EUROPEAN TRANSACTIONS ON TELECOMMUNICATIONS
Eur. Trans. Telecomms. 2009; **20**:389–401

(www.interscience.wiley.com) DOI: 10.1002/ett.1294

Mobile Networks

Performance analysis of a cellular mobile network with retrials and guard channels using waiting and first passage time measures

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SUMMARY

Most studies of modern cellular mobile networks concern performance measures directly computable from the stationary state probabilities such as the blocking probability and the mean traffic rates of the various kinds of calls. In this paper, we consider a cellular mobile system with retrials and guard channels for the handover calls, but we concentrate on performance measures related to the waiting and first passage times of the system. More concretely, we first build a Markovian model representing a station of the network and then we study the waiting time of a customer, the idle times of the guard channels and the time between successive lost calls. These measures shed light on the behaviour of the system and quantify the quality of service from both points of view of the customer and the administrator. Several numerical results illustrate the effect of the system parameters in its performance. Copyright © 2008 John Wiley & Sons, Ltd.

1. INTRODUCTION

In the studies of cellular mobile communication networks, it has been recognised that the proper modelling of customer behaviour is very important for the reliable evaluation of the quality of service (QoS) of a given system. Thus, fresh calls, initiated inside a given cell are distinguished from handover calls that appear due to the mobility of the customers. Since handover calls already use network resources, they should be prioritised with respect to fresh calls. Indeed, a premature termination of an ongoing call is considered to degrade the QoS much more severely than the experience of a busy signal at an attempt to initiate a call. To cope with this problem, several approaches have been proposed in the literature. One efficient method is the use of guard channels which are reserved only for the handover calls. The performance evaluation of systems with guard channels goes back at least to the pioneering paper of Guérin [1]. Subsequently, many authors (see, e.g. Yoon and Un [2], Chang *et al.* [3], Tran-Gia and Mandjes [4], Basharin and Merkulov [5], Pla and Casares-Giner [6], Gimenez-Guzman *et al.* [7]) have studied the performance of more involved systems with guard channels.

To represent accurately a cellular station, the phenomenon of repeated attempts should also be taken into account. Indeed, due to the increasing number of users and the system complexity of modern cellular networks, the impact of retrials is no longer negligible. For that reason, several papers (see, e.g. Tran-Gia and Mandjes [4], Ajmone Marsan *et al.* [8], Liu and Fapojuwo [9]) consider two-dimensional models to represent the behaviour of such systems. Indeed, for the accurate representation of a system with retrials, we need at least two state variables, one recording the number of busy channels and the other recording the number of calls in the retrial orbit. Although the computational complexity of these two-dimensional models is much greater than that of approximate one-dimensional models resulting from state aggregation, the corresponding results are far more reliable.

The two-dimensional continuous time Markov chains that appear in systems with retrials are in general harder to

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Received 20 April 2007
Revised 20 October 2007
Accepted 26 February 2008

analyse than their standard counterparts (see, e.g. Artalejo and Falin [10], Falin and Templeton [11]). This happens due to the retrial feature that implies linear transition rates with respect to the number of customers in orbit. Because of these mathematical difficulties, most studies for systems with retrials concern performance measures directly computable from the stationary probabilities of the number of customers in the system (see, e.g. the classified bibliographies of Artalejo [12] and Gómez-Corral [13]). Such measures include the blocking probability, the mean traffic rates of the various kinds of calls, etc. In particular, this is the rule in the recent studies for cellular mobile networks with guard channels and retrials.

Recently, Artalejo and Gómez-Corral [14] and Artalejo *et al.* [15] developed algorithmic procedures for the computation of the waiting time and busy period distributions in the main multiserver retrial queue. In the present paper, we aim to extend these ideas to provide an analysis for performance measures related to waiting and first-passage time distributions for cellular mobile networks with retrials and guard channels. More specifically, we study the waiting time of a customer, the idle time of the guard channels and the time between two successive lost calls. The conditional versions of these times given the state of the system are also discussed.

The paper is organised as follows. In Section 2, we describe the model for a single cellular station and introduce the associated Markov chain. In Section 3, we develop the algorithmic schemes for the waiting time analysis. We first obtain a linear system for the Laplace–Stieltjes transforms of the conditional waiting times and then proceed to the unconditional waiting time. We also study the moments (in particular, the mean and the variance) of the waiting time. In Section 4, we develop the algorithmic schemes for the other first-passage time measures of interest. Several numerical results are presented in Section 5. In particular, we present several tables and graphs that illustrate the influence of the system parameters in the QoS of a given system. Finally, in Section 6 we comment on the results and discuss possible extensions.

2. THE MODEL

We consider a cellular mobile network and we concentrate on a single station. The arrival processes for the fresh and the handover calls are assumed to be independent Poisson processes with rates λ_F and λ_H, respectively. There exist N channels for serving the calls. A number G of them are guard channels reserved only for handover calls, while the rest, $N-G$, are regular channels that can serve both fresh and handover calls. A handover call that finds all channels busy departs from the station and it is lost forever. A fresh call that finds at least $N-G$ occupied channels, joins the retrial orbit with probability θ or it is lost with probability $1-\theta$. The calls in the orbit conduct retrials. The interretrial times for the calls in the orbit are independent exponentially distributed random variables with parameter α. If a retrial call finds at least $N-G$ occupied channels, it returns to the orbit with probability θ or abandons the system with probability $1-\theta$ (this is the main difference with Tran-Gia and Mandjes [4] that allow retrial calls to use the guard channel). The completion rate for each call in service is μ. We denote by $\lambda=\lambda_F+\lambda_H$ the total arrival rate of non-retrial calls.

Let $C(t)$ and $N(t)$ be respectively the number of busy channels and the number of calls in orbit at time t. Then $\{(C(t), N(t))\}$ is a continuous time Markov chain with infinite state space $\{0,1,\ldots,N\}\times\{0,1,\ldots\}$. To numerically solve the system of the balance equations and the linear systems for the Laplace–Stieltjes transforms of several first-passage times of the model, we truncate the state space to $S=\{0,1,\ldots,N\}\times\{0,1,\ldots,K\}$. The orbit capacity K is chosen large enough to guarantee the numerical convergence of the mean values (for the exact convergence criterion see the first paragraph of Section 5). Let $p_{i,j}=\lim_{t\to\infty}\Pr[C(t)=i, N(t)=j]$, $(i,j)\in S$ denote the equilibrium probabilities of the system. Then, we have the balance equations

$$\begin{aligned}(\lambda+i\mu+j\alpha)p_{i,j} &= (1-\delta_{i,0})\lambda p_{i-1,j}+(i+1)\mu p_{i+1,j}\\ &\quad+(1-\delta_{j,K})(1-\delta_{i,0})(j+1)\alpha p_{i-1,j+1},\\ &\quad 0\leqslant i\leqslant N-G-1,\ 0\leqslant j\leqslant K \end{aligned} \tag{1}$$

$$\begin{aligned}&(\lambda_H+(1-\delta_{j,K})\theta\lambda_F+(N-G)\mu+j(1-\theta)\alpha)p_{N-G,j}\\ &\quad=\lambda p_{N-G-1,j}+(1-\delta_{j,0})\theta\lambda_F p_{N-G,j-1}\\ &\qquad+(N-G+1)\mu p_{N-G+1,j}\\ &\qquad+(1-\delta_{j,K})(j+1)\alpha p_{N-G-1,j+1}\\ &\qquad+(1-\delta_{j,K})(j+1)(1-\theta)\alpha p_{N-G,j+1},\quad 0\leqslant j\leqslant K\end{aligned} \tag{2}$$

$$\begin{aligned}&(\lambda_H+(1-\delta_{j,K})\theta\lambda_F+i\mu+j(1-\theta)\alpha)p_{i,j}\\ &\quad=\lambda_H p_{i-1,j}+(1-\delta_{j,0})\theta\lambda_F p_{i,j-1}+(i+1)\mu p_{i+1,j}\end{aligned}$$

