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Teoría electrodébil y potencial del Higgs en el Modelo Estándar y más all

Electroweak theory and Higgs potential in the Standard Model and beyon

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Resumen:

En este trabajo se van a tratar diversos modelos de simetría de prueba además del Modelo Estándard para estudiar en ellos el mecanismo de Higgs y cómo los bosones gauge vectoriales obtienen su masa mediante este mecanismo. Además estudiamos el patrón de masas de los escalares. Daremos especial importancia a la elección del estado de vacio de la teoría. Comprobaremos con ejemplos cómo esta elección resulta completamente arbitraria y no afecta de ninguna forma a las cantidades físicas de la teoría. En este trabajo, estas cantidades serán principalmente los autovalores de masa. Sin embargo, una elección apropiada de vacío si puede complicar o facilitar en gran medida los cálculos necesarios, así como alterar las combinaciones concretas de campos asociadas a nuestros autoestados físicos. También se discutirá el concepto de carga electrica, su relación con los generadores del grupo electrodébil $SU(2)_L \times U(1)_Y$. Veremos como las distintas elecciones de vacío pueden afectar a como entendemos esta cantidad pero nuevamente, los autovalores serán siempre independientes de esta elección.

Abstract:

During this work we will have a look at various symmetry toy models in addition to the Standard Model to study the effects of the Higgs mechanism and how this gives raise to masses for the gauge vector bosons. Moreover, we will explore the mass patron for the scalar bosons. We will give an special importance to the choice of the vacuum state in our theory. We will test on the different models how this election is completely arbitrary and does not affect any of the physical properties. The main property studied will be the mass eigenvalues. Nonetheless, an appropriate vacuum state choice can greatly facilitate or complicate the necessary calculations as well as the concrete field combinations that represent the physical eigenstates. Finally, we will discuss the concept of electric charge, how this quantity is related to the electroweak $SU(2)_L \times U(1)_Y$ generators. We will observe how the different vacuum state choices affect how we understand this quantity, but once again, the eigenvalues will be independent of this choice.

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1 Introduction

The Standard Model (SM) has been and still is one of the most successful theoretical models for particle physics and our understanding of the fundamental interactions of nature. The SM is built upon 3 main and brilliant ideas, the first of which is the quark model for hadrons. The concept of considering an intern structure for hadrons is analogous to one that had already revolutionized Physics many years prior to this, considering that atoms are not indivisible and are formed by protons, electrons and neutrons. The second idea is to build the theory around the symmetry principles that seem to rule nature as we experience it. To do this we build the Lagrangian of the system we want to study so that it is invariant under the observed symmetry. In the case of the electroweak (EW) interaction this work will focus on, this symmetry group is the $SU(2)_L \times U(1)_Y$ group were the precise meaning of this will be later explained with good detail. The last of these ideas is the concept of spontaneous symmetry breaking (SSB). SSB will allow us to give our particles a mass term while still maintaining the symmetry invariance. The consequences of this mechanism for the SM will be the main topic of discussion during this work [1].

During all this study, we will focus on a particular, yet general enough Lagrangian density which will then be concreted for each particular symmetry group, it will take the following form,

$$\mathscr{L} = -\frac{1}{4}(F_{\mu\nu})^2 + (D^\mu\phi)^\dagger D_\mu\phi - V(|\phi|)\,, \tag{1}$$

where each of the 3 terms will vary from one to the next model but always keeping this general form. We now proceed to give a brief explanation of each of the terms: The first term contains the gauge field dynamics and although this term will not be the main focus of this analysis it is a necessary term for our theory. The second term is what we will refer to as the kinetic term, and will be the term responsible for raising the masses of our gauge bosons, the precise definition of the covariant derivative D_μ for each symmetry model is a fundamental part of modern quantum field theories and will be specified for each distinct symmetry group used as it encodes a term that is directly responsible for "fixing" the covariance of the gauge transformations to keep the invariance of the complete Lagrangian. Finally, the third term is our potential, it will also be a fundamental part of our analysis as it will give birth to what we will call Goldstone bosons and the Higgs boson. The Higgs boson is named after the British physicist Peter Higgs, who was one of the first to discuss the existence of such particle [2]. As happens with the complete Lagrangian, this term will have a common general form for all the models but the involved fields will of course vary according to the treated symmetry, its form is:

$$V(\phi) = -\mu^2\phi^\dagger\phi + \lambda\left(\phi^\dagger\phi\right)^2\,, \tag{2}$$

with μ and λ positive parameters and ϕ the field related to the symmetry being studied [3]. A side note regarding notation: the usual Greek letter ϕ will be used for complex fields while φ will be used when the field is real valued except in section 2.2.2 where it will be a real triplet.

