	78.47	71.91	71.20	68.41	67.58	68.77	67.82
$\bar{\lambda}^i_{65}$%	72.65	13.73	2.62	1.11	4.36	2.51	3.03
$\bar{\delta}^d_{65}$%	---	2.41	3.47	3.39	5.67	2.13	5.79
Source: Own based on data from Friedberg *et al.* (2014)							

3.2.4.-Australia

The structure of the information for Australia is identical to that of the US, but the biometric data include only four states of dependence, as can be seen in Tables 18, 19 and 20.

Table 18: Life expectancy matrix in years at age 65 and percentage of life expectancy likely to be spent in each state (males)						
States	a	d_1	d_2	d_3	d_4	Total
a	10.32	2.99	1.38	0.98	1.95	17.62
	58.55%	16.96%	7.84%	5.56%	11.09%	100%
d_1	6.28	6.53	1.52	1.06	2.04	17.43
	36%	37.47%	8.71%	6.09%	11.73%	100%
d_2	3.26	4.54	5.66	1.25	2.26	16.97
	19.19%	26.76%	33.34%	7.38%	13.33%	100%
d_3	1.15	1.94	2.94	5.62	2.13	13.78
	8.35%	14.07%	21.3%	40.82%	15.46%	100%
d_4	0.35	0.69	1.13	2.7	5.17	10.04
	3.53%%	6.84%	11.26%	26.86%	51.5%	100%
Source: Own based on data from Hariyanto *et al.* (2014)						

Table 19: Life expectancy matrix in years at age 65 and percentage of life expectancy likely to be spent in each state (females)						
States	a	d_1	d_2	d_3	d_4	**Total**
a	11.39	2.32	1.71	1.52	4.07	21.01
	54.25%	11.03%	8.13%	7.22%	19.37%	100%
d_1	6.8	6.13	1.93	1.66	4.24	20.76
	32.76%	29.54%	9.28%	7.99%	20.43%	100%
d_2	3.44	4.27	5.98	1.96	4.58	20.23
	16.99%	21.12%	29.56%	9.69%	22.64%	100%
d_3	1.24	1.9	3.18	6.32	4.55	17.19
	7.21%	11.05%	18.49%	36.76%	26.47%	100%
d_4	0.39	0.71	1.29	3.4	8.22	14.01
	2.77%	5.04%	9.23%	24.27%	58.69%	100%
Source: Own based on data from Hariyanto *et al.* (2014)						

These data show that active people in Australia have a higher life expectancy than those in the US and are likely to spend a smaller percentage of their life expectancy in the same health state. To some extent it is understandable that the life expectancies deriving from the data for Australia are appreciably higher than those for the US because they are based on much more recent observations.

The data for the life expectancy of active people (17.62 and 21.01 years respectively for males and females) are surprisingly similar to those provided by the Australian Institute of Health and Welfare (2014) – to be found in Appendix A, Table A2 for 2003 (the reference year for the data in Hariyanto *et al*., 2004) – which are 17.6 years for men and 21.00 for women. However, there is some divergence regarding the time that active people are likely to spend in a state of dependence. In the Hariyanto *et al*. (2004) model, active life expectancy is 54.55% and 54.25% of total life expectancy for males and females respectively, whereas AIHW (2014) report values of 43.18% and 41.90% for males and females respectively.

Tables 18 and 19 clearly show that the so-called "male-female health-survival paradox" is verified for this biometric data set, i.e. women have greater longevity than men but are likely to spend more time in any state of dependence, irrespective of the initial health state.

From a demographic/epidemiological point of view, the data from Australia are coherent because life expectancy decreases when an individual's disability state worsens. However, the mortality ratios ($\bar{\delta}^{d}_{65}$) are lower than those shown for the case of the US and their time structure (Figure 13) seems well graduated and decreases with age.

Table 20: Further indicators of mortality and morbidity

Indicators/states	Males					Females				
	a	d_1	d_2	d_3	d_4	a	d_1	d_2	d_3	d_4
A^i (years)	82.62	82.44	81.97	78.78	75.04	86	85.74	85.22	82.19	79
Md^i (years)	82.77	82.66	81.87	78.53	72.9	86.77	86.65	85.87	82.88	77.98
Q^i_3 (years)	88.73	88.65	87.9	85.57	80.77	91.87	91.8	91.61	89.74	86.87
Q^i_1 (years)	76.78	76.65	75.88	71.69	67.97	80.88	80.67	79.78	74.74	70.63
IQR^i_{65} (years)	11.95	12	12.02	13.88	12.8	10.99	11.13	11.83	15	16.24
$\bar{M}^i_{65}$ (years)	82.10	79.54	79.91	75.04	68.61	87.56	85.70	85.65	80.90	68.31
$\bar{\lambda}^i_{65}$%	57.34	17.45	8.07	5.73	11.42	53.14	11.29	8.33	7.40	19.84
$\bar{\delta}^d_{65}$%	---	1.00	1.12	2.45	3.10	---	1.00	1.05	2.29	2.97

Source: Own based on data from Hariyanto *et al*. (2014)

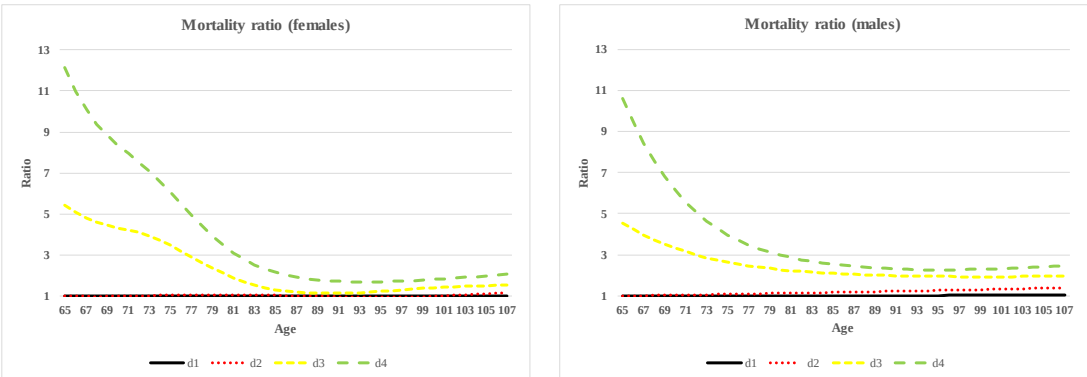


Figure 13: Comparison of mortality ratios: dependent/active people
Source: Own based on data from Hariyanto *et al*. (2014)

The synthetic LTC prevalence rates by age and gender are shown in Figure 14. As in the case of the US, the rate for the most severe dependence state d_4 would be highest for the very elderly (especially females). However, unlike for the US, the highest average prevalence rate for males ($\bar{\lambda}_{65}^{d_4}$) (see Table 23) would be in the most severe state of dependence at 19.87%, whereas for females it would be in the least severe state of dependence ($\bar{\lambda}_{65}^{d_1}$) at 17.45%. For both males and females, the lowest rate would be for dependent people in state d_3 ($\bar{\lambda}_{65}^{d_3}$). The total average LTC prevalence rate ($\bar{\lambda}_{65}^{d_j}$) would be 42.66% and 46.86% for males and females respectively, which would be far higher than those calculated for the US.

