The public investment rule in a simple endogenous growth model with public capital: active or passive?*

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ABSTRACT

In dynamic settings with public capital, it is common to assume that the government claims a constant fraction of public investment to total output each period, which is clearly a restrictive assumption. The goal of the paper is twofold: first, to find out a more *reasonable* rule for public investment, consistent with US data, than the constant-ratio rule; second, to analyze the impact of that rule on welfare and judge the public investment downsizing process held in US since the end of the sixties. Calibrating for US, the model simulation captures the public investment downsizing process held during 1960-2001, as well as the post-1970 slowdown in private factors productivity. Downsizing would be optimal whenever the public capital elasticity is approximately smaller than 0.09, a lower level than the general consensus in the literature. Thus, it is more likely that our result be consistent to Aschauer (1989) and Munnell (1990), which put forth that policymakers would have reduced the stock of public capital below its optimum level along this time.

Keywords: Public investment rule, policy coordination, transitional dynamics, endogenous growth, public capital elasticity.

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1. Introduction

Since the empirical paper of Aschauer (1989) and Munell (1990), many works have focused on the positive incidence of public investment on growth and welfare. From a theoretical point of view, Barro (1990) was an important breakpoint on that subject, considering an endogenous growth framework with public capital.[1] Among many others, Futagami et al. (1993), Glomm and Ravikumar (1994), Cassou and Lasing (1998,1999) and Turnovsky (1996, 2000) are variations of Barro (1990). In all them, public investment is considered to be a constant fraction of total output each period (the constant-ratio rule).

In a Barro-type setting, the competitive equilibrium allocation is not Pareto-efficient because of the externality driven by the public capital in the production process. However, the purpose of the paper is not to find the policy that would restore the efficient allocation. The goal of the paper is twofold: i) to find out a more *reasonable* rule for public investment, consistent with US data, than the constant-ratio rule; ii) to analyze the impact of that rule on welfare and judge the public investment downsizing process held in US since the end of the 60's.

Figure 1 shows the evolution of the public investment/output ratio from 1929 to 2001 in the US economy. It is clear that its path is far from being stationary. Omitting the war and the early post-war periods, we could distinguish four phases on its evolution. First, there is an upward sloping (upsizing) period from 1929 to approximately 1955, along which the ratio raised from an average of 4% in the 30's to an average between 5.5%-6% in the first half of the 50's. Second, there is a short period of time, between 1956 and 1966, in which the ratio fluctuates around 5.4%. Third, there is a downward sloping (downsizing) period from approximately 1967 to the beginning of the 80's, along which the ratio reduced from levels of 5.4% to levels close to 3%. Finally, there exists a relatively stable period from 1982 on, along which the ratio has been fluctuated around 3.3%. According to that, a constant-ratio rule would not be a realistic assumption: the convergence process during the upsizing and downsizing periods took enough time to consider that the economy just jumped from one stabilized period to another. In addition, we will find in Section 2 a positive and significative relationship between the public investment/output ratio and the current state of the economy.[2]

[INSERT FIGURE 1 ABOUT HERE]

This paper considers a more general and flexible rule for public investment (the active rule) that includes the constant-ratio rule as a particular one. The government targets a level of public investment as a percentage of output in the long-run, but along the transition it can adjust the ratio to the current state of the economy. We assume a log-linear functional form to capture this relationship, consistent to US data. Considering

[1] Endogenous growth models assign a key role to fiscal policy as a determinant of long-run economic growth, which constitutes an attraction to use these models to study fiscal policy implications [see Barro (1990), Rebelo (1991), Jones Manuelli and Rossi (1993), Turnovsky (1996, 2000), among many others]

[2] In our theoretical framework, the state variable will be the public to private capital stock ratio.

this active rule instead of the constant-ratio rule, we find that the model simulation fits much better the public investment downsizing and posterior stabilized process held during 1960-2001 in US, as well as the post-1970 slowdown in private factors productivity and economic growth.

Assuming an active rule, the government problem could be seen as a coordination problem between the short- and the long-term policy. While the relationship between short- and long-run policies has been widely studied in the monetary policy literature,[3] little work has been done regarding this subject for the fiscal policy, which is a contribution of the paper.[4] In general, depending on the short-run policy, the government would face with a continuum of alternative paths for the public investment/output ratio, leading all them to the same long-run target,[5] and must decide the optimal combination.

A public investment measure generates a particular trade-off between initial welfare, welfare along the transition and long-run welfare. In general, upsizing *sacrifice* initially consumption, welfare and private capital in favor of public capital. Whenever the public to private capital ratio is initially below its optimum, substituting the former by the latter would be propitious for the economy to start growing faster along the transition and even more than compensate the initial utility lost. A symmetric relationship is shown for downsizing, leading in general to an initial utility raise, followed by a welfare lost or gain, depending on whether the public to private capital ratio is below or above its optimum level.

Given a long-run policy, the short-run measure must follow to attain the optimal trade-off in welfare. Whenever upsizing is optimal,

the optimal path for public investment as a percentage of output shows the following shape: an initial big jump, but keeping the ratio below its final target, followed by a monotone and slow convergence process. That way the policy mimics the negative incidence on consumption along the former periods of the transition, while the positive effect of substituting private by public capital extends throughout the whole transition. On the other hand, whenever downsizing turns optimal, the optimal path is of the following kind: an initial important fall, overshooting its level, followed by a monotone and fast convergence process. That way, the short-run policy emphasizes the initial positive impact on consumption, while the effect on the long-run would remains almost unchanged because of the quick convergence. In both cases, the public investment/output ratio stays during the former periods of the transition below the welfare-maximizing ratio for a constant-ratio

[3]See Svensson (1999) and Taylor (1999), among many others. The monetary authority is assumed to follow a particular policy rule, with a *long-run target* (generally on inflation, nominal-GDP or money growth) as well as with a *short-run active rule* response to the state of the economy.

[4]For instance, to homogenize future members, the Maastricht Treaty on European Union imposed several *long-run* targets to be achieved by any country attempting to be a potential member of the Union. Thus each member must coordinate its short-run policy with the long-run target, in order to achieve it. As an additional example, the International Monetary Fund gives policy guidelines to countries with economic troubles in order to improve *long-run* sustained economic growth.

[5]The long-run target will be attained in just one period under the constant-ratio rule.

rule, but it ends up converging towards a higher level. Thus, the latter could be seen as a weighted average of the former.

The public capital elasticity is a controversial parameter to calibrate and is crucial to determine the optimal policy, so we condition the welfare analysis to its magnitude. We find that the optimal public investment path and the public capital elasticity are positive related, as in Barro (1990) and many others. For the benchmark economy, downsizing is optimal when the public capital elasticity is approximately lower than 0.09, a similar value to the estimated by Munell (1990). Moreover, being a public investment/output ratio of 0.033 an optimal choice, the public capital elasticity would need to be around 0.05. In both cases, these values are below the general consensus in the literature.[6] Thus, it is more likely that our result be consistent with, among many others, Aschauer (1989) and Munnell (1990), which put forth that policymakers have reduced the stock of public capital below its optimum level, in detriment also of the productivity of complementary private inputs.

The rest of the paper is organized as follows. Section 2 gives empirical evidences against the constant-ratio rule and in favor of the active rule for the US economy. Section 3 describes the framework of analysis. Section 4 exposes the competitive equilibrium and the balanced growth path conditions. Section 5 shows the way we design and handle the policy experiment. Section 6 simulates the model for the benchmark policy and exposes main results for a simple policy experiment. Section 7 shows the optimal policy results. Finally, section 8 ends with main conclusions and extensions.

2. A first exploration of data

In this section we briefly describe the evolution of some important macromagnitudes for the US economy during 1960-2001 and study the relationship between public investment as a percentage of real GDP, x, and the current state of the economy. We will consider these facts to support some assumptions made in our theoretical framework. We use yearly data for the US economy from 1930 to 2001.[7] Since our theoretical framework will be a non-stochastic endogenous growth setting, we take the public to private capital stock ratio, $k^g = K^g/K$, as the state variable of the economy.

Table 1 summarizes the evolution of main macroeconomic variables and public expenditure concepts from 1930 to 2001. We focus on the period 1960-2001. The real GDP raised an average rate of 4.4% per year in the 60's, and slowed down monotonically until

[6] Aschauer (1989) estimates an elasticity of 0.39, Munell (1990) gets a 0.1, Cazzavilan (1993) a 0.25, Lynde and Richmond (1993) a 0.2, Ai and Cassou (1993) get an elasticity of 0.2, etc.

[7] Source: Bureau of Economic Analysis, billions of dollars, yeared estimated series. The public investment measures the gross government fixed investment. The series of capital are the current-cost net stock of private and public fixed assets. The series of the private sector include equipment, software and structures. The series of the public sector include those of the general government (federal, state and local) and government enterprises.

an average of 2.8% in the 90's. In the 60's, private consumption represented a 61.8% of total output and a 67.1% in the 90's, while private investment remained pretty constant along this period (it slightly raised from 15.5% to 15.7%). On the other hand, as a percentage to output, public consumption and public investment diminished from 17.1% and 5.2% in the 60's to 15.5% and 3.3% in the 90's, respectively. However, the public sector size[8] increased from an average of 24.2% to 30.3%, mainly because of the net payment of interests and transfers. However, if we exclude the net payment of transfers, which is nothing but a way to redistribute resources among individuals, the ratio was fairly constant along these 40 years: the average was 18.6% in the 60's and 18.7% in the 90's.

[INSERT TABLE 1 ABOUT HERE]

Regarding the evolution of the public investment/output ratio -see Figure 1-, the convergence process during the upsizing (1945-1955) and downsizing (1965-1980) periods took enough time to consider that the economy just jumped from one stabilized period to another. Hence, a constant-ratio rule might not be appropriated to capture the evolution of the public investment/output ratio along any of these periods.

Public investment as a percentage of output might change in response to the current state of the economy (active rule) or might not (passive rule). For the US economy we run the following regression to study whether the public investment rule could be shown as active or passive,

$$\triangle\hat{x}_t = a + b\triangle\hat{k}_t^g + \sum_{i=1}^{p} c_i \triangle\hat{x}_{t-i} + \varepsilon_t, \tag{2.1}$$

where $\hat{x}_t = \ln(x_t/0.032)$ and $\hat{k}_t^g = \ln(k_t^g/0.28)$, being 0.032 and 0.28 the average of x_t and k_t^g in the 90's.[9] We test whether β is significative different from zero (i.e., the rule is active) or not (i.e., the rule is passive) for alternative periods of time.