In sec. 2 we will have a look at several toy models to start the study of the Higgs mechanism and the consequences of changing the vacuum state with simpler models than the SM. Next, in sec. 3, we are studying the SM to observe how the EW bosons get their masses. We are also exploring how we define the electric charge operator and how the choice of the vacuum state affects both the mass and the charge eigenvalues. In the Conclusions, we will give a brief summary of the obtained results, their relevance and some general aspects we can extract from this work. Finally, we will also highlight a couple of ideas that are beyond the scope of this work but that also hold great interest in relation to the discussed topics.

2 Simple models, the road to the SM

Before getting ourselves into all the complicated but also exciting study of the electroweak interaction and its symmetry group in the SM we will first tackle some simpler symmetry groups.

2.1 U(1) the QED group

The first example we will have a look at is the U(1) group which is Abelian, this property makes the calculations much simpler and therefore is a great way to start diving into the topic. This U(1) symmetry group is the gauge symmetry of the QED theory. For this symmetry group and example we will start a bit before just to give a general idea of how modern physical theories are built. First of all we have to identify the symmetry the situation we are trying to describe has, in this particular example an U(1) symmetry, after than we need to build a Lagrangian which is gauge invariant under such symmetry.

From the general Lagrangian density (1), we consider ϕ as a complex scalar field coupled with an electromagnetic field and to itself. It is easy to prove, as we will show, that this Lagrangian is indeed gauge invariant with the following precise definitions of its terms:

$$D_\mu \equiv \partial_\mu + iqA_\mu \,, \tag{3}$$

$$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu \,,$$

where A_μ is the usual EM potential 4-vector. We now check if, as anticipated our Lagrangian is invariant under a local U(1) transformation:

$$\phi(x) \to e^{i\theta(x)}\phi(x) \tag{4}$$

$$A_\mu(x) \to A_\mu(x) - \frac{1}{q}\partial_\mu\theta(x) \,.$$

The first term in Lagrangian (1) is trivially invariant as we suppose our gauge transformation $\theta(x)$ to follow the Schwartz's theorem [4]. The second term has carefully been defined to be invariant as the variation in $\partial_\mu\phi$ is neutralized by the variation in the added term thanks to the gauge field A_μ. And the potential term (2) is trivially invariant as the transformations in ϕ are always cancelled by the transformations in $\phi^\dagger$.

Once we have an invariant Lagrangian we can dive into the idea of spontaneously broken symmetry. It is easy to see that this potential does not take its lowest value at the origin but instead the minimum is found at,

$$|\phi_0| = \left(\frac{\mu^2}{2\lambda}\right)^{1/2} \,. \tag{5}$$

The potential at this point is what we will call the vacuum expectation value (vev). As we can see there is not a single point ϕ_0 which minimizes the potential but there is a continuous range of points, all of them placed on a circumference in the complex plane. We will now rewrite our Lagrangian (1) around the vacuum state, it is possible to choose any of the infinitely many vacuum states expressed by the condition of minimizing eq.(2). A clever choice of this state can simplify our work by making the explicit calculations easier, however, we will show that the final physical properties do not change by choosing the completely general representation of all the possible vacuum states,

$$\phi_0 = |\phi_0|\, e^{i\theta} \quad \longrightarrow \quad \begin{pmatrix} Re(\phi_0) \\ Im(\phi_0) \end{pmatrix} = |\phi_0| \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \equiv |\phi_0|\, \vec{n}. \tag{6}$$