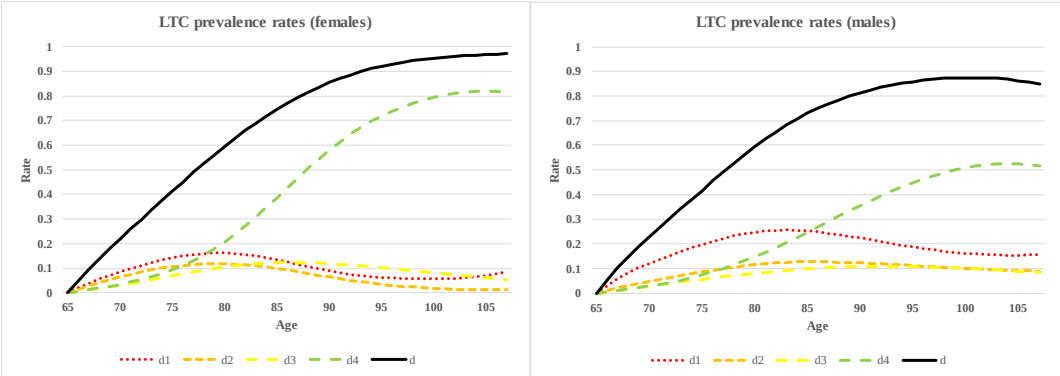


Figure 14: Comparison of mortality ratios: dependent/active people
Source: Own based on data from Hariyanto *et al.* (2014)

In the case of men, the least severe level of dependence d_1 would be predominant up to age 85, but from then on the highest prevalence rates would be for people at a more severe level of dependence (d_4). In the case of females, d_1 would be the predominant level of dependence up to age 78, while from then on the most severe level $d_4$would predominate.

In the case of women, the highest dependence rate (when all levels are included) would be reached at age 107, around 97.18%, while for men it would be at age 100, with a total rate of dependence of 87.47%.

With the above rates of prevalence by age, the total dependent population would exceed the active population from age 78 in both men and women.

In contrast to the case of the US, the range of values for IQR_{65}^{i} is very similar for both men and women. In the case of males, the increase in the level of activity limitation implies a decrease in the average age of death for those still alive at 65, while a decrease is also observed in the median and the adult weighted average mode of death. However, an increase in the value of IQR_{65}^{i} is also observed, which means a significant increase in heterogeneity as far as the death of individuals is concerned. This also occurs in the case of females but is even more pronounced than for males, showing a change in value from 10.99 (a) to 16.24 d_4.

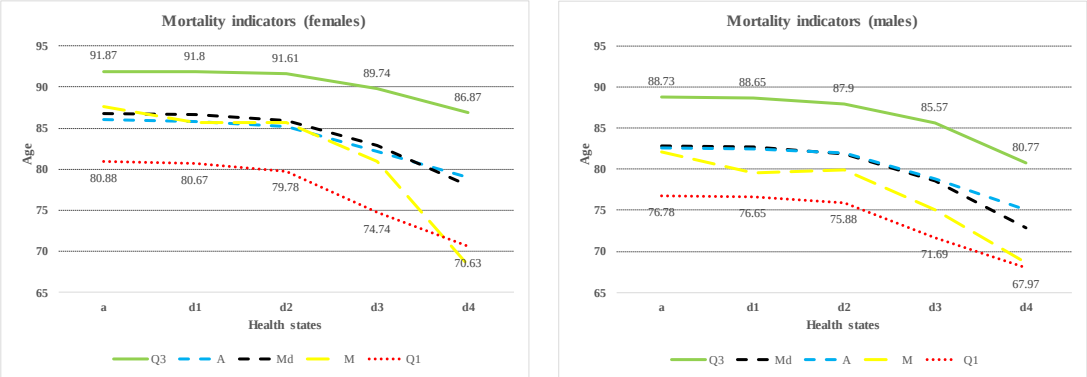


Figure 15: Mortality indicators.
Source: Own based on data from Hariyanto *et al.* (2014)

Finally, Figure 15 shows the mortality indicators for females (Graph 1) and males (Graph 2). It can be seen that for women in the first three states considered – a, d_1, d_2 – the values are very similar for the average age of death for people still alive at 65, and for the median and the adult weighted average mode age of death. However, when the most severe states of dependence are considered, the mode moves away from these values and, even in the most severe state of dependence, is below Q_1 (and well below the mean and median values). For men this behaviour is different, with the mode being close to the other two indicators only in the able state. From then on the divergence is more accentuated, although it does not fall below Q_1 even when the mean and median values separate from each other in this particular state.

3.2.5.-China

The structure of this information is identical to the cases of the US and Australia, although in the biometric data there are only two states of dependence, as can be seen in Tables 21 and 22.

The data shown for active life expectancy (14.24 years and 16.2 years respectively for men and women) are slightly lower than those presented for the 1940 cohort by Jiao (2019), according to whom life expectancy at age 65 would fluctuate between 16.45 and 16.65 years for women and 14.74 and 15.06 years for men. For 2015 Zhen *et al*. (2022) estimate a life expectancy of 14.36 and 17.81 years for men and women respectively.

Table 21: Life expectancy matrix in years at age 65 and percentage of life expectancy likely to be spent in each state

States	Males				Females			
	a	d_1	d_2	Total	a	d_1	d_2	Total
a	7.44	4.99	1.81	14.24	5.97	6.44	3.79	16.2
	52.26%	35.03%	12.71%	100%	36.86%	39.76%	23.38%	100%
d_1	4.88	6.51	2.13	13.52	4.18	7.55	4.01	15.74
	36.13%	48.18%	15.69%	100%	26.58%	48%	25.42%	100%
d_2	3.6	5.2	3.59	12.39	2.93	6.12	5.46	14.51
	29.07%	41.99%	28.94%	100%	20.18%	42.18%	37.64%	100%

Source: Own based on data from Cui *et al.* (2022)

There is more divergence in the time that active people are likely to spend in a state of dependence. In the Cui *et al*. (2022) model, active life expectancy is 52.26% and 36.86% of total life expectancy for males and females respectively, whereas Jiao (2019) and Zhen *et al*. (2022) report a range of values between 86.60% and 94.60% for males and 89.44% and 96.53% for females as a proportion of total life expectancy.

Table 22: Further indicators of mortality and morbidity						
Indicators/states	**Males**			**Females**		
	$\boldsymbol{a}$	$\boldsymbol{d_1}$	$\boldsymbol{d_2}$	$\boldsymbol{a}$	$\boldsymbol{d_1}$	$\boldsymbol{d_2}$
$\boldsymbol{A^i}$ (years)	79.24	78.52	77.39	81.19	80.74	79.5
$\boldsymbol{Md^i}$ (years)	78.82	77.91	76.82	80.98	80.8	79.71
$\boldsymbol{Q_3^i}$ (years)	84.8	83.98	83.59	86.83	86.74	85.88
$\boldsymbol{Q_1^i}$ (years)	73.73	72.66	70.86	75.76	75.46	72.93
$\boldsymbol{IQR_{65}^i}$ (years)	11.07	11.32	12.73	11.07	11.28	12.95
$\boldsymbol{\bar{M}_{65}^i}$ (years)	79.00	73.55	72.50	80.79	81.72	69.10
$\boldsymbol{\bar{\lambda}_{65}^i}$%	50.53	36.30	13.17	34.85	41.02	24.13
$\boldsymbol{\bar{\delta}_{65}^d}$%	---	1.26	3.50	---	2.07	4.69
Source: Own based on data from Cui *et al.* (2022)						

As we saw with the biometric data sets for other countries, Tables 19 and 20 again reflect the so-called "male-female health-survival paradox". The paradox is verified for this data set, although in this case the large proportion of time that both men and women in dependence level d_2 would likely spend in better health states is striking. In the case of men, 71.06% of life expectancy would be in a better level of activity limitation, while for women it would be 62.36%.

