Instituto Complutense de Análisis Económico

Disentangling permanent and transitory monetary shocks with a non-linear Taylor rule

J. A. Lafuente
Universitat Jaume I

R. Pérez
Universidad Complutense
and ICAE

J. Ruiz
Universidad Complutense
and ICAE

Abstract

This paper provides an estimation method to decompose monetary policy innovations into persistent and transitory components using the non-linear Taylor rule proposed in Andolfatto et al. [Journal of Monetary Economics 55 (2008) 406–422]. In order to use the Kalman filter as the optimal signal extraction technique we use a convenient reformulation for the state equation by allowing expectations play in significant role in explaining the future time evolution of monetary shocks. This alternative formulation allows us to perform the maximum likelihood estimation for all the parameters involved in the monetary policy. Empirical evidence on US monetary policy making is provided for the period 1980-2011. We compare our empirical estimates with those obtained based on the particle filter. While both procedures lead to similar quantitative and qualitative findings, our approach has much less computational cost and is capable to obtain regime-switching probabilities.

Keywords Monetary shocks, Kalman filter, Particle filter, Taylor rule

JEL Classification C22; F31.

Working Paper nº 1819
September, 2018

UNIVERSIDAD
COMPLUTENSE
MADRID

ISSN: **2341-2356**
WEB DE LA COLECCIÓN: http://www.ucm.es/fundamentos-analisis-economico2/documentos-de-trabajo-del-icae

Disentangling permanent and transitory monetary shocks with a non-linear Taylor rule[1]

J. A. Lafuente
Universitat Jaume I

R. Pérez
Universidad Complutense and ICAE

J. Ruiz
Universidad Complutense and ICAE

July 2018

Abstract

This paper provides an estimation method to decompose monetary policy innovations into persistent and transitory components using the non-linear Taylor rule proposed in Andolfatto et al. [Journal of Monetary Economics 55 (2008) 406–422]. In order to use the Kalman filter as the optimal signal extraction technique we use a convenient reformulation for the state equation by allowing expectations play in significant role in explaining the future time evolution of monetary shocks. This alternative formulation allows us to perform the maximum likelihood estimation for all the parameters involved in the monetary policy. Empirical evidence on US monetary policy making is provided for the period 1980-2011. We compare our empirical estimates with those obtained based on the particle filter. While both procedures lead to similar quantitative and qualitative findings, our approach has much less computational cost and is capable to obtain regime-switching probabilities.

JEL Classification: C22; F31.

Keywords: Monetary shocks, Kalman filter, Particle filter, Taylor rule

---

[1] Corresponding author: Juan Angel Lafuente, Departamento de Finanzas y Contabilidad, Facultad de Ciencias Jurídicas y Económicas, 12071 Castellón (SPAIN). E-mail: lafuen@uji.es. We would like to thank Oscar Jordà, Alfonso Novales, participants in the seminar on Econometrics at UC Davis and participants in the 4th International Conference on Computational and Financial Econometrics for their helpful comments and suggestions. Financial support from the Bank of Spain (program of excellence on Education and Research, 2016, grant PR71/15-20229) and the Spanish Ministry of Education through grant ECO2015-67305-P is gratefully acknowledged. The usual disclaimer applies.

1. Introduction

State space models are useful for many economic applications. As it is well-known, under normality, the classical Kalman filter provides the minimum-variance estimate of the current state taking into account the most recent signal. This prediction is just the conditional expectation. However, under non-linearity and/or non-normality, the filtering procedure developed by Kalman (1960) becomes non-optimal. Two alternatives has been developed in the literature to deal with this aspect: a) the use of first order Taylor series expansion to get linearized equations (transition and/or observation) and b) the use of simulations techniques based on sequential estimation of conditional densities through lot of replications. The first alternative leads to biased estimators. As to the second approach, the seminal papers of Fernández- Villaverde and Rubio-Ramirez (2005 and 2007) show how to deal with the likelihood-based estimation of non-linear DSGE models with non-normal shocks using a sequential Monte-Carlo method (particle filter). This procedure requires a heavy computational burden.

This paper rethinks about the non-optimality of the Kalman filter by revisiting the signal extraction problem proposed in Andolfatto et al. (2008). These authors consider a non-linear Taylor rule where regime shifts reflect the updating of the central bank's inflation target. Such rule could be useful not only to analyze monetary policy making through the lens of a Taylor rule but also in the context of New-keynesian models that incorporate imperfect monetary policy credibility and/or changes in the Central Bank’s inflation target (see, for example, Kozicki and Tinsley, 2005, Ireland, 2007, Coogley et al., 2010, Aruoba and Schorfheide, 2011 and Milani and Treadwell, 2012). The paper contributes to the literature by providing an optimal use of the Kalman filter to estimate persistent and transitory monetary shocks when permanent shifts in the inflation target takes place. Therefore, we focus on how to estimate a Taylor rule where central banks' smoothing of interest rates is time varying as a consequence of time-varying inflation targeting.

We consider a new state-space representation requires the use of state-contingent matrices and expectations play a significant role in monetary policy making. Our procedure has two clear advantages over the standard particle filter: a) the possibility of performing a maximum likelihood estimation of the parameters involved in the monetary policy, and therefore, the estimation of conditional time-varying probabilities of regime switching, b) a remarkable lower computational cost. Moreover, it could be incorporated into simulation algorithms for DSGE models in a straightforward manner.

In order to provide an empirical comparison between our estimation procedure and the particle filter we estimate permanent and transitory monetary shocks from quarterly US data covering the period 1980–2011. We find that the evidence of a regime change in US monetary policy making during the period 1984 to 1999 is weak. However, after the Great Moderation, September eleven, the recession that started in March 2001 and the subprime crisis are three events clearly affecting inflation targets in terms of the long-term nominal anchor. The point estimates for all the parameters involved are close similar. Moreover, Monte Carlo simulations reveal that a) the probability distribution of the discrepancy between the current inflation target and its long-term mean is statistically similar in most of cases (83%) and b) Mean squared errors to predict deviations of inflation from the long-run target are lower when using our estimation procedure.

The rest of the paper is organized as follows: The next section reminds the non-linear Taylor rule in which we focus. Section III describes the reformulation of the state-space representation proposed. Section IV presents empirical evidence for the US, and compares our empirical findings with those based the particle filter. Finally, section V summarizes and provides concluding remarks.