Expanding our potential around this minima as follows, we obtain:

$$\phi(x) = \phi_0 + \frac{1}{\sqrt{2}}\left(\varphi_1(x) + i\varphi_2(x)\right) , \tag{7}$$

where φ_1 and φ_2 are real valued fields, inserting eq. (7) in eq. (2) and cutting our expansion to terms quadratic in φ_i we build our square mass matrix as the Hessian matrix of this quadratic form,

$$M_h^2 = \begin{pmatrix} \frac{\partial^2 V}{\partial \varphi_1^2} & \frac{\partial^2 V}{\partial \varphi_1 \varphi_2} \\ \frac{\partial^2 V}{\partial \varphi_1 \varphi_2} & \frac{\partial^2 V}{\partial \varphi_2^2} \end{pmatrix} = \mu^2 \begin{pmatrix} 2\cos^2(\theta) & \sin(2\theta) \\ \sin(2\theta) & 2\sin^2(\theta) \end{pmatrix}. \tag{8}$$

As we can see, this matrix is symmetrical and therefore is diagonalizable by rotations, this means our basis change matrix P is therefore an orthogonal matrix and $P^{-1} = P^t$. We now find the eigenvalues and eigenvectors to see that the physical properties do not depend of the choice of our vacuum state,

$$D_h = P^t M_h^2 P, \tag{9}$$

$$P = \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix}, \tag{10}$$

$$D_h = \begin{pmatrix} 2\mu^2 & 0 \\ 0 & 0 \end{pmatrix}, \tag{11}$$

as we can see in eq. (10), the field that gets a mass squared equal to $2\mu^2$ is not φ_1 or φ_2, it is a combination of both which we will call h.

$$\text{Eigenvalue } 2\mu^2 \quad \longrightarrow \quad \vec{w}_{2\mu^2} = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} = \vec{n} \quad \longrightarrow \quad h = \vec{n}\cdot\vec{\varphi} = \cos\theta\varphi_1 + \sin\theta\varphi_2, \tag{12}$$

where $\vec{\varphi} = (\varphi_1, \varphi_2, \varphi_3)^t$. This massive scalar field is what we call the Higgs boson of our theory, hence this would be a 1 Higgs model. The orthogonal combination which we can call π remains massless as our Goldstone boson [5].

$$\text{Eigenvalue } 0 \quad \longrightarrow \quad \vec{w}_0 = \begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix} \perp \vec{n} \quad \longrightarrow \quad \pi = \vec{w}_0 \cdot \vec{\varphi} = -sin\theta\varphi_1 + \cos\theta\varphi_2, \tag{13}$$

as intuition suggests, the eigenvectors of M_h^2 are such that they rotate axis x and y the angle θ that appears in the vacuum state. This is easily seen in fig. 1 where the red arrows represents the π field while the green ones represents the Higgs boson.

This result is of extreme importance as it proves that the physical properties which are quantifiable are completely independent of the mathematical elections such as the vacuum state. For this reason, it would be completely impossible to distinguish if this result was obtained using one or another vacuum state. Nonetheless, if we had chosen our vacuum state ϕ_0 in eq.(6) to be purely real by fixing $\theta = 0$ and had then built the square mass matrix with an analogous expansion, we would have directly gotten the diagonalized D_h matrix(11). This proves that a clever choice of our vacuum can simplify the computations but never change the final result. If, on the same symmetry model we now have a look at the gauge bosons masses, we can also find that the vacuum state choice is still irrelevant to describe the physics of the situation. For this study we will now focus on the kinetic term,

$$\left(D^\mu\phi\right)^\dagger D_\mu\phi = \frac{1}{2}\left(\partial_\mu\varphi_1\right) + \frac{1}{2}\left(\partial_\mu\varphi_2\right) + \sqrt{2}q\phi_0 A_\mu\partial^\mu\varphi_1 + q^2\phi_0^2 A_\mu A^\mu + \mathcal{O}(\Phi_i^3), \tag{14}$$

