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Degree of polarization of non-uniformly partially polarized beams: a proposal

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Two overall parameters are proposed to characterize the degree of polarization and its uniformity across the transversal section of non-uniformly partially polarized beams. Such parameters can be measured according to the well-known standard methods used to obtain the Stokes parameters, with the addition of a CCD camera to determine the beam intensity at each point of the observation plane.

1. Introduction

As is well known, the standard degree of polarization P of a general beam at a certain plane is defined in terms of the Stokes parameters [1, 2]. According to the value of P, beams can be classified as totally ($P = 1$), partially ($0 < P < 1$) or non-polarized ($P = 0$) fields. The experimental procedure to determine P involves measurements of the *integrated beam intensity over the full detection area* for different orientations of a polarizer and a quarter-wave plate. But this method only makes sense if the radiation is assumed to have uniform polarization properties over its cross-sectional area. In this connection, the current ISO proposal concerning laser beam polarization [2] takes into account only this case. However, beams with spatially distributed polarization states are attracting increasing interest [3–7]. For this kind of fields, the overall parameter P is not adequate to properly characterize their polarization properties (for example, for radially totally polarized (RTP) beams [3, 4] $P = 0$).

To characterize non-uniform totally polarized (NUTP) beams, a generalized parameter $\mathbf{P}$ was recently introduced [8, 9] on the basis of the Stokes–Mueller formalism extended to describe the intensity moments of a beam [10–13]. Such a parameter was shown to represent a measure of the uniformity of the polarization state of the field over those cross-sectional regions where the beam intensity is important. However, it was noted that, without any prior knowledge, no conclusion could be inferred from a particular value of $\mathbf{P}$ concerning whether the field is actually partially polarized or exhibits a spatial dependence of the polarization over the beam profile (NUTP beams).

It would then be useful to handle meaningful overall measurable parameters that allow us to characterize in a simple way both, the degree of polarization and its uniformity across the transversal section of a general beam. This is the aim of the present proposal.

2. Formalism and definitions

Let us consider quasimonochromatic optical fields, with E_x and E_y representing the components of the electric vector in two mutually orthogonal directions at right angles to the direction of propagation. As is well-known [1, 2] the standard parameter P is defined as

$$P = \sqrt{\frac{(s_1^2 + s_2^2 + s_3^2)}{s_0^2}} \tag{1}$$

where s_i are the Stokes parameters expressed as follows

$$s_0 = J_{xx} + J_{yy} \tag{2}$$

$$s_1 = J_{xx} - J_{yy} \tag{3}$$

$$s_2 = J_{xy} + J_{yx} \tag{4}$$

$$s_3 = i(J_{yx} - J_{xy}) \tag{5}$$

In these equations $J_{ij}, i, j = x, y$, are the elements of the so-called coherency matrix

$$J_{xx} = \langle E_x E_x^* \rangle \tag{6}$$

$$J_{yy} = \langle E_y E_y^* \rangle \tag{7}$$

$$J_{xy} = \langle E_x E_y^* \rangle \tag{8}$$

where the symbol * means the complex conjugate, $J_{xy} = J_{yx}^*$ and the sharp brackets denote a temporal average.

The Stokes parameters $s_i, i = 0, 1, 2, 3$, can be measured in different ways. For example, we can use a polarizer and a quarter-wave plate, and measure the beam intensity, integrated over the full detection area, for several orientations of the transmission axis of the polarizer, namely [1],

$$s_0 = I_{0^\circ} + I_{90^\circ} \tag{9}$$

$$s_1 = I_{0^\circ} - I_{90^\circ} \tag{10}$$

$$s_2 = I_{45^\circ} - I_{135^\circ} \tag{11}$$

$$s_3 = I_{45^\circ,\pi/2} - I_{135^\circ,\pi/2} \tag{12}$$

In Equations 9–11 the subscripts indicate the angle that the transmission axis of the polarizer makes with the x-axis. To get s_3 the intensity is measured after the beam propagates successively through the quarter-wave plate (whose fast axis makes an angle 0° with the x-axis) and the polarizer oriented so as to transmit the component in the azimuth 45° and 135° (see Equation 12).

This procedure makes sense for uniformly polarized fields, for which the parameter P provides a meaningful characterization. However, in a general case, the polarization state is a function of the transversal variables (x, y) across the beam profile. Consequently, the degree of polarization is a spatial function too and should be written in the form

$$P(x,y) = \sqrt{\frac{(s_1^2(x,y) + s_2^2(x,y) + s_3^2(x,y))}{s_0^2(x,y)}} \tag{13}$$

where the Stokes parameters must be measured at each point. This can be done easily by using a CCD camera at the observation plane (instead of a photodetector). Equations 9–12 remain valid as well as the above method to determine the Stokes parameters $s_i(x,y)$.