2. The Econometric Problem

Consider the following Taylor rule with time-varying inflation targeting (Andolfatto et al. (2008)):

$$i_t = (1-\rho)\left[r^* + \pi^* + \alpha(\pi_t - \pi_t^*) + \beta\left(y_t - y_t^*\right)\right] + \rho i_{t-1} + u_t \ , \tag{1}$$

where r^* is the long-run equilibrium real interest rate, π_t^* denotes the inflation target,

$y_t - y_t^*$ is the output gap, ρ is the parameter accounting for monetary policy inertia and u_t represents the monetary shocks, which can be interpreted as errors underlying the central bank's control over the policy instrument. We suppose that the time evolution of this shock can be represented as follows:

$$u_{t+1} = \phi\, u_t + e_{t+1},\ 0, < \phi << 1,\ e_{t+1} \sim N(0, \sigma_e^2). \tag{2}$$

Following Andolfatto et al. (2008) a second disturbance to monetary policy is considered. This noise represents the change in the proper rate of inflation the central bank should pursue as a consequence of changes in the economic outlook. We express these shifts as $z_t = \pi_t^* - \pi^*$, so that z_t represents the deviation of the current target (π_t^*) from its long term (time-invariant) mean (π^*). Increases of z_t in the range of positive values mean that monetary policy stance becomes more expansionary, because the central bank relaxes its short run inflation target. On the contrary, decreases of z_t in the range of negative values represent a tightening of monetary policy. It is expected that these shifts will exhibit significant duration:

$$z_{t+1} = \begin{cases} z_t, & \text{with probability } p \\ g_{t+1}, & \text{with probability } 1-p \end{cases} \tag{3}$$

with $g_{t+1} \sim N(0, \sigma_g^2)$.

Combining the definition of z_t with (1), the Taylor rule can be rearranged as follows:

$$i_t = (1-\rho)\left[r^* + \pi^* + \alpha(\pi_t - \pi^*) + \beta\left(y_t - y_t^*\right)\right] + \rho i_{t-1} + \underbrace{(1-\rho)(1-\alpha)z_t + u_t}_{\varepsilon_t}, \tag{4}$$

Therefore, monetary shocks in the above Taylor rule (ε_t) are a combination of a persistent ($(1-\rho)(1-\alpha)z_t$) and a transitory (u_t) innovation.

Researchers interested in incorporating the above monetary rule as a plausible representation scheme for monetary policy making into a DSGE model consider that agents need to learn about the decisions by the central bank in two ways: a) they should solve a signal extraction problem to break down the aggregate shock into the permanent and the transitory components, and b) they should act as econometricians in order to estimate parameters ϕ , σ_e^2, σ_g^2 and p . The next section explain how to deal with both aspects.

3. State-space representation and maximum likelihood estimation

Andolfatto et al. (2008) propose the following state-space representation for the monetary shocks in the above-mentioned Taylor rule:

$$\begin{bmatrix} z_{t+1} \\ u_{t+1} \end{bmatrix} = \begin{bmatrix} p & 0 \\ 0 & \phi \end{bmatrix} \begin{bmatrix} z_t \\ u_t \end{bmatrix} + \begin{bmatrix} N_{t+1} \\ e_{t+1} \end{bmatrix}, \text{ where } N_{t+1} = \begin{cases} (1-p)\, z_t, \text{ with prob. } p \\ g_{t+1} - p z_t, \text{ with prob. } 1-p \end{cases},$$
$$\hat{\varepsilon}_t = \left[(1-\rho)(1-\alpha) \quad 1 \right] \begin{bmatrix} z_t \\ u_t \end{bmatrix}, \tag{5}$$

where the observable signal, $\hat{\varepsilon}_t$, is the OLS estimate of the error term in the Monetary Authority's reaction function (equation 4).

As pointed out by Andolffato et al. (2008) the use of the Kalman filter is not fully optimal because z_t is a mixture of a Bernoulli process and a Gaussian noise. To overcome the absence of non-normality let us consider an alternative formulation of the time evolution of z_t that requires a state-space representation with state-contingent matrices in the state equation[2]. This alternative formulation (LPR-representation, hereafter) is as follows:

[2] It is very well known that the state space representation of a dynamic system is not unique. In the problem at hand here a simpler and more intuitive representation is as follows:

$$x_t = A_{s,t} x_{t-1} + B_{s,t} \upsilon_t, \text{ where } x_t = \begin{bmatrix} z_t \\ u_t \end{bmatrix},\ \upsilon_t = \begin{bmatrix} g_t \\ u_t \end{bmatrix},\ A_{s,t} = \begin{bmatrix} s_t & 0 \\ 0 & \phi \end{bmatrix},\ B_{s,t} = \begin{bmatrix} 1-s_t & 0 \\ 0 & 1 \end{bmatrix},$$
$$\varepsilon_t = \left[(1-\rho)(1-\alpha) \quad 1 \right] x_t .$$

However, under this representation, numerical problems are very likely to arise because matrix $A_{s,t}$ either has three zero elements when S_t=0 or a unit eigenvalue when S_t=1, which implies nonstationarity concerns. However, our state-space representation overcomes these problems.

$$\begin{bmatrix} z_{t+1} \\ u_{t+1} \end{bmatrix} = \begin{bmatrix} \varphi & 0 \\ 0 & \phi \end{bmatrix} \begin{bmatrix} z_t \\ u_t \end{bmatrix} + \begin{bmatrix} \varpi_{S_{t+1}} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} E_t z_{t+1} \\ E_t u_{t+1} \end{bmatrix} + \begin{bmatrix} \delta_{S_{t+1}} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} g_{t+1} \\ e_{t+1} \end{bmatrix}, \tag{6}$$

$$\hat{\varepsilon}_t = \begin{bmatrix} (1-\rho)(1-\alpha) & 1 \end{bmatrix} \begin{bmatrix} z_t \\ u_t \end{bmatrix}, \tag{7}$$

where:

$$|\varphi| \in (0,1), \quad \varpi_{S_{t+1}} = \begin{cases} \dfrac{1-\varphi}{p}, \text{ if } S_{t+1} = 1, \text{ with prob. } p \\ -\dfrac{\varphi}{p}, \text{ if } S_{t+1} = 0, \text{ with prob. } 1-p \end{cases}$$

and (P)

$$\delta_{S_{t+1}} = \begin{cases} 0, \text{ if } S_{t+1} = 1, \text{ with prob. } p \\ 1, \text{ if } S_{t+1} = 0, \text{ with prob. } 1-p \end{cases}$$

Proposition 1: *If $\varpi_{S_{t+1}}$ and $\delta_{S_{t+1}}$ are defined as in (7), the dynamics of z_t is observationally equivalent to (5) from the perspective of conditional mean.*

Proof. From (6), we have that

$$z_{t+1} = \varphi z_t + \varpi_{S_{t+1}} E_t z_{t+1} + \delta_{S_{t+1}} g_{t+1}, \tag{8}$$

and, therefore, the conditional expectation of z_{t+1} is:

$$E_t z_{t+1} = \frac{\varphi}{1 - p\varpi_{S_{t+1}=1} - (1-p)\varpi_{S_{t+1}=0}} z_t. \tag{9}$$

From (8), with probability *p*, $S_{t+1} = 1$, and $z_{t+1} = z_t$; then:

$$\varphi z_t + \varpi_{S_{t+1}=1} E_t z_{t+1} + \delta_{S_{t+1}=1} g_{t+1} = z_t \Rightarrow \varphi z_t + \varpi_{S_{t+1}=1} \frac{\varphi}{1 - p\varpi_{S_{t+1}=1} - (1-p)\varpi_{S_{t+1}=0}} z_t + \delta_{S_{t+1}=1} g_{t+1} = z_t.$$

This equation holds when:

$$\delta_{S_{t+1}=1} = 0, \quad \text{and} \quad \varphi \left[1 + \frac{\varpi_{S_{t+1}=1}}{1 - p\varpi_{S_{t+1}=1} - (1-p)\varpi_{S_{t+1}=0}} \right] = 1. \tag{10}$$