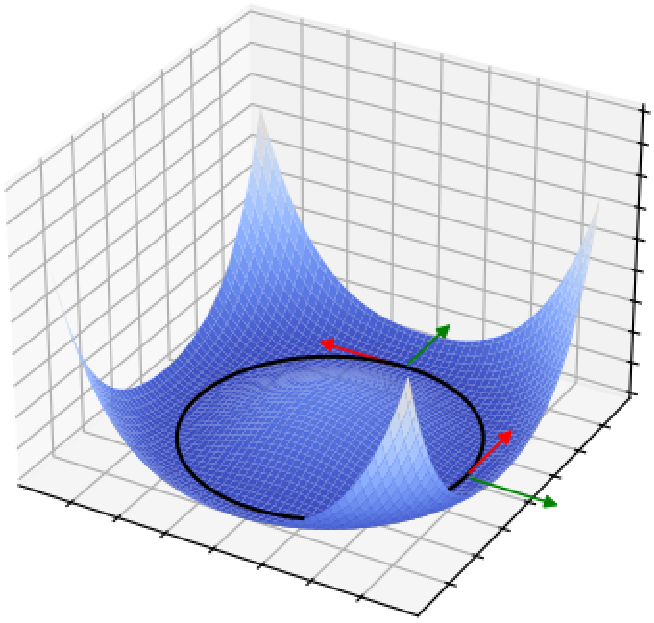

Figure 1: Potential for the U(1) SSB.

where Φ_i includes both $\varphi_{1,2}$ and A_μ. This result is exactly the same that we could have obtained with probably less work if had chosen a particular vacuum state. As we can trivially see by looking at eq.(14), the phase we included in our vacuum state to generalize it does not appear anywhere since every field in the kinetic term is multiplied by its conjugate. This is a direct consequence of the gauge invariance on which we built our Lagrangian. We ignored the cubic and above terms in eq.(14) which would be responsible for all the different phenomena and self interactions between our bosons because, we are looking for a mass term for our gauge boson. A mass term has the form of $\frac{1}{2}m^2 A_\mu A^\mu$ so comparing with the expression above we can immediately find that our mass squared is:

$$m_A^2 = 2q^2|\phi_0|^2. \tag{15}$$

With this first model, we have found one scalar Higgs boson and one massive gauge boson, this is not the result we are looking for to explain the electroweak unification, so we will proceed to look into more complex symmetry groups.

2.2 Non-Abelian examples

After having tackled the easiest Abelian group example with a lot of detail we will now move on to a non-Abelian example which allows for a more complex and rich phenomenology, nonetheless the steps to be taken are the same as in the Abelian example. As a brief reminder, we build a gauge invariant Lagrangian density according to eq.(1) and define a covariant derivative. Then we give our vacuum a non-zero expectation value and explore how this gives rise to a SSB process and therefore the creation of Goldstone bosons and mass eigenstates for our theory particles.

2.2.1 Complex doublet representation of SU(2)

Lie groups have infinitely many representations, to start with we will have a look at the fundamental representation of SU(2), this means that ϕ will now be a complex doublet under this group. As said before, we will skip some of the previous steps. In order for the Lagrangian to be gauge invariant our covariant derivative has to take a different form than in the previous case.

$$D_\mu \phi = \left(\partial_\mu - ig\tilde{A}_\mu\right)\phi, \tag{16}$$

where $\tilde{A}_\mu$ is a compact matrix way of expressing our gauge fields,

$$\tilde{A}_\mu \equiv \frac{\sigma^a}{2} A^a_\mu, \tag{17}$$

and σ^a are the Pauli matrices. $\tilde{A}$ follows this transformation law [6],

$$\tilde{A}_\mu(x) \to U\tilde{A}_\mu(x)U^\dagger + \frac{1}{g}\partial_\mu U(x)U^\dagger(x)\,. \tag{18}$$

Proceeding as in the Abelian example, we allow ϕ to get a vev which we can characterize as follows,

$$\phi_0 = \frac{v}{\sqrt{2}}\begin{pmatrix} e^{i\beta} sin(\theta) \\ e^{i\alpha} cos(\theta)\end{pmatrix} \quad \longrightarrow \quad \begin{pmatrix} Re(\phi_0)_1 \\ Im(\phi_0)_1 \\ Re(\phi_0)_2 \\ Im(\phi_0)_2 \end{pmatrix} = \frac{v}{\sqrt{2}}\begin{pmatrix} \sin\theta\cos\beta \\ \sin\theta\sin\beta \\ \cos\theta\cos\alpha \\ \cos\theta\sin\alpha \end{pmatrix} \equiv \frac{v}{\sqrt{2}}\vec{n}, \tag{19}$$

this relation reduces itself to the standard vacuum state when $\alpha = \theta = 0$. Hence, our field expansion around this general point can be expressed as:

$$\phi(x) = \phi_0 + \frac{1}{\sqrt{2}}\begin{pmatrix} \varphi_1(x) + i\varphi_2(x) \\ \varphi_3(x) + i\varphi_4(x)\end{pmatrix}. \tag{20}$$

