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On Neutrosophic Sets and Topology

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Abstract

Recently, F.Smarandache generalized the Atanassov's intuitionistic fuzzy sets and other kinds of sets to neutrosophic sets. Also, this author defined the notion of neutrosophic topology on the non-standard interval. One can expect some relation between the intuitionistic fuzzy topology on an IFS and the neutrosophic topology. We show in this work that this is false.

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1. On neutrosophic Topology

1.1. Introduction.

The neutrosophic logic is a formal frame trying to measure the truth, indeterminacy, and falsehood.

Smarandache²⁷ remarks the differences between neutrosophic logic (NL) and intuitionistic fuzzy logic (IFL) and the corresponding neutrosophic sets and intuitionistic fuzzy sets. The main differences are:

a) Neutrosophic Logic can distinguish between absolute truth and relative truth, because

$NL(\text{absolute truth})=1^+$ while $NL(\text{relative truth})=1$. This has application in philosophy. That is why the unitary standard interval $[0,1]$ used in IFL has been extended to the unitary non-standard interval $]0,1^+[$ in NL.

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Similar distinctions for absolute or relative falsehood, and absolute or relative indeterminacy are allowed in NL.

b) In NL there is no restriction on T, I, F other than they are subsets of $]0, 1[$, thus:

$$-0 \leq \inf T + \inf I + \inf F \leq \sup T + \sup I + \sup F \leq 3^+.$$

This non-restriction allows paraconsistent, dialetheist, and incomplete information to be characterized in NL (i.e. the sum of all three components if they are defined as points, or sum of superior limits of all three components if they are defined as subsets can be >1 , for paraconsistent information coming from different sources, or <1 for incomplete information), while that information cannot be described in IFL because in IFL the components T (truth), I (indeterminacy), F (falsehood) are restricted either to $t+i+f=1$ if T, I, F are all reduced to the points t, i, f respectively, or to $\sup T + \sup I + \sup F = 1$ if T, I, F are subsets of $[0, 1]$.

c) In NL the components T, I, F can also be non-standard subsets included in the unitary non-standard interval $]0, 1[$, not only standard subsets, included in the unitary standard interval $[0, 1]$ as in IFL.

In various recent papers^{26,28,29,30}, F. Smarandache generalizes intuitionistic fuzzy sets (IFSs) and other kinds of sets to neutrosophic sets (NSs). In²⁹ some distinctions between NSs and IFSs are underlined.

The notion of intuitionistic fuzzy set defined by K.T. Atanassov¹ has been applied by Çoker⁸ for study intuitionistic fuzzy topological spaces. This concept has been developed by many authors (Bayhan and Çoker⁴, Çoker^{5,6}, Çoker and Eş⁷, Eş and Çoker¹⁰, Gürçay, Çoker and Eş¹¹, Hanafy¹², Hur, Kim and Ryou¹³, Lee and Lee¹⁴, Lupiáñez^{15,16,17,18,19}, Turan and Çoker³¹).

A few years ago raised some controversy over whether the term "intuitionistic fuzzy set" was appropriate or not⁹. At present, it is customary to speak of "Atanassov's intuitionistic fuzzy set".

F. Smarandache also defined the notion of neutrosophic topology on the non-standard interval²⁶.

One can expect some relation between the intuitionistic fuzzy topology on an IFS and the neutrosophic topology. We show in this Section that this is false. Indeed, the complement of an IFS A is not the complement of A in the neutrosophic operation, the union and the intersection of IFSs do not coincide with the corresponding operations for NSs, and finally an intuitionistic fuzzy topology is not necessarily a neutrosophic topology.

1.2. Basic definitions.

First, we present some basic definitions:

Definition 1¹: Let X be a non-empty set. An intuitionistic fuzzy set (IFS for short) A , is an object having the form $A = \{ \langle x, \mu_A, \gamma_A \rangle / x \in X \}$ where the functions $\mu_A: X \rightarrow I$ and $\gamma_A: X \rightarrow I$ denote the degree of membership (namely $\mu_A(x)$) and the degree of nonmembership (namely $\gamma_A(x)$) of each element $x \in X$ to the set A , respectively, and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for each $x \in X$.

