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Measurement of graded-index media using digitized images of light paths

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Abstract
In a stratified graded-index medium the variation of the refractive index can be measured directly from the analysis of light paths. Only Snell's law and a simple set-up based on digitized images and a personal computer are needed.

1. Introduction

One of the most startling phenomenon at first sight, even for experienced physicists, is the propagation of a light beam in a graded-index optical medium. In such media the refractive index changes from one layer to another, bending the light and producing so-called mirages. Several simple set-ups can be used to display such phenomena [1–3], but quantitative measurements certainly add some value to the experiment, and procedures for making these can be found in the literature [4, 5]. In this paper we propose a simple method that uses digitized images to obtain the profile of the refractive index of a stratified medium directly from measurements of the trajectories of light beams inside the medium. The method gives insight into an interesting physical phenomenon and because of its simplicity it can be translated into a complete experiment for undergraduate students. The experiment would start with the design and preparation of the set-up, followed by the data acquisition using the computer and the camera as a measurement instrument and finishing with the data fitting and the discussion of results.

2. Theory and experimental procedure

The measurement procedure uses the basic theoretical laws of propagation in stratified graded-index media. When light propagates through such media it follows curved trajectories due to refraction in the layers with different refractive indices. If we choose the y-axis as the stratification direction (i.e. $n = n(y)$), the ray paths are confined to planes parallel to the y-axis. The refractive index $n = n(y)$ and the angle $\theta(y)$ between the trajectory and the y-axis are related through a generalized Snell law

$$n(y)\sin[\theta(y)] = K, \qquad (1)$$

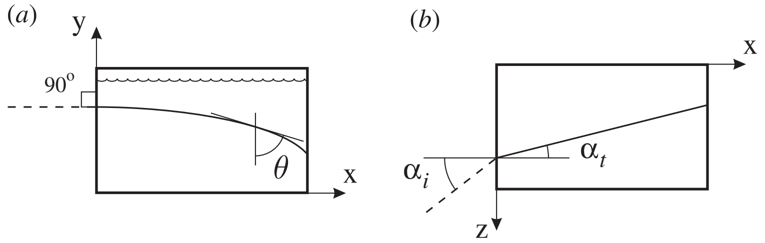

Figure 1. Schematic diagram of the trajectories used to measure the variation of the refractive index. (*a*) Side view with a ray that follows equation (1) in the xy plane; (*b*) top view of a beam entering parallel to the xz plane.

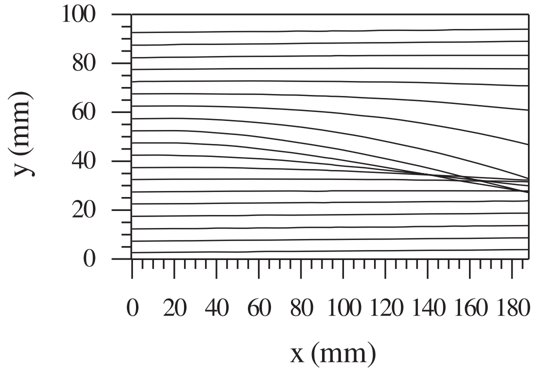

Figure 2. Light paths for different heights fitted to polynomials.

where K is a constant for each individual trajectory (we can consider the medium as composed of very thin layers $1 \ldots i$, using Snell's law again and again, so that for each ray $n_1 \sin\theta_1 = \cdots = n_i \sin\theta_i = K$). The above equation allows us to obtain $n = n(y)$ for a given ray if we know $\theta(y)$ and K. As a particular case, for the points where $\theta(y) = 90°$ we have $K = n(y)$.

Let us now consider the particular set-ups shown in figures 1(*a*) and (*b*): a light ray parallel to plane xz enters from vacuum into a parallelepiped of a stratified graded-index medium $n = n(y)$. If observation were along the z-axis (perpendicular to the stratification direction), we would see the typical curved trajectories shown in figure 1(*a*). From the images of such curves only $\theta(y)$ can be obtained; if we want a full measurement of the refractive index we need the value of the constant K for each of these trajectories. For this purpose we can use the set-up shown in figure 1(*b*) and Snell's law. In this configuration the light ray is seen as a straight line when observing in the stratification direction (figure 1(*b*)). Now, if the incident ray is parallel to the xz-plane, and the incidence angle is α_i, we can use Snell's law at the input point

$$\sin(\alpha_i) = n(y)\sin[\alpha_t(y)], \tag{2}$$

where $\alpha_t(y)$ is the angle of refraction at height y. Since in stratified media the trajectories are confined to planes, $\alpha_t(y)$ is also the angle of the projected trajectories, so it can be measured from them. Finally, from equations (2) and (1) the value of K is obtained (at the input plane $\theta = 90°$).