Again from (8), but with probability 1-*p*, $S_{t+1} = 0$, and $z_{t+1} = g_{t+1}$; then:

$$\varphi z_t + \varpi_{S_{t+1}=0} E_t z_{t+1} + \delta_{S_{t+1}=0} g_{t+1} = g_{t+1} \Rightarrow$$

$$\varphi z_t + \varpi_{S_{t+1}=0} \frac{\varphi}{1 - p\varpi_{S_{t+1}=1} - (1-p)\varpi_{S_{t+1}=0}} z_t + \delta_{S_{t+1}=0} g_{t+1} = g_{t+1}.$$

This equation holds when:

$$\delta_{S_{t+1}=0} = 1, \quad \text{and} \quad 1 + \frac{\varpi_{S_{t+1}=0}}{1 - p\varpi_{S_{t+1}=1} - (1-p)\varpi_{S_{t+1}=0}} = 0 \tag{11}$$

Equations in (10) and (11) define a system for the variables $\left\{\varpi_{S_{t+1}=1}, \varpi_{S_{t+1}=0}\right\}$, with the following solution:

$$\varpi_{S_{t+1}=1} = \frac{1-\varphi}{p}; \quad \varpi_{S_{t+1}=0} = -\frac{\varphi}{p}. \square$$

Note that the representation that we propose is a function of the parameter φ. Next, we demonstrate that there is a unique value of φ in terms of probability p that yields the same conditional variance as in (5) for the z_t process.

Proposition 2: *The LPR-representation yields the same conditional variance as in (5) for the z_t process if $\varphi = p/2$.*

Proof: In accordance with equation (4), the conditional variance of z_t is as follows:

$$\begin{aligned} \operatorname{var}_t(z_{t+1}) &= E_t\left(z_{t+1} - E_t z_{t+1}\right)^2 \underbrace{=}_{E_t z_{t+1} = p z_t} E_t\left(z_{t+1} - p z_t\right)^2 = \\ &= p(1-p) z_t^2 + (1-p)\sigma_g^2 \ . \end{aligned} \tag{12}$$

Using our representation we have:

$$\begin{aligned} \operatorname{var}_t(z_{t+1}) &= E_t\left(z_{t+1} - E_t z_{t+1}\right)^2 \underbrace{=}_{E_t z_{t+1} = p z_t} E_t\left(z_{t+1} - p z_t\right)^2 \\ &= E_t\left(\varphi z_t + \varpi_{S_{t+1}} E_t z_{t+1} + \delta_{S_{t+1}} g_{t+1} - p z_t\right)^2 \\ &= E_t\left[\left(\varphi - (1 - \varpi_{S_{t+1}}) p\right) z_t + \delta_{S_{t+1}} g_{t+1}\right]^2 \\ &= p(2\varphi - 1)^2 z_t^2 + (1-p) E_t\left[-p z_t + g_{t+1}\right]^2 \\ &= p(2\varphi - 1)^2 z_t^2 + (1-p)\left(p^2 z_t^2 + \sigma_g^2\right) \end{aligned} \tag{13}$$

Substituting $\varphi = p/2$ into equation (13), is straightforward to get expression in (12). $\square$

Our state-space formulation, which is characterized by having Gaussian innovations, is:

$$\xi_{t+1} = F\xi_t + B_{(S_{t+1})} E_t \xi_{t+1} + U_{(S_{t+1})} \upsilon_{t+1}, \tag{14}$$

$$\hat{\varepsilon}_t = H' \xi_t, \tag{15}$$

where:

$$\xi_{t+1} = \left[z_{t+1} \;\; u_{t+1} \right]', \qquad \upsilon_{t+1} = \left[g_{t+1} \;\; e_{t+1} \right]', \qquad F = \begin{bmatrix} \frac{p}{2} & 0 \\ 0 & \phi \end{bmatrix}, \qquad B_{(S_{t+1})} = \begin{bmatrix} \varpi_{S_{t+1}} & 0 \\ 0 & 0 \end{bmatrix},$$

$$U_{(S_{t+1})} = \begin{bmatrix} \delta_{S_{t+1}} & 0 \\ 0 & 1 \end{bmatrix}, \; E\left[\upsilon_t \upsilon_t' \right] = \begin{bmatrix} \sigma_g^2 & 0 \\ 0 & \sigma_e^2 \end{bmatrix}, \; H' = \left[(1-\rho)(1-\alpha) \quad 0 \quad 1 \right]$$

and $\varpi_{S_{t+1}}$ and $\delta_{S_{t+1}}$ are defined as in (P).

Equations (14) and (15) define a state-space system (see Hamilton, 1994, chapter 13), where (14) is the state equation and (15) is the observation equation.

For each of the two relevant histories, $S_t = k$ ($k = \{0, 1\}$), the equations for the Kalman filter are[3]:

$$\begin{aligned}
K_t^{(k)} &= P_{t|t-1}^{(k)} H \left(H' P_{t|t-1}^{(k)} H \right)^{-1}, \\
\hat{\xi}_{t+1|t}^{(k)} &= (I - B_{(S_t = k)})^{-1} F \left[\hat{\xi}_{t|t-1}^{(k)} + K_t^{(k)} \left(\hat{\varepsilon}_t - H' \hat{\xi}_{t|t-1}^{(k)} \right) \right], \\
P_{t+1|t}^{(k)} &= F P_{t|t-1}^{(k)} F' - F K_t^{(k)} P_{t|t-1}^{(k)} F' + U_{(S_t = k)} Q U'_{(S_t = k)}.
\end{aligned}$$

Next, we describe how to get the log-likelihood function to be maximized with respect to the parameters ϕ, p, σ_g^2, and σ_e^2:

Step 1: Computing the density functions for each history:

The conditional density function of $\hat{\varepsilon}_t$ to $Y_{t-1} \equiv \left(\hat{\varepsilon}_1, \hat{\varepsilon}_2, \ldots, \hat{\varepsilon}_{t-1} \right)'$ is:

$$\begin{aligned}
f\left(\hat{\varepsilon}_t \mid Y_{t-1}, S_t = k; \theta \right) &= \left(2\pi \right)^{-1/2} \left| \omega_t^{(k)} \right|^{-1/2} \exp\left(-\frac{1}{2} \mu_t^{(k)\prime} \left[\omega_t^{(k)} \right]^{-1} \mu_t^{(k)} \right) \\
&= \left(2\pi \right)^{-1/2} \left| \omega_t^{(k)} \right|^{-1/2} \exp\left(-\frac{1}{2} \left[\mu_t^{(k)} \right]^2 / \omega_t^{(k)} \right),
\end{aligned}$$

[3] We derive in the appendix 1 the equations for the Kalman filter using our state-space representation.

where: $\omega_t^{(k)} = H' P_{t|t-1}^{(k)} H$; $\hat{\mu}_t^{(k)} = \hat{\varepsilon}_t - H' \hat{\xi}_{t|t-1}^{(k)}$. and $\theta \equiv (\phi,\ p,\ \sigma_g^2,\ \sigma_e^2)'$.