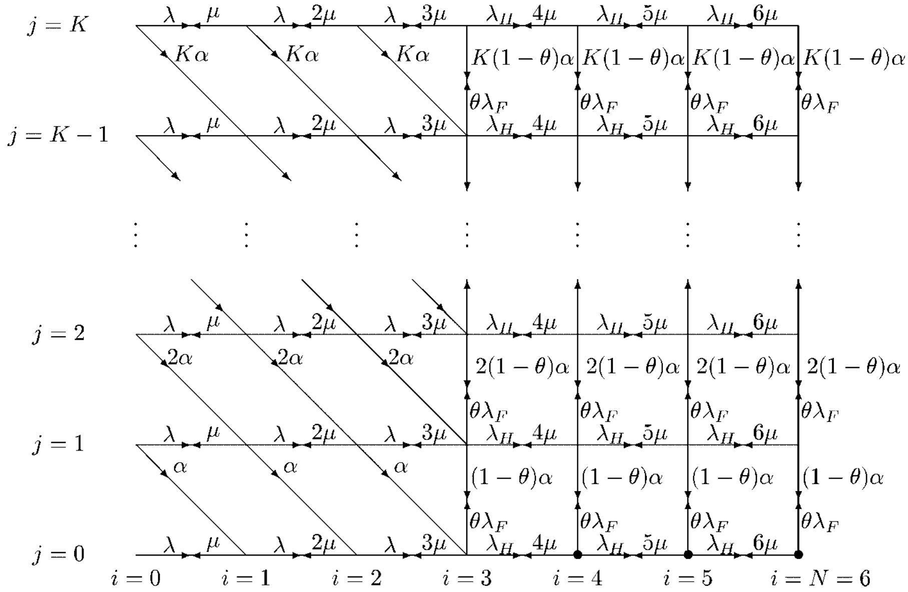

Figure 1. State space and transitions for $N = 6$ and $G = 3$.

$$+(1-\delta_{j,K})(j+1)(1-\theta)\alpha p_{i,j+1},$$
$$N-G+1 \leqslant i \leqslant N-1,\ 0 \leqslant j \leqslant K \qquad (3)$$

$$\begin{aligned}((1-\delta_{j,K})\theta\lambda_{\mathrm{F}} + N\mu + j(1-\theta)\alpha)p_{N,j} &= \lambda_{\mathrm{H}} p_{N-1,j} + (1-\delta_{j,0})\theta\lambda_{\mathrm{F}} p_{N,j-1} \\ &\quad + (1-\delta_{j,K})(j+1)(1-\theta)\alpha p_{N,j+1}, \quad 0 \leqslant j \leqslant K\end{aligned} \qquad (4)$$

where $\delta_{x,y}$ is the Kronecker's delta which is 1 for $x = y$ and 0 otherwise. By partitioning the state space as $S = \cup_{j=0}^{K} l(j)$, where $l(j) = \{(i, j),\ 0 \leqslant i \leqslant N\}$ denotes the jth level of the process, we have that $\{(C(t), N(t))\}$ is a continuous time finite quasi-birth-death (QBD) process. An illustration of its transition diagram, where the levels correspond to the horizontal lines is presented in Figure 1 (for the special case $N = 6$ and $G = 3$).

The transition rate matrix of $\{(C(t), N(t))\}$ is block tridiagonal and the computation of the stationary probabilities can be done at a low cost using one of the standard algorithms (see, e.g. the algorithms in Latouche and Ramaswami [16], Chapter 10, in particular the algorithm of Gaver *et al.* [17]). Moreover, the blocks of the QBD are tridiagonal matrices so we can also use the algorithms of Servi [18] which exploit further this special structure. We do not proceed further in the stationary analysis of the system, as our focus is on the waiting and first-passage time measures that we present in the next sections. For implementation details of the corresponding algorithms, we point to the papers of Servi [18] and Domenech-Benlloch *et al.* [19] who carry out the stationary analysis of similar models.

Equating the rates between the set of states with orbit level $\geqslant j$ with the set of states with orbit level $< j$, we obtain

$$j\alpha \sum_{i=0}^{N-G-1} p_{i,j} + j(1-\theta)\alpha \sum_{i=N-G}^{N} p_{i,j} = \theta\lambda_{\mathrm{F}} \sum_{i=N-G}^{N} p_{i,j-1},$$
$$1 \leqslant j \leqslant K$$

Summing for all $1 \leqslant j \leqslant K$, we obtain the cross-orbit equation

$$\begin{aligned}\alpha \sum_{i=0}^{N-G-1} \sum_{j=1}^{K} j p_{i,j} + (1-\theta)\alpha \sum_{i=N-G}^{N} \sum_{j=1}^{K} j p_{i,j} \\ = \theta\lambda_{\mathrm{F}} \sum_{i=N-G}^{N} \sum_{j=0}^{K-1} p_{i,j}\end{aligned} \qquad (5)$$

which we use in the study of the waiting time distribution of a customer.

DOI: 10.1002/ett

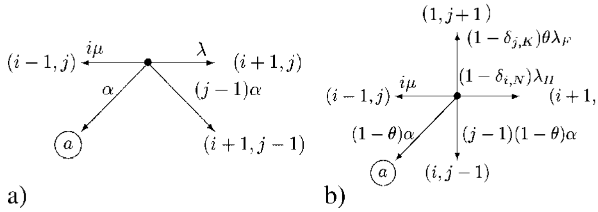

Figure 2. Transitions out of $(i, j) \in S^a$. a) $0 \leqslant i \leqslant N-G-1$, $1 \leqslant j \leqslant K$. b) $N-G \leqslant i \leqslant N, 1 \leqslant j \leqslant K$.

3. THE WAITING TIME OF A CALL

We consider the system at an arbitrary time t and suppose that it is at state (i, j), $j > 0$. We mark one of the j retrial calls and denote by $W_{i,j}$ a random variable representing its waiting time in the system (excluding service). Let also $W^*_{i,j}(s) = E[e^{-sW_{i,j}}]$ be the corresponding Laplace–Stieltjes transform. To study $W^*_{i,j}(s)$, we introduce an auxiliary Markov chain on the state space $S^a = S \cup \{a\}$, where a represents an absorbing state. The absorption occurs when the waiting time of the tagged customer expires. The transition rates $q^a_{(i,j)(i',j')}$ of the auxiliary chain are identical to the transitions rates $q_{(i,j)(i',j')}$ of the original chain with the exception of the rates

$$\begin{aligned}
q^a_{(i,j),(i+1,j-1)} &= (j-1)\alpha, \quad 0 \leqslant i \leqslant N-G-1, \\
&\quad 1 \leqslant j \leqslant K \\
q^a_{(i,j),a} &= \alpha, \quad 0 \leqslant i \leqslant N-G-1,\ 1 \leqslant j \leqslant K \\
q^a_{(i,j),(i,j-1)} &= (j-1)(1-\theta)\alpha, \quad N-G \leqslant i \leqslant N, \\
&\quad 1 \leqslant j \leqslant K \\
q^a_{(i,j),a} &= (1-\theta)\alpha, \quad N-G \leqslant i \leqslant N, 1 \leqslant j \leqslant K
\end{aligned} \tag{6}$$

The dynamics of this auxiliary absorbing Markov chain where the absorption corresponds to the termination of a waiting time are illustrated in the transition diagram given in Figure 2.