From a demographic/epidemiological standpoint, the data for China are apparently coherent because life expectancy decreases when an individual's disability state worsens. However, the mortality ratios ($\bar{\delta}_{65}^d$) are lower than those shown for the US and higher than those for Australia, although the comparison is not entirely reliable because there are only two levels of dependence as opposed to 6 and 4 in the US and Australia respectively. The mortality ratio by age (Figure 16, Graph 1 for females and Graph 2 for males) shows large irregularities for some sections that are very difficult to understand and justify. Inexplicable jumps occur for both males and females.

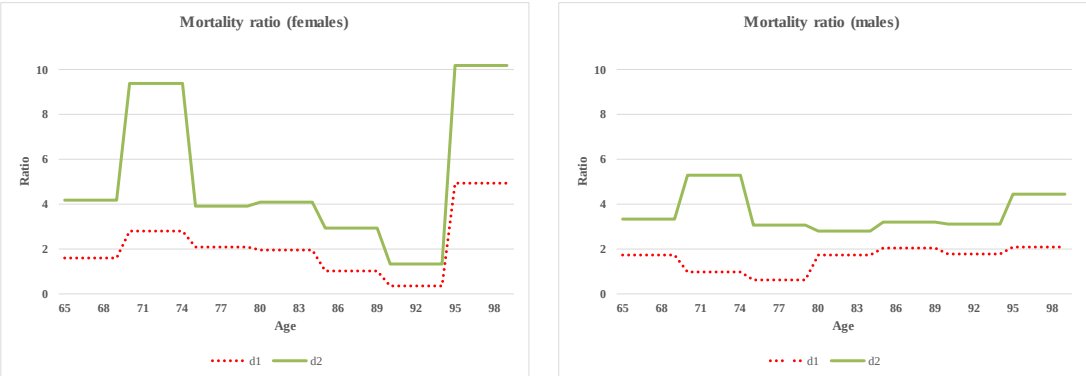


Figure 16: Comparison of mortality ratios: dependent/active people
Source: Own based on data from Cui *et al.* (2022)

The synthetic LTC prevalence rates by age and gender are shown in Figure 17. The highest average rate for males and females ($\bar{\lambda}_{65}^{d_1}$) (see Table 20) would be for the least severe state of dependence at 36.60% and 41.42% for males and females respectively. The total average rate ($\bar{\lambda}_{65}^{d_j}$) would be 49.47% and 65.15% for males and females respectively, which is far higher than the figures calculated in the case of Australia.

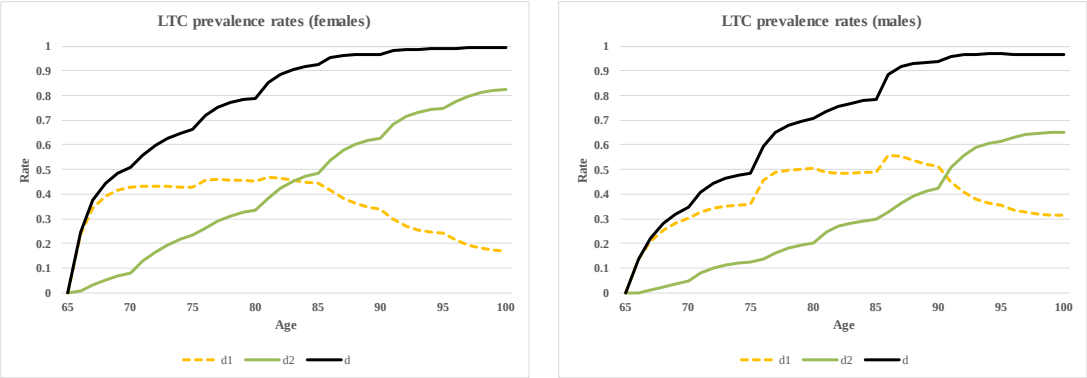


Figure 17: Comparison of mortality ratios: dependent/active people
Source: Own based on data from Cui *et al.* (2022)

In the case of men, the least severe level of dependence $d_1$would be predominant up to age 90, while for women it would be the most prevalent level up to age 82, with d_2 predominating from then on.

In the case of women, those aged 83 and over would already have an LTC prevalence rate of over 90%, while for men this level would be reached at age 87. The highest dependence rate (for all levels) would be attained at age 100, being 96.18% and 99.44% for males and females respectively.

According to the previous rates of LTC prevalence by age, the total number of dependent people would be greater than the active population after age 76 for men and age 70 for women.

Like we saw in the case of Australia, the range of values for the IQR_{65}^{i} is very similar for both men and women (Table 22). The increase in the level of activity limitation means a decrease in the average age of death for people still alive at age 65, and also for the median and the mode. However, an increase in the IQR_{65}^{i} value is observed, which means a significant increase in heterogeneity as regards individuals' deaths.

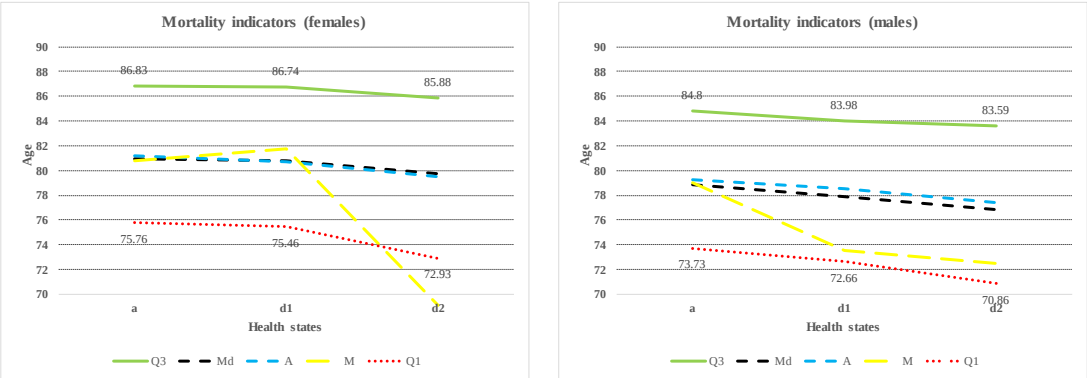


Figure 18: Mortality indicators.
Source: Own based on data from Cui *et al.* (2022)

Finally, Figure 18 shows the mortality indicators for females (Graph 1) and males (Graph 2). We can see that the main difference between them is the value of the mode in the active state. The average age of death for people still alive at 65 and the median and the mode have similar values. However, although the value of the mode for women remains close to the other two indicators in d_1, in d_2 it decreases sharply and reaches a value lower than Q_1. The divergence in the case of men can already be observed in d_1, but in d_2 the mode stays clearly above the level of Q_1. The explanation for the different mode values by gender lies in the average number of deaths in each state. It should be remembered

that the value is calculated as an average of the mode levels reached in each of the states separately (formulas 19 to 23 in the Technical Appendix).

3.2.6.-Summary

The analysis carried out in the previous sub-sections showed that the biometric data sets have several weaknesses that we summarize in Table 23.