Definition 2²: Let X be a non-empty set, and the IFSs $A = \{ \langle x, \mu_A, \gamma_A \rangle / x \in X \}$, $B = \{ \langle x, \mu_B, \gamma_B \rangle / x \in X \}$.

Let

$$A^c = \{ \langle x, \gamma_A, \mu_A \rangle / x \in X \}$$

$$A \cap B = \{ \langle x, \mu_A \wedge \mu_B, \gamma_A \vee \gamma_B \rangle / x \in X \}$$

$$A \cup B = \{ \langle x, \mu_A \vee \mu_B, \gamma_A \wedge \gamma_B \rangle / x \in X \}.$$

Definition 3⁶: Let X be a non-empty set. Let $0_\square = \{ \langle x, 0, 1 \rangle / x \in X \}$ and $1_\square = \{ \langle x, 1, 0 \rangle / x \in X \}$.

Definition 4⁶: An intuitionistic fuzzy topology (IFT for short) on a non-empty set X is a family τ of IFSs in X satisfying:

(a) $0_\square, 1_\square \in \tau$,

(b) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,

(c) $\bigcup G_j \in \tau$ for any family $\{G_j\}_{j \in J} \subset \tau$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS for short) and any IFS in τ is called an intuitionistic fuzzy open set (IFOS for short) in X .

Definition 5³⁰: Let T, I, F be real standard or non-standard subsets of the non-standard unit interval $]0, 1[$, with $\sup T = t_{\sup}$, $\inf T = t_{\inf}$

$$\sup I = i_{\sup}, \inf I = i_{\inf}$$

$$\sup F = f_{\sup}, \inf F = f_{\inf} \text{ and } n_{\sup} = t_{\sup} + i_{\sup} + f_{\sup}, n_{\inf} = t_{\inf} + i_{\inf} + f_{\inf},$$

T, I, F are called neutrosophic components. Let U be an universe of discourse, and M a set included in U . An element x from U is noted with respect to the set M as $x(T, I, F)$ and belongs to M in the following way: it is t % true in the set, i % indeterminate (unknown if it is) in the set, and f % false, where t varies in T , i varies in I , f varies in F . The set M is called a neutrosophic set (NS).

Remark. All IFS is a NS.

Definition 6²⁷: Let S_1 and S_2 be two (unidimensional) real standard or non-standard subsets, then we defines:

$$S_1 \oplus S_2 = \{x \mid x = s_1 + s_2, \text{ where } s_1 \in S_1 \text{ and } s_2 \in S_2\},$$

$$S_1 \ominus S_2 = \{x \mid x = s_1 - s_2, \text{ where } s_1 \in S_1 \text{ and } s_2 \in S_2\},$$

$$S_1 \odot S_2 = \{x \mid x = s_1 \cdot s_2, \text{ where } s_1 \in S_1 \text{ and } s_2 \in S_2\}.$$

Definition 7³⁰. One defines, with respect to the sets A and B over the universe U :

1. Complement: if $x(T_1, I_1, F_1) \in A$, then

$$x(\{1^+\} \ominus T_1, \{1^+\} \ominus I_1, \{1^+\} \ominus F_1) \in C(A).$$

2. Intersection: if $x(T_1, I_1, F_1) \in A$, $x(T_2, I_2, F_2) \in B$, then

$$x(T_1 \odot T_2, I_1 \odot I_2, F_1 \odot F_2) \in A \cap B.$$

3. Union: if $x(T_1, I_1, F_1) \in A$, $x(T_2, I_2, F_2) \in B$, then

$$x(T_1 \oplus T_2 \ominus T_1 \odot T_2, I_1 \oplus I_2 \ominus I_1 \odot I_2, F_1 \oplus F_2 \ominus F_1 \odot F_2) \in A \cup B.$$

1.3. Results.

Proposition 1. Let A be an IFS in X , and $j(A)$ be the corresponding NS. We have that the complement of $j(A)$ is not necessarily $j(A^c)$.