Step 2: Computing the marginal density function of $\hat{\varepsilon}_t$ conditional to Y_{t-1} :

$$f\left(\hat{\varepsilon}_t \mid Y_{t-1};\theta\right) = \sum_{k=0}^{1} f\left(\hat{\varepsilon}_t \mid Y_{t-1}, S_t = k;\theta\right) P\left[S_t = k\right].$$

Step 3: Obtaining the log-likelihood function of $\hat{\varepsilon}$:

$$\ln L\left(\theta\right) = \sum_{t=1}^{T} \ln f\left(\hat{\varepsilon}_t \mid Y_{t-1};\theta\right).$$

Once the parameters have been estimated, the probability of a regime change in the current period conditional on a given shock can be estimated as follows:

$$\Pr\left[S_t = 0 \mid \hat{\varepsilon}_t\right] = \frac{\Pr\left[S_t = 0\right] \cdot f\left(\hat{\varepsilon}_t \mid Y_{t-1}, S_t = 0;\hat{\theta}\right)}{f\left(\hat{\varepsilon}_t \mid Y_{t-1};\hat{\theta}\right)},$$

where $\hat{\theta}$ denotes the vector of estimated parameters.

4. Empirical Evidence

In this section we show how to use our estimation method to provide empirical evidence on monetary policy making for the US through the lens of a Taylor rule. We use quarterly data from the EcoWin Economic & Financial database, and in particular we collect information on interest rates, inflation and GDP for the sample period covering 1980-2011 (first quarter). We estimate the output gap by subtracting a non-linear trend from real GDP using the Hoddrick-Prescott filter. Figure 1 depicts the time evolution for inflation, output-gap and interest rates. Shadows highlight the time periods corresponding to the tenure of the three chairmen of the Fed involved in the sample: Volcker, Greenspan and Bernanke, respectively.

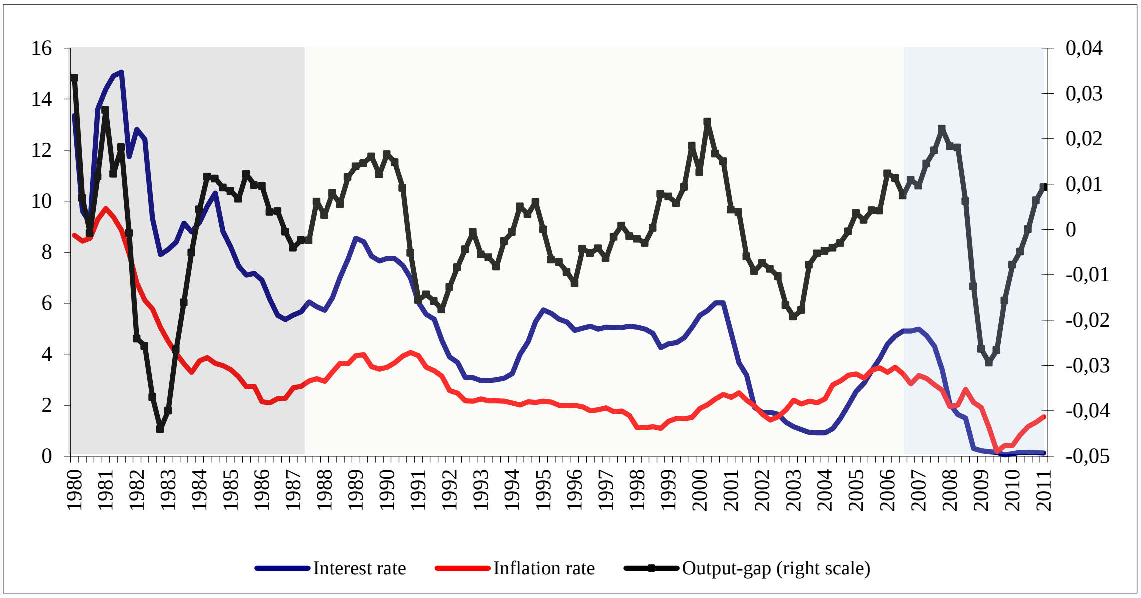

Figure 1: Time series for Inflation, Output-Gap abd Interest Rate for US economy.

A least square regression of the following Taylor rule:

$$i_t = \beta_0 + \beta_1 i_{t-1} + \beta_2 \pi_t + \beta_3 \left(y_t - y_t^* \right) + \varepsilon_t \tag{16}$$

yields the following parameter estimates (standard deviations in brackets):

$$\begin{array}{c} i_t = \underset{[0.0014]}{0.0007} + \underset{[0.0346]}{0.8722} i_{t-1} + \underset{[0.0635]}{0.1710} \pi_t + \underset{[0.0534]}{0.1503} \left(y_t - y_t^* \right) + \hat{\varepsilon}_t \end{array} \tag{17}$$

$$\overline{R}^2 = 0.9463, \ \hat{\sigma}_\varepsilon^2 = 0.0001.$$

$$\hat{\alpha} = g_1(\hat{\boldsymbol{\beta}}) = \frac{\hat{\beta}_2}{1-\hat{\beta}_1} = 1.3376; \ std(\hat{\alpha}) = \left(\nabla g_1 \cdot \text{cov}(\hat{\boldsymbol{\beta}}) \cdot \nabla g_1' \right)^{1/2} = 0.3046,$$

$$\hat{\beta} = g_2(\hat{\boldsymbol{\beta}}) = \frac{\hat{\beta}_3}{1-\hat{\beta}_1} = 1.1763; \ std(\hat{\beta}) = \left(\nabla g_2 \cdot \text{cov}(\hat{\boldsymbol{\beta}}) \cdot \nabla g_2' \right)^{1/2} = 0.4923,$$

where $\boldsymbol{\beta} = [\beta_0, \beta_1, \beta_2, \beta_3]'$, and $\nabla g_i, \ i = 1,2$ denotes the gradient of the function $g_i(\cdot), \ i = 1,2$.

Consistent with previous empirical research, a significant point estimate of the lagged policy rate is detected, suggesting very slow partial adjustment in US monetary policy making. Also, the estimated response for the deviation of the short-run inflation target from its long-run counterpart is consistent with the Taylor principle, that is, the nominal interest rate raises more than point-for-point when inflation exceeds the target inflation rate.

As for the nature of regime switching detected from the estimated monetary shocks, the maximization of the likelihood function yields the following point estimates (standard deviations are in brackets): $\hat{p} = 0.8626[0.0247]$, $\hat{\sigma}_g = 0.4416[0.1024]$, $\hat{\sigma}_e = 0.0030[0.0004]$ and $\hat{\phi} = 0.5636[0.0962]$. These parameters are the estimated probability of regime change (p), the estimated volatility of permanent and transitory shocks (σ_e^2 and σ_g^2, respectively) and the AR(1) parameter that corresponds to the time evolution of the transitory shock (ϕ). The probability of regime change for the US is around 13% (that is, $1-\hat{p}$), which implies a mean duration of shifts of around seven quarters. Also, as expected, the volatility of the shocks in the two regimes differs significantly. In particular, the volatility of transitory shocks is clearly lower than that of corresponding to permanent shocks. Moreover, the estimated coefficient ϕ is positive, statistically significant at the 1% significance level and clearly lower than one, a finding that is consistent with the assumptions made.