Table 23: Main weaknesses of the biometric data sets selected

Data set	Weaknesses
Artís *et al.* (2007)	There is only one level of dependence. US data are used to model the mortality of dependent individuals. The amount of time likely to be spent in a dependent state is nowhere near the figures estimated in official statistics.
Albarrán-Lozano *et al.* (2021)	This data set does not contain any information about active people. It deals only with dependent people. Anomalies in life expectancy are detected depending on the level of dependence. Mortality ratios for females present irregularities that are difficult to explain.
Esquivel *et al.* (2021)	Lower life expectancy for the active population and a relatively small amount of time that active people are likely to spend in a state of dependence. Incomplete definition for the classification of dependent individuals and no breakdown by gender. High recovery rates. Low mortality ratios with inexplicable irregularities. Lack of graduation in incidence and mortality rates. Very high prevalence rates for people in the least severe level of dependence.
Robinson (1996)	Different levels of heterogeneity at the same level of dependence for males and females. Big differences in the amount of time likely to be spent in a dependent state when compared to the official statistics and/or similar investigations.
Friedberg *et al*. (2014)	Same characteristics as the data in Robinson (1996) above. Lower life expectancy for active and dependent people and some compression of morbidity when official statistics present opposing trends.
Hariyanto *et al*. (2014a, 2014b)	People in any state of core activity limitation can only improve by one category over a one-year interval. Very high prevalence rates.
Cui *et al*. (2022)	Only two states of dependence. Unhealthy life expectancy very high in comparison with other recent studies. High recovery rates. The mortality ratio presents sizeable irregularities for sections that are very difficult to understand and justify. We see inexplicable jumps for both males and females. Lack of graduation in incidence and mortality rates. Very high prevalence rates.

Source: Own elaboration

The above table shows how difficult it is to build the biometric data sets needed to carry out an actuarially fair valuation[7] of the prices for LTCI and LCA contracts.

It has long been known in LTC insurance that constructing bases of experience adapted to Markov or semi-Markov models is a delicate exercise due to the low volumes observed by insurers up to now and also due to the evolving nature of the field. The construction process becomes more complex as the number of considered states increases and when reversible states exist (Guibert and Planchet, 2019).

Choosing which mortality table to use has a crucial impact on the pricing, reserving and management of LTC portfolios. Tomas and Planchet (2013) and Planchet and Tomas (2016) show that the construction of such tables is a difficult exercise given that the mortality law for LTC claimants consists of a mixture of pathologies, LTC portfolios are relatively small and the estimation of crude death rates is very volatile.

4.-Discussion

From an LTC insurance perspective, HLE and life expectancy with activity limitation are key indicators and we are therefore very interested in the biometric assumptions made by the LTC insurance scheme's insurer or sponsor. Including information by breaking down life expectancy by health state is invaluable for illustrating the actuarial calculations used to price LTCI in general and LCAs in particular.

According to the equivalence principle used in insurance, the expected present values of premiums and benefits should be equal. Roughly speaking, therefore, premiums and benefits will balance on average (Norberg, 2002). The main assumptions to be made when pricing LTCI are (Denuit *et al*., 2019; Dickson *et al*., 2019; Pitacco, 2014, 2016; SAS, 2020; SOA, 2016):

- longevity (the yearly rates of survival by age and health state);
- morbidity/incidence rates (transition rates by age and gender from the healthy state to the different states of dependence);
- recovery rates (transition rates by age and gender to better health states);
- specific, detailed insurance data on health and living with illness. However, where the LTCI market is small or the national commercial insurance data needed for estimating premiums are unavailable, national health statistics can be a viable starting point for insurance ratemaking purposes, even if they cover the general population, are aggregate and reported at irregular intervals (Baione and Levantesi, 2014 and 2018);
- lapse rates (although they may not apply to mandatory public schemes[8] or contracts where premiums are paid as an upfront (lump sum) payment);
- the interest rate (technical interest rate or wage bill growth depending on how LTC is financed: full funding or pay-as-you-go);
- the loading for expenses on acquisition (with agents' commission being the biggest expense in this category and medical examination costs in the case of LCAs, given that entitlement to LTC benefits depends on the result of a mandatory medical

[7] A pricing scheme is "actuarially fair" if each insured individual pays a price for coverage that is equivalent to the risk they pose of having to draw from the insurance pool, given the available information at the inception of the contract.

[8] Compulsory LTCI has been introduced in Luxembourg, Singapore, Germany, Japan and the Republic of Korea. The Netherlands uses a mix of compulsory LTC and health insurance (Barber *et al*., 2021; SAS, 2020; Pla-Porcel *et al*., 2016).

assessment when the insured person makes a transition from one health state to another), collection and administration;

- inflation protection (especially relevant when the benefits claimed involve reimbursement or are linked to inflation and/or the indexation rate); and
- indexation of benefits in payment.

Why, then, are we so interested in LCAs? The uncertainty surrounding the need for LTC services presents a significant challenge in retirement planning. Preparing for the possible costs of future impairment and LTC is a task that everyone has to confront as they get older. The probability of losing physical functioning increases dramatically with age and is therefore a highly relevant consideration for the population aged 65 and above. According to the biometric data set for Australia that we analysed, for example, between 42.66% (males) and 46.86% (females) of the over-65s will become care dependent to some extent. As regards the very elderly, the proportion of people aged 85 and above requiring LTC is expected to be between 73.88% (males) and 79.71% (females).

To give another example, according to Robinson's (1996) model that was used in the American insurance industry, for a healthy 65-year-old man, the probabilities of needing LTC services when he reaches the ages of 70, 80 and 90 are 8.39%, 30.91% and 60.75% respectively, whereas for a healthy woman at the same age the probabilities are 10.19%, 31.32% and 64.37%. These figures are fairly close to those provided by AALTCI, 2022. Using a more restricted definition of the situation of dependence[9] than that used under Robinson's (1996) care transition model (CTM), AALTCI (2022) reports that for a woman (man) who is currently age 65, the probabilities of needing LTC at the ages of 70, 80 and 90 are 5.60% (5.30%), 27.20% (24.30%) and 58.30% (51.10%) respectively.

LCAs have several advantages. First, combining LTCI and life annuities would reduce adverse selection for the annuity portion, resulting in lower premiums and allowing the relaxation of underwriting standards for the LTCI portions. Second, LCAs would allow people to delay the purchase of LTCI until closer to retirement, which would have the further benefit of allowing insurers to better account for trends in disability, longevity, investment returns and other sources of risk in designing and pricing policies. The cash-for-care benefit in a combined annuity has the potential to simplify choices and reduce uncertainty about future benefits, which are two frequently cited reasons for explaining the low market penetration of conventional LTCI. Finally, if the individual has an existing health issue, they might find it easier to be approved for an annuity with an LTC rider instead of a stand-alone LTCI. This is a very important point, given that Cornell *et al*. (2016) show that in the target age range (50-71 years), approximately 30% of those whose wealth meets minimum industry standards for suitability for LTCI would have their applications rejected for this type of insurance at the underwriting stage[10].

In short, an LCA is different from a stand-alone LTCI in various ways. With LTCI, the individual is buying an insurance policy specifically for LTC and they might pay a premium upfront or at regular intervals. Once they need LTC, the policy can be paid out monthly or on a lump-sum basis to help with the costs. LTCI doesn't have the growth component that an LCA would have. Another key difference is that if the policyholder

[9] Care need is defined as being unable to perform two or more of the six ADLs (bathing, continence, dressing, eating, toileting and transferring) or being "severely cognitively impaired".

[10] "*Your money pays for LTCI, but it's your health that really buys it. Insurers decline nearly half of those who apply after the age of 70*" explains Jesse Slome, director of the American Association for LTCI. (AALTCI, 2022)

does not ultimately need LTC, they will not have their premiums returned unless they purchase a return-of-premium rider. With an LCA, the annuitant could still receive annuitized payments even if they do not use the benefits from the LTC rider. In other words, this is a form of guaranteed income that the annuitant can use for LTC if needed or for other expenses in retirement.