Proof. If $A = \langle x, \mu_A, \gamma_A \rangle$ is $x(\mu_A(x), 1 - \mu_A(x) - \nu_A(x), \nu_A(x)) \in j(A)$.

Then ,

$$\text{for } 0 = \langle x, 0, 1 \rangle \text{ is } x(0, 0, 1) \in j(0_-)$$

$$\text{for } 1 = \langle x, 1, 0 \rangle \text{ is } x(1, 0, 0) \in j(1_-)$$

$$\text{and for } A^c = \langle x, \gamma_A, \mu_A \rangle \text{ is } x(\gamma_A(x), 1 - \mu_A(x) - \nu_A(x), \mu_A(x)) \in j(A^c).$$

Thus, $1 = 0_-$ and $j(1_-) \neq C(j(0_-))$ because $x(1, 0, 0) \in j(1_-)$ but $x(\{1^+\}, \{1^+\}, \{0^+\}) \in C(j(0_-))$.

Proposition 2. Let A and B be two IFSs in X , and $j(A)$ and $j(B)$ be the corresponding NSs. We have that $j(A) \cup j(B)$ is not necessarily $j(A \cup B)$, and $j(A) \cap j(B)$ is not necessarily $j(A \cap B)$.

Proof. Let $A = \langle x, 1/2, 1/3 \rangle$ and $B = \langle x, 1/2, 1/2 \rangle$ (i.e. $\mu_A, \nu_A, \mu_B, \nu_B$ are constant maps).

Then, $A \cup B = \langle x, \mu_A \vee \mu_B, \gamma_A \wedge \gamma_B \rangle = \langle x, 1/2, 1/3 \rangle$ and $x(1/2, 1/6, 1/3) \in j(A \cup B)$. On the other hand, $x(1/2, 1/6, 1/3) \in j(A), x(1/2, 0, 1/2) \in j(B)$,

$$x(1, 1/6, 5/6) \in j(A) \oplus j(B), x(1/4, 0, 1/6) \in j(A) \odot j(B) \text{ and } x(3/4, 1/6, 2/3) \in j(A) \cup j(B). \text{ Thus } j(A \cup B) \neq j(A) \cup j(B).$$

Analogously, $A \cap B = \langle x, \mu_A \wedge \mu_B, \gamma_A \vee \gamma_B \rangle = \langle x, 1/2, 1/2 \rangle$ and $x(1/2, 0, 1/2) \in j(A \cap B)$, but $x(1/4, 0, 1/6) \in j(A) \cap j(B)$. Thus, $j(A \cap B) \neq j(A) \cap j(B)$.

Definition 8²⁶: Let's construct a neutrosophic topology on $NT =]0, 1^+[$, considering the associated family of standard or non-standard subsets included in NT , and the empty set which is closed under set union and finite intersection neutrosophic. The interval NT endowed with this topology forms a neutrosophic topological space.

Proposition 3. Let (X, τ) be an intuitionistic fuzzy topological space. Then, the family $\{j(U) \mid U \in \tau\}$ is not necessarily a neutrosophic topology.

Proof. Let $\tau = \{1_-, 0_-, A\}$ where $A = \langle x, 1/2, 1/2 \rangle$ then $x(1, 0, 0) \in j(1_-)$, $x(0, 0, 1) \in j(0_-)$ and $x(1/2, 0, 1/2) \in j(A)$. Thus $\{j(1_-), j(0_-), j(A)\}$ is not a neutrosophic topology, because this family is not closed by finite intersections, indeed, $x(1/2, 0, 0) \in j(1_-) \cap j(A)$, and this neutrosophic set is not in the family.

2. Other neutrophic topologies

2.1. Introduction.

F. Smarandache^{26,30} also defined various notions of neutrosophic topologies on the non-standard interval. One can expect some relation between the intuitionistic fuzzy topology on an IFS and the neutrosophic topology. We show in

this chapter that this is false. Indeed, the union and the intersection of IFSSs do not coincide with the corresponding operations for NSs, and an intuitionistic fuzzy topology is not necessarily a neutrosophic topology on the non-standard interval, in the various senses defined by Smarandache.