[INSERT TABLE 2 ABOUT HERE]

Table 2 summarizes main results from estimations, from which we can infer the following facts: i) results are quit sensitive to the sample considered; ii) however, during the convergence periods, the passive rule hypothesis is clearly rejected in favor of the alternative active rule; iii) but, if we focus on the stabilized period 1982-2001, the rule behaves as a passive one, with the parameter b not being statistically different from zero.

[8] It is measured as the current public expenditure (general public consumption+net transfers+net paid of interest) as a percentage of current GDP.

[9] Time series are non-stationary and non-cointegrated, so we take first differences. We add some dynamics of x in order to rid off any significative autorregresive structure in the residuals, and thus make estimations more efficient. When considering the whole sample and the subsample of 1929-1966, we consider a dummy variable, $WWII$, which takes 1 for 1942-1945 and -1 for 1946-1948, and zero otherwise.

3. The theoretical framework

We describe in this section the theoretical framework. It is an endogenous growth setting with public and private capital, and three economic agents: households, firms and a government.

3.1. Firms

There exists a continuum of identical firms producing the single commodity good in the economy. Private capital, $\tilde{k}_t$, and labor, $\tilde{l}_t$, are lent by households to the firms to produce $\tilde{y}_t$ units of output.

The total amount of physical capital used by all firms in the economy, $\tilde{K}_t$, is taken as a proxy for the index of knowledge available to each firm [as in Romer (1986)]. Additionally, public capital, $\tilde{K}_t^g$, affects the production process of all individual firms. Except for these externalities, the private production technology is a standard *Cobb-Douglas* function presenting constant returns to scale in the private inputs and increasing returns in the aggregate. For any firm,

$$\tilde{y}_t = f(\tilde{l}_t, \tilde{k}_t, \tilde{K}_t, \tilde{K}_t^g) = F\tilde{l}_t^{1-\alpha}\tilde{k}_t^{\alpha}\tilde{K}_t^{\phi}\left(\tilde{K}_t^g\right)^{\varphi}, \ \varphi,\ \alpha \in (0,1),\ \phi \geq 0, \tag{3.1}$$

where α is the share of private capital in gross output, φ and ϕ are the elasticities of output with respect to public capital and the knowledge index, respectively, and F is a technological scale factor.

Since firms are identical, from (3.1), aggregate output, $\tilde{Y}_t$, is produced according to,

$$\tilde{Y}_t = F\tilde{L}_t^{1-\alpha}\tilde{K}_t^{\alpha+\phi}\left(\tilde{K}_t^g\right)^{\varphi}, \tag{3.2}$$

where $\tilde{L}_t$ is aggregate labor.

During period t, each firm pays the competitive-determined wage $\tilde{w}_t$ on the labor it hires and the rate r_t on the capital it rents. The profit maximizing problem of the typical firm turns out to be static,

$$\underset{\{l_t,k_t\}}{Max}\ f(\tilde{l}_t, \tilde{k}_t, \tilde{K}_t, \tilde{K}_t^g) - \tilde{w}_t\tilde{l}_t - r_t\tilde{k}_t.$$

Optimally leads to the usual marginal productivity conditions:

$$r_t = f_{\tilde{k}}' = \alpha F\tilde{l}_t^{1-\alpha}\tilde{k}_t^{\alpha-1}\tilde{K}_t^{\phi}\left(\tilde{K}_t^g\right)^{\varphi} = \alpha\frac{\tilde{y}_t}{\tilde{k}_t} = \alpha\frac{\tilde{Y}_t}{\tilde{K}_t}, \tag{3.3}$$

$$\tilde{w}_t = f_{\tilde{l}}' = (1-\alpha)F\tilde{l}_t^{-\alpha}\tilde{k}_t^{\alpha}\tilde{K}_t^{\phi}\left(\tilde{K}_t^g\right)^{\varphi} = (1-\alpha)\frac{\tilde{y}_t}{\tilde{l}_t} = (1-\alpha)\frac{\tilde{Y}_t}{\tilde{L}_t}, \tag{3.4}$$

where we have considered that each firm treats its own contribution to the aggregate capital stock as given, rents the same quantity of private inputs and produces the same amount of output.

3.2. Households

The representative consumer chooses the fraction of time to spend as leisure. She is the owner of the physical capital, and allocates her resources between consumption, $\tilde{C}_t$, and investment in physical capital, $\tilde{I}_t^k$. Private capital accumulates over time according to

$$\tilde{K}_{t+1} = (1-\delta^k)\tilde{K}_t + \tilde{I}_t^k, \tag{3.5}$$

where $\tilde{K}_{t+1}$ denotes the stock of physical capital at the end of time t and δ^k is the depreciation factor for private capital, between zero and one. Zero population growth is assumed and the time endowment is normalized to one. Decisions are made every period to maximize the discounted aggregate value of the time separable utility function,[10]

$$\begin{aligned} \sum_{t=0}^{\infty}\beta^t u(\tilde{C}_t, h_t) &= \sum_{t=0}^{\infty}\beta^t \frac{\left[\tilde{C}_t^{\rho}(1-h_t)^{1-\rho}\right]^{1-\theta}-1}{1-\theta}, \ \rho\in[0,1],\ \theta>0,\ \theta\neq 1, \qquad (3.6)\\ &= \sum_{t=0}^{\infty}\beta^t\left[\rho\ln\tilde{C}_t + (1-\rho)\ln(1-h_t)\right],\ \rho\in[0,1],\ \theta=1, \end{aligned}$$

where h_t is the fraction of time devoted to production, β is the discount factor, between zero and one, $1/\theta$ is the elasticity of substituting consumption intertemporally and ρ characterizes the importance of consumption relative to leisure.

Her budget constraint is

$$\tilde{C}_t + \tilde{K}_{t+1} + \tilde{T}_t \leq \tilde{w}_t h_t(1-\tau_t^h) + \tilde{K}_t\left[1-\delta^k + r_t\left(1-\tau_t^k\right)\right], \tag{3.7}$$

every period, where τ_t^k and τ_t^h are the tax rates applied to capital and labor income, respectively, and $\tilde{T}_t$ is a net transfer made by households to the public sector.

The representative household faces a discrete dynamic programing problem, in which corner solutions are avoided and restrictions hold with equality due to the special form of the instantaneous utility function and the fact that consumption and leisure are normal goods. Optimal conditions are standard: the consumption-saving decision (3.8), the consumption-leisure choice (3.9), the budget constraint (3.7),

$$\frac{\tilde{C}_{t+1}}{\tilde{C}_t} = \left\{\beta\left(\frac{1-h_{t+1}}{1-h_t}\right)^{(1-\rho)(1-\theta)}\left[1-\delta^k + r_{t+1}\left(1-\tau_{t+1}^k\right)\right]\right\}^{\frac{1}{1-\rho(1-\theta)}}, \tag{3.8}$$

$$\frac{\rho}{1-\rho} = \frac{\tilde{C}_t}{\tilde{w}_t(1-h_t)(1-\tau_t^h)}, \tag{3.9}$$

border constraints, $\tilde{C}_t > 0$ and $\tilde{K}_{t+1} > 0$, $h_t \in (0,1)$, and the transversality conditions

$$\lim_{t\to\infty}\beta^t\tilde{K}_{t+1}\frac{\partial u(\tilde{C}_t,h_t)}{\partial\tilde{C}_t} = \lim_{t\to\infty}\beta^t\tilde{K}_{t+1}^g\frac{\partial u(\tilde{C}_t,h_t)}{\partial\tilde{C}_t} = 0, \tag{3.10}$$

[10] A CES representation is assumed for the single period utility function, capturing cross-substitution between leisure and consumption [King et al. (1988)].

that places a limit on the accumulation of private and public capital.

3.3. The public sector

The government is characterized as a fiscal authority. We consider a broad classification of public expenses: unproductive public expenses, $\tilde{C}_t^g$, which do not directly affect the productive process or consumers' welfare, and public investment, $\tilde{I}_t^g$, which positively affects production. Public capital is accumulated according to

$$\tilde{K}_{t+1}^g = \tilde{I}_t^g + (1-\delta^g)\tilde{K}_t^g, \tag{3.11}$$

where δ^g is the public capital depreciation factor, between zero and one.

The empirical analysis conducted in Section 2 supports the following assumptions: $A1$) the government claims a constant fraction, g, of $\tilde{C}_t^g$ to output each period,

$$g = \tilde{C}_t^g / \tilde{Y}_t, \;\; g \in [0,1); \tag{3.12}$$

$A2$) the public investment/output ratio, x, follows an active rule during the convergence period; $A3$) the rule behaves as a passive one during a stationary period.

In general, the conduct of x can be captured by a function f of state variables, $s \in S \subseteq \Re_+^n$, and a set of policy parameters, $q \in Q \subseteq \Re^m$:

$$f : S\text{x}Q \longrightarrow [0, 1-g], \tag{3.13}$$

such that: i) $f(s;q)$ is continuous in $S\text{x}Q$; ii) there exists a policy parameter $\bar{x} \in q$ such that $f(\bar{s};q) = \bar{x}$, where $\bar{s}$ is the long-run equilibrium level of s; iii) there exists a policy $\hat{q}$ such that $f(s;\hat{q}) = \bar{x}$ for all $s \in S$; iv) for $q \neq \hat{q}$ and $s \neq \bar{s}$, $\triangle f/\triangle s \neq 0$ and $\triangle f^2/\triangle s \triangle q \neq 0$. According to i)-iv), the rule is passive whenever $q = \hat{q}$; moreover, the rule behaves as a passive one along the long-run equilibrium path. Otherwise, the rule will be active, with x converging towards $\bar{x}$, the *long-run policy instrument*. The degree of response of x to the current state of the economy depends on the remaining parameters in q, the *short-run policy instruments*.