As we did in the Abelian example, we have 2 main things to do, firstly, we have to find the Goldstone bosons and the Higgs boson by expanding our potential around its minima and secondly, we will find our gauge bosons masses by examining the kinetic term $(D^\mu\phi)^\dagger D_\mu\phi$. We turn our attention to the potential term $V(\phi)$, we expand the potential in eq.(2) according to eq.(20), then we follow the steps taken in the Abelian example. We only take into account terms quadratic in φ_i and build our mass squared matrix as the Hessian matrix. In this example our matrix is of course a 4×4 square matrix since we have 4 real fields in our potential expansion. This adds some numerical and computational difficulty but the conceptual frame is exactly the same as in the U(1) example.

$$M_h^2 = \\ \mu^2\begin{pmatrix} 2\cos^2(\beta)\sin^2(\theta) & \sin(2\beta)\sin^2(\theta) & \cos(\alpha)\cos(\beta)\sin(2\theta) & \sin(\alpha)\cos(\beta)\sin(2\theta) \\ \sin(2\beta)\sin^2(\theta) & 2\sin^2(\beta)\sin^2(\theta) & \cos(\alpha)\sin(\beta)\sin(2\theta) & \sin(\alpha)\sin(\beta)\sin(2\theta) \\ \cos(\alpha)\cos(\beta)\sin(2\theta) & \cos(\alpha)\sin(\beta)\sin(2\theta) & 2\cos^2(\alpha)\cos^2(\theta) & \sin(2\alpha)\cos^2(\theta) \\ \sin(\alpha)\cos(\beta)\sin(2\theta) & \sin(\alpha)\sin(\beta)\sin(2\theta) & \sin(2\alpha)\cos^2(\theta) & 2\sin^2(\alpha)\cos^2(\theta) \end{pmatrix} \tag{21}$$

The matrix is both real and symmetric so it can be diagonalized by orthogonal transformations which can be seen as 4 dimensional rotations in this case.

$$D_h = P^t M^2_{SU(2)} P, \tag{22}$$

$$D_h = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 2\mu^2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \tag{23}$$

where P can again be seen as the matrix that represents the rotation of the eigenvectors from the general vacuum state to the usual one that is obtained by fixing $\theta = \alpha = 0$. The visual representation is not as direct as in the U(1) case for obvious dimensional reasons as we would need a 5 dimensional plot. Nonetheless, we can characterize the eigenspaces for the 2 different eigenvalues we have with no issue at all the following way.

$$\text{Eigenvalue } 2\mu^2 \quad \longrightarrow \quad \vec{w}_{2\mu^2} = \begin{pmatrix} \sin\theta\cos\beta \\ \sin\theta\sin\beta \\ \cos\theta\cos\alpha \\ \cos\theta\sin\alpha \end{pmatrix} = \vec{n}, \tag{24}$$

$$\text{Eigenvalue } 0 \quad \longrightarrow \quad \vec{w}_0 \text{ such that } \vec{w}_0 \cdot \vec{n} = 0. \tag{25}$$

Note that the eigenvector in eq.(24) is nothing but the normalized real and imaginary components of the vacuum state in eq.(19). This is an analogous result to the one we obtained in the U(1) example, where the massive scalar field h was pointing in same direction as the vacuum, we therefore, can call this particular field combination the Higgs boson,

$$h = \vec{n} \cdot \vec{\varphi}, \tag{26}$$

$$\pi = \vec{w}_0 \cdot \vec{\varphi}, \tag{27}$$

with $\vec{\varphi} = (\varphi_1, \varphi_2, \varphi_3, \varphi_4)^t$. The space perpendicular to $\vec{n}$ where the massless π fields live is described by eq.(25). This equation has infinitely many solutions but only 3 will be lineally independent as 3 is the dimension of the massless eigenspace and this gives us the freedom to define the basis eigenvector as we wish. What this mass squared matrix tells us is that three Goldstone bosons arise from this symmetry group, this implies that we should expect to get three gauge bosons which acquire mass according to Goldstone's theorem. On the other hand we are still left with 1 massive scalar particle, the Higgs boson. In this case we have 3 different group generators for our symmetry so we will find a mass for all the associated fields. To confirm this fact, we study the kinetic term.