In those countries for which the data sets have been analysed, the private LTCI market either does not exist or is very small. In Portugal and Spain, for example, the markets for LTCI are still underdeveloped. In Spain, the number of individual LTCI policies in force is almost negligible, less than 60,000 in 2021 (DGSFP, 2022). Private LTCI is not currently available in Australia (NSPAC, 2013; Sherris, 2022), and in China the LTC insurance market is still at an early stage of development. Policy and regulation need to be improved before the market can enter a phase of high growth (Swiss Re, 2022). The private LTCI market in the US had a promising start at the beginning of the 80s, but is now in a downturn.

A series of papers by Brown and Finkelstein (2007, 2008, 2009) provided evidence of the relatively high loading factors used in the private LTCI market in the US, particularly for men. The authors concluded that, for the typical policies purchased, the premiums were marked up substantially higher than the expected benefits. As we will see, however, the evolution of the market does not appear to bear this out. The LTCI premium rates developed by insurers from at least 10 years ago have generally turned out to be underpriced, and therefore many premium rate increases have been filed in this area (AAA, 2018). Given that liabilities from LTCI (which depend on future morbidity and mortality) are very volatile, it is not surprising that premiums (especially in the form of regular premium payments) on many policies in the US private market are subject to increases based on claims experience.

By 2014 the average policy premiums (for the same benefits) had increased by 215% of the premiums charged in 2000 (AAA, 2016). The variability of insurance premiums means that some of the spending risk of unknown LTC expenditure is transferred back to the policyholder. This means they participate in the risk of increased LTC costs or in claims that exceed those anticipated in the underwriting process (Pfau and Finke, 2016). Indeed, in the US the issue of increased LTC premiums is a nationwide trend[11]. Most insurance companies underestimated the cost of paying claims and overestimated the number of people cancelling policies. When original LTCI policy forms were issued in the 1980s and 1990s, morbidity assumptions were often based on statistics for the general population, while lapse and mortality assumptions were based on experience with non-LTC insurance products. Inadequate medical underwriting was another source of deviation (AAA, 2018).

Insurance companies are forced to adjust premiums to compensate for inaccurate pricing assumptions to ensure there are enough reserves to pay LTC claims under each plan. If rates were not increased, the insurance carriers could run into financial trouble, leaving them unable to pay claims. The result of all this along with other reasons that go beyond the scope of this paper is that the number of individual LTCI policies sold in the US declined to 49,000 in 2020 (USDT, 2020). This was after they increased from 380,000 in 1990 to a peak of 754,000 in 2002. The number of companies offering policies fell and many faced financial problems (OECD, 2020). According to the National Association of

[11] The history of requested and approved LTCI rate increases for companies currently underwriting LTCI in the US is constantly being updated and can be consulted online: https://longtermcareinsurancepartner.com/resources/long-term-care-insurance-rate-increase-history

Insurance Commissioners (NAIC), the number of insurers offering LTCI coverage has fallen from just over 100 in 2004 to around a dozen in 2021. Between 2012 and 2021 the number of individuals with an active or "in-force" stand-alone LTCI policy[12] declined from 7.2 million to 5.3 million (AALTCI, 2022). The continuous reduction in the number of LTCI policies sold and the declining size of the market could to some extent be a result of the very high prevalence of bad experiences undergone by policy-holders due to premium increases, which in turn has led to a lessening of trust in insurance companies (Courbage and Nicolas, 2021).

Now that we have analysed the data sets, we will briefly consider the most relevant aspects to be taken into account for each country if we were to determine the actuarially fair price of an LCA by looking only at the biometric aspect of pricing.

Spain: Judging by the data in Artís *et al*. (2007), it is obvious that women would have to pay a higher price than men for each benefit unit because more of them would become dependent (higher prevalence rates in the case of women) and payouts would be more expensive (higher life expectancy in women). The main drawback would be that distinctions could not be made by levels of dependence, and the biometric data would need to be updated and deal exclusively with the case of Spain.

The biometric data in Albarrán-Lozano *et al*. (2021) have the disadvantage of lacking information on the probabilities of transition from active to dependent and are therefore not very useful for assessing an LCA. Nevertheless, including them in this work is interesting since they could be used to value "special-rate annuities", and in particular the "enhanced pension annuity (EPA)"[13].

Portugal: Using the data in Esquivel *et al*. (2021) it is not possible to distinguish between men and women. The data show a low life expectancy (compared to that of the general population) as a healthy (active) and dependent individual, a small amount of time likely to be spent in a state of dependence, and a preponderance of the least severe level of dependence $d_1$compared to the rest, which means that using these biometric data would result in a clear underestimation of LCA premiums.

USA: Using the biometric data in Friedberg *et al*. (2014) would result in a clear decrease in the price of LCAs with respect to the Robinson (1996) model, given that life expectancies are lower for both active and dependent people, and also the number of active and dependent survivors would be lower.

Australia: Prices would be much higher than in the US. This would be due to greater life expectancy for both active and dependent individuals, along with a greater proportion of life expectancy spent in states of dependence.

China: Using the data in Cui *et al*. (2022) would result in a clear overestimation of LCA premiums because of the very high prevalence rates reported.

To conclude this section, we should ask ourselves whether it should be compulsory to disclose information about the HLE and the other longevity risk and morbidity indicators used in the technical bases for computing LTCI and/or LCA premiums?

In the world of science and engineering, a black box is a device, system or object that can be viewed solely in terms of its input and output without the user knowing how it works.

[12] American Association for Long-Term Care Insurance, 2022, https://www.aaltci.org/

[13] The enhanced life annuity pays an income to a person with lower life expectancy, in particular because of a personal history of medical conditions. The "enhancement" in the annuity benefit (compared to a standard-rate life annuity, same premium) naturally comes from the use of a higher mortality assumption.

Individuals or couples who are interested in purchasing LTCI or an LCA in particular might feel they are up against something similar. For them, a black box could be the actuarial calculation done to compute the premium for an LCA. The actuarial black box can be defined as a situation in which the actuarial analysis has not been adequately explained to its users (Gutterman, 1996). This might be due to the use of a complicated actuarial model, a lack of understanding of what the actuary does, or simply poor communication.

The market for individual LTC products would greatly benefit from the elimination of as many of these black boxes as possible. We could start to eliminate them by disclosing more information about the biometric assumptions made to compute the premiums and by spending more time with possible purchasers to explain the meaning of concepts such us life expectancy, healthy life expectancy, and longevity and morbidity risks. Fuino *et al*. (2022) find that factors relating to the awareness and understanding of LTC are extremely relevant when purchasing LTCI. Boyer *et al*. (2020) suggest that to spark an interest in buying LTCI policies it is essential to make the population aware of the frequency and severity of the risk insured, and also to provide a clear explanation of the benefits that come from having an LTCI policy.

In short, our analysis and its associated data could provide very useful information to help individuals or couples to understand the need to be protected against the cost of requiring LTC services. It would also make the computation of the actuarial factors more transparent (Ventura-Marco *et al*., 2022). This information could also be embedded in retirement calculators and other tools used by financial advisors (Hurwitz *et al*., 2022). This is in line with the proposal to increase the transparency of complex products and strategies during the decumulation phase (Bär and Gatzert, 2022).

5.-Conclusions and future research

From the beginning of this research, the authors were fully aware of how difficult it would be to compare the health and demographic indicators deriving from the various data sets analysed. Nevertheless, we believe that the paper's aims have been reasonably achieved given that the five basic research questions have been answered.

