Flexible optical system for separable fractional Fourier transform

José A. Rodrigo   Tatiana Alieva   María L. Calvo

Universidad Complutense de Madrid,

Facultad de Ciencias Físicas, Ciudad Universitaria s/n, Madrid 28040, Spain.

Abstract. In this work we propose a flexible symmetric configuration for the separable fractional FT composed by three generalized lenses. The distances between the lenses are fixed. The fractional angles are acieved by means of lens rotations.

Keywords: Separable fractional Fourier Transform, Optical system design.
PACS:42.15.E9, 42.30.d

INTRODUCTION

Nowadays the fractional Fourier transform is actively used in optical signal processing. For many applications such as noise reduction, shift variant convolution, neural networks, etc. the fractional angle have to be adapted mostly in real time. Therefore, the flexible optical scheme which permits easily vary the fractional angle is required. Here we propose a flexible symmetric configuration for the separable fractional FT composed by three generalized lenses. The distances between the lenses are fixed. The generalized lenses are implemented by a phase operating spatial light modulators (SLMs) or by the assembled sets of cylindrical lenses. The fractional angle is changed according to the corresponding modulation of SLMs or by the appropriate rotation of some lenses.

The kernel of the separable fractional FT is parametrized by the following ray transformation matrix

$$\mathbf{T}_{frFT} = \begin{bmatrix} \mathbf{X}_{frFT} & \mathbf{Y}_{frFT} \\ -\mathbf{Y}_{frFT} & \mathbf{X}_{frFT} \end{bmatrix}, \tag{1}$$

where

$$\mathbf{X}_{frFT} = \begin{bmatrix} \cos\gamma_x & 0 \\ 0 & \cos\gamma_y \end{bmatrix}, \tag{2}$$

$$\mathbf{Y}_{frFT} = \begin{bmatrix} \sin\gamma_x & 0 \\ 0 & \sin\gamma_y \end{bmatrix}. \tag{3}$$

CP860, *Information Optics: 5th International Workshop,*
edited by G. Cristóbal, B. Javidi, and S. Vallmitjana

OPTICAL SYSTEM DESIGN

The basic elements for the construction of the flexible fractional FT system are generalized lenses and free space intervals, described by corresponding ray transformation matrices $\widetilde{\mathbf{L}}_n$ and $\widetilde{\mathbf{Z}}$:

$$\widetilde{\mathbf{L}}_n = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{L}_n & \mathbf{I} \end{bmatrix}, \tag{4}$$

$$\widetilde{\mathbf{Z}} = \begin{bmatrix} \mathbf{I} & z\mathbf{I} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}. \tag{5}$$

It has been shown in [1] that the following composition of these elements is able to perform optical separable fractional FT

$$\mathbf{T} = \widetilde{\mathbf{L}}_1\, \widetilde{\mathbf{Z}}\, \widetilde{\mathbf{L}}_2\, \widetilde{\mathbf{Z}}\, \widetilde{\mathbf{L}}_1, \tag{6}$$

Note that the system is symmetric: the last lens equals to the first one and the distances between the lenses are identical.

In order to achieve the fractional FT for angles γ_x, γ_y at the output plane of the system the lens matrices have to in the form (dimensionless variables):

$$\mathbf{L}_1 = \begin{bmatrix} \cot\left(\frac{\gamma_x}{2}\right) - \frac{1}{z} & 0 \\ 0 & \cot\left(\frac{\gamma_y}{2}\right) - \frac{1}{z} \end{bmatrix}, \tag{7}$$

$$\mathbf{L}_2 = \frac{1}{z^2} \begin{bmatrix} \sin\gamma_x - 2z & 0 \\ 0 & \sin\gamma_y - 2z \end{bmatrix}. \tag{8}$$

Where z is the free propagation distance. These lenses can be implemented by the phase operating SLMs, that permits quasi-automatic real time control of fractional angles.

For the separable fractional FT for the angles $\gamma_x = -\gamma_y = \alpha$, Eqs. (7,8), $\mathbf{L}_1(\alpha, -\alpha)$ and $\mathbf{L}_2(\alpha, -\alpha)$ can be implemented by means of a well-known particular generalized lens $\widetilde{\mathscr{L}}$ [1, 3, 4]. This generalized lens is constructed using two cylindrical and one spherical lenses assembled together, defined by the lens submatrix:

$$\mathscr{L} = p\sin 2\omega \begin{bmatrix} -\sin 2\Omega & \cos 2\Omega \\ \cos 2\Omega & \sin 2\Omega \end{bmatrix} + p_3\mathbf{I}. \tag{9}$$

The matrix $\widetilde{\mathscr{L}}$ is given by $\widetilde{\mathscr{L}} = \widetilde{\mathscr{L}_1}\widetilde{\mathscr{L}_2}\widetilde{\mathscr{L}_3}$, where the cylindrical lenses $\widetilde{\mathscr{L}_1}$ and $\widetilde{\mathscr{L}_2}$ can be rotated by angles $\varphi_1 = \Omega + \omega$ and $\varphi_2 = \Omega - \omega$ respectively, and the convergent spherical lens $\widetilde{\mathscr{L}_3}$ remains fixed. The parameters $p = p_1 = -p_2$ and p_3 are powers of the corresponding lenses.