2.2. Basic definitions. First, we present some basic definitions:

Definition 9³⁰. Let $\mathbf{J} \in \{T, I, F\}$ be a component. Most known N-norms are:

The algebraic product N-norm: $N_{n-\text{algebraic}} \mathbf{J}(\mathbf{x}, \mathbf{y}) = \mathbf{x} \cdot \mathbf{y}$

The bounded N-norm: $N_{n-\text{bounded}} \mathbf{J}(\mathbf{x}, \mathbf{y}) = \max\{0, \mathbf{x} + \mathbf{y} - 1\}$

The default (min) N-norm: $N_{n-\text{min}} \mathbf{J}(\mathbf{x}, \mathbf{y}) = \min\{\mathbf{x}, \mathbf{y}\}$

N_n represents the intersection operator in neutrosophic set theory. Indeed $\mathbf{x} \wedge \mathbf{y} = (T_n, I_n, F_n)$.

Definition 10³⁰. Let $\mathbf{J} \in \{T, I, F\}$ be a component. Most known N-conorms are:

The algebraic product N-conorm: $N_{c-\text{algebraic}} \mathbf{J}(\mathbf{x}, \mathbf{y}) = \mathbf{x} + \mathbf{y} - \mathbf{x} \cdot \mathbf{y}$

The bounded N-conorm: $N_{c-\text{bounded}} \mathbf{J}(\mathbf{x}, \mathbf{y}) = \min\{1, \mathbf{x} + \mathbf{y}\}$

The default (max) N-conorm: $N_{c-\text{max}} \mathbf{J}(\mathbf{x}, \mathbf{y}) = \max\{\mathbf{x}, \mathbf{y}\}$

N_c represents the union operator in neutrosophic set theory. Indeed $\mathbf{x} \vee \mathbf{y} = (T_v, I_v, F_v)$

2.3 Results.

Proposition 1. Let \mathbf{A} and \mathbf{B} be two IFSSs in \mathbf{X} , and $\mathbf{j}(\mathbf{A})$ and $\mathbf{j}(\mathbf{B})$ be the corresponding NSs. We have that $\mathbf{j}(\mathbf{A}) \cup \mathbf{j}(\mathbf{B})$ is not necessarily $\mathbf{j}(\mathbf{A} \cup \mathbf{B})$, and $\mathbf{j}(\mathbf{A}) \cap \mathbf{j}(\mathbf{B})$ is not necessarily $\mathbf{j}(\mathbf{A} \cap \mathbf{B})$, for any of three definitions of intersection of NSs.

Proof. Let $\mathbf{A} = \langle \mathbf{x}, 1/2, 1/3 \rangle$ and $\mathbf{B} = \langle \mathbf{x}, 1/2, 1/2 \rangle$ (i.e. $\mu_A, \nu_A, \mu_B, \nu_B$ are constant maps).

Then, $\mathbf{A} \cup \mathbf{B} = \langle \mathbf{x}, \mu_A \vee \mu_B, \gamma_A \wedge \gamma_B \rangle = \langle \mathbf{x}, 1/2, 1/3 \rangle$ and $\mathbf{x}(1/2, 1/6, 1/3) \in \mathbf{j}(\mathbf{A} \cup \mathbf{B})$. On the other hand, $\mathbf{x}(1/2, 1/6, 1/3) \in \mathbf{j}(\mathbf{A}), \mathbf{x}(1/2, 0, 1/2) \in \mathbf{j}(\mathbf{B})$.

Then, we have that:

- 1) for the union operator defined by the algebraic product N-conorm $\mathbf{x}(3/4, 1/6, 2/3) \in \mathbf{j}(\mathbf{A}) \cup \mathbf{j}(\mathbf{B})$.
- 2) for the union operator defined by the bounded N-conorm $\mathbf{x}(1, 1/6, 5/6) \in \mathbf{j}(\mathbf{A}) \cup \mathbf{j}(\mathbf{B})$.
- 3) for the union operator defined by the default (max) N-conorm $\mathbf{x}(1/2, 1/6, 1/2) \in \mathbf{j}(\mathbf{A}) \cup \mathbf{j}(\mathbf{B})$.