The methodology used to obtain the life expectancy matrix based on an individual's initial health state plus the analysis carried out has enabled us to show various health and demographic indicators to help assess the coherence and quality of the different data sets analysed. These indicators are rarely presented when authors build their biometric data sets nor when they are used to calculate LTCI or LCA premiums, nor when they are used in research articles to estimate the future demand for LTC services in a particular country.

The data sets analysed confirm the existence of the so-called "male-female health-survival paradox". Intensive research performed in recent decades has made it clear that the extent of and trends in mortality differences between women and men are caused by a complex combination of biological factors and acquired risks. In contrast to the situation regarding mortality differences, there is still no conclusive understanding of the reasons for the contradictory picture of higher female morbidity rates (Di Lego *et al*., 2020).

The construction of biometric data sets is a difficult task, particularly when more than one state of dependence is considered along with the recovery assumption. Bearing in mind that insurers have so far dealt with only a low volume of cases, that the nature of the situation is constantly evolving, that the mortality law for LTC claimants consists of a mixture of pathologies, and that the estimation of crude death rates is very volatile, it is not surprising that we found a number of flaws in the data sets analysed.

The main weaknesses are the following:

- Low number of dependence states (Artis *et al.*, 2007; Cui *et al*., 2022).
- Big differences in the amount of time likely to be spent in a dependent state when compared to official statistics and/or similar investigations (Artis *et al*, 2007; Esquivel *et al*., 2021; Robinson, 1996; Friedberg *et al*., 2014; Cui *et al*., 2022). The use of very different methodologies (prevalence-rate versus incidence-rate models) and varying definitions of disability and age in the samples across studies is one of the reasons for the differences in the observed results[14].
- Anomalies in life expectancy depending on the level of dependence (Albarrán-Lozano *et al*., 2021).
- High recovery rates (Esquivel *et al*., 2021; Cui *et al*., 2022).
- Lack of graduation in incidence and mortality rates (Esquivel *et al*., 2021; Cui *et al*., 2022).
- Mortality ratios with large irregularities that are very difficult to understand and justify (Albarrán-Lozano *et al*. 2021; Esquivel *et al*., 2021; Cui *et al*., 2022).
- Very high prevalence rates (Hariyanto *et al*., 2014a, 2014b; Esquivel *et al*., 2021 (people in the least severe level of dependence); Cui *et al*., 2022).
- Low life expectancy for the active population (Esquivel *et al*., 2021; Friedberg *et al*., 2014).

In short, despite the fact that all the biometric data sets we have analysed have their own problems, we would say that the most coherent from a demographic and epidemiological standpoint are those in Hariyanto *et al*. (2014a, 2014b) and Robinson (1996). On the basis of all the information we have analysed, the data sets with the most consistency problems would be Esquivel *et al*. (2021) and Cui *et al*. (2022).

For all these reasons it is no surprise that insurance companies are becoming less willing to provide LTCI and LCA contracts, since there is a big problem with the biometric aspect when it comes to calculating actuarially fair premiums with any degree of certainty. In those countries (except the US) whose data sets we have analysed, the market for this type of product is either non-existent or underdeveloped.

From the point of view of potential purchasers of this type of insurance product, the disclosure of the summary health/longevity measures used in the technical bases to compute LTCI and LCA premiums would help them to understand the need to be protected against the cost of requiring LTC services and also make the computation of the actuarial factors transparent. This information could also be embedded in the retirement calculators and other tools used by financial advisors, given that many people are simply unaware of longevity (and morbidity) risks.

Finally, it could be said that LTCI is one of the least secure insurance products for the insured person since the periodic increases in premiums clearly distort the traditional concept of insurance. Seen from this perspective, LCAs are certainly better for the annuitant because the premium is generally paid as a lump-sum when the individual is close to retirement.

[14] The complexity of this issue clearly goes beyond the scope of this paper.

Future research could extend the analysis carried out in this paper to other biometric data sets that we initially excluded for various reasons, such us the difficulty of collecting all the necessary data or because they did not use a Markovian structure.

6.-References

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Technical Appendix

Life expectancy[15]

The curtate future lifetime random variable is defined as the integer part of future lifetime and is denoted by K_x for an individual aged x.

The expected value of K_x is denoted by e_x, so that $e_x = \mathbb{E}[K_x]$, and is referred to as the curtate expectation of life (even though it represents the expected curtate lifetime). So,

$$e_x = \mathbb{E}[K_x] = \sum_{k=0}^{w-x-1} k \cdot \overbrace{{}_{k/}q_x}^{\mathbb{P}[K_x = k]} \qquad \textbf{[1.]}$$

with ${}_{k/}q_x$ being the probability that an individual aged x survives k years and then dies within that year, and where ω is the maximum lifespan, and:

$${}_{k/}q_x = {}_kp_x - {}_{k+1}p_x \qquad \textbf{[2.]}$$

where ${}_kp_x$ is the probability that the individual aged x survives to age $x + k$.

It is easy to see that:

$$e_x = \mathbb{E}[K_x] = \sum_{k=1}^{w-x-1} {}_kp_x \qquad \textbf{[3.]}$$

Given that in our framework the curtate future lifetime random variable can be disaggregated as the sum of the curtate future lifetime of health states:

$$K_x = \sum_{i \in S'} K_x^i \qquad \textbf{[4.]}$$

with $\mathcal{S}' = \mathcal{S} \backslash \{f\} = \{a, d_1, \cdots, d_n\}$, and consequently the probability that the individual aged x survives to age $x + k$ can be disaggregated also by health state:

$${}_kp_x = \sum_{i \in S'} {}_kp_x^i \qquad \textbf{[5.]}$$

we get:

$$e_x = \sum_{k=1}^{w-x-1} \sum_{i \in S'} {}_kp_x^i = \sum_{i \in S'} e_x^i \qquad \textbf{[6.]}$$

For the complete life expectancy (Gerber, 1997; Dickson *et al*., 2020):

$$e_x' \equiv \mathbb{E}[T_x] = \mathbb{E}[K_x + R_x] = \mathbb{E}[K_x] + \mathbb{E}[R_x] = e_x + 1/2 \qquad \textbf{[7.]}$$

where T_x is the random variable of future lifetime at age x for this individual, and $R_x = T_x - K_x$ is the random variable of the fractional part of the future lifetime, independent of K_x, with the assumption of uniform distribution, i.e. $R_x \sim U(0,1)$.

[15] For the sake of completeness, we include the development of life expectancy over the years from an actuarial point of view.

The life expectancy matrix depending on an individual's initial health state

The discrete multistate model (Figure 1, Section 2.2.1.) can be expressed as an $(n+2)$-state-age non-homogeneous discrete-time Markov chain. The yearly transition probabilities forming the stochastic one-year transition matrix (M_{x+k}) are shown in Table A1:

Table A1: $\boldsymbol{M_{x+k}}$

Starting status, i	**Ending status, j**					
	$\boldsymbol{a}$	$\boldsymbol{d_1}$	$\boldsymbol{d_2}$	… ….	$\boldsymbol{d_n}$	$\boldsymbol{f}$
$\boldsymbol{a}$	p_{x+k}^{aa}	$p_{x+k}^{ad_1}$	$p_{x+k}^{ad_2}$	… ….	$p_{x+k}^{ad_n}$	p_{x+k}^{af}
$\boldsymbol{d_1}$	$p_{x+k}^{d_1a}$	$p_{x+k}^{d_1d_1}$	$p_{x+k}^{d_1d_2}$	… ….	$p_{x+k}^{d_1d_n}$	$p_{x+k}^{d_1f}$
$\boldsymbol{d_2}$	$p_{x+k}^{d_2a}$	$p_{x+k}^{d_2d_1}$	$p_{x+k}^{d_2d_2}$	… ….	$p_{x+k}^{d_2d_n}$	$p_{x+k}^{d_2f}$
… …	… ..	… ..	… ..	… ….	… ..	… ..
$\boldsymbol{d_n}$	$p_{x+k}^{d_na}$	$p_{x+k}^{d_nd_1}$	$p_{x+k}^{d_nd_2}$	…….	$p_{x+k}^{d_nd_n}$	$p_{x+k}^{d_nf}$
$\boldsymbol{f}$	0	0	0	0	0	1

Source: Own

p_{x+k}^{aa} is the probability that an active person aged $x+k$, $k \in \{0,1,\cdots,w-(x)\}$ will reach age $x+k+1$ in the same state.