Tax revenues finance total public expenses each period. We just consider a proportional tax on total income as the way to collect taxes. Hence, $\tau_t = \tau_t^k = \tau_t^h$ and $T_t = 0$ for all t. The government budget constraint is:

$$\tilde{C}_t^g + \tilde{I}_t^g = \tau_t \tilde{Y}_t \Leftrightarrow g + x_t = \tau_t. \tag{3.14}$$

4. Competitive equilibrium and the balanced growth path

Given $\tilde{K}_0, \tilde{K}_0^g > 0$, the *competitive equilibrium* is a set of prices $\tilde{p}_t = \{r_t, \tilde{w}_t\}_{t=0}^{\infty}$, a set of allocations $\left\{\tilde{C}_t, h_t, \tilde{L}_t, \tilde{K}_{t+1}, \tilde{I}_t^k, \tilde{Y}_t, \tilde{C}_t^g, \tilde{K}_{t+1}^g, \tilde{I}_t^g\right\}_{t=0}^{\infty}$ and a fiscal policy $\tilde{\pi}_t = \{x_t, g, \tau_t\}_{t=0}^{\infty}$, such that, given $\tilde{p}_t$ and $\tilde{\pi}_t$: *i*) $\{\tilde{C}_t, h_t, \tilde{K}_{t+1}\}_{t=0}^{\infty}$ maximize households' welfare [i.e., (3.7)-(3.10) hold]; *ii*) $\{\tilde{K}_{t+1}, \tilde{L}_t\}_{t=0}^{\infty}$ satisfy the profit-maximizing conditions [(3.3)-(3.4) hold], and $\tilde{I}_t^k$ accumulates according to (3.5); *iii*) $\{\tilde{C}_t^g, \tilde{K}_{t+1}^g, \tilde{I}_t^g\}_{t=0}^{\infty}$ evolve according to (3.12)-(3.13); *iv*) the budget constraint of the public sector (3.14) and the technology constraint (3.2) to produce $\tilde{Y}_t$ hold; *v*) markets clear every period,[11]

$$\tilde{L}_t = h_t, \tag{4.1}$$

$$\tilde{Y}_t = \tilde{C}_t + \tilde{C}_t^g + \tilde{I}_t^k + \tilde{I}_t^g. \tag{4.2}$$

A *balanced growth path, bgp,* is defined as an equilibrium path along which aggregate variables either stay constant or grow at a constant rate. Barro (1990) and Jones and Manuelli (1997), among many others, have shown that cumulative inputs must show constant returns to scale in the productive process (i.e., $\alpha+\varphi+\phi=1$) and r_t be constant and high enough for the equilibrium displaying positive steady-growth (hereinafter, variables with bar "$-$" denotes values along the *bgp*). From equilibrium conditions, it is easy to show that $\tilde{Y}_t$, $\tilde{C}_t$, $\tilde{K}_t$, $\tilde{K}_t^g$, $\tilde{C}_t^g$ and $\tilde{X}_t$ must all grow at the same constant rate along the *bgp*, denoted by $\bar{\gamma}$ hereinafter, while bounded variables, such as the tax rate, r_t and h_t, must be constant. From now on, we will focus on the special case in which $\alpha+\varphi+\phi=1$.

From (3.8), a positive long-term growth rate is achieved whenever

$$\bar{\gamma} = \left\{\beta\left[1-\delta^k+(1-\bar{\tau})\bar{r}\right]\right\}^{\frac{1}{1-\rho(1-\theta)}} - 1 > 0 \Leftrightarrow \bar{r} > \frac{1-\beta(1-\delta^k)}{(1-\bar{\tau})\beta}. \tag{4.3}$$

However, although $\bar{\gamma}$ will then be positive, it cannot get so high as to allow households to follow a chain-letter action [(3.10) must hold on the *bgp*], i.e.,

$$\lim_{t\to\infty} \frac{\rho(1-\bar{h})^{(1-\rho)(1-\theta)}\tilde{K}_0\left(1+\bar{\gamma}\right)\beta^t\left(1+\bar{\gamma}\right)^t}{\left[\tilde{C}_0(1+\bar{\gamma})^t\right]^{1-\rho(1-\theta)}} = 0 \Leftrightarrow \beta(1+\bar{\gamma})^{\rho(1-\theta)} < 1, \tag{4.4}$$

which is a necessary condition to ensure time-aggregate utility (3.6) to be bounded.

5. Calibration, the policy rule and the government problem

The economy is assumed to start on the *bgp* associated to the benchmark calibration, with a public capital stock, K_0^g, of 100.[12] We first calibrate the economy. Second, we assume a particular functional form for the policy rule (3.13) consistent with data and *A*1)-*A*3). Next, we expose the procedure to solve the competitive equilibrium for the dynamics of level variables. Finally, we outline the government problem and the way we handle it.

[11] See Appendix (part 1).

[12] The initial state is $K_0^g = 100$ and $K_0 = 100/\bar{k}_0^g$. In this setting, the optimal policy is invariant to this initial condition.

5.1. The benchmark calibration

The calibration matches the initial steady-state of the model with main macroeconomic properties of the US economy at the beginning of the 60's - see Table 3. The time unit is one quarter. The set of all parameters is denoted by Φ. On its initial *bgp*, we want the model to show an average proportion of working time, $\bar{h}$, of 0.33, a 4% annual growth rate ($\bar{\gamma} = 0.0098$ for quarterly data) and a public capital/private capital ratio, $\bar{k}^g$, of 0.34 -see Table 1.

According to Table 1, we set g and $\bar{x}$ equal to 0.18 and 0.054, respectively. Some technological parameters are standard in the literature: $\delta^k = 0.025$ and $\alpha = 0.36$. However, the broad empirical literature discussing the productive nature of public capital shows controversial conclusions, different data sources and econometric techniques leading to rather different estimations of φ.[13] For example, the public capital elasticity varies from 0.06 in Ratner (1983) to the 0.39 in Aschauer (1989). Munnell (1990) uses data for 48 states in the post-war US economy and estimates the public capital elasticity to be about 0.1, while Lynde and Richmond (1993) use time series techniques, accounting for non-stationarity in the data, estimating φ equal to 0.2. For the benchmark economy, we choose $\varphi = 0.15$ and $\phi = 0.49$, so that $\alpha + \varphi + \phi = 1$. Because of its importance, we will consider alternative values of φ when carrying out the policy analysis.

Mehra and Prescott (1985) suggest a relative risk aversion parameter between 1 and 2, and we pick $\theta = 1.5$. Finally, being $\beta = 0.99$, ρ, F and δ^g are chosen to maintain $\bar{h}$, $\bar{\gamma}$ and $\bar{k}^g$ at the values mentioned above.

[INSERT TABLE 3 AROUND HERE]

5.2. The log-linear public investment rule

We assume a specific functional form for $f(\cdot)$ in (3.13) to solve the competitive equilibrium. The following log-linear specification is consistent with $A1$)-$A3$):

$$\begin{aligned} x_t &= \bar{x}\left(k_t^g/\bar{k}^g\right)^{\eta}, \\ \ln(x_t) &= \ln(\bar{x}) + \eta \ln\left(k_t^g/\bar{k}^g\right), \end{aligned} \tag{5.1}$$

being $q = \{\bar{x}, \eta\}$. In addition, this specification shows several advantages. First, it is easy to deal with when solving the model for the competitive equilibrium; second, it fits pretty well to data;[14] third, the parameters have a straightforward interpretation: $\bar{x}$ is the *long-run policy instrument* and η is the elasticity between x and k^g (i.e., *the short-run policy instrument*). If $\eta = 0$, the rule is passive and $x_t = \bar{x}$ every period. Otherwise, the

[13] Section 4 of Glomm and Ravikumar (1997) and Munnell (1992) show a selective review of these empirical studies.

[14] We have considered several specifications (linear, polynomial,...) and the higher adjusted R^2 is that of the log-linear specification.

rule turns active. Additionally, the rule is called pro-cyclical whenever $\eta > 0$, while it is counter-cyclical if $\eta < 0$.[15]

5.3. Solving for the dynamics of level variables

The competitive equilibrium cannot be analytically solved, so a numerical solution is required. However, numerical techniques are designed to solve the transitional dynamics of variables with a well defined steady-state, which is not the case for level variables in an endogenous growth framework. An alternative approach is to deal with normalized variables. Hereinafter, Z_t denotes the normalized level of $\tilde{Z}_t$, $Z_t = \tilde{Z}_t/(1+\bar{\gamma})^t$, which grows at a zero rate along the *bgp*. But the steady-state of Z_t is not well defined, and standard numerical methods applied directly to normalized variables cannot be used either. The standard approach is to solve the equilibrium for stationary ratios, but that strategy precludes the possibility of analyzing welfare issues. In the Appendix (part 2), we describe a procedure that combines the dynamics of stationary ratios - $c_t = \tilde{C}_t/\tilde{K}_t$, $k_t^g = \tilde{K}_t^g/\tilde{K}_t$, $y_t = \tilde{Y}_t/\tilde{K}_t$ and $k_{t+1} = \tilde{K}_{t+1}/\tilde{K}_t$ - and equilibrium conditions to recover the equilibrium path for normalized variables, starting from an initial state of the economy.[16]

5.4. The government problem

The government is benevolent in the sense that its objective function is to maximize the welfare of the representative consumer, given competitive equilibrium conditions. Given the tax system and the public consumption to output ratio, g, the government makes decision on its investment plans. Under the log-linear policy rule (5.1), a public investment policy is given by the pair $q = \{\bar{x}, \eta\}$. The welfare maximizing policy will be denoted by $q^+ = \{\bar{x}^+, \eta^+\}$. A standard search method is used to numerically handle this control problem [as in Jones et al. (1993)].

For each policy and simulation, we check the following conditions: i) K_{t+1}, K_{t+1}^g, C_t, Y_t must be positive; ii) h_t, τ_t must belong to (0,1); iii) x_t has to be inside $(0, 1-g)$; iv) the *npg* condition (4.4) must hold; v) x_t must converge[17] towards $\bar{x}$ in a *reasonable* number of periods (i.e., 200 periods or 50 years); vi) the government is not allowed to destroy infrastructures, unless they will be restored in the current period, i.e., $I_t^g - \delta^g K_t^g \geq 0$. These conditions limit, in a reasonable sense, the set of investment policies available to the government.

For any pair $\{\bar{x}, \eta\}$: a) we solve numerically the balanced growth path equilibrium; b) we recover time series of C_t and h_t from their log-linear approximations, as it is shown in the Appendix (part 2); c) we check conditions i)-vi) and move to the next steps whenever

[15] We use the same terminology than in the cycle literature.

[16] Novales et al. (1999) describes an alternative method to solve for the dynamics of level variables in an endogenous growth setting.

[17] We accept convergence whenever $|\bar{x} - x_t| < 0.001$.

they are satisfied; d) we evaluate total welfare -see Appendix (part 3)-.[18,19]

$$\sum_{t=0}^{\infty}\left\{\frac{\left[\beta(1+\bar{\gamma})^{\rho(1-\theta)}\right]^{t}\left[C_t^{\rho}(1-h_t)^{1-\rho}\right]^{1-\theta}}{1-\theta}-\frac{\beta^t}{1-\theta}\right\}; \tag{5.2}$$

e) the process is repeated for any feasible policy, and the one maximizing (5.2) is the welfare-maximizing choice.