Thus $\mathbf{j}(\mathbf{A} \cup \mathbf{B}) \neq \mathbf{j}(\mathbf{A}) \cup \mathbf{j}(\mathbf{B})$, with the three definitions

Analogously, $\mathbf{A} \cap \mathbf{B} = \langle \mathbf{x}, \mu_A \wedge \mu_B, \gamma_A \vee \gamma_B \rangle = \langle \mathbf{x}, 1/2, 1/2 \rangle$ and $\mathbf{x}(1/2, 0, 1/2) \in \mathbf{j}(\mathbf{A} \cap \mathbf{B})$.

And, we have that:

- 1) for the intersection operator defined by the algebraic product N-norm $\mathbf{x}(1/4, 0, 1/6) \in \mathbf{j}(\mathbf{A}) \cap \mathbf{j}(\mathbf{B})$.
- 2) for the intersection operator defined by the bounded N-norm $\mathbf{x}(0, 0, 0) \in \mathbf{j}(\mathbf{A}) \cap \mathbf{j}(\mathbf{B})$.
- 3) for the intersection operator defined by the default (min) N-norm $\mathbf{x}(1/2, 0, 1/3) \in \mathbf{j}(\mathbf{A}) \cap \mathbf{j}(\mathbf{B})$.

Thus $\mathbf{j}(\mathbf{A} \cap \mathbf{B}) \neq \mathbf{j}(\mathbf{A}) \cap \mathbf{j}(\mathbf{B})$, with the three definitions.

Definition 11^{26,30}. Let us construct a neutrosophic topology on $\mathbf{NT} =]0, 1[$, considering the associated family of standard or non-standard subsets included in \mathbf{NT} , and the empty set which is closed under set union and finite intersection neutrosophic. The interval \mathbf{NT} endowed with this topology forms a neutrosophic topological space. There exist various notions of neutrosophic topologies on \mathbf{NT} , defined by using various N-norm/N-conorm operators.

Proposition 2. Let (\mathbf{X}, τ) be an intuitionistic fuzzy topological space. Then, the family $\{\mathbf{j}(\mathbf{U}) \mid \mathbf{U} \in \tau\}$ is not necessarily a neutrosophic topology on \mathbf{NT} (in the three defined senses).

Proof. Let $\tau = \{1_-, 0_-, \mathbf{A}\}$ where $\mathbf{A} = \langle \mathbf{x}, 1/2, 1/2 \rangle$ then $\mathbf{x}(1, 0, 0) \in \mathbf{j}(1_-)$, $\mathbf{x}(0, 0, 1) \in \mathbf{j}(0_-)$ and $\mathbf{x}(1/2, 0, 1/2) \in \mathbf{j}(\mathbf{A})$. Thus $\tau^* = \{\mathbf{j}(1_-), \mathbf{j}(0_-), \mathbf{j}(\mathbf{A})\}$ is not a neutrosophic topology, because this family is not closed by finite intersections, for any neutrosophic topology on \mathbf{NT} . Indeed,

- 1) For the intersection defined by the algebraic product N-norm, we have that $\mathbf{x}(1/2, 0, 0) \in \mathbf{j}(1_-) \cap \mathbf{j}(\mathbf{A})$, and this neutrosophic set is not in the family τ^* .

2) For the intersection defined by the bounded N-norm, we have also that $x(1/2,0,0) \in j(1-) \cap j(A)$, and this neutrosophic set is not in the family τ^* .

3) For the intersection defined by the default (min) N-norm, we have also that $x(1/2,0,0) \in j(1-) \cap j(A)$, and this neutrosophic set is not in the family τ^* .

3. Interval neutrosophic sets and topology

3.1 Introduction.

Also, Wang, Smarandache, Zhang, and Sunderraman³² introduced the notion of interval neutrosophic set, which is an instance of neutrosophic set and studied various properties. We study in this chapter relations between interval neutrosophic sets and topology.

3.2. Basic definitions.

First, we present some basic definitions. For definitions on non-standard Analysis, see the book²⁴:

Definition 12³². Let X be a space of points (objects) with generic elements in X denoted by x . An interval neutrosophic set (INS) A in X is characterized by truth-membership function T_A , indeterminacy-membership function I_A and falsity-membership function F_A . For each point x in X , we have that $T_A(x), I_A(x), F_A(x) \in [0,1]$.