$$(D^\mu\phi)^\dagger D_\mu\phi = \left(\partial^\mu\phi - ig\tilde{A}^\mu\phi\right)^\dagger \left(\partial_\mu\phi - ig\tilde{A}_\mu\right). \tag{28}$$

If we focus on the term which is quadratic in the gauge fields, which is the one of interest to find the corresponding masses, and we use the $\tilde{A}_\mu$ definition in eq.(17) explicitly, we get,

$$(D^\mu\phi)^\dagger D_\mu\phi = \frac{1}{4}g^2\phi_0^\dagger A^{a\mu}A_\mu^b\sigma^a\sigma^b\phi_0 + ... \tag{29}$$

where we ignored every term except for the term quadratic in A_μ. This expression is not symmetric in a, b but this is an easily solvable issue if we use the known commutation and anticommutation relations for sigma matrices [7].

$$A^{a\mu}A_\mu^b\sigma^a\sigma^b = A^{a\mu}A_\mu^b\frac{1}{2}\left(\left[\sigma^a,\sigma^b\right] + \left\{\sigma^a,\sigma^b\right\}\right) = A^{a\mu}A_\mu^b\left(i\varepsilon^{abc}\sigma^c + \delta^{ab}\right) = A^{a\mu}A_\mu^a, \tag{30}$$

where the term involving the Levi-Civita pseudo-tensor is 0 after being contracted with $A^{a\mu}A_\mu^b$. Thanks to this simplification, our mass term for gauge bosons reduces to a diagonal matrix and is hence immediate to write the boson masses.

$$\Delta\mathscr{L} = \frac{1}{2}g^2 A^{a\mu} A^a_\mu \frac{v^2}{4}, \tag{31}$$

$$(m_{Aa})^2 = \frac{g^2 v^2}{4}. \tag{32}$$

As we can see, all 3 gauge bosons get a mass as we had already predicted, but not only they get a mass but all 3 gauge bosons get the same mass m_A. This means that our SSB mechanism broke all three generators (directions) of our SU(2) symmetry equally.

2.2.2 Real triplet representation of SU(2)

So far we have always chosen our fields to transform as the fundamental representation of the group we are studying. Lets briefly take a look at what happens if, without changing the underlying symmetry group, we choose our fields to transform as the real triplet representation of SU(2). Now ϕ is a 3 component real vector under this group. This means that we have to make some little adjustments to the normalization factors in the Lagrangian described in eq.(1). This changes are made to keep the theory consistent in case we describe a complex valued field in terms of two real valued fields, which is something we could always be doing. After this consideration, the Lagrangian we will be using for this section, and only during this section is as follows,

$$\mathscr{L} = -\frac{1}{4}(F_{\mu\nu})^2 + \frac{1}{2}\left(D^\mu\phi\right)^t D_\mu\phi - V(|\phi|)\,, \tag{33}$$

with,

$$V(\phi) = \frac{\mu^2}{2}\phi^t\phi + \frac{\lambda}{4}\left(\phi^t\phi\right)^2. \tag{34}$$

After this subtle but important clarification, we can go back to the same path of action we are already very used to. We build the covariant derivative that ensures the Lagrangian invariance,

$$\left(D_\mu\phi\right)_a = \partial_\mu\phi_a + g\epsilon_{abc}A^b_\mu\phi_c. \tag{35}$$

The vacuum gets a non zero vev and the easiest way to represent this general vacuum state is with the use of the spherical coordinates. We are using α and β as our 2 angles to not get confused with the already existing ϕ symbols for the fields. This vacuum state reduces to a vector pointing in the z direction with the choice of $\alpha = 0$. It is worth noting that we changed the normalization factor for the vev as a consequence of the aforementioned changes related to the fields being real during this section of the study,

$$\phi_0 = v\begin{pmatrix} sin(\alpha)cos(\beta) \\ sin(\alpha)sin(\beta) \\ cos(\alpha) \end{pmatrix} \equiv v\vec{n}. \tag{36}$$