Figure 2 depicts the time evolution for the probability of regime change conditional to a given monetary shock, as well as the permanent component of the monetary shock, that is, the deviation of the current inflation target from its long-term mean ($\hat{z}_{t|t-1}$).

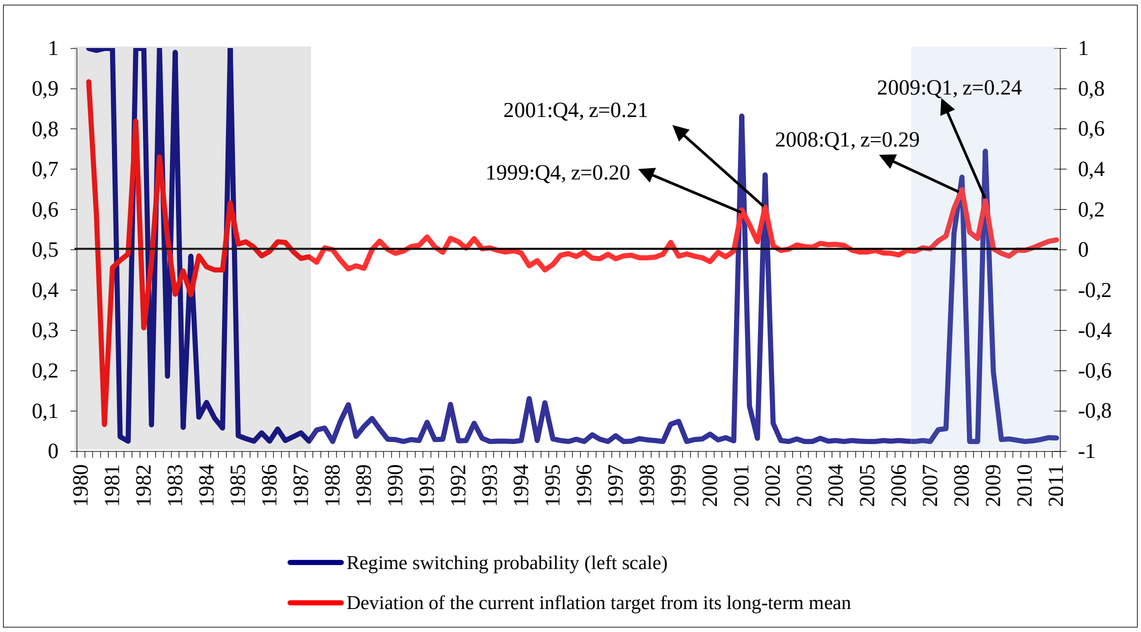

Figure 2. Monetary policy with time-varying inflation target during the Volcker-Greenspan-Bernanke period.

Our empirical findings show an extremely high probability of regime change at the beginning of the eighties. This is consistent with historical monetary policy making in the US[4]: in the period following the Great Inflation, Fed operating procedures were modified. On October 1979, targeting of non-borrowed reserves directly replaced Fed funds rate targeting, but after the meeting of the Federal Open Market Committee in October 1982, the Fed abandoned non-borrowed targeting and concluded that short-run control of monetary aggregates was less strict than interest control. After the Great Moderation, the probability of regime change approaches unity just in March and December 2001. On 26 November 2001 the National Bureau of Economic Research announced that the US economy had been in recession since 1 March 2001. However, as Mostaghimi (2004) notes, there was some speculation that even though US monetary authorities had anticipated the severity of the problems in the US economy in 2000, they hesitated to act promptly because of the prolonged US presidential election process. Another probable regime change detected is immediately after the unexpected shock of 9/11 event, which undoubtedly accelerated the decline in consumer confidence first noted in August 2001. After the terrorist attack, the Fed took up the challenge of maintaining and managing countercyclical policy in a stable price environment. To face the crisis, target federal funds rates was lowered quickly, and US monetary policy was easy during the period 2002 to 2006.

It is also observed two potential regime changes in the first quarter of 2008 and 2009, which are both related to the subprime mortgage crisis. The initial signals for the crisis in financial markets can be dated in June-July 2007 (problems at the Bear Stearns hedge fund); next, economic growth weakened and the recession officially started in December 2007. In March 2008 Bear Stearns collapsed, while Lehman Brothers followed in September 2008. By late 2008, nominal interest rates were close to the zero bound, but financial markets were not responding as expected. The Fed took additional measures. On march 18, 2009 the press release made by the Fed stated: “to provide greater support to mortgage lending and housing markets, the Committee decided today to increase the size of the Federal Reserve’s balance sheet further by purchasing up to an additional $750 billion of agency mortgage-backed securities, bringing its total purchases of these securities to up to $1.25 trillion this year, and to increase its

[4]See Orphanides (2003) for a detailed analysis of US monetary policy and the usefulness of the Taylor-rule framework to interpret it.

purchases of agency debt this year by up to $100 billion to a total of up to $200 billion. Moreover, to help improve conditions in private credit markets, the Committee decided to purchase up to $300 billion of longer-term Treasury securities over the next six months".

As to the time-varying estimates of the difference between the current and the long-term targeted rates, Figure 2 suggests that the short-run inflation target has been close to a constant since 1984, and extremely more volatile (relative to the post-1984 period) in the early 1980s. Such extreme realizations are at odds with a variety of estimates previously reported in the literature (e.g., Ireland (2007), Cogley, Primiceri, and Sargent (2010), and Aruoba and Schorfheide (2011)), probably reflecting that the 1980-1984 period, roughly corresponding to the Volcker disinflation, is difficult to model with the rule under scrutiny. However, after the Great Moderation, the regime changes detected in monetary policy making are matched with substantial updates in the current inflation target.

It is also remarkable that our empirical evidence suggests that, during the period 1994-2000, the monetary policy implemented by the Federal Reserve was, in general, based on short-run inflation targets below the long-term target. This path for flexible inflation targeting is consistent with no accommodative monetary policy, in line with the Fed's policy during this period. The economic environment at the beginning of the past decade was sharply affected by the terrorist attack of September 11, 2001. During the period covering 1999-2001 our estimates reveal two significant updates of inflation target, in the fourth quarter of 1999 and 2001, respectively. This two "regime shifts" are motivated not only by geopolitical uncertainties derived from the terrorist attack, but also by the weak recovery of US economy after the moderate recession between March and November 2001. For the period 2001-2004, the estimated discrepancy between the current inflation target and the long-term inflation target is, on average, positive, revealing that inflation did not appear as a serious concern in the short-run for the Federal Open Market Committee during this period. Therefore, the maximum sustainable employment arises now as the only relevant goal in this period. Both aspects explain the aggressive response of the Fed in 2002 and 2003. As pointed out by Bernanke (2010), the discrepancy between the actual federal funds rates and the values implied by the Taylor rule during this time period is the most commonly cited evidence that monetary policy was too easy in order to prevent further bubbles in financial markets. However, our empirical findings suggest that the Fed managed the federal fund rates in accordance with short and long-run inflation targets. However, we can observe that the period 2004-2006 is characterized by negative differences between current inflation targets and the long-term inflation target. This suggests that, as a difference with the previous period (2001-2004), the Fed should now face the classical trade-off between employment and inflation in monetary policy making. And to prevent for inflationary pressures that might cause US economic growth, especially encouraged by the aggressive response of the Fed after 2001, just in June 2004 the Federal Market Committee began to raise the target rate, reaching 5.25% in June 2006. In 2008 and 2009 two clear changes in inflation targeting are detected, in a similar way as described for 1999 and 2000. After 2008, the estimated departures of current inflation targets are positive, on average, suggesting that employment becomes again the key short-run objective for the Fed. We can conclude that our empirical evidence on flexible inflation targeting suggests that US monetary policy was implemented accordingly with the macroeconomic conditions after the Great Moderation.