Being on a state (i, j), we now condition on the time to the next event and the next state, that is we apply first-step analysis in the auxiliary chain to obtain the Laplace–Stieltjes transforms $W^*_{i,j}(s)$ (for a more thorough treatment see Appendix where the main mathematical result is reported and the application to the specific auxiliary chain is described in detail). Then, the transforms $\{W^*_{i,j}(s)\}$ are given as the smallest non-negative solution of the system

$$\begin{aligned}
&(s+\lambda+i\mu+j\alpha)W^*_{i,j}(s) - \lambda W^*_{i+1,j}(s) - i\mu W^*_{i-1,j}(s) \\
&\quad -(j-1)\alpha W^*_{i+1,j-1}(s) = \alpha, \quad 0 \leqslant i \leqslant N-G-1, \\
&\quad 1 \leqslant j \leqslant K
\end{aligned} \tag{7}$$

$$\begin{aligned}
&(s+\lambda_{\mathrm{H}}+(1-\delta_{j,K})\theta\lambda_{\mathrm{F}}+i\mu+j(1-\theta)\alpha)W^*_{i,j}(s) \\
&\quad -\lambda_{\mathrm{H}}W^*_{i+1,j}(s) - (1-\delta_{j,K})\theta\lambda_{\mathrm{F}}W^*_{i,j+1}(s) - i\mu W^*_{i-1,j}(s) \\
&\quad -(j-1)(1-\theta)\alpha W^*_{i,j-1}(s) = (1-\theta)\alpha, \\
&\quad N-G \leqslant i \leqslant N-1,\ 1 \leqslant j \leqslant K
\end{aligned} \tag{8}$$

$$\begin{aligned}
&(s+(1-\delta_{j,K})\theta\lambda_{\mathrm{F}}+N\mu+j(1-\theta)\alpha)W^*_{N,j}(s) \\
&\quad -(1-\delta_{j,K})\theta\lambda_{\mathrm{F}}W^*_{N,j+1}(s) - N\mu W^*_{N-1,j}(s) \\
&\quad -(j-1)(1-\theta)\alpha W^*_{N,j-1}(s) = (1-\theta)\alpha, \quad 1 \leqslant j \leqslant K
\end{aligned} \tag{9}$$

The coefficient matrix of the unknowns for the linear system (7)–(9) is block tridiagonal. This enables to get the solution of the system at a low computational cost, using standard numerical linear algebra algorithms (see, e.g. the block LU factorisation algorithm for block tridiagonal systems in Datta [20], Subsection 6.4, p. 233 or in Ciarlet [21], Subsection 4.3, pp. 142–145). We can also exploit this system to develop a recursive algorithm for computing the moments $E[W^n_{i,j}]$ of the conditional waiting times up to any desired order n. By differentiating $(n+1)$ times (7)–(9), and evaluating at $s = 0$, we arrive at

$$\begin{aligned}
&(\lambda+i\mu+j\alpha)E\left[W^{n+1}_{i,j}\right] - \lambda E\left[W^{n+1}_{i+1,j}\right] - i\mu E\left[W^{n+1}_{i-1,j}\right] \\
&\quad -(j-1)\alpha E\left[W^{n+1}_{i+1,j-1}\right] = (n+1)E\left[W^n_{i,j}\right], \\
&\quad 0 \leqslant i \leqslant N-G-1,\ 1 \leqslant j \leqslant K
\end{aligned} \tag{10}$$

$$\begin{aligned}
&(\lambda_{\mathrm{H}}+(1-\delta_{j,K})\theta\lambda_{\mathrm{F}}+i\mu+j(1-\theta)\alpha)E\left[W^{n+1}_{i,j}\right] \\
&\quad -\lambda_{\mathrm{H}}E\left[W^{n+1}_{i+1,j}\right] - (1-\delta_{j,K})\theta\lambda_{\mathrm{F}}E\left[W^{n+1}_{i,j+1}\right] \\
&\quad -i\mu E\left[W^{n+1}_{i-1,j}\right] - (j-1)(1-\theta)\alpha E\left[W^{n+1}_{i,j-1}\right] \\
&\quad = (n+1)E\left[W^n_{i,j}\right], \quad N-G \leqslant i \leqslant N-1,\ 1 \leqslant j \leqslant K
\end{aligned} \tag{11}$$

$$\begin{aligned}
&((1-\delta_{j,K})\theta\lambda_{\mathrm{F}}+N\mu+j(1-\theta)\alpha)E\left[W^{n+1}_{N,j}\right] \\
&\quad -(1-\delta_{j,K})\theta\lambda_{\mathrm{F}}E\left[W^{n+1}_{N,j+1}\right] - N\mu E\left[W^{n+1}_{N-1,j}\right]
\end{aligned}$$

$$-(j-1)(1-\theta)\alpha E\left[W_{N,j-1}^{n+1}\right] = (n+1)E\left[W_{N,j}^{n}\right], \quad 1 \leqslant j \leqslant K \tag{12}$$

The expectations $E[W_{i,j}^{n}]$ for $n = 0$ are trivially equal to 1. Hence, from them we obtain the expectations for $n = 1$ by solving the system (10)–(12), then we proceed for $n = 2$, and so on. In general, each iteration allows us to compute the $(N+1)K$ unknowns $E[W_{i,j}^{n+1}]$ in terms of the moments of one order less. Note also that the coefficient matrix of the unknowns is identical to the matrix of the system (7)–(9) for $s = 0$. Therefore, we can exploit the special structure as we have described above.

We now concentrate on the study of the unconditional waiting time of a customer. Denote by W a random variable representing the waiting time of a tagged fresh call and by $W^*(s) = E[e^{-sW}]$ the corresponding Laplace–Stieltjes transform. Since the fresh calls arrive according to a Poisson arrival process, the probability that a fresh call finds the system at state (i, j) coincides with the stationary probability $p_{i,j}$ that the system is in state (i, j) at an arbitrary epoch (PASTA property). A fresh call that finds the system at a state (i, j) with $N - G \leqslant i \leqslant N$ and $0 \leqslant j \leqslant K - 1$ will join the retrial orbit with probability θ and then its waiting time is distributed as the random variable $W_{i,j+1}$; otherwise, the waiting time of the call is 0, so we have that $\Pr[W = 0] = 1 - \theta \sum_{i=N-G}^{N} \sum_{j=0}^{K-1} p_{i,j}$. Therefore, we obtain

$$W^*(s) = 1 - \theta \sum_{i=N-G}^{N} \sum_{j=0}^{K-1} p_{i,j} + \theta \sum_{i=N-G}^{N} \sum_{j=0}^{K-1} p_{i,j} W_{i,j+1}^{*}(s) \tag{13}$$

Thus, once the system (7)–(9) has been solved, we can compute $W^*(s)$. Moreover, for the corresponding moments we have

$$E[W^n] = \delta_{n,0} + (1-\delta_{n,0})\theta \sum_{i=N-G}^{N} \sum_{j=0}^{K-1} p_{i,j} E[W_{i,j+1}^{n}] \tag{14}$$

that is, the nth moment of the unconditional waiting time is expressed in terms of conditional moments of the same order. However, we can derive an alternative expression for $W^*(s)$ which is advantageous for the computation of the unconditional moments. To this end, we multiply Equations (7)–(8) and (9) by $jp_{i,j}$ and $jp_{N,j}$, respectively, and we replace $(\lambda + i\mu + j\alpha)p_{i,j}$, $(\lambda_{\mathrm{H}} + (1-\delta_{j,K})\theta\lambda_{\mathrm{F}} + (N-G)\mu + j(1-\theta)\alpha)p_{N-G,j}$, $(\lambda_{\mathrm{H}} + (1-\delta_{j,K})\theta\lambda_{\mathrm{F}} + i\mu + j(1-\theta)\alpha)p_{i,j}$ and $((1-\delta_{j,K})\theta\lambda_{\mathrm{F}} + N\mu + j(1-\theta)\alpha)p_{N,j}$ by the right sides of Equations (1)–(4). Summing for all i, j the resulted equations and cancelling equal terms, after some algebra, yields

$$s \sum_{i=0}^{N} \sum_{j=1}^{K} jp_{i,j} W_{i,j}^{*}(s) + \theta\lambda_{\mathrm{F}} \sum_{i=N-G}^{N} \sum_{j=0}^{K-1} p_{i,j} W_{i,j+1}^{*}(s) = \alpha \sum_{i=0}^{N-G-1} \sum_{j=1}^{K} jp_{i,j} + (1-\theta)\alpha \sum_{i=N-G}^{N} \sum_{j=1}^{K} jp_{i,j} \tag{15}$$