To characterize the (overall) degree of polarization of a general field we define a parameter $\tilde{P}$ as follows

$$\tilde{P} = \frac{\int\int I(x,y)P(x,y)\,\mathrm{d}x\,\mathrm{d}y}{\int\int I(x,y)\,\mathrm{d}x\,\mathrm{d}y} \tag{14}$$

where $I(x,y)$ is the intensity of the direct (free-propagating) laser beam at each point of the observation plane. We will call $\tilde{P}$ the weighted degree of polarization. It should be noted that $\tilde{P}$ computes mainly those regions where the beam intensity is significant. In fact, the existence of the intensity factor $I(x, y)$ in the definition of $\tilde{P}$ minimizes the contribution of the beam wings, thus reducing certain harmful effects (camera offset, small signal-to-noise ratio, background, etc). Also note that, for uniformly partially polarized beams, the standard parameter P and the weighted degree of polarization have the same value. It should also be remarked that, in general, $\tilde{P}$ is not invariant under propagation through first-order optical systems.

To get deeper insight into the physical meaning of $\tilde{P}$, let us first point out that, like P, it satisfies the inequality

$$0 \leq \tilde{P} \leq 1 \tag{15}$$

Those beams whose parameter $\tilde{P}$ is close to the unity will be mostly totally polarized (at least in the regions with significant intensity), even though they have spatially distributed polarization states. The opposite case $\tilde{P} = 0$ means that the beam is non-polarized over the whole profile.

It is important to note that, in a sense, $\tilde{P}$ is complementary to the parameter $\mathbf{P}$ defined in [9]. To clarify this let us consider a RTP field [3, 4], whose polarization state is linear at each point and oriented along radial lines. Such beam can be experimentally synthesized by means of interferometric procedures or in concentric-circular-grating surface emitting semiconductor lasers. For this field $\mathbf{P} = 0$ and $\tilde{P} = 1$. In fact, this beam is totally polarized throughout its transversal section ($\Rightarrow \tilde{P} = 1$), but the azimuth of its linear polarization states takes any value across the beam profile ($\Rightarrow \mathbf{P} = 0$). Also note that, as it was mentioned before, the standard parameter P equals zero.

Parameter $\tilde{P}$ enables us to classify again the beams as totally ($\tilde{P} = 1$), partially ($0 < \tilde{P} < 1$) or non-polarized ($\tilde{P} = 0$) fields, but now this classification scheme also applies for non-uniform polarization distributions.

The dispersion of the values of the degree of polarization $P(x,y)$ from one point to another in those regions where the beam intensity is important can be easily evaluated by means of the following parameter:

$$\tilde{\sigma}_{\mathrm{p}}^2 = \frac{\int\int I(x,y)[P(x,y) - \tilde{P}]^2\,\mathrm{d}x\,\mathrm{d}y}{\int\int I(x,y)\,\mathrm{d}x\,\mathrm{d}y} \tag{16}$$

similar to the variance of $P(x, y)$ (the intensity $I(x, y)$ acting as a density function). It is not difficult to show that

$$0 \leq \tilde{\sigma}_{\mathrm{p}} \leq 1/2 \tag{17}$$

This parameter provides, in fact, an overall characterization of the uniformity of the degree of polarization across the transversal profile. Thus, for example, for RTP fields, $\tilde{\sigma}_{\mathrm{p}} = 0$ (these beams are totally polarized everywhere). Of course, when we handle uniformly polarized beams, $\tilde{\sigma}_{\mathrm{p}} = 0$ too.

Let us finally point out that the effect of the finite size of the pixels of the CCD camera on the measured value of $\tilde{P}$ and $\tilde{\sigma}_{\mathrm{p}}$ is negligible, provided the pixel array has enough resolution (say, $> 30 \times 30$ pixels) in the region where the beam intensity is significant.

3. Conclusions

Two overall parameters have been proposed to characterize the degree of polarization and its uniformity over the cross-sectional area of non-uniformly partially polarized beams. The attention is focussed on the regions where the beam intensity is significant. The above parameters can be experimentally determined following well-known procedures used to measure the Stokes parameters, with a final CCD array to get the intensity at each point of the observation plane. Moreover, the standard parameter P (the intensities are integrated over the full detection region) and the parameter $\tilde{P}$ proposed here give the same value for the usually assumed case of uniformly polarized beams. Consequently, we feel that the parameters $\tilde{P}$ and $\tilde{\sigma}_{\mathrm{p}}$ can be of use in the corresponding ISO normative, which is being currently discussed.

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