Remark. All INS is clearly a NS.

When X is continuous, an INS A can be written as

$$A = \int_{x \in X} \langle T(x), I(x), F(x) \rangle / x, \quad x \in X$$

When X is discrete, an INS A can be written as

$$A = \sum_{i=1}^n \langle T(x_i), I(x_i), F(x_i) \rangle / x_i, \quad x_i \in X$$

Definition 13³² a) An interval neutrosophic set A is empty if $\inf T_A(x) = \sup T_A(x) = 0$, $\inf I_A(x) = \sup I_A(x) = 1$, $\inf F_A(x) = \sup F_A(x) = 0$ for all x in X .

b) Let $\underline{0} = \langle 0, 1, 1 \rangle$ and $\underline{1} = \langle 1, 0, 0 \rangle$.

Definition 14³² (Complement) Let C_N denote a neutrosophic complement of A .

Then C_N is a function $C_N: N \rightarrow N$ and C_N must satisfy at least the following three axiomatic requirements:

1. $C_N(\underline{0}) = \underline{1}$ and $C_N(\underline{1}) = \underline{0}$ (boundary conditions).
2. Let A and B be two interval neutrosophic sets defined on X , if $A(x) \leq B(x)$, then $C_N(A(x)) \geq C_N(B(x))$, for all x in X . (monotonicity).
3. Let A be an interval neutrosophic set defined on X , then $C_N(C_N(A(x))) = A(x)$, for all x in X . (involutivity).

Remark. There are many functions which satisfy the requirement to be the complement operator of interval neutrosophic sets. Here we give one example.

Definition 15. (Complement C_{N1}) The complement of an interval neutrosophic set A is denoted by A_- and is defined by

$$T_{A_-}(x) = F_A(x);$$

$$\inf I_{A_-}(x) = 1 - \sup I_A(x); \quad \sup I_{A_-}(x) = 1 - \inf I_A(x);$$

$$F_{A_-}(x) = T_A(x); \quad \text{for all } x \text{ in } X.$$

Definition 16³² (N-norm) Let I_N denote a neutrosophic intersection of two interval neutrosophic sets A and B . Then I_N is a function $I_N: N \times N \rightarrow N$ and I_N must satisfy at least the following four axiomatic requirements:

1. $I_N(A(x), \underline{1}) = A(x)$, for all x in X . (boundary condition).
2. $B(x) \leq C(x)$ implies $I_N(A(x), B(x)) \leq I_N(A(x), C(x))$, for all x in X . (monotonicity).
3. $I_N(A(x), B(x)) = I_N(B(x), A(x))$, for all x in X . (commutativity).
4. $I_N(A(x), I_N(B(x), C(x))) = I_N(I_N(A(x), B(x)), C(x))$, for all x in X . (associativity).

Remark. Here we give one example of intersection of two interval neutrosophic sets which satisfies above N-norm axiomatic requirements. Other different definitions can be given for different applications

Definition 17. (Intersection I_{N1}) The intersection of two interval neutrosophic sets **A** and **B** is an interval neutrosophic set **C**, written as $C = A \cap B$, whose truth-membership, indeterminacy-membership, and false-membership are related to those of **A** and **B** by

$$\begin{aligned} \inf T_C(x) &= \min(\inf T_A(x); \inf T_B(x)), & \sup T_C(x) &= \min(\sup T_A(x); \sup T_B(x)), \\ \inf I_C(x) &= \max(\inf I_A(x); \inf I_B(x)), & \sup I_C(x) &= \max(\sup I_A(x); \sup I_B(x)), \\ \inf F_C(x) &= \max(\inf F_A(x); \inf F_B(x)), & \sup F_C(x) &= \max(\sup F_A(x); \sup F_B(x)); \text{ for all } x \text{ in } X. \end{aligned}$$

Definition 18³². (N-conorm) Let U_N denote a neutrosophic union of two interval neutrosophic sets **A** and **B**. Then U_N is a function $U_N: N \times N \rightarrow N$ and U_N must satisfy at least the following four axiomatic requirements:

1. $U_N(A(x), 0) = A(x)$, for all x in X . (boundary condition).
2. $B(x) \leq C(x)$ implies $U_N(A(x), B(x)) \leq U_N(A(x), C(x))$, for all x in X . (monotonicity).
3. $U_N(A(x), B(x)) = U_N(B(x), A(x))$, for all x in X . (commutativity).
4. $U_N(A(x), U_N(B(x), C(x))) = U_N(U_N(A(x), B(x)), C(x))$, for all x in X . (associativity).