5. An alternative approach: the particle filter

As a robustness check we now explore differences between the above empirical findings and those based on the use of the particle filter in order to estimate directly the state-space form (4)[5]. The next table shows the estimated parameters with the two alternatives using 20,000 particles.

	$\hat{p}$	$\hat{\phi}$	$\hat{\sigma}_g$	$\hat{\sigma}_e$
Kalman filter, (*LPR-representation*),	0.8626 (0.0247)	0.5636 (0.0962)	0.4416 (0.1024)	0.0030 (0.0004)
Particle Filter	0.8656 (0.0131)	0.5613 (0.0096)	0.4573 (0.0392)	0.0031 (0.0001)

Table 1. Estimates of structural parameters using the Particle Filter, and the Kalman filter with the LPR-representation, respectively.

For each estimation procedure, the confidence interval at conventional significance levels contains the point estimate obtained with the alternative approach.

[5] Appendix 2 describes technical details for the implementation of the particle filter to our estimation problem.

However, the particle filter exhibits higher accuracy. Figure 3 shows the time evolution of the estimated discrepancies between the current inflation target and its long term counterpart using both procedures.

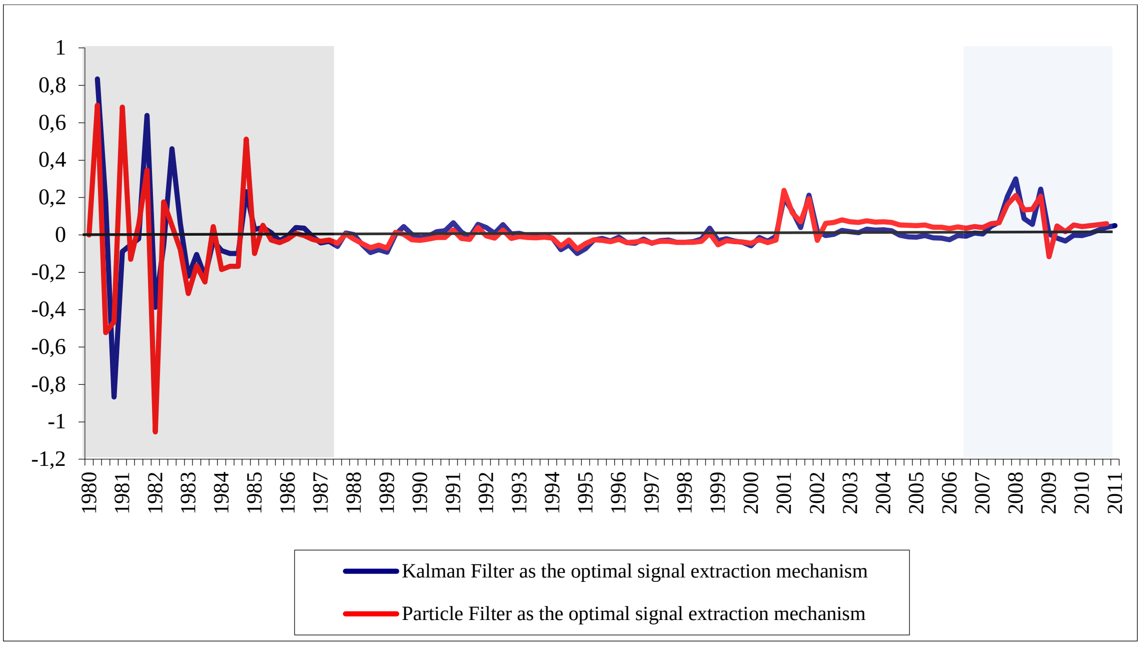

Figure 3. Deviation estimated of the current inflation target from its long-term mean.

Interestingly enough, the time evolution of $\hat{z}_{t|t-1}$ is quite similar under both methodologies. To statistically assess whether both procedures lead to the same probability distribution of the variable z_t we perform a Monte Carlo simulation in order not only to test null hypothesis of equality between the two distributions but also to compute the Mean Squared Error in forecasting theoretical z_t values[6]. In particular, we proceed as follows: considering a sample size equal to 125 (the same number of observations as in the data sample), we simulated $\{N_{t+1}, e_{t+1}, z_{t+1}, u_{t+1}, \varepsilon_t\}_{t=1}^{125}$ with initial conditions $z_0 = u_0 = 0$. Using these simulated time series we generate shocks $\{\varepsilon_t\}_{t=1}^{125}$ in accordance with equation (6). Now we consider the estimated parameters using the Kalman filter with the *LPR-representation* to generate theoretical values of z_t. After that, we make simulation exercises for shocks to generate conditional estimates of z_t either using the Particle Filter or the Kalman Filter. Let us denote each of these time series as $\{\hat{z}_{t|t-1}^{PF}\}_{t=2}^{125}$ and $\{\hat{z}_{t|t-1}^{KF}\}_{t=2}^{125}$, respectively. We can now perform a Kolmogorov-

[6] The Mean Square Error computed is $\left((1/T)\sum_{t=1}^{T}(z_t - \hat{z}_{t|t-1})^2\right)$, where z_t is the theoretical value of inflation-target and $\hat{z}_{t|t-1}$ is the estimated value either using either the Kalman-filter with the *LPR-representation* or the Particle-Filter.

Smirnov test for the null hypothesis of equality of distributions between $\left\{\hat{z}_{t|t-1}^{PF}\right\}_{t=2}^{125}$ and $\left\{\hat{z}_{t|t-1}^{KF}\right\}_{t=2}^{125}$ at the 5% significance level. The percentage of rejections with 1,000 replications is about 17%, neither so high nor negligible, as expected from the visual inspection of Figure 3.

However, as to the mean squared error to fit the theoretical differences between the current inflation target and the long-term target, we obtain the following median values:

	$MSE^{(PF)}$	$MSE^{(KF)}$
Median(MSE)	0.0260	0.0132

Table 4. Testing the fit of each methodology:

$$MSE^{(j)} = \left(\frac{1}{124}\right)\sum_{t=2}^{125}(z_t - \hat{z}_{t|t-1}^{(j)})^2,\ j \equiv PF, KF$$

Therefore, our simulation experiment shows that our estimation procedure has a better predictive ability to forecast the discrepancy between the short and long-run inflation targets.