$p_{x+k}^{ad_j}$ is the probability that an active person aged $x+k$ will reach age $x+k+1$ in state of dependence d_j, $j \in \{1,2,\dots n\}$.

p_{x+k}^{af} is the probability that an active person aged $x+k$ will die during the year.

$p_{x+k}^{d_id_j}$ is the probability that a person ($i \in \{1,2,\dots n\}$-level dependent) aged $x+k$ will reach age $x+k+1$ in state of dependence $d_j, j \in \{1,2,\dots n\}$.

The rows and columns of the matrix embedded in Table A1 relate to the starting and ending health status respectively of each individual in the cohort.

As we are working with Markov processes, we apply classic recurrent Chapman-Kolmogorov equations to obtain the corresponding multiyear transition probabilities.

A key element when it comes to computing the value of LCAs is the probability that an active person aged x will reach age $x+k$ in any state of $j \in \{1,2,\dots,n\}$ dependence ($_kp_{x_e+A}^{rd_j}$).

This probability can be obtained from the vector (or matrix row) of the multiyear transition probabilities ($_kM_x$), given the initial state a at age x. This vector is the product of the annual probabilities of transition among the different states, which can also be expressed as an annual transition matrix for each age attained. At time t, for the group of healthy people aged x, the resulting probabilities are:

$$\left({}_kp_x^{aa},\ {}_kp_x^{ad_1},\ {}_kp_x^{ad_2}, \dots\dots\dots,\ {}_kp_x^{ad_n},\ {}_kp_x^{af} \right) = {}_kM_x = \overbrace{u_1 \cdot M_x \cdot M_{x+1} \cdot \dots\dots \cdot M_{x+k-2}}^{{}_{k-1}M_x} \cdot M_{x+k-1} \qquad \textbf{[8.]}$$

where:

vector row u_1, with value one in the first position and zero in the rest of the positions, sets the true initial state a at age x. The individual could have become dependent in any year within the corresponding age period $(x, x+k]$.

${}_kp_x^{ad_j}$ is the probability that a healthy person aged x will reach age $x+k$ in level $j \in \{1,2,\dots,n\}$ of dependence:

$$ {}_{k-1}p_a^{aa} \cdot p_{x+k-1}^{ad_j} + \sum_{i=1}^{n} {}_{k-1}p_x^{ad_i} \cdot p_{x+k-1}^{d_i d_j} \qquad \textbf{[9.]} $$

${}_kp_x^{aa}$ is the probability that a healthy individual aged x will reach age $x+k$ in the same state:

$$ {}_{k-1}p_x^{aa} \cdot p_{x+k-1}^{aa} + \sum_{j=1}^{n} {}_{k-1}p_x^{ad_j} \cdot p_{x+k-1}^{d_j a} \qquad \textbf{[10.]} $$

and finally, ${}_kp_x^{af}$ is the probability that an active person aged x will not reach age $x+k$. The individual could have died (active or dependent) in any year within the period:

$$ {}_{k-1}p_x^{aa} \cdot p_{x+k-1}^{af} + \sum_{j=1}^{n} {}_{k-1}p_x^{ad_j} \cdot p_{x+k-1}^{d_j f} \qquad \textbf{[11.]} $$

Under the framework described above, the life expectancy of active people disaggregated into healthy and unhealthy life years can be computed as:

$$ \begin{aligned} e_x^a &= \\ \sum_{k=0}^{w-x} {}_kp_x^{aa} + \sum_{k=1}^{w-x} {}_kp_x^{ad_1} + \cdots \dots \dots + \sum_{k=1}^{w-x} \cdot {}_kp_x^{ad_{n-1}} + \sum_{k=1}^{w-x} {}_kp_x^{ad_n} &= \\ \overbrace{e_x^{aa}}^{healthy\ life\ years} + \underbrace{\sum_{j=1}^{n} e_x^{ad_j}}_{\substack{unhealthy\ life\ years \\ with\ j-level\ LTC}} \end{aligned} \qquad \textbf{[12.]} $$

If the individual aged (x) is in state (i), the vector row in this case with value one in the position corresponding to state (i) and zero in the rest of the positions, i.e. their health state is not considered able, then life expectancy can be expressed as:

$$ e_x^{d_i} = \overbrace{\sum_{k=0}^{w-x} {}_kp_x^{d_i d_i}}^{In\ the\ same\ state} + \overbrace{\sum_{k=1}^{w-x} {}_kp_x^{d_i a}}^{Total\ recovery} + \underbrace{\sum_{k=1}^{w-x} \sum_{\substack{j=1 \\ j \neq i}}^{n} {}_kp_x^{d_i d_j}}_{Transitions\ to\ otherstates} \qquad \textbf{[13.]} $$

$$ = $$

$$\overbrace{e_x^{d_i d_i}}^{In\ the\ same\ state} + \underbrace{e_x^{d_i a}}_{\substack{Free\ of \\ activity \\ limitation}} + \overbrace{\sum_{j=1}^{i-1} e_x^{d_i d_j}}^{\substack{\text{In better} \\ \text{states of} \\ \text{dependence}}} + \underbrace{\sum_{j=i+1}^{n} e_x^{d_i d_j}}_{\substack{\text{In worse} \\ \text{states of} \\ \text{dependence}}}$$

where:

${}_k p_x^{d_i d_i}$ is the probability that a person with level d_i dependence aged (x) will reach age $(x+k)$ in the same state of dependence. The probability that a person will remain continuously in the same state for (k) length of time, occupancy probability term ${}_k p_x^{\frac{d_i d_i}{}}$, is included here:

$$\begin{aligned} {}_k p_x^{d_i d_i} &= \\ {}_k p_x^{\frac{d_i d_i}{}} + \sum_{s=1}^{k} {}_{s-1} p_x^{d_i a} \cdot p_{x+s-1}^{a d_i} \cdot {}_{k-(x+s)} p_{x+s}^{\frac{d_i d_i}{}} & \\ + \sum_{\substack{j=1 \\ j \neq i}}^{n} \sum_{s=1}^{k} {}_{s-1} p_x^{d_i d_j} \cdot p_{x+s-1}^{d_j d_i} \cdot {}_{k-(x+s)} p_{x+s}^{\frac{d_i d_i}{}} & \end{aligned} \qquad \textbf{[14.]}$$

${}_k p_x^{d_i d_j}$ is the probability that a person with level d_i dependence aged (x) will reach age $(x+k)$ in level j of dependence.

$\sum_{j=1}^{i-1} e_x^{d_i d_j}$ indicates how many years of their total remaining life expectancy the individual can expect to live in better states of dependence.