But now we can use the cross orbit Equation (5) and obtain

$$s \sum_{i=0}^{N} \sum_{j=1}^{K} jp_{i,j} W_{i,j}^{*}(s) + \theta\lambda_{\mathrm{F}} \sum_{i=N-G}^{N} \sum_{j=0}^{K-1} p_{i,j} W_{i,j+1}^{*}(s) = \theta\lambda_{\mathrm{F}} \sum_{i=N-G}^{N} \sum_{j=0}^{K-1} p_{i,j} \tag{16}$$

Using Equations (16) and (13), we get

$$W^*(s) = 1 - \frac{s}{\lambda_{\mathrm{F}}} \sum_{i=0}^{N} \sum_{j=1}^{K} jp_{i,j} W_{i,j}^{*}(s) \tag{17}$$

Differentiating Equation (17) n times and evaluating at $s = 0$, gives

$$E[W^n] = \delta_{n,0} + \frac{n}{\lambda_{\mathrm{F}}} \sum_{i=0}^{N} \sum_{j=1}^{K} jp_{i,j} E\left[W_{i,j}^{n-1}\right] \tag{18}$$

For computational purposes, Equation (18) is more convenient than (14) since the nth moment of the unconditional waiting time is expressed in terms of conditional moments of smaller orders. In particular, for $n = 1$ we obtain $E[W] = \frac{1}{\lambda_{\mathrm{F}}} \sum_{i=0}^{N} \sum_{j=1}^{K} jp_{i,j}$, which can be independently derived by applying Little's formula for the number of calls in orbit. For $n = 2$, we obtain the formula $E[W^2] = \frac{2}{\lambda_{\mathrm{F}}} \sum_{i=0}^{N} \sum_{j=1}^{K} jp_{i,j} E[W_{i,j}]$ which expresses the second moment of the unconditional waiting time in terms of the mean values of the conditional waiting times.

4. TIME TO GUARD CHANNEL ACTIVATION AND TIME TO NEXT LOST CALL

The same methodology that we presented in the previous section may be used for obtaining other performance measures of interest that are related to first passage times of the underlying Markov chain $\{(C(t), N(t))\}$. We consider two such measures: the time to gth guard channel activation and the time to next lost call. Suppose that at an arbitrary time t we find the system at state (i, j). Let $X_{i,j}$ be a generic random variable representing the remaining time till the next activation of the gth guard channel, given that the current state of the system is (i, j) and denote by $X^*_{i,j}(s)$ the corresponding Laplace–Stieltjes transform. To study $X^*_{i,j}(s)$, we consider the Markov chain that results from $\{(C(t), N(t))\}$ by converting the states $\{(i, j) : N-G+g \leqslant i \leqslant N,\ 0 \leqslant j \leqslant K\}$ to absorbing. Indeed, the gth guard channel is activated the first time that the original process hits this set. Using first-step analysis for this modified chain (for details see Appendix), we obtain that $\{X^*_{i,j}(s)\}$ is the smallest non-negative solution of the system

$$\begin{aligned}&(s+\lambda+i\mu+j\alpha)X^*_{i,j}(s)-\lambda X^*_{i+1,j}(s)-i\mu X^*_{i-1,j}(s)\\&\quad -j\alpha X^*_{i+1,j-1}(s)=0, \quad 0\leqslant i\leqslant N-G-1, 0\leqslant j\leqslant K\end{aligned} \tag{19}$$

$$\begin{aligned}&(s+\lambda_{\mathrm{H}}+(1-\delta_{j,K})\theta\lambda_{\mathrm{F}}+i\mu+j(1-\theta)\alpha)X^*_{i,j}(s)\\&\quad-\lambda_{\mathrm{H}}X^*_{i+1,j}(s)-(1-\delta_{j,K})\theta\lambda_{\mathrm{F}}X^*_{i,j+1}(s)\\&\quad-i\mu X^*_{i-1,j}(s)-j(1-\theta)\alpha X^*_{i,j-1}(s)=0,\\&\quad N-G\leqslant i\leqslant N-G+g-1,\ 0\leqslant j\leqslant K\end{aligned} \tag{20}$$

$$X^*_{i,j}(s)=1, \quad N-G+g\leqslant i\leqslant N,\ 0\leqslant j\leqslant K \tag{21}$$

This system is also block tridiagonal and can be solved efficiently at a low computational cost, as in the case of the conditional waiting time Laplace–Stieltjes transforms. Regarding the moments $E[X^n_{i,j}]$, we develop a recursive algorithm for computing them, following the same procedure that yields Equations (10)–(12) from Equations (7)–(9). More concretely, by differentiating $(n+1)$ times Equations (19)–(21) and evaluating at $s=0$, we obtain a linear system for $E[X^{n+1}_{i,j}]$ with identical coefficient matrix as Equations (19)–(21) for $s=0$. However, the right sides are now $(n+1)E[X^n_{i,j}]$ and 0, for Equations (19)–(20) and (21), respectively.

We now let $X_{g\text{-idle}}$ be a generic random variable representing the distribution of the idle time of the gth guard channel; that is the time between a de-activation of the gth guard channel and its next re-activation. The stationary probability that the system is at a state (i, j) just after a de-activation of the gth guard channel is 0 for states with $i \neq N-G+g-1$. For states $(N-G+g-1, j)$, this probability is easily seen to be $p_{N-G+g,j}/\sum_{j=0}^{K} p_{N-G+g,j}$. Hence, the corresponding Laplace–Stieltjes transform $X^*_{g\text{-idle}}(s)$ is expressed as

$$X^*_{g\text{-idle}}(s)=\frac{1}{\sum_{j=0}^{K}p_{N-G+g,j}}\sum_{j=0}^{K}p_{N-G+g,j}X^*_{N-G+g-1,j}(s) \tag{22}$$

An alternative expression for $X^*_{g\text{-idle}}(s)$ can be obtained by multiplying Equations (19)–(20) by $p_{i,j}$ and replacing $(\lambda+i\mu+j\alpha)p_{i,j}$ and $(\lambda_{\mathrm{H}}+(1-\delta_{j,K})\theta\lambda_{\mathrm{F}}+i\mu+j(1-\theta)\alpha)p_{i,j}$ by the corresponding right sides of Equations (1)–(3). Summing for all $0\leqslant i\leqslant N-G+g-1$, $0\leqslant j\leqslant K$ and cancelling equal terms, yields

$$\begin{aligned}&s\sum_{i=0}^{N-G+g-1}\sum_{j=0}^{K}p_{i,j}X^*_{i,j}(s)+\mu(N-G+g)\\&\quad\sum_{j=0}^{K}p_{N-G+g,j}X^*_{N-G+g-1,j}(s)=\lambda_{\mathrm{H}}\sum_{j=0}^{K}p_{N-G+g-1,j}\end{aligned} \tag{23}$$

Equating the rates into and out of the set of states $\{(N-G+g, j), 0\leqslant j\leqslant K\}$, we obtain

$$\lambda_{\mathrm{H}}\sum_{j=0}^{K}p_{N-G+g-1,j}=\mu(N-G+g)\sum_{j=0}^{K}p_{N-G+g,j} \tag{24}$$

By substituting Equation (24) into Equation (23) and using Equation (22), we obtain that

$$X^*_{g\text{-idle}}(s)=1-\frac{s}{(N-G+g)\mu\sum_{j=0}^{K}p_{N-G+g,j}}\sum_{i=0}^{N-G+g-1}\sum_{j=0}^{K}p_{i,j}X^*_{i,j}(s) \tag{25}$$

Finally, differentiating Equation (25) n times and setting $s = 0$, yields

$$E[X^n_{g\text{-idle}}] = \delta_{n,0} + \frac{n}{(N-G+g)\mu \sum_{j=0}^{K} p_{N-G+g,j}} \sum_{i=0}^{N-G+g-1} \sum_{j=0}^{K} p_{i,j} E\left[X^{n-1}_{i,j}\right] \tag{26}$$

which expresses the nth order moment for the idle time of the guard channel in terms of conditional moments of $(n-1)$th order. This is clearly more advantageous than differentiating Equation (22) n times, because there the nth order moment for the idle time requires the computation of the nth order conditional moments.