5. Conclusions

This paper proposes an estimation procedure to decompose monetary shocks into permanent and transitory components using an inertial Taylor rule and the monetary innovations scheme proposed in Andolfatto et al. (2008). Our estimation procedure is based on a convenient reformulation of the state-space model representation that allows us an optimal use of the Kalman filter. This way we show how to perform the maximum likelihood estimation of the parameters involved in the time evolution of persistent and transitory monetary shocks, including the conditional probability of regime change. Researches interested in using new Keynesian DSGE models could take advantages of our estimation procedure in order to incorporate imperfect knowledge of the monetary policy rule implemented by the Central Bank.

We provide empirical evidence on US historical monetary policy making through the lens of a Taylor during the period 1980-2011 (first quarter). Consistent with previous findings, the evidence for a regime change in the inflation target during the nineties is extremely weak. However, September eleven, the recession that started in March 2001 and the subprime crisis were significant events that affected US monetary policy making in the last decade. We check the robustness of our empirical findings on flexible inflation targeting by comparing our estimations with those obtained using the particle filter. It is showed that the estimated deviations of the short-run inflation target from its long-run counterpart are remarkably similar over time. However, our estimation procedure is associated with lower mean squared errors in order to forecast theoretical difference between the short and long-run targeted inflation rates.

Our estimation procedure allows the comparison of our conditional probabilities of time varying inflation targeting with those obtained with a regime-switching approach where, with constant long-term inflation target, responses to output gap and inflation are time-varying as in the recent paper of Klingelhöfer, and Sun (2017). In case of both estimated probabilities being close to one for a given time period, it might be interesting to assess whether regime change is jointly due to, not only a new targeting regime but also the updating of responses. We leave this extension as a topic for further research.

References

Andolfatto, D., Hendry, S. and K. Moran (2008), "Are inflation expectations rational?", *Journal of Monetary Economics* 55, 406-422.

Aruoba, SB. and F. Schorpheide (2011), "Sticky prices versus monetary frictions: An estimation of policy trade-offs", *American Economic Journal: Macroeconomics* 3, 60-90.

Bernanke, B. (2010), "Monetary Policy and the Housing Bubble", Speech at the Annual Meeting of the American Economic Association, Atlanta Georgia, January.

Cogley, T., G. E. Primiceri, and T. Sargent (2010), "Inflation-Gap Persistence in the

U.S., *American Economic Journal: Macroeconomics*, 2, 43-69.

Fernández-Villaverde, J. and J.F Rubio-Ramírez (2007), "Estimating macroeconomic models: A likelihood approach", *Review of Economic Studies*, 1-46.

Fernández-Villaverde, J. and J. F. Rubio-Ramírez (2005), "Estimating dynamic equilibrium economies: linear versus nonlinear likelihood", *Journal of Applied Econometrics* 20, 891–910.

Hamilton, J.D. (1994), Time Series Analysis, Princeton University Press, Princeton, NJ.

Ireland, P. (2007), "Changes in Federal Reserve's inflation target: Causes and consequences", *Journal of Money Credit and Banking* 39, 1851-1882.

Klingelhöfer, J. and R. Sun, (2017), "China's regime-switching monetary policy", *Economic Modelling*, forthcoming

Kozicki, S. and P. Tinsley (2005), "Permanent and transitory policy shocks in an empirical macro model with asymmetric information", *Journal of Economics Dynamics and Control* 29, 1985-2015.

Milani, F. and J. Treadwell (2012), "The effect of monetary policy "news" and "surprises", *Journal of Money Credit and Banking* 44, 1667-1692.

Mostaghimi, M. (2004), "Monetary policy, composite leading economic indicators and predicting the 2001 recession", *Journal of Forecasting* 23, 463-477.

Orphanides, A. (2003), "Historical monetary policy analysis and the Taylor rule", *Journal of Monetary Economics* 50, 983-1022.

Taylor J.B. (1993), "Discretion versus policy rules in practice". Canergie-Rochester Conference on Public Policy, 39, 195-214.

Appendix 1

This appendix describes how to get equations for the Kalman filter using our state-space representation with Gaussian innovations.

Following Hamilton (1994), we consider the following state-space system:

$$\underbrace{\xi_{t+1}}_{r\times 1} = \underbrace{F}_{r\times r}\underbrace{\xi_t}_{r\times 1} + \underbrace{B}_{r\times r}\underbrace{E_t\xi_{t+1}}_{r\times 1} + \underbrace{U}_{r\times r}\underbrace{\upsilon_{t+1}}_{r\times 1}, \tag{A.1}$$

$$\underbrace{y_t}_{n\times 1} = \underbrace{H'}_{n\times r}\underbrace{\xi_t}_{r\times 1} + \underbrace{w_t}_{n\times 1}, \tag{A.2}$$

with

$$E(\upsilon_t\upsilon_\tau') = \begin{cases} Q, & \text{for } t=\tau \\ 0, & \text{otherwise} \end{cases} \tag{A.3}$$

$$E(w_t w_\tau') = \begin{cases} R, & \text{for } t=\tau \\ 0, & \text{otherwise.} \end{cases} \tag{A.4}$$

We assume that $\{y_1, y_2, ..., y_T\}$ are observable variables and that, *B*, *U*, *H*, *Q* and *R* are known with certainty.

The Kalman Filter calculates the forecasts $\hat{\xi}_{t+1|t}$ recursively, and, associated with each of these forecasts, the Kalman Filter computes the Mean Squared Error matrix: $P_{t+1|t} \equiv E\left[(\xi_{t+1} - \hat{\xi}_{t+1|t})(\xi_{t+1} - \hat{\xi}_{t+1|t})'\right]$.

The forecasting of y_t is as follows:

$$\hat{y}_{t|t-1} \equiv E(y_t \mid \mathscr{Y}_{t-1}) = H'E(\xi_t \mid \mathscr{Y}_{t-1}) = H'\hat{\xi}_{t|t-1}, \text{ where } \mathscr{Y}_{t-1} = \left(y_t', y_{t-1}', ..., y_1'\right)'.$$

The associated Mean Squared Error is:

$$E\left[(y_t - \hat{y}_{t|t-1})(y_t - \hat{y}_{t|t-1})'\right] = H'P_{t|t-1}H + R.$$

Next we update ξ_t taking into account the information set available at time *t* as follows:

$$\begin{aligned}\hat{\xi}_{t|t} \equiv \hat{E}(\xi_t \mid \mathscr{Y}_t) = \hat{\xi}_{t|t-1} + \left\{E\left[(\xi_t - \hat{\xi}_{t|t-1})(y_t - \hat{y}_{t|t-1})'\right]\right\} \times \\ \left\{E\left[(y_t - \hat{y}_{t|t-1})(y_t - \hat{y}_{t|t-1})'\right]\right\}^{-1}(y_t - \hat{y}_{t|t-1}) \\ = \hat{\xi}_{t|t-1} + P_{t|t-1}H(H'P_{t|t-1}H + R)^{-1}(y_t - H'\hat{\xi}_{t|t-1}).\end{aligned} \tag{A.5}$$

with Mean Squared Error:

$$P_{t|t} \equiv E\left[(\xi_t - \hat{\xi}_{t|t})(\xi_t - \hat{\xi}_{t|t})'\right] = E\left[(\xi_t - \hat{\xi}_{t|t-1})(\xi_t - \hat{\xi}_{t|t-1})'\right] - \left\{E\left[(\xi_t - \hat{\xi}_{t|t-1})(y_t - \hat{y}_{t|t-1})'\right]\right\} \times$$

$$\left\{E\left[(y_t - \hat{y}_{t|t-1})(y_t - \hat{y}_{t|t-1})'\right]\right\}^{-1} \times$$

$$\left\{E\left[(y_t - \hat{y}_{t|t-1})(\xi_t - \hat{\xi}_{t|t-1})'\right]\right\}$$

$$= P_{t|t-1} - P_{t|t-1} H (H' P_{t|t-1} H + R)^{-1} H' P_{t|t-1}. \qquad \text{(A.6)}$$