6. A policy experiment

We address several issues in this section: i) the ability of the model to fit the downsizing period 1960-2001; ii) the shape of x_t under the log-linear policy rule; iii) the welfare trade-off due to a particular public investment policy.

6.1. Simulating the benchmark economy

We assume the economy exhibits initially balanced growth with public investment/output ratio of 0.054. Its long-run target changes and the economy moves to a new and stable *bgp*. The benchmark policy sets $q^b = \left\{\bar{x}^b; \eta^b\right\} = \{0.032; 0.52\}$ in (5.1), those parameters estimated in (2.1) for the 1960-2001 period. The constant-ratio rule would set $q^o = \{0.032; 0\}$ in (5.1).

The downsizing process in public investment held during the 1960-2001 period drove the economy to a gradual reduction in the public to private capital ratio (see Section 2). This fact might had to do, at least partially, with the post-1970 slowdown in private factors productivity and economic growth. In terms of total output, private investment substituted public capital initially, but it ended falling towards its initial level due to, among other reasons, the mentioned reduction in private factors productivity. Finally, there was a positive income effect in consumption, that increased its fraction to output. For the benchmark policy, the model simulation captures pretty well all these facts and some others commented below.

Figure 2 compares the simulated paths of x and k^g under q^o and q^b with the observed time series along the 1960-2001 period. The downsizing and posterior stabilized process is much better fitted by the simulation under the active policy than under the passive one. The former shows the pronounced downsizing trend of public investment from mid-60's to mid-70's, as well as the slowdown in its downsizing process from mid-70's to mid-80's and, finally, its stabilization from mid-80's on.

[18] Appendix 3 shows how we evaluate this infinite sum.

[19] The feasible set of welfare levels is bounded because: i) the numerical procedure imposes the competitive equilibrium to be on the stable manifold, hence C_t and h_t eventually stabilize; ii) $\bar{\gamma}$ is bounded from above by (4.4). Moreover, since utility is continuous and strictly concave and the choice set is convex, there exists at most one interior solution to the government problem.

[INSERT FIGURE 2 ABOUT HERE]

[INSERT TABLE 4 ABOUT HERE]

Table 4 compares initial and final values of main ratios in the simulation with their observed annual averages in 1955-1967 and 1995-2001. The initial values of x, k^g and γ obviously coincide, since we match them in the model calibration. According to data, k^g falls from an average of 0.35 to 0.28, while it does until 0.24 in the simulation; the annual growth rate falls from an average of 4.2% to 2.8%, instead of the 3.1% shown in the simulation. On the other hand, the ratios C/Y and I^k/Y are significative larger and smaller, respectively, than those predicted by the simulation. However, if we account as private investment those purchases in durable goods done by households (like cars, houses,...), the difference is almost insignificant. Nevertheless, the simulation captures several important facts for the period on concern: i) C/Y is significative higher than I^k/Y; ii) through the end of the simulation, the level of C/Y is slightly higher than the initial one, and iii) I^k/Y increases initially but it returns to its starting value by the end of the sample.

6.2. Alternative shapes for the public investment/output ratio

The public sector commits to follow the policy rule (5.1), so the public investment/output ratio path depends on $\{\bar{x}, \eta\}$. Table 5 and Figure 3 summarize their main properties.

In general, $\bar{k}^g$ and $\bar{x}$ are directly related. A higher level of $\bar{x}$ increases the long-run income tax rate, which disincentives the accumulation of private capital, at the same time the government enforces to accumulate more public capital. Thus, given $\bar{x}_0$ and $\bar{k}^g_0$, the ratios $\bar{x}/\bar{x}_0$ and $\bar{k}^g_0/\bar{k}^g$ move in opposite directions. Using the same argument, x_t and k^g_{t+1} are also positive related. However, the relationship between x_t and k^g_t depends on the sign of η.

From (5.1), the initial impact on x is measured by:

$$\frac{x_1}{x_0} = \frac{\bar{x}}{x_0}\left(k^g_1/\bar{k}^g\right)^\eta = \frac{\bar{x}}{\bar{x}_0}\left(\bar{k}^g_0/\bar{k}^g\right)^\eta, \tag{6.1}$$

where $x_0 = \bar{x}_0$ and $k^g_1 = \bar{k}^g_0$, their initial steady-state levels, and $\bar{x}$ and $\bar{k}^g$ are the final ones.[20] Let's suppose a *long-run downsizing policy*, $\bar{x} < \bar{x}_0$. From (6.1), it is easy to show that a counter-cyclical policy, $\eta < 0$, provokes an initial large negative impact on x, overshooting the long-run target $\bar{x}$ in the first period (i.e., $x_1 < \bar{x}$).[21] On the other hand, if the short-run policy is pro-cyclical, the bigger η, the further x_1 above $\bar{x}$ is going to be.[22]

[20] $k^g_1 = \bar{k}^g_0$ because k^g_1 is a result of a decision taken on the previous period, in which the policy had not changed.

[21] Since $\bar{x}/\bar{x}_0 < 1$, then $\bar{k}^g_0/\bar{k}^g > 1$, but $\left(\bar{k}^g_0/\bar{k}^g\right)^\eta < 1$ because $\eta < 0$. Hence, $x_1/x_0 < \bar{x}/\bar{x}_0$ and $x_1 < \bar{x}$, since $x_0 = \bar{x}_0$.

[22] Moreover, it could even be the case that x will raise initially, to then start converging towards $\bar{x}$.

After that initial impact, a similar argument can be used to explain the evolution of x_t along the transition, and its convergence is monotone towards its steady-state. Therefore, a *long-run downsizing policy* keeps x below its steady-state level when combined with a counter-cyclical measure, while x remains above its long-run target when combined with a pro-cyclical policy.

A symmetric pattern is shown for *long-run upsizing*, $\bar{x} > \bar{x}_0$: if $\eta < 0$, then $x_t > \bar{x}$, while $x_t < \bar{x}$ whenever $\eta > 0$. Thus, a *long-run upsizing policy* combined with a counter-cyclical measure keeps the public investment/output ratio above its final steady-state level, while the ratio remains below its long-run target when combined with a pro-cyclical policy.

[INSERT TABLE 5 ABOUT HERE]

[INSERT FIGURE 3 ABOUT HERE]

Regarding the transitional dynamics, the larger η, the slower the convergence speed. Moreover, a big enough level of η makes x never converge to its steady-state. The intuition of that result is similar to that in the cycle literature, when arguing why a pro-cyclical policy enlarges cycles.

6.3. A simple policy experiment

Starting with an initial public investment/output ratio of 0.054, its average in the 60's, the government implements either a long-run upsizing policy, setting $\bar{x} = 0.10$, or a downsizing one, setting $\bar{x} = 0.02$. A continuity argument suggests that policies inside this range must fall between these extremes. For each $\bar{x}$, we consider alternative values of η, -2, 0 and 0.5.[23] Figure 4 shows the path of main macroeconomic ratios for the combined policies.

[INSERT FIGURE 4 ABOUT HERE]

In general, under the upsizing (downsizing) policy, the public investment/output ratio and the income tax rate increase (fall) initially for *reasonable* levels of η.[24] In general, it initially disincentives (incentives) labor and private capital accumulation, but instead it incentives (disincentives) the accumulation of public capital. Hence, the public to private capital ratio raises (falls) initially. Finally, private consumption falls (raises), due to the negative (positive) income effect of the policy measure.

We have already seen that the public investment/output ratio overshoots its long-run target when combined with a counter-cyclical measure. Because of that reason, the

[23] For the benchmark economy, a policy with η approximately larger than 0.75 is *unfeasible*, in the sense that x needs more than 200 periods to converge, while with a η approximately lower than -3 is also *unfeasible*, in the sense that $I^g - \delta^g K^g < 0$.

[24] As it was commented in section 5.2, a very large level of η would make x fall initially, although $\bar{x} > x_0$. However, this possibility leads to unfeasible policies for the benchmark economy.

counter-cyclical measure emphasizes the effect of the long-run policy along the former periods of the transition. Moreover, since it helps the economy to move faster towards its steady-state trajectory, the long-run impact remains almost unchanged. A symmetric behavior is shown for a pro-cyclical policy.

6.4. The welfare trade-off

In principle, the initial impact on welfare is uncertain, since private consumption and leisure move in opposite directions. With respect to stay on the initial balanced growth path, Figure 5 shows the relative welfare gain for the ten initial periods, the next hundred and from the two hundred periods on (the long-run welfare) for the following policies: $\{0.02; -2\}$, $\{0.02; 0.5\}$, $\{0.10; -2\}$ and $\{0.10; 0.5\}$. According to it, the effect on private consumption prevails, and welfare falls initially for the upsizing policy, while it raises for the downsizing.

[INSERT FIGURE 5 ABOUT HERE]

This figure reveals an additional interesting fact. A public investment policy produces a particular trade-off between initial welfare, welfare along the transition and long-run welfare. Upsizing policies, such as $\{0.10; -2\}$ and $\{0.10; 0.5\}$, crowd-out private resources in favor of public infrastructure, so they diminish welfare, at least along the former periods of the transition. However, whenever the public to private capital ratio is initially below its optimum level, substituting the former by the latter might impulse the economy to growing faster after a certain number of periods, which might even more than compensate the initial utility lost. On the other hand, this substitution would never be able to compensate the initial lost in welfare. According to Figure 4, our benchmark economy is identified with the first group and then upsizing will be optimal.[25] A symmetric relationship is shown for downsizing, leading to an initial utility raise and a posterior welfare lost.

Given the long-run policy, the short-run measure must follow to attain the optimal trade-off in welfare. Depending on the long-run target, the optimal short-run policy might be different. For instance, relative to a counter-cyclical measure, a pro-cyclical policy reduces the welfare lost along the former periods of the transition due to an upsizing process. In the long-run, welfare is pretty the same since they share common long-run targets. Hence, a long-run upsizing policy must be combined with a pro-cyclical measure in order to maximize *aggregate* welfare. On the other hand, by symmetry, a long-run downsizing policy must be combined with a counter-cyclical measure to maximize *aggregate* welfare. We will come back to that point latter.

[25] We will see this result in the next section.

7. The optimal public investment policy

The government commits to follow the log-linear policy rule (5.1) and decides two policy parameters, $\bar{x}$ and η. Thus, the government faces with a continuum of short-run policies, leading all them to the same long-run target. While the long-run instrument, $\bar{x}$, affects the steady-state of the economy, the short-run policy tool, η, might affect its transitional dynamics. The government pursues to combine them in an optimal way.