Another related measure of interest is the distribution of the stationary time till the next gth guard channel activation when we observe the system at an arbitrary time. Denoting by X a generic random variable representing this time, we have that the Laplace–Stieltjes transform is given by

$$X^*(s) = \sum_{i=0}^{N} \sum_{j=0}^{K} p_{i,j} X^*_{i,j}(s) \tag{27}$$

Moments of the time to the gth guard channel activation can be determined routinely from Equation (27) *via* differentiation.

Now consider the system at an arbitrary time t and suppose that it is at state (i, j). Let $Y_{i,j}$ be a generic random variable representing the remaining time till the next lost call given that the current state is (i, j). These lost calls are due to either impatience or non-availability of channels and orbit space. We introduce an appropriate auxiliary Markov chain, with an absorbing state b representing the loss of a call. Thus, the auxiliary Markov chain has the state space $S^b = S \cup \{b\}$ and its transition rates $q^b_{(i,j),(i',j')}$ are identical to the transitions rates $q_{(i,j),(i',j')}$ of the original chain with the exception of the rates

$$\begin{aligned} q^b_{(i,j),(i,j-1)} &= 0, \quad N-G \leqslant i \leqslant N,\ 0 \leqslant j \leqslant K \\ q^b_{(i,j),b} &= \delta_{i,N}\lambda_{\mathrm{H}} + \delta_{j,K}\theta\lambda_{\mathrm{F}} + (1-\theta)\lambda_{\mathrm{F}} + j(1-\theta)\alpha, \\ &\quad N-G \leqslant i \leqslant N,\ 0 \leqslant j \leqslant K \\ q^b_{(i,j),(i,j)} &= -(\lambda + i\mu + j(1-\theta)\alpha), \\ &\quad N-G \leqslant i \leqslant N,\ 0 \leqslant j \leqslant K \end{aligned} \tag{28}$$

Using first-step analysis (for details see Appendix), we obtain the Laplace–Stieltjes transforms $\{Y^*_{i,j}(s)\}$ as the smallest non-negative solution of the system

$$\begin{aligned} &(s + \lambda + i\mu + j\alpha)Y^*_{i,j}(s) - \lambda Y^*_{i+1,j}(s) - i\mu Y^*_{i-1,j}(s) \\ &\quad - j\alpha Y^*_{i+1,j-1}(s) = 0, \quad 0 \leqslant i \leqslant N-G-1,\ 0 \leqslant j \leqslant K \end{aligned} \tag{29}$$

$$\begin{aligned} &(s + \lambda + i\mu + j(1-\theta)\alpha)Y^*_{i,j}(s) - \lambda_{\mathrm{H}} Y^*_{i+1,j}(s) \\ &\quad - (1-\delta_{j,K})\theta\lambda_{\mathrm{F}} Y^*_{i,j+1}(s) - i\mu Y^*_{i-1,j}(s) \\ &\quad = \delta_{j,K}\theta\lambda_{\mathrm{F}} + (1-\theta)\lambda_{\mathrm{F}} + j(1-\theta)\alpha, \\ &\quad N-G \leqslant i \leqslant N-1,\ 0 \leqslant j \leqslant K \end{aligned} \tag{30}$$

$$\begin{aligned} &(s + \lambda + N\mu + j(1-\theta)\alpha)Y^*_{N,j}(s) \\ &\quad - (1-\delta_{j,K})\theta\lambda_{\mathrm{F}} Y^*_{N,j+1}(s) - N\mu Y^*_{N-1,j}(s) \\ &\quad = \lambda_{\mathrm{H}} + \delta_{j,K}\theta\lambda_{\mathrm{F}} + (1-\theta)\lambda_{\mathrm{F}} + j(1-\theta)\alpha, \quad 0 \leqslant j \leqslant K \end{aligned} \tag{31}$$

Using the standard procedure that we have described for the derivation of Equations (10)–(12) from Equations (7)–(9), we obtain a linear system for $E[Y^{n+1}_{i,j}]$ having identical coefficient matrix with Equations (29)–(31) for $s = 0$. However, the right sides are $(n+1)E[Y^n_{i,j}]$ and $(n+1)E[Y^n_{N,j}]$ for Equations (29)–(30) and (31), respectively.

We now define a generic variable $Y_{\text{interlost}}$ that represents the time between two consecutive lost calls (interlost call time). The stationary probability that the system is at a state (i, j) just after the loss of a call, $p^{\text{lost}+}_{i,j}$, is given as the fraction of the rate towards the state (i, j) due to lost calls over the total rate of transitions due to lost calls (see, e.g. Chao *et al.* [22], Subsection 2.2) Hence, it is given by

$$p^{\text{lost}+}_{i,j} = 0, \quad 0 \leqslant i \leqslant N-G-1,\ 0 \leqslant j \leqslant K \tag{32}$$

$$\begin{aligned} p^{\text{lost}+}_{i,j} &= \frac{1}{C}\big((\delta_{j,K}\theta\lambda_{\mathrm{F}} + (1-\theta)\lambda_{\mathrm{F}})p_{i,j} \\ &\quad + (1-\delta_{j,K})(j+1)(1-\theta)\alpha p_{i,j+1}\big), \\ &\quad N-G \leqslant i \leqslant N-1,\ 0 \leqslant j \leqslant K \end{aligned} \tag{33}$$

$$\begin{aligned} p^{\text{lost}+}_{N,j} &= \frac{1}{C}\big((\lambda_{\mathrm{H}} + \delta_{j,K}\theta\lambda_{\mathrm{F}} + (1-\theta)\lambda_{\mathrm{F}})p_{N,j} \\ &\quad + (1-\delta_{j,K})(j+1)(1-\theta)\alpha p_{N,j+1}\big), \quad 0 \leqslant j \leqslant K \end{aligned} \tag{34}$$

where C is the stationary mean rate of lost calls per unit time, which is expressed as

$$C = \sum_{i=N-G}^{N-1} \sum_{j=0}^{K} (\delta_{j,K}\theta\lambda_{\mathrm{F}} + (1-\theta)\lambda_{\mathrm{F}}) p_{i,j} + (1-\theta)\alpha \sum_{i=N-G}^{N} \sum_{j=0}^{K-1} (j+1) p_{i,j+1} + \sum_{j=0}^{K} (\lambda_{\mathrm{H}} + \delta_{j,K}\theta\lambda_{\mathrm{F}} + (1-\theta)\lambda_{\mathrm{F}}) p_{N,j} \quad (35)$$

Then, the Laplace–Stieltjes transform $Y^*_{\text{interlost}}(s)$ is computed in terms of the conditional transforms as

$$Y^*_{\text{interlost}}(s) = \sum_{i=0}^{N} \sum_{j=0}^{K} p^{\text{lost}+}_{i,j} Y^*_{i,j}(s) \quad (36)$$

By multiplying Equations (29)–(30) and (31) by $p_{i,j}$ and $p_{N,j}$, respectively, using the balance equations and summing over all i, j, we obtain an alternative expression for $Y^*_{\text{interlost}}(s)$. We omit the details as they are very similar to the derivation of Equation (25). Finally, we obtain that

$$Y^*_{\text{interlost}}(s) = 1 - \frac{s}{C} \sum_{i=0}^{N} \sum_{j=0}^{K} p_{i,j} Y^*_{i,j}(s) \quad (37)$$

By differentiating Equation (37) n times and setting $s = 0$, we can compute the nth moment of $Y_{\text{interlost}}$ in terms of the $(n-1)$th moments of the conditional times to the next lost call, as

$$E[Y^n_{\text{interlost}}] = \delta_{n,0} + \frac{n}{C} \sum_{i=0}^{N} \sum_{j=0}^{K} p_{i,j} E\left[Y^{n-1}_{i,j}\right] \quad (38)$$

In particular, for $n = 1$ and $n = 2$, we obtain $E[Y_{\text{interlost}}] = \frac{1}{C}$ and $E[Y^2_{\text{interlost}}] = \frac{2}{C} \sum_{i=0}^{N} \sum_{j=0}^{K} p_{i,j} E[Y_{i,j}]$, respectively.

