

MATRIX SUMMABILITY METHODS AND WEAKLY UNCONDITIONALLY CAUCHY SERIES

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ABSTRACT. We study new sequence spaces determined by series in normed spaces and a matrix summability method, giving new characterizations of weakly unconditionally Cauchy series. We obtain characterizations for the completeness of a normed space, and a version of the Orlicz-Pettis theorem via matrix summability methods is also proved.

1. Introduction. Let X be a real normed space. A series $\sum_i x_i$ in X is said to be unconditionally convergent (uc) if $\sum_i x_{\pi(i)}$ converges for every permutation π of \mathbf{N} . We say that $\sum_i x_i$ is weakly unconditionally Cauchy (wuc) if, for every permutation π of \mathbf{N} , we have that the sequence $(\sum_{i=1}^n x_{\pi(i)})_n$ is weakly Cauchy. It is a well-known fact, see [5], that $\sum_i x_i$ is wuc if and only if $\sum_i |f(x_i)| < \infty$ for every $f \in X^*$, where X^* denotes the dual space of X . The following results are also well known, see [3, 5, 6]:

Let X be a Banach space, and let $\sum_i x_i$ be a series in X . Then:

1. $\sum_i x_i$ is uc if and only if $\sum_i a_i x_i$ is convergent for every $(a_i)_i \in l_\infty$.
2. $\sum_i x_i$ is wuc if and only if $\sum_i a_i x_i$ is convergent for every $(a_i)_i \in c_0$.
3. There exists a series $\sum_i x_i$ wuc and not uc in X if and only if X has a copy of c_0 .

The following concepts and definitions can be found in [4].

A matrix method of limit is defined by a matrix $A = (\alpha_{ij})_{(i,j) \in \mathbf{N} \times \mathbf{N}}$ of real entries in the following way: If $(x_i)_i$ is a sequence in a normed space X , we say that $A \lim_i x_i = x_0$ if, for every $i \in \mathbf{N}$, the series $\sum_j \alpha_{ij} x_j$ is convergent and $\lim_i \sum_j \alpha_{ij} x_j = x_0$.

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