Since the public capital elasticity, φ, is a controversial parameter to calibrate[26] and is crucial to determine the optimal policy,[27] we condition the welfare analysis to its magnitude. Table 6 summarizes the optimal policy for the benchmark economy and alternative values of φ. For each φ, we show the optimal policy under the constant-ratio rule, $\{\bar{x}^{0+}; 0\}$, the active rule, $\{\bar{x}^{+}; \eta^{+}\}$, and the public investment/output ratio maximizing the long-run economic growth rate, $\bar{x}^{*}$. Welfare, growth and steady-states are shown in relative terms to the initial balanced growth path.

[INSERT TABLE 6 ABOUT HERE]

As expected, $\bar{x}^{+}$, $\bar{x}^{0+}$ and $\bar{x}^{*}$ are positive related with φ (see Figure 6) as in Barro (1990), Glomm and Ravikumar (1994) and Turnovsky (1996, 2000). Thus, starting with an initial public investment/output ratio, 0.054 in our case, and taxing income proportionally, downsizing would be optimal in economies where public capital is not *important enough* in the productive process. In our benchmark economy and according to Figure 6, downsizing turns optimal when the public capital elasticity is approximately lower than 0.09, a similar value to that estimated by Munell (1990). Moreover, the public capital elasticity would need to be around 0.05 for a long-run public investment/output ratio of 0.032, the level in the 90's, being an optimal choice. These values are in both cases below the general consensus in the literature. Thus, it is more likely that our result be consistent with, among many others, Aschauer (1989) and Munnell (1990), which put forth that policymakers have reduced the stock of public capital below its optimum level, in detriment also of the productivity of complementary private inputs and economic growth.

[INSERT FIGURE 6 ABOUT HERE]

The growth maximizing policy sets $\bar{x}^{*}$ equal to $(1-g)\varphi$, a standard result in the literature. Under a constant-ratio rule, the welfare-maximizing public investment/output ratio $\bar{x}^{0+}$ is always lower than $\bar{x}^{*}$, as in Futagami et al. (1993). The existence of transitional dynamics generates in this setting a trade-off between consumption along the former periods of the transition and growth in the long-run, that makes the optimal public investment/output ratio be lower than the one maximizing growth. For instance, a

[26] See section 5.1.

[27] See, among others, Barro (1990), Glomm and Ravikumar (1994) and Turnovsky (1996, 2000).

faster convergence would reduce this trade-off, hence the welfare maximizing policy would be closer to the growth maximizing one.

On the other hand, an active rule allows along the transition for a non-constant path of the public investment/output ratio and the tax rate, hence being able to achieve a better trade-off for aggregate welfare. Figure 7 compares the public investment/output ratio path under $\{\bar{x}^{0+}; 0\}$ and $\{\bar{x}^{+}; \eta^{+}\}$ for $\varphi = 0.15$ -upsizing is optimal- and $\varphi = 0.07$ -downsizing is optimal. Along the former periods of the transition, the optimal ratio stays below $\bar{x}^{0+}$, but it ends up converging towards a level $\bar{x}^{+}$ between $\bar{x}^{0+}$ and $\bar{x}^{*}$. It would be suboptimal that $\bar{x}^{+}$ being higher than the one maximizing growth, since that would induce lower growth and consumption along the final balanced growth path. In any case, as expected, $\bar{x}^{0+}$ could be seen as a weighted average of the optimal public investment/output ratio path under an active policy.

[INSERT FIGURE 7 ABOUT HERE]

Upsizing has two main effects on the economy: i) the effect of raising public capital and ii) the effect of raising taxes. The latter is always harmful for welfare, since now the private sector would have less resources to consume. Thus, whenever upsizing is optimum, the optimal path would never overshoot its long-run target, but would stay below its long-run target, since otherwise the harmful impact on consumption and welfare would be emphasized. Moreover, the public to private capital ratio must be initially below its optimum, and its increment should more than compensate the harmful impact of raising taxes. According to our results, upsizing must be combined with a pro-cyclical policy and convergence is monotone and slow. This combination extends the positive effect of substituting public capital by private capital throughout the whole transition, at the same time the tax raise is very smooth in order to mimic the negative incidence on consumption.

On the other hand, downsizing induces a positive income effect on private consumption and welfare, since now more resources are available to the private sector. Whenever downsizing is optimal, the optimal short-run policy would make the public investment output ratio fall below its final level, enhancing this way the positive effect on consumption and welfare. This effect should more than compensate the *possible* negative impact of falling public capital. However, if initially the public to private capital ratio is high enough, the effect of falling public capital is in fact positive. According to our results, downsizing must be combined with a counter-cyclical policy and convergence is monotone and fast. Precisely, this combination emphasizes consumption and welfare along the former periods of the transition and, since convergence is fast, the *possible* negative impact of falling public capital is less likely.

We close this section remarking the effect of the initial public investment/output ratio on the optimal policy. Obviously, concluding that downsizing or upsizing is optimal depends on this inial level. It is widely accepted in the literature that several items in public consumption can be considered as productive,[28] so the initial public investment/output ratio could be higher than the benchmark 0.054. Under a constant-ratio rule, the optimal policy is robust to the initial public investment/output ratio. Moreover, the relationship between $\bar{x}^{o+}$, $\bar{x}^{+}$ and $\bar{x}^{*}$ is also robust to x_0, being $\bar{x}^{*} \geq \bar{x}^{+} \geq \bar{x}^{o+}$ independently on the nature of the optimal policy.

8. Conclusions and extensions

In dynamic settings with public capital, it is common to assume that the government claims a constant fraction of public investment to total output each period. We first show that this is not a realistic assumption for US to validate the downsizing process held in this country since the end of the sixties. We relax this assumption and consider a more flexible rule for public investment, consistent to US data. Given a long-run target for the public investment/output ratio, the government can adjust this ratio along the transition to the current state of the economy.

In comparison to the constant-ratio rule, a model simulation exercise fits better the downsizing process in public investment as well as the post-1970 slowdown in private factors productivity and economic growth. This downsizing process starts with a public investment output ratio of 0.054 and ends with 0.032. In our policy analysis, this process would be close to the optimal one whenever the public capital elasticity would be around 0.05, which leads us to support the idea that policymakers have reduced the stock of public capital below its optimum level along this time. Since downsizing has been a general trend in the last 40 years in most Occidental Economies, it would be interesting to extend the analysis made in this paper to these economies.

The attempt made in the paper to study a more realistic rule for public investment could be connected with two issues: 1) the coordination between short- and long-term policies; 2) the characterization of the optimal public investment path. The former was already commented in the Introduction and was used to motivate the paper. Because of its interest and complexity, the latter is left for a future extension. The idea is that a more flexible public investment rule than that considered in the paper might approach the public investment/output ratio path, x_t, to the one obtained by solving the Ramsey problem. S_t denotes the set of state variables at time t, so the Ramsey path can be expressed as $x_t = g[S_t(\Phi)]$, Φ being the fundamentals of the economy and $g(\cdot)$ showing a generally unknown, continuous and monotone function in S_t. The approach carried in the paper assumes ex-ante that $x_t = f[S_t(\Phi, Q); Q]$, where Q is a set of policy parameters and $f(\cdot)$ is a well known function. Given $f(\cdot)$, the government decides on Q, so the optimization problem reduces to a parameterized maximization problem. On the other hand, in the Ramsey framework, the government has to choose the optimal x_t, given S_t, every period, which is a much more tedious problem to deal with. Moreover, since $g(\cdot)$

[28] For example, see Easterly and Rebelo (1993).

is unknown, it is difficult to make predictions and simulate the model without solving previously the Ramsey problem.

On the other hand, coordinating short- and long-run fiscal policies is an issue of utmost interest that requires extending the analysis made in this paper in several directions. It is clear that the optimal public investment policy depends on the cost of raising resources to finance public expenditures, and the tax base becomes crucial. Hence, alternative financing structures should be considered in our analysis. Another extension of the paper regards the particular shape of the public investment rule. More interesting policy rules could be considered. For instance, the public sector could have several short-run policy tools at its disposal, allowing the government to select among a higher set of public investment plans. In addition, since politicians increasingly make decisions according to the evolution of *real* macroeconomic variables, specifying a reasonable policy rule would be a convenient way to describe the performance of the public expenditure plans.

9. Appendix

9.1. Part 1: Competitive equilibrium conditions

In terms of stationary ratios, $c_t = \tilde{C}_t/\tilde{K}_t$, $k_t^g = \tilde{K}_t^g/\tilde{K}_t$, $y_t = \tilde{Y}_t/\tilde{K}_t$ and $k_{t+1} = \tilde{K}_{t+1}/\tilde{K}_t$, competitive equilibrium conditions can be reduced to a system of seven equations in c, k^g, y, k, r, h and τ,

$$\begin{aligned}
k_{t+1}\frac{c_{t+1}}{c_t} &= \left\{\beta\left(\frac{1-h_{t+1}}{1-h_t}\right)^{(1-\rho)(1-\theta)}\left[1-\delta^k+(1-\tau_{t+1})r_{t+1}\right]\right\}^{\frac{1}{1-\rho(1-\theta)}}, && (9.1)\\
\frac{\rho}{1-\rho} &= \frac{c_t}{(1-\tau_t)(1-\alpha)y_t}\frac{h_t}{1-h_t}, && (9.2)\\
y_t(1-g) &= c_t + k_{t+1} - (1-\delta^k) + \bar{x}y_t\left(k_t^g/\bar{k}^g\right)^{\eta}, && (9.3)\\
r_t &= \alpha y_t, && (9.4)\\
y_t &= F h_t^{1-\alpha}\left(k_t^g\right)^{\varphi}, && (9.5)\\
k_{t+1}k_{t+1}^g &= \left(1-\delta^g\right)k_t^g + \bar{x}y_t\left(k_t^g/\bar{k}^g\right)^{\eta}, && (9.6)\\
\tau_t &= g + \bar{x}\left(k_t^g/\bar{k}^g\right)^{\eta}, && (9.7)
\end{aligned}$$

where (9.1) corresponds to (3.8); (9.2) comes from combining (3.9) with (3.4) and (4.1); (9.3) combines (4.2) with (3.5), (3.12) and (5.1); (9.4) comes directly from (3.3); (9.5) combines (3.2) with (4.1); (9.6) combines (5.1) with (3.11); finally, (9.7) combines (3.14) with (3.12) with (5.1).