If we are interested in Y, the stationary time till the next lost call, we have that its Laplace–Stieltjes transform is given by

$$Y^*(s) = \sum_{i=0}^{N} \sum_{j=0}^{K} p_{i,j} Y^*_{i,j}(s) \quad (39)$$

Its corresponding moments are derived in the usual way (as the derivations of Equations (10)–(12) and (14) from Equations (7)–(9) and (13), respectively).

5. NUMERICAL RESULTS

In this section, we present a numerical study that illustrates the behaviour of the performance measures that we considered in the previous sections. In what follows, we deal with models having a fixed unitary completion rate for each call in the cell, that is $\mu = 1.0$. In order to fix the truncation, we increase successively the orbit capacity K until we get that two successive values of the mean (for the measure under evaluation) differ in less than 10^{-6}. Moreover, numerical results show that this criterion provides truncation levels such that the probability of a blocked fresh call going to orbit with no orbit space to be smaller than 10^{-7}.

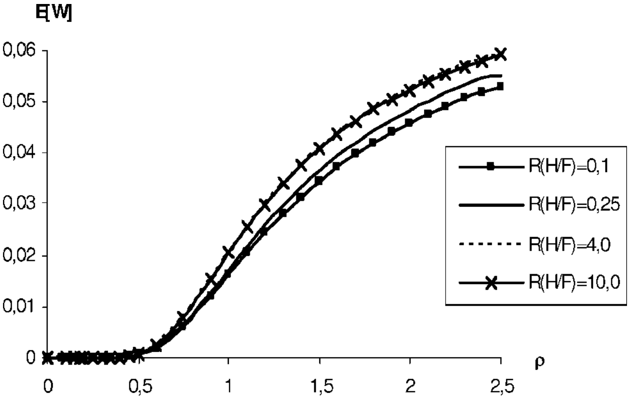

Figure 3. $E[W]$ *versus* ρ.

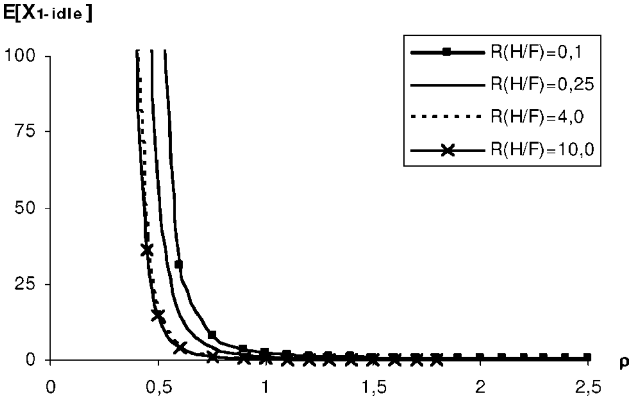

Figure 4. $E[X_{1\text{-idle}}]$ *versus* ρ.

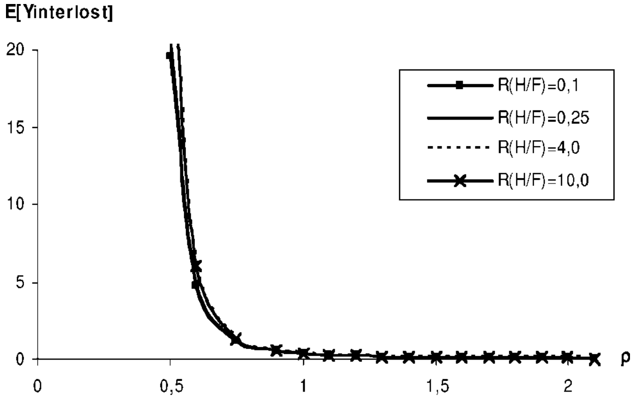

Figure 5. $E[Y_{\text{interlost}}]$ *versus* ρ.

First, we present the expected waiting, idle and interlost times in terms of the normalised traffic intensity $\rho = \lambda / N\mu$, for a cell having $N = 16$ channels, one guard channel, with a persistence parameter $\theta = 0.75$ for those blocked fresh calls. We consider also four scenarios by varying the rate of handover over fresh calls; more precisely $R(\text{H/F}) \equiv \lambda_{\text{H}}/\lambda_{\text{F}} \in \{0.1, 0.25, 4.0, 10.0\}$. The retrial rate was fixed as $\alpha = 38.0$.

Table 1. Performance measures *versus* N and ρ.

		$N = 8$, $G = 2$	$N = 16$, $G = 4$	$N = 32$, $G = 8$
$P\{Block\|H\}$	$\rho = 0.5$	0.0002087	0.0000024	2.2×10^{-10}
	$\rho = 1.0$	0.0048281	0.0004165	0.0000017
	$\rho = 2.0$	0.0427692	0.0128011	0.0007034
$P\{W = 0\}$	$\rho = 0.5$	0.8929466	0.9490692	0.9842394
	$\rho = 1.0$	0.6235145	0.6331760	0.6317818
	$\rho = 2.0$	0.3828517	0.3556379	0.3335281
$E[W]$	$\rho = 0.5$	0.0058826	0.0024729	0.0006686
	$\rho = 1.0$	0.0215029	0.0194425	0.0178657
	$\rho = 2.0$	0.0372343	0.0379679	0.0384027
$E[X_{1}\text{-idle}]$	$\rho = 0.5$	9.7010927	10.394344	16.994545
	$\rho = 1.0$	1.4151438	0.7423506	0.3730654
	$\rho = 2.0$	0.4228243	0.2036854	0.0974974
$E[X_{G}\text{-idle}]$	$\rho = 0.5$	86.134561	6985.6254	65726262.7
	$\rho = 1.0$	6.8162541	70.033485	12834.868
	$\rho = 2.0$	1.2374283	3.4447525	40.919164
$E[Y_{\text{interlost}}]$	$\rho = 0.5$	2.6479755	3.1282498	5.8419122
	$\rho = 1.0$	0.5359557	0.2007968	0.1093221
	$\rho = 2.0$	0.1017750	0.0511414	0.0254231
C	$\rho = 0.5$	0.3776467	0.3196675	0.1711768
	$\rho = 1.0$	2.7811935	4.9801570	6.1164952
	$\rho = 2.0$	9.8255908	19.553621	39.334204

Note that mean waiting times increase with traffic intensity while expected idle or interlost call times have a decreasing behaviour, which agrees with the intuitive expectations. In addition, for cells operating under heavy traffic conditions; that is $\rho > 1$, the rate of handover over fresh calls shows little influence over the measures. Hence, in the rest of the numerical results we will consider models having $\lambda_{\text{H}} = 0.25\lambda_{\text{F}}$.

Table 1 shows performance measures of models with persistence probability $\theta = 0.75$ and retrial intensity $\alpha = 60.0$. We vary the normalised traffic intensity ρ and the number of channels, reserving one fourth of them for handover calls.

Table 2. Performance measures *versus* G and α.