Next, we forecast ξ_{t+1} given the current set of available information as follows:

$$\hat{\xi}_{t+1|t} \equiv \hat{E}(\xi_{t+1} \mid \mathscr{Y}_t) = F\,\hat{E}(\xi_t \mid \mathscr{Y}_t) + B\,\hat{E}(E_t(\xi_{t+1}) \mid \mathscr{Y}_t) + U\,\hat{E}(\upsilon_{t+1} \mid \mathscr{Y}_t) = F\hat{\xi}_{t|t} + B\hat{\xi}_{t+1|t}$$

where, given that υ_{t+1} and w_t are Gaussian, we use that $\hat{\xi}_{t+1|t} = E_t(\xi_{t+1})$.

Rearranging the above equation we have $\hat{\xi}_{t+1|t} = (I - B)^{-1} F \hat{\xi}_{t|t}$. (A.7)

Substituting (A.7) into (A.9):

$$\hat{\xi}_{t+1|t} = (I - B)^{-1} F \hat{\xi}_{t|t-1} + (I - B)^{-1} F\, K_t (y_t - H'\hat{\xi}_{t|t-1}), \qquad \text{(A.8)}$$

where

$$K_t = P_{t|t-1} H (H' P_{t|t-1} H + R)^{-1} \qquad \text{(A.9)}$$

Taking into account not only that $\xi_{t+1} = F\xi_t + B\,E_t(\xi_{t+1}) + U\,\upsilon_{t+1}$, but also that $E_t(\xi_{t+1}) = \hat{\xi}_{t+1|t} = F\hat{\xi}_{t|t} + B\hat{\xi}_{t+1|t}$, we obtain the expression for the forecasting error: $\xi_{t+1} - \hat{\xi}_{t+1|t} = F(\xi_t - \hat{\xi}_{t|t}) + U\,\upsilon_{t+1}$.

Thus, the Mean Squared Error associated to $\hat{\xi}_{t+1|t}$ can be obtained as follows:

$$P_{t+1|t} = E\left[(F(\xi_t - \hat{\xi}_{t|t}) + U\,\upsilon_{t+1})(F(\xi_t - \hat{\xi}_{t|t}) + U\,\upsilon_{t+1})'\right] = F\,P_{t|t}\,F' + \tilde{Q}.$$

(A.10)

Substituting (A.6) into (A.8):

$$P_{t+1|t} = F\,P_{t|t-1}\,F' - F\,K_t\,H'\,P_{t|t-1} F' + \tilde{Q}. \qquad \text{(A.11)}$$

Summarizing, given $\hat{\xi}_{1|0}$ and $P_{1|0}$, the Kalman Filter computes recursively $\hat{\xi}_{t+1|t}$ and $P_{t+1|t}$ using the equations (A.8), (A.9) and (A.11).

Appendix 2

The particle filter is an alternative to overcome non-normality. In this appendix, we describe how to evaluate the likelihood function of monetary innovations using a Sequential Monte Carlo Filter when the *AHM-representation* is considered.

The Andolfatto et al. (2008) specification is:

$$z_{t+1} = p\, z_t + N_{t+1} \tag{A.12}$$

$$\varepsilon_t = (1-\rho)(1-\alpha)z_t + u_t \tag{A.13}$$

where $\begin{cases} N_{t+1} = \begin{cases} (1-p)z_t, \text{ with prob. } p \\ g_{t+1} - p\, z_t, \text{with prob. } 1-p, \text{ where } g_{t+1} \sim N(0,\sigma_g^2) \end{cases} \\ u_{t+1} = \phi\, u_t + e_{t+1}, \text{ where } e_{t+1} \sim N(0,\sigma_e^2) \end{cases}$

Assuming that $z_0 = 0$, we proceed as follows:

Step 1: Evaluate the probability of $u_{t|t-1}$**:**

i) We draw a random sample of size I =10000 from the uniform distribution in (0,1) and from a Normal distribution with zero mean and σ_g^2 variance. We call each observation of these two initial samples as U_i^1 and $x_i^1, i = 1,2,\dots,I$. Now, we use these two samples to generate a new sample the we denote $N^{1|0}$ as follows:

$$N_i^{1|0} = \begin{cases} 0 \; if \; U_i^1 \le p \\ x_i^1 \; if \; U_i^1 > p \end{cases} \quad i = 1,2,\dots,I$$

where $1-p$ is the probability of a regime change. We use the sample $N^{1|0}$ to generate an additional sample that we denote $z^{1|0}$ as follows:

$$z_i^{1|0} = pz_0 + N_i^{1|0}, \qquad i = 1,2,\dots,I$$

Without loss of generality, we assume $z_0 = 0$

ii) Next, we use the estimated value for the first element of the noise vector ε_t, that we denote as $\widehat{\varepsilon_1}$, to generate a random sample for the innovation u_t as follows:

$$u_i^{1|0} = \widehat{\varepsilon_1} - (1-\rho)(1-\alpha)z_i^{1|0}, \qquad i = 1,2,\dots,I$$

iii) We evaluate the relative weight for each observation $u_i^{1|0}$:

$$q_{u_i^{1|0}} = \frac{p\left(u_i^{1|0}\right)}{\sum_{i=1}^{I} p\left(u_i^{1|0}\right)}, \qquad i = 1,2,\dots,I$$

where the probability $p\left(u_i^{1|0}\right)$ corresponds to a Gaussian distribution with zero mean and $\frac{\sigma_e^2}{1-\phi}$ variance.

iv) We update the initial sample $z^{1|0}$ by performing a weighted sampling with replacement in accordance with the above-mentioned weights.

v) We repeat the process described in i) to v) for each estimated component of the noise vector ε_t.

Step 2: Using the Law of the Large Numbers:

$$p\left(\varepsilon_t|\varepsilon_{t-1}\right) \approx \frac{1}{I}\sum_{i=1}^{I} p\left(u_{i,t}|u_{i,t-1}\right), i = 1,2,\dots,I$$

where the conditional distribution of $u_{i,t}$ is $N\left(\phi u_{i,t}|u_{i,t-1}, \sigma_e^2\right)$. Once the conditional probabilities for monetary innovations are computed, we can evaluate the likelihood function as: $p\left(\varepsilon_{1,}\varepsilon_{2,\dots,}\varepsilon_T\right) = \prod_{i=1}^{T}\left[\frac{1}{I}\sum_{i=1}^{I} p\left(u_{i,t}|u_{i,t-1}\right)\right]$, where T denotes the sample size.

Step 3: We maximize the likelihood with respect to the parameters parameters ϕ, σ_e^2, σ_g^2 and p.