9.2. Part 2: Solving the transitional dynamics for normalized variables

A log-linear based approach is used to solve for the dynamics of stationary ratios [Uhlig (1999)]. $V(t)$ includes the beginning-of-period state variables, just k_t^g in our model; $Q(t)$ is the vector of real variables $(y_t, c_t, r_t, h_t, k_{t+1}, \tau_t)$. Their values on the *bgp* are denoted by $\bar{V}$ and $\bar{Q}$, and $\hat{v}(t)$ and $\hat{q}(t)$ denote log-deviations of $V(t)$ and $Q(t)$ around $\bar{V}$ and $\bar{Q}$, respectively.

First, we consider the benchmark calibration and solve (9.1)-(9.7) for the *bgp*, getting $\left(\bar{y}, \bar{c}, \bar{r}, \bar{h}, \bar{k}, \bar{\tau}, \bar{k}^g\right)$.

Second, we log-linearize (9.1)-(9.7) around the *bgp*. He proposes a procedure where optimal conditions are log-linearized without the need of differentiating. A variable U^a can be approximated as:

$$\left(\frac{U}{\bar{U}}\right)^a = \exp\left(a\ln\left(\frac{U}{\bar{U}}\right)\right) = \exp(au) \simeq (1+a\hat{u}) \Rightarrow U^a \simeq \bar{U}^a(1+a\hat{u}). \qquad (9.8)$$

In addition, we can assume $\hat{v}_1\hat{v}_2 \simeq 0$ if variables are close enough to their steady-state values. Log-linearized versions of (9.1)-(9.7) are (all variables are in log-deviations about the steady-state):

$$
\begin{aligned}
\hat{c}_{t+1} - \hat{c}_t + \hat{k}_{t+1} + \frac{\tilde{\rho}\tilde{\theta}\bar{h}}{1-\bar{h}}(\hat{h}_{t+1} - \hat{h}_t) + \frac{\tilde{\theta}\delta^k \bar{r}\left[(1-\bar{\tau})\,\hat{r}_{t+1} - \bar{\tau}\hat{\tau}_{t+1}\right]}{1-\delta^k \bar{r}(1-\bar{\tau})} &= 0, && (9.9)\\
\hat{c}_t - \hat{y}_t + \frac{\bar{\tau}}{1-\bar{\tau}}\hat{\tau}_t + \frac{1}{1-\bar{h}}\hat{h}_t &= 0, && (9.10)\\
\bar{y}\,(1-g-\bar{x})\,\hat{y}_t - \bar{c}\hat{c}_t - \bar{k}\hat{k}_{t+1} - \bar{x}\bar{y}\eta\hat{k}^g_t &= 0, && (9.11)\\
\hat{r}_t - \hat{y}_t &= 0, && (9.12)\\
\hat{y}_t - (1-\alpha)\hat{h}_t - \varphi\hat{k}^g_t &= 0, && (9.13)\\
\bar{k}\bar{k}^g\left(\hat{k}_{t+1} + \hat{k}^g_{t+1}\right) - (1-\delta^g)\bar{k}^g\hat{k}^g_t - \bar{x}\bar{y}\hat{y}_t - \bar{x}\bar{y}\eta\hat{k}^g_t &= 0, && (9.14)\\
\bar{x}\eta\bar{k}^g_t - \bar{\tau}\hat{\tau}_t &= 0, && (9.15)
\end{aligned}
$$

where $\tilde{\theta} = \frac{1}{1-\rho(1-\theta)}$ and $\tilde{\rho} = (1-\rho)(1-\theta)$. Steady-state conditions have been used to simplify these expressions. Conditions (9.9)-(9.15) can be grouped into the following matrix system:

$$
\begin{aligned}
A\hat{v}(t+1) + B\hat{v}(t) + C\hat{q}(t) &= 0, && (9.16)\\
F\hat{v}(t+2) + G\hat{v}(t+1) + H\hat{v}(t) + J\hat{q}(t+1) + K\hat{q}(t) &= 0, && (9.17)
\end{aligned}
$$

where the Euler condition (9.9) is (9.17), (9.10)-(9.15) are grouped into (9.16), and matrices A, B, C,..., are functions of all structural and policy parameters.

Third, assuming the following log-linear law of motion for $Q(t)$ and $V(t)$,

$$
\begin{aligned}
\hat{v}(t+1) &= P\hat{v}(t), && (9.18)\\
\hat{q}(t) &= S\hat{v}(t), && (9.19)
\end{aligned}
$$

the system (9.16)-(9.17) are directly solved by the undetermined coefficients method, imposing the eigenvalues of P to be inside the unit circle.

Fourth, starting with (K_0, K^g_0), and $k^g_0 = K^g_0/K_0$, we get C_0 and h_0 from (9.19)

$$
\begin{pmatrix} Y_0 \\ C_0 \\ r_0 \\ h_0 \\ k_1 \\ \tau_0 \end{pmatrix} = \begin{pmatrix} K_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & K_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} Q(0), \qquad (9.20)
$$

with $Q(0) = \exp\left[S\left(\ln k^g_0 - \ln \bar{k}^g\right) + \ln \bar{Q}\right] = \left(c_0\ y_0\ r_0\ h_0\ k_1\ \tau_0\right)'$. K_1 is easily recovered from k_1,

$$
K_1 = \frac{K_0 k_1}{\bar{k}}. \qquad (9.21)
$$

Next, normalized private investment is given by,

$$I_0^k = \bar{k}K_1 - K_0\left(1 - \delta^k\right). \tag{9.22}$$

From (9.3), Y_0 is given by

$$Y_0 = \frac{C_0 + I_0^k}{1 - g - \bar{x}\left(\frac{k_0^g}{\bar{k}^g}\right)^{\eta}} \tag{9.23}$$

Finally,

$$C_0^g = gY_0, \tag{9.24}$$

$$I_0^g = \bar{x}\left(\frac{k_0^g}{\bar{k}^g}\right)^{\eta} Y_0, \tag{9.25}$$

$$K_1^g = \left(I_0^g + K_0^g(1 - \delta^g)\right)/\bar{k}. \tag{9.26}$$

Values for next periods are obtained in a recursive way. The resulting time series are stable since stability conditions are imposed when solving the system (9.16)-(9.17).

9.3. Part 3: Computing welfare

We truncate the infinite sum in (5.2) at period T^*. T^* is chosen so that equilibrium time series are close enough to the *bgp*, i.e., $\left|X_{T^*} - \bar{X}\right| < 10^{-3}$, with $T^* < 200$ (50 years). Time series $\{C_t, h_t\}_{t=0}^{T^*}$ are used to evaluate welfare up to period T^*:

$$\sum_{t=0}^{T^*}\left\{\frac{\left[\beta(1+\bar{\gamma})^{\rho(1-\theta)}\right]^t\left[C_t^{\rho}(1-h_t)^{1-\rho}\right]^{1-\theta}}{1-\theta} - \frac{\beta^t}{(1-\theta)}\right\}. \tag{9.27}$$

After period T^* the economy is considered to be on the final *bgp* [this strategy is similar to that used in Jones et al. (1993)]. Since $\beta(1+\bar{\gamma})^{\rho(1-\theta)} < 1$ by (4.4), the term

$$\sum_{t=T^*+1}^{\infty}\left\{\frac{\left[\beta(1+\bar{\gamma})^{\rho(1-\theta)}\right]^t\left[C_{T^*}^{\rho}(1-\bar{h})^{1-\rho}\right]^{1-\theta} - \beta^t}{1-\theta}\right\} \tag{9.28}$$

$$= \left[C_{T^*}^{\rho}(1-\bar{h})^{1-\rho}\right]^{1-\theta}\frac{\left[\beta(1+\bar{\gamma})^{\rho(1-\theta)}\right]^{T^*+1}}{(1-\theta)\left[1-\beta(1+\bar{\gamma})^{\rho(1-\theta)}\right]} - \frac{\beta^{T^*+1}}{(1-\theta)(1-\beta)}$$

approximates aggregate utility after period T^*. Notice that (9.27) and (9.28) must be computed simultaneously, because C_{T^*} depends on the whole transitional dynamics up to period T^*.

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Appendix of tables

Table 1: Main Macromagnitudes and Public Sector Expenditure of US(1)

USA	30	40	50	60	70	80	90(2)
Real GDP growth rate (%)	1.3	6.0	4.2	4.4	3.3	3.0	2.8
Private consumption(3)	76.9	60.6	62.5	61.8	62.4	64.3	67.1
Gross Private Investment(3)	8.1	10.6	15.8	15.5	16.7	16.9	15.7
Gross Public Investment(3),(4)	4.1	8.7	5.4	5.2	3.7	3.6	3.3
Public Consumption(3)	10.6	19.0	16.1	17.1	17.5	17.0	15.5
Public Capital/Private Capital (%)	21.6	39.3	32.9	35.5	34.2	29.1	28.0
General public expenditures(4)							
Current expenditure(3)	14.5	24.0	21.9	24.2	28.2	30.4	30.3
Consumption expenditures(3)	10.6	19.0	16.1	17.1	17.5	17.0	15.5
Transfer payments (net)(3)	2.3	3.2	4.4	5.4	8.7	10.0	11.2
Net interest paid(3)	1.4	1.4	1.4	1.4	1.6	2.9	3.3
Current expenditure − Transfers(3)	12.0	20.4	17.4	18.6	19.1	19.9	18.7

(1) anual averages; (2) 2000 and 2001 are included; (3) percentage to real GDP;
(4) Includes federal, local and public enterprises.

Table 2: Estimation results

	29-01	29-60	60-01	80-01
α	$-0.02^{(**)}$ (0.04)	$-0.02^{(**)}$ (0.04)	$-0.00^{(**)}$ (0.01)	$-0.00^{(**)}$ (0.01)
β	1.14 (0.63)	1.26 (0.80)	0.52 (0.27)	$-0.06^{(**)}$ (0.89)
δ_1	0.50 (0.17)	0.49 (0.18)	$0.26^{(*)}$ (0.18)	$0.26^{(**)}$ (0.21)
δ_2	−0.18 (0.08)	$-0.19^{(*)}$ (0.10)	$-0.24^{(*)}$ (0.16)	–
$WWII$	−1.37 (0.16)	−1.36 (0.17)	–	–
R^2	0.71	0.73	0.16	0.07
DW	2.29	2.31	1.78	1.92

Note: Standard deviations are in parenthesis. DW is Durbin-Watson statistic, rejecting the existence of AR(1) structure in the residuals. The correlogram of residuals shows lack of structure as well. (*) Non-significative at 5%, but it is at 10%; (**) Non-significative at 10%.