Remark. Here we give one example of union of two interval neutrosophic sets which satisfies above N-conorm axiomatic requirements. Other different definitions can be given for different applications.

Definition 19. (Union U_{N1}) The union of two interval neutrosophic sets **A** and **B** is an interval neutrosophic set **C**, written as $C = A \cup B$, whose truth-membership, indeterminacy-membership, and false-membership are related to those of **A** and **B** by

$$\begin{aligned} \inf T_C(x) &= \max(\inf T_A(x); \inf T_B(x)), & \sup T_C(x) &= \max(\sup T_A(x); \sup T_B(x)), \\ \inf I_C(x) &= \min(\inf I_A(x); \inf I_B(x)), & \sup I_C(x) &= \min(\sup I_A(x); \sup I_B(x)), \\ \inf F_C(x) &= \min(\inf F_A(x); \inf F_B(x)), & \sup F_C(x) &= \min(\sup F_A(x); \sup F_B(x)), \text{ for all } x \text{ in } X. \end{aligned}$$

3.3. Results.

Proposition 1. Let **A** an IFS in X , and $j(A)$ be the corresponding INS. We have that the complement of $j(A)$ is not necessarily $j(A)-$.

Proof. If $A = \langle x, \mu_A, \gamma_A \rangle$ is $j(A) = \langle \mu_A, 0, \gamma_A \rangle$. Then, for $0 = \langle x, 0, 1 \rangle$ is $j(0) = j(\langle x, 0, 1 \rangle) = \langle 0, 0, 1 \rangle \neq \underline{0} = \langle 0, 1, 1 \rangle$ for $1 = \langle x, 1, 0 \rangle$ is $j(1) = j(\langle x, 1, 0 \rangle) = \langle 1, 0, 0 \rangle = \underline{1}$.

Thus, $1 =$ complement of 0 and $j(1) = \underline{1} \neq C_N(j(0))$ because $C_N(\underline{1}) = \underline{0} \neq j(0)$.

Definition 20²⁶. Let us construct a neutrosophic topology on $NT =]0, 1[$, considering the associated family of standard or non-standard subsets included in NT , and the empty set which is closed under set union and finite intersection neutrosophic. The interval NT endowed with this topology forms a neutrosophic topological space.

Proposition 2. Let (X, τ) be an intuitionistic fuzzy topological space. Then, the family of INSs $\{j(U) | U \in \tau\}$ is not necessarily a neutrosophic topology.

Proof. Let $\tau = \{1-, 0-, A\}$ where $A = \langle x, 1/2, 1/2 \rangle$, then $j(1-) = \underline{1}$, $j(0-) = \langle 0, 0, 1 \rangle \neq \emptyset$, and $j(A) = \langle 1/2, 0, 1/2 \rangle$. Thus $\{j(1-), j(0-), j(A)\}$ is not a neutrosophic topology, because the empty INS is not in this family.

4. Neutrosophic paraconsistent Topology

4.1. Introduction

The history of paraconsistent logic is not very long. It was designed by S. Jaskowski in 1948. Without knowing the work of this author, N.C. A. da Costa, from 1958, using different methods and ideas, began to make statements about this type of logic. After other logicians have developed independently, new systems of paraconsistent logic, as Routley, Meyer, Priest, Asenjo, Sette, Anderson and Benalp, Wolf (with da Costa himself), At present there is a thriving movement dedicated to the study of paraconsistent logic in several countries. In the philosophical aspect has meant, in some cases, a real opening of horizons, for example, in the treatment of the paradoxes, in efforts to treat rigorously dialectical thinking, in fact possible to develop a set theory inconsistent. .. Because of this, there is growing interest in understanding the nature and scope.