		$\alpha = 6.4$	$\alpha = 12.8$	$\alpha = 25.6$
$P\{Block\| H\}$	$G = 1$	0.0634369	0.0599687	0.0559070
	$G = 2$	0.0156616	0.0147524	0.0137327
	$G = 4$	0.0010989	0.0010388	0.0009716
	$G = 8$	0.0000115	0.0000112	0.0000107
	$G = 12$	5.66×10^{-7}	5.62×10^{-7}	5.55×10^{-7}
$P\{W = 0\}$	$G = 1$	0.7145336	0.7301140	0.7484181
	$G = 2$	0.6542202	0.6742930	0.6968072
	$G = 4$	0.5478882	0.5726265	0.6002429
	$G = 8$	0.3795352	0.3987875	0.4236120
	$G = 12$	0.2828968	0.2881660	0.2970587
$E[W]$	$G = 1$	0.0888086	0.0472120	0.0249608
	$G = 2$	0.1148849	0.0602045	0.0314327
	$G = 4$	0.1672994	0.0860590	0.0442581
	$G = 8$	0.2867068	0.1445344	0.0730185
	$G = 12$	0.4064536	0.2035201	0.1020097
$E[X_{1}\text{-idle}]$	$G = 1$	0.9227296	0.9797103	1.0554263
	$G = 2$	0.7713366	0.8238032	0.8909170
	$G = 4$	0.5836342	0.6231486	0.6730372
	$G = 8$	0.4112785	0.4295927	0.4550137
	$G = 12$	0.3444955	0.3498953	0.3591917
$E[X_{G}\text{-idle}]$	$G = 1$	0.9227296	0.9797103	1.0554263
	$G = 2$	3.9281404	4.1740776	4.4886735
	$G = 4$	56.810350	60.102411	64.258781
	$G = 8$	5398.3382	5571.2075	5811.1576
	$G = 12$	110315.65	111132.25	112538.15
$E[Y_{\text{interlost}}]$	$G = 1$	0.3850578	0.3609707	0.3441990
	$G = 2$	0.3137508	0.2998338	0.2875845
	$G = 4$	0.2187271	0.2126168	0.2067277
	$G = 8$	0.1277294	0.1266857	0.1253823
	$G = 12$	0.0900990	0.0899692	0.0897490
C	$G = 1$	2.6280663	2.7703076	2.9052957
	$G = 2$	3.1872427	3.3351799	3.4772378
	$G = 4$	4.5719063	4.7032970	4.8372808
	$G = 8$	7.8290453	7.8935449	7.9756069
	$G = 12$	11.098898	11.114914	11.142184

Table 3. Performance measures *versus* ρ and θ.

	$\theta = 0.01$	$\theta = 0.25$	$\theta = 0.5$	$\theta = 0.75$	$\theta = 0.99$
$P\{Block\|H\}$	3.5×10^{-12}	3.5×10^{-12}	3.6×10^{-12}	3.6×10^{-12}	3.8×10^{-12}
	0.017264	0.017962	0.019230	0.022151	0.042949
	0.112921	0.117467	0.125597	0.143694	0.226279
	0.294914	0.302539	0.315031	0.337623	0.373674
$P\{W = 0\}$	0.999999	0.999999	0.999999	0.999999	0.999999
	0.998748	0.967736	0.931820	0.884549	0.702548
	0.995108	0.873833	0.733473	0.550604	0.359748
	0.992136	0.799228	0.584140	0.333980	0.013292
$E[W]$	3.1×10^{-14}	9.5×10^{-13}	2.5×10^{-12}	5.9×10^{-12}	1.7×10^{-11}
	0.000021	0.000683	0.001988	0.005528	0.068082
	0.000082	0.002698	0.008004	0.023396	0.066437
	0.000132	0.004351	0.012986	0.038558	1.247878
$E[X_{1\text{-idle}}]$	1.75×10^{10}	1.74×10^{10}	1.73×10^{10}	1.70×10^{10}	1.64×10^{10}
	3.557599	3.416940	3.187479	2.758955	1.392696
	0.490983	0.469562	0.435122	0.372451	0.213706
	0.149425	0.144084	0.135892	0.122617	0.104716
$E[Y_{\text{interlost}}]$	4.29×10^{9}	4.62×10^{9}	5.31×10^{9}	7.32×10^{9}	9.08×10^{10}
	0.755665	0.770514	0.798829	0.871436	2.187374
	0.100784	0.101312	0.102259	0.104376	0.113697
	0.030302	0.030339	0.030398	0.030501	0.030647
C	2.3×10^{-10}	2.1×10^{-10}	1.8×10^{-10}	1.3×10^{-10}	1.1×10^{-10}
	1.323336	1.297834	1.251831	1.147531	0.457169
	9.922163	9.870454	9.779041	9.580713	8.795280
	33.000043	32.960456	32.896655	32.785513	32.629879

We can observe that the blocking probability for handovers, $P\{Block|H\}$, increases with the traffic intensity and decreases when increasing the number of channels. The probability $P\{W = 0\}$ of getting a successful connection upon arrival decreases for increasing traffic intensities. For models operating under low traffic conditions (i.e. $\rho = 0.5$), the probability increases when the number of channels increases; according to that, $E[W]$, the expected waiting time for a fresh call to get a free channel, increases for increasing traffic rates and, when varying N shows different behaviours depending on the traffic intensity, which can be explained in terms of the mean occupancy level of the orbit (not reported on the table), that shows large values in models having more traffic congestion or a greater number of channels. When increasing traffic rates, the mean length of the period with unused guard channels, $E[X_{1\text{-idle}}]$, G-idle times and Interlost call expected times show a decreasing behaviour while the mean rate of lost calls per unit time, C (see Equation (35)), increases. When the total number of channels increases, the mean length of the period between total use of guard channels, $E[X_{G\text{-idle}}]$, also increases. For the rest of measures, this behaviour depends on the traffic conditions.

In Table 2, we present results when varying the retrials intensity and the number of reserved channels. We consider scenarios having $N = 16$ channels, persistence probability of $\theta = 0.75$ and the arrival rate of fresh calls is fixed as $\lambda_F = 12.8$ in order to keep $\rho = 1$.

Handovers blocking probability is decreasing both for increasing retrial rates and for increasing number of reserved channels. $P\{W = 0\}$ decreases when we increase the number of reserved channels and increases for increasing retrial rates, note that larger retrial rates also imply more lost calls consequently the system is less congested and it is easier to get an available line. Results for the expected waiting time are intuitively expected, the measure is decreasing with α and increases if we reserved more channels. $E[X_{1\text{-idle}}]$ decreases with increasing number of guard channels while $E[X_{G\text{-idle}}]$ increases when the number of reserved channels increases. The retrial intensity shows light influence on the incremental variation of both measures. $E[Y_{\text{lostcall}}]$ decreases for increasing retrial rates or increasing number of guard channels. Accordingly, C increases for increasing retrial rates and also when increasing the number of guard channels.

Table 4. Performance measures *versus* α and θ.