Table 3: Benchmark calibration

F	α	φ	ϕ	δ^k	δ^g
0.403	0.36	0.15	0.49	0.025	0.016
β	θ	$\bar{x}_0$	g	ρ	
0.99	1.5	0.054	0.18	0.38	

Table 4: Public Investment/output ratio transitional dynamics				
	$\bar{x} < \bar{x}_0$ (long-run downsizing)		$\bar{x} > \bar{x}_0$ (long-run upsizing)	
	InI	Tr	InI	Tr
$\eta << 0$	unfeasible: in general because I^g cannot be smaller than $-\delta^g K^g$		unfeasible: in general because either C or I^k turns negative or smaller than $-\delta^k K$	
$\eta < 0$	negative and overshooting $\bar{x}$	fast and smooth convergence towards $\bar{x}$	positive and overshooting $\bar{x}$	fast and smooth convergence towards $\bar{x}$
$\eta = 0$	negative	x=$\bar{x}$ in just one period	positive	x=$\bar{x}$ in just one period
$\eta > 0$	negative, but never overshooting $\bar{x}$	slow and smooth convergence towards $\bar{x}$	positive, but never overshooting $\bar{x}$	slow and smooth convergence towards $\bar{x}$
$\eta >> 0$	unfeasible: in general because: i) C<0, ii) I^k<$-\delta^k K$ or iii) x requires a long time to get $\bar{x}$		unfeasible: in general because: i) I^g cannot be smaller than $-\delta^g K^g$; ii) x requires a long time to achieve $\bar{x}$	

Note: InI: Initial impact; Tr: transitional dynamics

Table 5: Real and simulated values for the benchmark economy. Initial and final values.								
	x	g	C/Y	$C/Y^{(*)}$	I^k/Y	$I^k/Y^{(**)}$	k^g	γ
Real: $1960^{(1)}$	0.054	0.18	0.63	0.53	0.16	0.24	0.35	0.042
Simulation: $1960^{(3)}$	0.054	0.18	0.56	0.56	0.21	0.21	0.35	0.042
Real: $2001^{(2)}$	0.032	0.18	0.67	0.59	0.16	0.24	0.28	0.028
Simulation: $2001^{(s)}$	0.034	0.18	0.58	0.58	0.21	0.21	0.24	0.031

(1) average from 1955-67; (2) average from 1995-2001; (3) values corresponding to the initial steady-state; (4) value in the simulation at 2001; (*) excluding expenditure in durable goods; (**) including expenditure in durable goods.

Table 6: Optimal policies for alternative levels of public capital

						Initial impacts (%)						steady-state for normalized variables (3)						
θ	xi (1)	η	γ (2)	Wel (3)	speed(4)	C_t	h_t	Y_t	K^g_{t+1}	K_{t+1}	τ	C/Y	Ik/Y	Ig/Y	Cg/Y	tax	h	Kg/K
0.050	0.040	-1.00	0.960	3.78	10.52	3.21	0.31	0.41	-1.38	0.18	-10.65	0.010	0.004	-0.014	0.000	-0.014	0.000	-0.093
0.050	0.031	0.00	0.104	3.12	22.88	3.10	-0.16	-0.11	-1.27	0.09	-9.83	0.016	0.006	-0.023	0.000	-0.023	0.000	-0.150
0.050	0.042	0.00	0.966	2.30	22.82	1.60	-0.06	-0.05	-0.67	0.04	-5.13	0.008	0.003	-0.012	0.000	-0.012	0.000	-0.080
0.070	0.045	-2.20	-0.808	2.01	6.25	2.90	0.88	0.99	-1.29	0.30	-10.51	0.007	0.002	-0.009	0.000	-0.009	0.000	-0.060
0.070	0.043	0.00	-1.185	0.57	23.69	1.51	-0.12	-0.06	-0.58	0.05	-4.70	0.008	0.003	-0.011	0.000	-0.011	0.000	-0.073
0.070	0.059	0.00	0.083	-0.61	23.59	-0.68	0.04	0.02	0.27	-0.02	2.14	-0.004	-0.001	0.005	0.000	0.005	0.000	0.034
0.10	0.07	0.40	2.82	0.991	45.68	-1.19	0.27	0.20	0.39	-0.03	3.48	-0.011	-0.004	0.015	0.000	0.015	0.000	0.104
0.10	0.06	0.00	2.03	0.347	24.50	-1.25	0.13	0.06	0.44	-0.05	3.85	-0.007	-0.002	0.009	0.000	0.009	0.000	0.062
0.10	0.08	0.00	3.54	-1.082	24.37	-3.84	0.33	0.16	1.40	-0.13	11.97	-0.021	-0.007	0.028	0.000	0.028	0.000	0.198
0.15	0.10	0.22	16.02	7.920	34.84	-5.00	1.16	0.75	1.51	-0.17	14.72	-0.037	-0.012	0.049	0.000	0.049	0.001	0.355
0.15	0.09	0.00	14.76	6.289	25.13	-5.64	0.81	0.31	1.77	-0.24	17.09	-0.030	-0.010	0.040	0.000	0.040	0.001	0.286
0.15	0.12	0.00	17.12	3.811	24.92	-9.57	1.12	0.41	3.13	-0.37	29.49	-0.052	-0.017	0.069	0.000	0.069	0.001	0.516
0.20	0.14	0.16	35.20	20.953	32.14	-9.08	2.29	1.29	2.46	-0.37	26.53	-0.062	-0.019	0.082	0.000	0.082	0.003	0.618
0.20	0.13	0.00	33.53	18.271	25.05	-10.10	1.73	0.51	2.83	-0.49	30.34	-0.054	-0.017	0.071	0.000	0.071	0.003	0.526
0.20	0.17	0.00	36.86	14.092	24.75	-15.62	2.19	0.51	4.61	-0.69	47.86	-0.084	-0.027	0.112	0.000	0.112	0.003	0.888
0.25	0.17	0.14	58.21	40.381	30.97	-12.80	3.61	1.86	3.11	-0.59	37.18	-0.087	-0.027	0.113	0.000	0.113	0.005	0.881
0.25	0.15	0.00	55.44	36.065	24.52	-13.88	2.74	0.54	3.47	-0.76	41.45	-0.075	-0.023	0.097	0.000	0.097	0.005	0.737
0.25	0.21	0.00	60.80	28.812	24.07	-21.90	3.46	0.20	5.86	-1.08	67.09	-0.119	-0.039	0.157	0.000	0.157	0.005	1.321

(1): For elasticity, it first shows the optimal ratio under an active rule, next under a passive, finally the one maximizing long-run growth.

(2): Difference between the final and initial steady-state.

(3): Along the initial bgp, the equivalent difference (%) in consumption required to get the welfare level in the final bgp.

(4): :it measures the number of periods to cover half of the distance between the initial and the final steady-state

Appendix of figures

Figure 1: Public Investment/output ratio for the US economy from 1929 to 2001
(logarithmic scale)

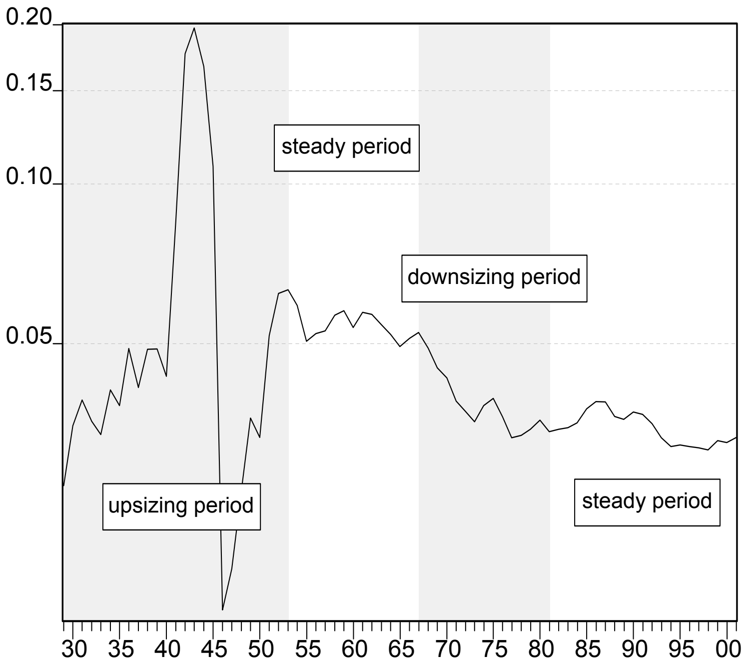
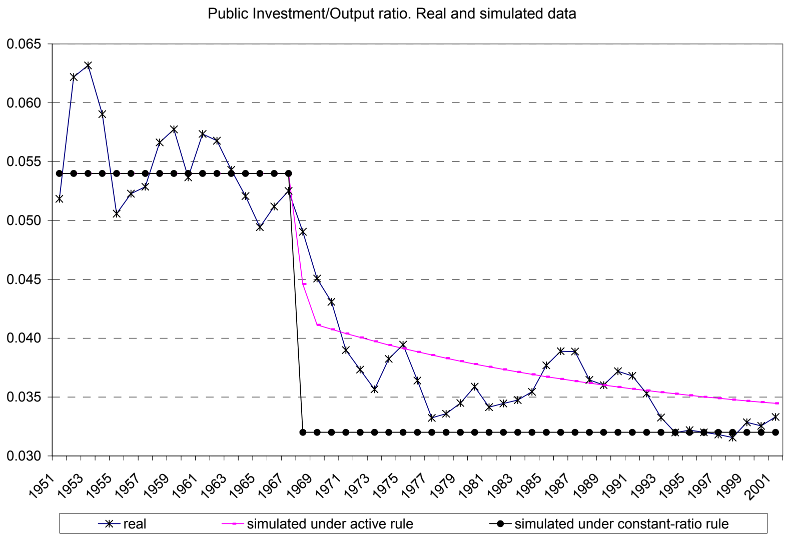