3. Results

We have used the proposed scheme to measure the index of refraction of a stratified medium made with a solution of sugar in water. We followed the recipe given by Mak [3]. Using a funnel we carefully poured a concentrated water–sugar solution (350 g of sugar in 700 cm^3 of water) under 700 cm^3 of water. A total height of 9.5 cm (y-axis) was reached inside a methacrylate container with a base 18.8 cm (x-axis) by 8.8 cm (z-axis). At about one-half of the height there was a transition region. Since the water–sugar solution was poured very slowly, the pure water remained on top while the water–sugar solution had full concentration at the bottom. In fact, the transition region was so thin that it had to be disturbed in order to have a smoother gradient. Once the graded-index medium was ready the beam from a He–Ne laser was used as the ray that propagates through the medium. The images of the different trajectories were taken with a camera in a darkened room and then processed in a personal computer. In order to have digitized images we used a CCD camera (Pulnix TM-765) connected to a frame grabber card in the computer. Since the key point is to obtain the Cartesian coordinates of the light trajectories any other available source for digitized images, such as webcams or scanned pictures, can be used. The laser was moved up and down to vary the height at which the beam enters the medium. Finally, a very simple program was used to filter the noise and to reduce the images to simple curves. The program uses the images as matrices. Each element of the image matrix stores a value proportional to the irradiance at a given pixel. The program goal is to extract a very bright line from an image with a low level of noise (background noise, secondary scattering and reflections from the container walls, mainly). Therefore as a first step the program puts to zero the value of all pixels with an irradiance lower than a given value between the maximum irradiance and the average value (the line we want to measure is very bright, but it contributes with very few pixels to the total image). As a second step, in order to eliminate isolated bright points, the program calculates the average row for each column using all the pixels with non-zero irradiance. All the pixels more than a few rows away from these values are put to zero. More refinement could be considered, but those two steps are usually enough.

In order to obtain the images we first placed the camera over the container to obtain images similar to that shown in figure 1(b), while α_i was measured using a protractor. From the slope of each trajectory and from equation (2) we obtained the refractive index $n(y)$. The second set of measurements was obtained from the curved trajectories similar to that shown in figure 1(a). In order to minimize the 'mirage of the mirage' effect, light was propagated close to one side of the container. Each of these trajectories was fitted to a polynomial and the value of $\theta(y)$ at each point was obtained from the slope. The polynomial fits are shown in figure 2. Substituting $\theta(y)$ in equation (1) and using the corresponding value of K for each trajectory we obtained $n[y(x)]$.

The experimental results are summarized in figure 3. The refractive indices obtained from the first group of measurements and equation (2) for $\alpha_i = 39.5° \pm 1.0°$ are represented as open circles. In the same figure we have plotted the curves of the variation of the refractive index obtained from the second type of measurement and equation (1). Note that by construction each one of these curves starts at an open circle that corresponds to the value of $n(y)$ at the input point, where $n(y) = K$. As we can see from this figure both groups of measurements give similar results. The variation of the refractive index is stronger in the central region of the medium than in the upper and lower layers, in which n remains almost constant but with different values. This is consistent with the preparation recipe, where the transition layer created is relatively thin.

To verify these results we have compared them with values listed in physical data handbooks [6, 7]. The difference between values of n for water without sugar and water with our higher concentration given in references is similar to the results presented here. Nevertheless the absolute values differ slightly. We have obtained a value of $n = 1.359$ for pure water and $n = 1.408$ for the concentrated solution, while [6, 7] give $n = 1.333$ and 1.381,

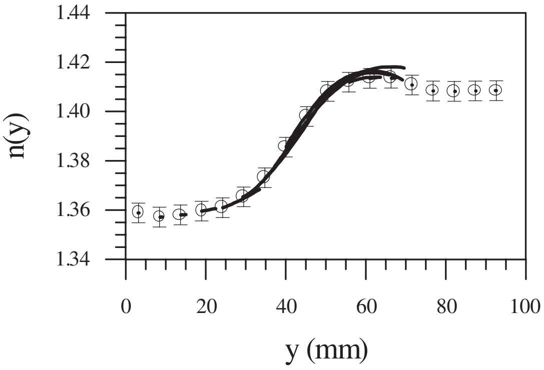

Figure 3. Variation of the refractive index for different heights y. Open circles represent indices measured using the set-up shown in figure 1(*b*) and equation (2). Error bars only include the error due to α_t. The results obtained from set-up 1(*a*) and equation (1) are shown as solid curves.

respectively. Note that the outlined procedure gives a high accuracy in the measurement of $\alpha_t(y)$ (inside the media, using digitized images), while it is not so easy to measure α_i (outside the media, using a protractor). This problem could be solved with an additional measurement of the water refractive index using a goniometer and a hollow prism. With this method we obtained $n = 1.336$, that could be used to correct the values of figure 3.

As a side note we would like to mention that once we have a container filled with sugar and water it could be a good opportunity to notice the strong polarization effects produced by scattering and by the optical activity of sugar.

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