		$\theta = 0.1$	$\theta = 0.5$	$\theta = 0.99$
$P\{Block \mid H\}$	$\alpha = 12.0$	0.017681	0.020543	0.044408
	$\alpha = 20.0$	0.017599	0.020003	0.044364
	$\alpha = 60.0$	0.017432	0.018808	0.041442
	$\alpha = 120.0$	0.017355	0.018207	0.037957
$E[W]$	$\alpha = 12.0$	0.001138	0.009157	0.144015
	$\alpha = 20.0$	0.000686	0.005671	0.115842
	$\alpha = 60.0$	0.000230	0.001988	0.068082
	$\alpha = 120.0$	0.000115	0.001013	0.045684
$\sigma[W]$	$\alpha = 12.0$	0.014051	0.044709	0.360536
	$\alpha = 20.0$	0.008492	0.028153	0.292809
	$\alpha = 60.0$	0.002870	0.010310	0.177669
	$\alpha = 120.0$	0.001444	0.005391	0.123223
$E[X_1\text{-idle}]$	$\alpha = 12.0$	3.347220	2.979841	1.344890
	$\alpha = 20.0$	3.488675	3.061927	1.346273
	$\alpha = 60.0$	3.522710	3.260514	1.445623
	$\alpha = 120.0$	3.538630	3.370136	1.584081
$\sigma[X_1\text{-idle}]$	$\alpha = 12.0$	4.609996	4.125010	2.459290
	$\alpha = 20.0$	4.625072	4.197352	2.462851
	$\alpha = 60.0$	4.658630	4.386450	2.571516
	$\alpha = 120.0$	4.675143	4.497605	2.716272
$E[Y_{\text{interlost}}]$	$\alpha = 12.0$	0.766754	0.848991	3.419501
	$\alpha = 20.0$	0.763641	0.824575	2.831129
	$\alpha = 60.0$	0.758998	0.787514	1.891724
	$\alpha = 120.0$	0.757321	0.773664	1.505547
$\sigma[Y_{\text{interlost}}]$	$\alpha = 12.0$	1.635068	1.786775	5.722531
	$\alpha = 20.0$	1.630265	1.749563	4.959173
	$\alpha = 60.0$	1.621752	1.683770	3.622459
	$\alpha = 120.0$	1.618204	1.655392	3.011165
C	$\alpha = 12.0$	1.304198	1.177867	0.292440
	$\alpha = 20.0$	1.309515	1.212745	0.353215
	$\alpha = 60.0$	1.317525	1.269817	0.528618
	$\alpha = 120.0$	1.320442	1.292549	0.664210

In Table 3, we display performance measures *versus* persistence parameter, θ, and traffic intensity, ρ, the cell has $N = 16$ channels, one of them reserved for handover calls, and, in accordance to the realistic idea that $\lambda_F < \alpha$, the retrial parameter was chosen as $\alpha = 40.0$. Each cell contains information, from top to bottom, of the corresponding measure for $\rho \in \{0.1, 0.8, 1.5, 3.0\}$.

The blocking probability for handovers exhibits an increasing behaviour in terms of the persistence probability and also increases when the intensity of calls increases. We observe that for increasing persistence parameters the probability of no waiting and the expected idle times are decreasing but, mean waiting times and mean interlost call times are increasing. This can be explain by observing that larger persistence parameters give systems with larger orbits and more patient customers in. When the traffic rate increases, we get increasing waiting times and increasing lost call rates, but we observe that the probability of no waiting, the expected idle time and the expected interlost call time are decreasing. However, the persistence parameter is of little influence on the rate C and on the expected interlost call times, for systems operating under high traffic conditions.

In Table 4, in addition to blocking probability and lost calls rate we present mean and standard deviation for W, $X_{1\text{-idle}}$ and $Y_{\text{interlost}}$ for different values of the persistence parameter and the retrial rate. The traffic intensity is $\rho = 0.8$ and the cell has $N = 16$ channels, one of them reserved for handovers.

Notice that waiting time moments are increasing for increasing persistence probabilities and decrease for increasing retrial rates. Idle time moments decrease with the persistence parameter, and increase slowly in terms of the retrial intensity. Interlost call time moments are larger for larger persistence probabilities and decrease with the retrial rate. The handovers blocking probability increases with the persistence parameter and decreases in terms of α. The rate of lost calls per unit time is decreasing for increasing persistence parameters and is increasing as a function of the retrial rate.

6. CONCLUSION AND EXTENSIONS

In this paper, we have complemented earlier studies of cellular mobile networks with retrials and guard channels by providing an approach for the computation of first-passage time performance measures. This include the waiting time of a call, the time to a guard channel activation and the time till the next lost call. These measures are valuable both from the point of view of the customers and of the administrator of the network. They allow, in combination with the classical performance measures in terms of the stationary state probabilities (e.g. blocking probabilities, mean traffic rates, etc.), a better quantification of the QoS of a cellular mobile network.

The methods presented, show that the proposed performance measures are easily computable. Indeed, the approach is based on the definition of certain auxiliary absorbing Markov chains, where the absorption corresponds to the event under investigation (the termination of a waiting time, a guard channel activation or the loss of a call). Then, using first-step analysis, we obtain block tridiagonal linear systems that can be solved easily with well-known algorithms.

This approach can be also applied in cellular mobile networks with more involved call handling schemes (see, e.g. Basharin and Merkulov [5] and Liu and Fapojuwo [9]). For example, it is possible to study a model that takes into account the portion of the blocked handover calls that retry later for establishing a connection. We should then consider a separate retrial orbit for these handover calls since their behaviour is quite different from the fresh calls. Indeed, it is plausible to assume higher retrial rates and higher abandonment probabilities for the handover calls. For modelling a station in this framework, we should define a continuous time Markov chain $\{(C(t), N(t), M(t))\}$ with state space $\{0, 1, \ldots, N\} \times \{0, 1, \ldots\}^2$, where $C(t)$ is the number of occupied channels and $N(t)$, $M(t)$ are, respectively, the numbers of fresh and handover calls in retrial orbit. The dynamics of the system becomes much more involved as we have a three-dimensional state-description. Nevertheless, the corresponding random variables $W_{i,j,k}$, $X_{i,j,k}$ and $Y_{i,j,k}$ can be studied similarly by defining the associated auxiliary absorbing Markov chains and performing first-step analysis.

APPENDIX : FIRST-STEP ANALYSIS

The technique of first-step analysis that was repeatedly used in the main body of the paper for the derivation of first-passage time Laplace–Stieltjes transforms is based on the following result that can be found in Kulkarni [23], Theorem 6.21, p. 301.

Theorem A. *Let* $\{X(t)\}$ *be a continuous time Markov chain on a countable state space S, parametrised as* $S = \{0, 1, 2, \ldots\}$*, with transition rate matrix* $Q = (q_{i,j})$*. Define* $T = \min\{t \geqslant 0 : X(t) = 0\}$ *to be the first passage time to 0 and* $\phi_i(s) = E[e^{-sT}|X(0) = i]$ *the Laplace–Stieltjes transform of T given that the process is initially at state i. Then* $\{\phi_i(s) : i \geqslant 1\}$ *is the smallest non-negative solution of the system*

$$s\phi_i(s) = \sum_{j=1}^{\infty} q_{i,j}\phi_j(s) + q_{i,0},\ i \geqslant 1 \tag{A1}$$

In the case of the absorbing auxiliary Markov chain where the modified rates are given by Equation (6), we are interested in the first passage time to a. First, we consider a state (i, j) with $0 \leqslant i \leqslant N - G - 1$, $1 \leqslant j \leqslant K$. Then the only non-zero rates out of (i, j) are the rates $q^a_{(i,j),(i+1,j)} = \lambda$, $q^a_{(i,j),(i-1,j)} = i\mu$, $q^a_{(i,j),(i+1,j-1)} = (j-1)\alpha$, $q^a_{(i,j),a} = \alpha$ and $q^a_{(i,j),(i,j)} = -(\lambda + i\mu + j\alpha)$. By applying Equation (A1), we obtain immediately Equation (7). The Equations (8) and (9) are obtained similarly by considering states (i, j) with $N - G \leqslant i \leqslant N - 1$, $1 \leqslant j \leqslant K$ and (N, j), $1 \leqslant j \leqslant K$, respectively.

This theorem is also applied to the auxiliary chains associated with $X^*_{i,j}(s)$ and $Y^*_{i,j}(s)$. It yields the set of Equations (19)–(21) and (29)–(31), respectively.

ACKNOWLEDGEMENTS

The authors thank the referees for their suggestions on the earlier version of this paper. The support of the research project MTM2005-01248 is gratefully acknowledged.

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DOI: 10.1002/ett