Figure 2: Downsizing in US. Real and simulated series



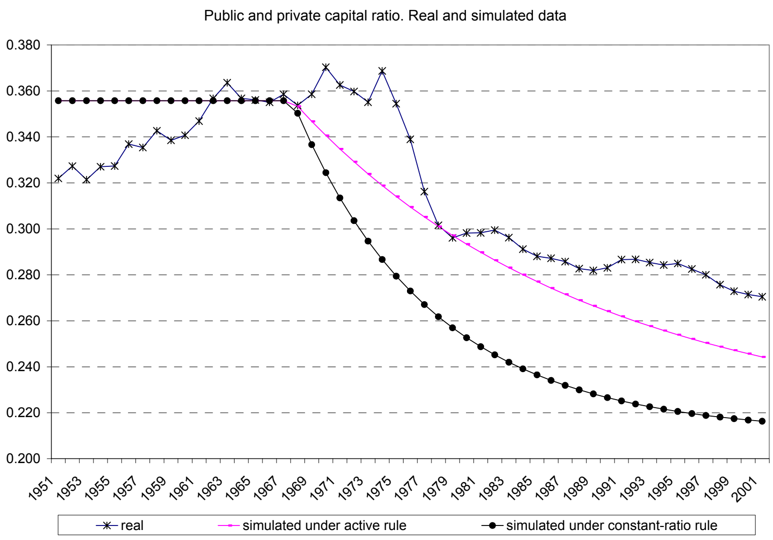
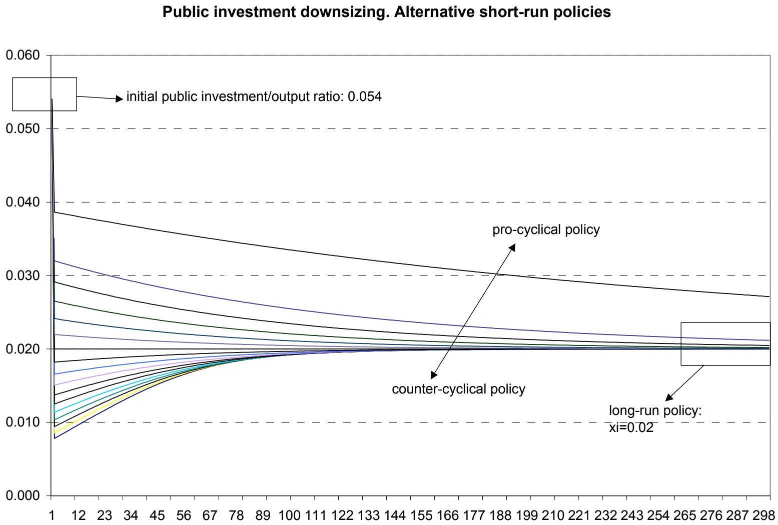


Figure 3: The public investment/output ratio path



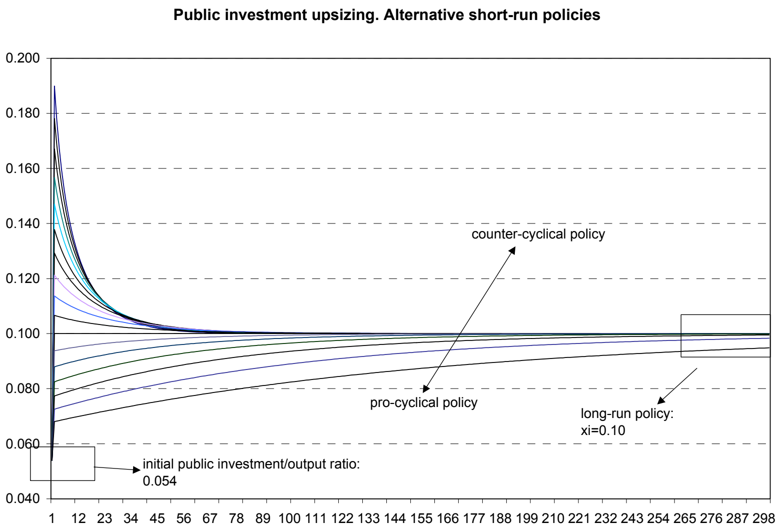


Figure 4: Transitional dynamics of main ratios. Upsizing and downsizing experiment

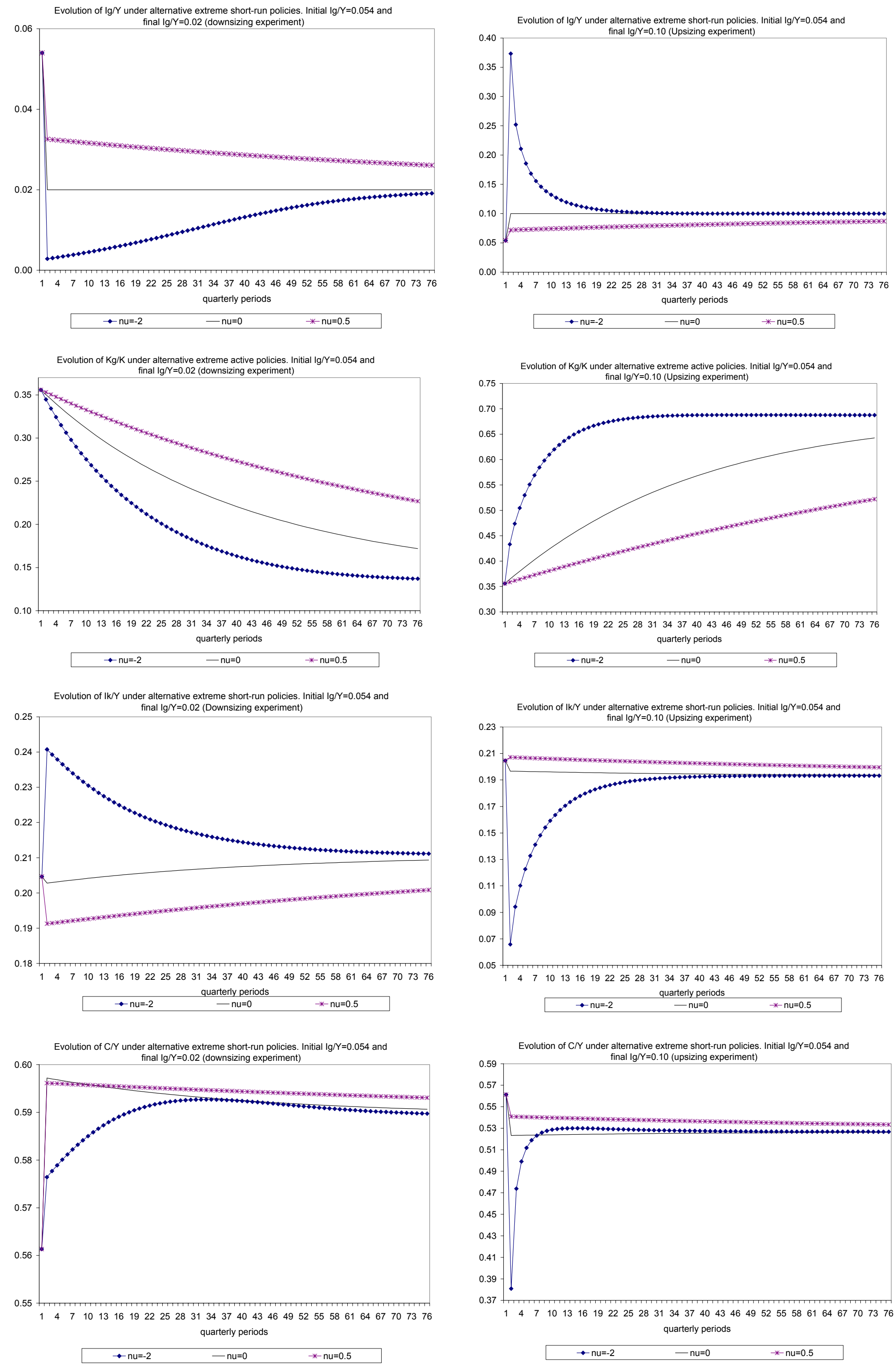


Figure 5: The welfare trade-off

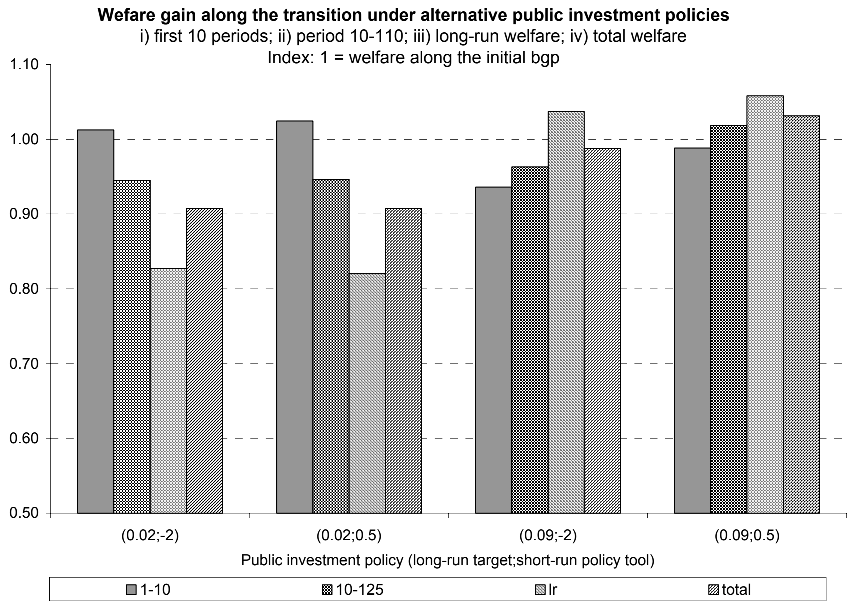


Figure 6: Optimal policies and the public capital elasticity

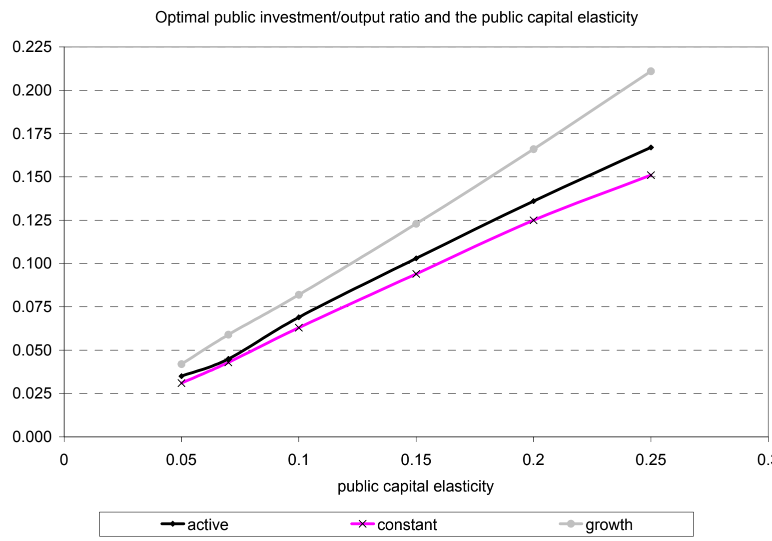


Figure 7: The optimal public investment/output ratio path