$\sum_{j=i+1}^{n} e_x^{d_i d_j}$ indicates how many years of their total remaining life expectancy the individual can expect to live in worse states of dependence.

The median age at death, Md.

When the value of Md is found between two complete single ages x and $x+1$, its value needs to be interpolated as $Md = x + \gamma$, where γ is a function of the number of people surviving in the same health state between ages x and $x+1$. Assuming linearity in this interval, age Md is located as:

$$Md = x + \frac{(l_x - \frac{l_{x_e}}{2})}{(l_x - l_{x+1})} \qquad \textbf{[15.]}$$

Alternatively, and more appropriately for actuarial approach, the median age at death can be defined as the age at which the survival function is equal to one half (Canudas-Romo, 2008), and using the discrete distribution of deaths is

$$Md = \left\{x \; such \; that \; S_{x_e}(x) = {}_{x-x_e}p_{x_e} = \frac{1}{2}\right\} \qquad \textbf{[16.]}$$

The value of Md with decimal precision points can also be estimated by linear interpolation between two complete single ages x and $x + 1$

$$Md = x + \frac{(S_{x_e}(x) - 0.5)}{(S_{x_e}(x) - S_{x_e}(x+1))} \qquad \textbf{[17.]}$$

The interquartile range (IQR)

It is computed by the mathematical difference between the third and first quartiles of the data:

$$Q_3 - Q_1 \qquad \textbf{[18.]}$$

where Q_1 and Q_3 are, respectively, the first and third age quartiles, values which will have to be interpolated with a discrete distribution by analogous procedures to the median (van Raalte & Caswell, 2013). IQR is not sensitive to outlier data. It is sensitive to transfers between quartiles but not to transfers within quartiles. In our context, it indicates the table-specific difference between the 25th and 75th percentiles in survivorship. The larger range in this measure indicates more variability and uncertainty, whereas a smaller range signals greater regularity in lifespans.

The adult modal age at death, *M*.

In our case the cohort is aged $x_e = 65$ or older and the health state is able (or with j-level activity limitation).

$$M = \{x \; such \;\; that \; x > x_e \; and \; d_x > \; d_a \;\; \forall \; a > x_e\} \qquad \textbf{[19.]}$$

where d_x is the number of deaths between the exact ages of x and $x + 1$.

Alternatively, and also more appropriately for actuarial purposes, the modal age at death can be defined as

$$M = \left\{x \; such \;\; that \; Max\left[d_{x_e}(x)\right] \;\; \forall \; x > x_e\right\} \qquad \textbf{[20.]}$$

where $d_{x_e}(x)$ is the lifespan for the cohort aged x_e, i.e. the life table density function describing the distribution of deaths for a cohort starting from age x_e. To obtain the expression for the modal age at death with decimal precision its value is estimated in the range $[x - 1, x + 1]$ by the parabola (a quadratic polynomial approximation) which has the right areas below it to produce the observed values $d_{x_e}(x - 1)$, $d_{x_e}(x)$ and $d_{x_e}(x + 1)$, that is (Canudas-Romo, 2010)

$$M = x + \frac{d_{x_e}(x) - d_{x_e}(x-1)}{\left(d_{x_e}(x) - d_{x_e}(x-1)\right) + \left(d_{x_e}(x) - d_{x_e}(x+1)\right)} \qquad \textbf{[21.]}$$

For the discrete distribution of deaths and for practical purposes consider that $d_{x_e}(x) = {}_{x-x_e/}q_{x_e}$, the deferred probability of death from x_e.

For a given age, the resulting value of the adult modal age at death, M, depends on the distribution of individuals at that age between active (healthy) (a) and dependent ($j \in \{1,2,\dots,n\}$-level dependent)), so in reality there is clearly more than one mode. Given a starting age and an initial health state (able or dependent), there is a mode for each state. In states with a high level of dependence, a biased mode is usually found at ages very close to the starting age that is significantly far from life expectancy and from the median, denoting a highly asymmetric probability distribution.

If we decompose the deferred probabilities of death for a given age (x_e) and the initial health state ($i \in \{a, d_1, d_2, \cdots, d_n\}$) in the deferred probabilities of death of the different states of activity considered in which death occurs ($j \in \{a, d_1, d_2, \cdots, d_n\}$), a modal value is obtained for each one of them ($M_{x_e}^{ij}$). Thus it can be calculated as the weighted adult modal age at death ($\bar{M}_{x_e}^{i}$).

The weighted adult modal age at death

It can be expressed as:

$$\bar{M}_{x_e}^{i} = \sum_{j} M_{x_e}^{ij} \cdot Q_{x_e|i}^{jf} \qquad \textbf{[22.]}$$

where the weightings are the percentages of deaths in the different states:

$$Q_{x_e|i}^{jf} = \sum_{x>x_e} {}_{x-xe/}p_{x_e|i}^{jf} \qquad \textbf{[23.]}$$

Other indicators of mortality and morbidity.

The implicit (synthetic) LTC prevalence rate

Given that the evolution of the hypothetical population broken down by health states can be easily computed from the vector (or matrix row) of the multiyear transition probabilities (${}_kM_x$), the implicit (synthetic) LTC prevalence rate[16], λ_{x+k}^{j}, which is the ratio between the number of dependent people with dependence level j (l_{x+k}^{j}) and the number of individuals aged $x+k$ (l_{x+k}), it is easy to get:

$$\lambda_{x+k}^{j} = \frac{l_{x+k}^{j}}{l_{x+k}} = \frac{{}_kp_x^{ad_j}}{{}_kp_x^{aa} + \sum_{i=1}^{n} {}_kp_x^{ad_n}} = \frac{{}_kp_x^{ad_j}}{{}_kM_x \cdot 1(n) - {}_kp_x^{af}} = \frac{{}_kp_x^{ad_j}}{1 - {}_kp_x^{af}} \qquad \textbf{[24.]}$$

where $1(n)$ is the n-dimensional row vector whose n components are 1.

And under the assumption that in the first year (at the inception of the system and/or at the beginning of the insurance contract) there are only healthy people, **the average LTC prevalence rate**, $\bar{\lambda}_x^{j}$, is:

$$\bar{\lambda}_x^{j} = \frac{\sum_{k=1}^{w-x} l_{x+k}^{j}}{\sum_{k=0}^{w-x} l_{x+k}} \qquad \textbf{[25.]}$$

[16] This multistate disease prevalence might be referred to as a "synthetic prevalence", analogous to the life table cohort (Barendregt, 2002).

Obviously, the active prevalence rate is: $\bar{\lambda}_x^a = 1 - \sum_{j=1}^{n} \bar{\lambda}_x^j$.

The mortality ratio

The mortality ratio, δ_{x+k}^j, of dependent people with dependence level j aged $x + k$, is the ratio between the mortality rates for dependent people and active people (the extra-mortality added for dependent people), which in general terms decreases with age. In our notation it can be expressed as:

$$\delta_{x+k}^j = \frac{p_{x+k}^{d_j f}}{p_{x+k}^{af}} \qquad \textbf{[26.]}$$

Depending on data availability, this ratio could be expressed as dependent persons/general population.

The average mortality ratio ($\bar{\delta}_x^j$) is:

$$\bar{\delta}_x^j = \frac{\sum_{k=1}^{w-x} l_{x+k}^j \cdot \delta_{x+k}^j}{\sum_{k=1}^{w-x} l_{x+k}^j} = \frac{\sum_{k=1}^{w-x} \lambda_{x+k}^j \cdot \delta_{x+k}^j}{\bar{\lambda}_x^j} \qquad \textbf{[27.]}$$