It is easy to see that $\mathbf{L}_2$ can be implemented using lens $\mathscr{L}$ with the following parameters: $\Omega = -\frac{\pi}{4}$, $2\omega = \alpha = 2\varphi_1 + \frac{\pi}{2}$, $p = -\frac{1}{z^2}$ and $p_3 = -\frac{2}{z}$. Different values of angle α for $\mathbf{L}_2(\alpha,-\alpha)$ are obtained by only rotation of cylindrical lens $\widetilde{\mathscr{L}_1}$, and the angle $\varphi_1+\varphi_2=-\frac{\pi}{2}$ is fixed.

The lens $\mathbf{L}_1(\alpha,-\alpha)$ can also be implemented by means of $\mathscr{L}$ set up Eq.(9), but only for certain values of α such that $\cot\left(\frac{\alpha}{2}\right)=p\sin 2\omega$. In this case $\Omega=-\frac{\pi}{4}$, $2\omega=2\varphi_1+\frac{\pi}{2}$ and $p_3=-\frac{1}{z}$. In order to obtain a full angular domain $\alpha\in(0,2\pi]$, the lens power p must be adjusted.

The operation curves for the rotation angles φ_1 and φ_2 are represented for lens power $p=2m^{-1}$ in Fig.1. Notice that Fig.1.a and Fig.1.b correspond to generalized lens $\widetilde{\mathbf{L}}_1$ and $\widetilde{\mathbf{L}}_2$ respectively, which are performed by means of $\widetilde{\mathscr{L}}$ lens set. Rotation angles φ_1(continuous line) and φ_2 (dashed line) are given as a function of the fractional angle α. For the lens power $p=2m^{-1}$ all angular domain is covered except certain α values which are indicated by thick line in Fig.1.a. This angular domain enlarges when lens power p is increasing.

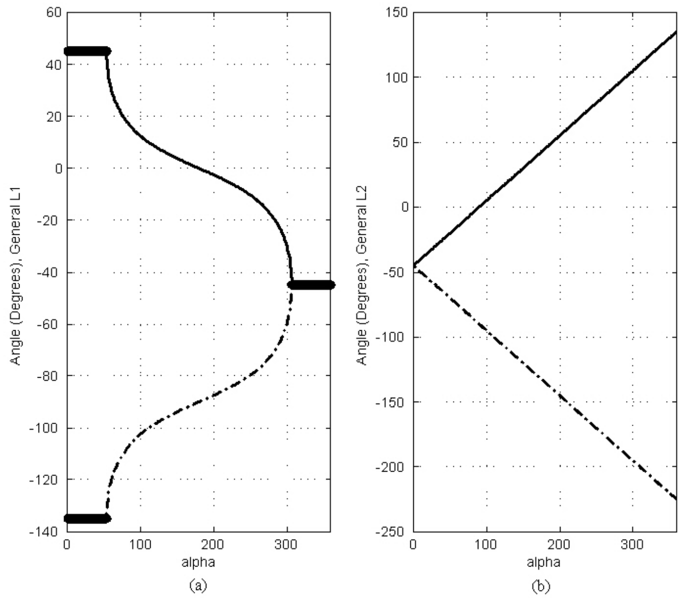

FIGURE 1. Operation curves for the rotation angles φ_1 (continuous line) and φ_2 (dashed line) are representated for lens power $p=2m^{-1}$. Figure (a) and (b) corresponds to generalized lens $\widetilde{\mathbf{L}}_1$ and $\widetilde{\mathbf{L}}_2$ respectively.

NUMERICAL SIMULATION

The Figure 2 demonstrates the action of the proposed system obtained by numerical simulations. Special program calculates step by step the light propagation through the system including phase modulation associated with the thin lenses and 2 free space paraxial diffraction intervals. The intensity distributions of the fractionally Fourier transformed image for the different angles γ_x and γ_y are displayed.

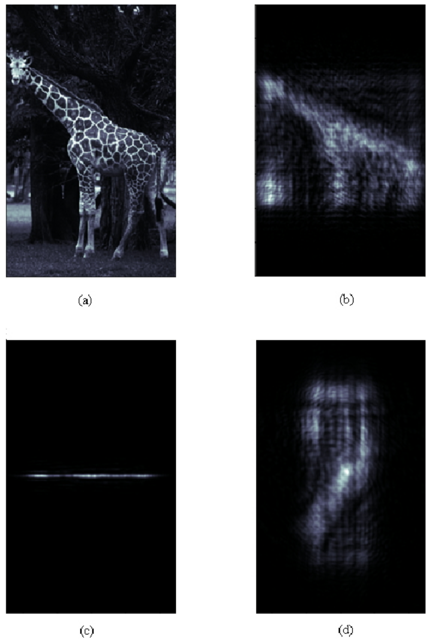

FIGURE 2. Intensity distribution for the fractional FT transfrom for angles $\gamma = 2\gamma_x = -\gamma_y$ for $\gamma = 0$, original image (a), $\gamma = \frac{\pi}{4}$ (b), $\gamma = \frac{\pi}{2}$ (c) and $\gamma = \frac{3\pi}{4}$ (d).

CONCLUSIONS

A flexible symmetric configuration for the separable fractional FT composed by three generalized lenses and two free space intervals has been obtained. The distances between the lenses are fixed. The generalized lenses are implemented by an assembled sets of cylindrical lenses. The fractional parameter is changed by the cylindrical lens rotation. The analogue lens system simulated in this paper covers the range of angles α about 250 degrees. Notice that in order to cover all angular combinations, except some singular cases (γ_x or γ_y is zero), the generalized lenses have to be implemented by a SLM.

ACKNOWLEDGMENTS

Partial financial support from Spanish Ministry of Education and Science, project TEC2005-02180 is acknowledged.

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