Jaskowski deductive logic led her to refer to several problems that caused the need for paraconsistent logic:

1) The problem of organizing deductive theories that contain contradictions, as in the dialectic: "The principle that no two contradictory statements are both true and false is the safest of all."

2) To study theories that there are contradictions engendered by vagueness: "The contemporary formal approach to logic increases the accuracy of research in many fields, but it would be inappropriate to formulate the principle of contradiction of Aristotle thus: "Two contradictory propositions are not true". We need to add: "in the

same language" or "if the words that are part of those have the same meaning". This restriction is not always found in daily use, and also science, we often use terms that are more or less vague.

3) To study directly some postulates or empirical theories whose basic meanings are contradictory. This applies, for example, the physics at the present stage.

Various authors^{22,23} worked on "paraconsistent Logics", that is, logics where some contradiction is admissible. We remark the theories exposed by Da Costa⁸, Routley and other²⁵, and Peña^{20,21}.

Smarandache defined also the neutrosophic paraconsistent sets³⁹ and he proposed a natural definition of neutrosophic paraconsistent topology.

A problem that we consider is the possible relation between this concept of neutrosophic paraconsistent topology and the previous notions of general neutrosophic topology and intuitionistic fuzzy topology. We show in this chapter that neutrosophic paraconsistent topology is not an extension of intuitionistic fuzzy topology.

4.2. Basic definitions

First, we present some basic definitions:

Definition 21³⁰. Let \mathbf{M} be a non-empty set. A general neutrosophic topology on \mathbf{M} is a family Ψ of neutrosophic sets in \mathbf{M} satisfying the following axioms:

- (a) $0_{\sim} = x(0,0,1)$, $1_{\sim} = x(1,0,0) \in \Psi$
- (b) If $A, B \in \Psi$, then $A \cap B \in \Psi$
- (c) If a family $\{A_j | j \in J\} \subset \Psi$, then $\bigcup A_j \in \Psi$.

Definition 22²⁹. A neutrosophic set $x(\mathbf{T}, \mathbf{I}, \mathbf{F})$ is called paraconsistent if $\inf(\mathbf{T}) + \inf(\mathbf{I}) + \inf(\mathbf{F}) > 1$.

Definition 23. (Smarandache). For neutrosophic paraconsistent sets $0_{\sim} = x(0,1,1)$ and $1_{\sim} = x(1,1,0)$.

Remark. If we use the unary neutrosophic negation operator for neutrosophic sets⁴⁰, $n_N(x(\mathbf{T}, \mathbf{I}, \mathbf{F})) = x(\mathbf{F}, \mathbf{I}, \mathbf{T})$ by interchanging the truth \mathbf{T} and falsehood \mathbf{F} components, we have that $n_N(0_{\sim}) = 1_{\sim}$.

Definition 24. (Smarandache) Let \mathbf{X} be a non-empty set. A family Φ of neutrosophic paraconsistent sets in \mathbf{X} will be called a neutrosophic paraconsistent topology if:

- (a) 0_{\sim} and $1_{\sim} \in \Phi$
- (b) If $A, B \in \Phi$, then $A \cap B \in \Phi$
- (c) Any union of a subfamily of paraconsistent sets of Φ is also in Φ .

4.3. Results.

Proposition 1. The neutrosophic paraconsistent topology is not an extension of intuitionistic fuzzy topology.

Proof. We have that $0_{\sim} = x(0,1,1)$ and $1_{\sim} = x(1,1,0)$ are members of all intuitionistic fuzzy topology, but

$$x(0,0,1) \in j(0_{\sim}) \neq 0_{\sim} \text{ and } x(1,0,0) \in j(1_{\sim}) \neq 1_{\sim}.$$

Proposition 2. A neutrosophic paraconsistent topology is not a general neutrosophic topology.

Proof. Let the family $\{1,0\}$. Clearly it is a neutrosophic paraconsistent topology, but $0_{\sim}, 1_{\sim}$ are not in this family.

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