We start by checking the scalar masses that arise from the potential in eq.(34) to find the massless Goldstone bosons, and the massive field h. As usual, we expand the potential around our vacuum state represented by eq.(36),

$$\phi = \phi_0 + \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{pmatrix} = v\vec{n} + \vec{\varphi}. \tag{37}$$

With this expansion we build the Hessian matrix and diagonalize it to get the eigenvalues. We will not show the full not diagonalized matrix this time as it does not give any additional value that has not been already discussed before, and will only make the looks of this work messier. The full matrix reduces to the diagonal form for the usual vacuum choice. The eigenvalues, which have the physical information, do not change with the change of the vacuum state. This only affects the eigenvectors. Exactly as it happened in the first U(1) example, which can be seen in fig. 1, the different elections of the vacuum state only induce a rotation in the corresponding space.

$$D_h = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2\mu^2 \end{pmatrix}. \tag{38}$$

As in the previous SU(2) example, we can characterize the eigenvectors in terms of the massive eigenvector and its orthogonal eigenspace. This is particularly easy in this example as we can see that eigenvector corresponding to the massive eigenvalue, is non other than the vector $\vec{n}$ that appears in the vacuum state. Hence, we can give the full description of the eigenvalues in the following way,

$$\text{Eigenvalue } 2\mu^2 \quad \longrightarrow \quad \vec{w}_{2\mu^2} = \vec{n}, \tag{39}$$

$$\text{Eigenvalue } 0 \quad \longrightarrow \quad \vec{w}_0 \text{ such that } \vec{w}_0 \cdot \vec{n} = 0, \tag{40}$$

and $\vec{w}_0$ references each of the 2 lineally independent eigenvectors associated with the massless eigenvalue. As in the doublet representation, we can define the physical fields with this vectors,

$$h = \vec{n} \cdot \vec{\varphi}, \tag{41}$$

$$\pi = \vec{v}_0 \cdot \vec{\varphi}. \tag{42}$$

We have found 2 massless Goldstone bosons in this model, so we should expect to get 2 massive gauge bosons as a result. Leaving 1 massless gauge boson in our model. We build a covariant derivative to study the kinetic term and hence the gauge boson masses that arise from the SSB,

$$\left(D_\mu \phi\right)_a = \partial_\mu \phi_a + g \epsilon_{abc} A_\mu^b \phi_c. \tag{43}$$

As we can see in the Lagrangian we are using in eq.(33) the kinetic term now adopts a slightly different form,

$$\Delta\mathcal{L} = \frac{1}{2}\left(D^\mu \phi^t\right)_a \left(D_\mu \phi\right)_a = \frac{g^2}{2}\left(\epsilon_{abc} A_\mu^b (\phi_0)_c\right)\left(\epsilon_{ade} A_\mu^d (\phi_0)_e\right) + \dots \tag{44}$$

where again we just wrote explicitly the relevant term to find the masses of the gauge bosons, ignoring any other terms except for the one which is quadratic in A_μ. With eq.(44) we can build the square mass matrix as done before.

$$M_A^2 = v^2 g^2 \begin{pmatrix} \sin^2(\alpha)\sin^2(\beta) + \cos^2(\alpha) & -\sin^2(\alpha)\sin(\beta)\cos(\beta) & -\sin(\alpha)\cos(\alpha)\cos(\beta) \\ -\sin^2(\alpha)\sin(\beta)\cos(\beta) & \sin^2(\alpha)\cos^2(\beta) + \cos^2(\alpha) & -\sin(\alpha)\cos(\alpha)\sin(\beta) \\ -\sin(\alpha)\cos(\alpha)\cos(\beta) & -\sin(\alpha)\sin(\beta)\cos(\alpha) & \sin^2(\alpha) \end{pmatrix}. \tag{45}$$

A first and easy check we can perform is to substitute the particular case $\alpha = 0$ which we said relates to choosing the vacuum pointing in the z direction. This election directly makes our matrix diagonal and leaves our gauge fields A^1 and A^2 with an equal mass of $m_{1,2} = vg$. The gauge boson in the direction in which we chose the vacuum state, remains massless since the SSB has not broken the group generator in that direction. This can be easily visualized for the general case in fig. 2,

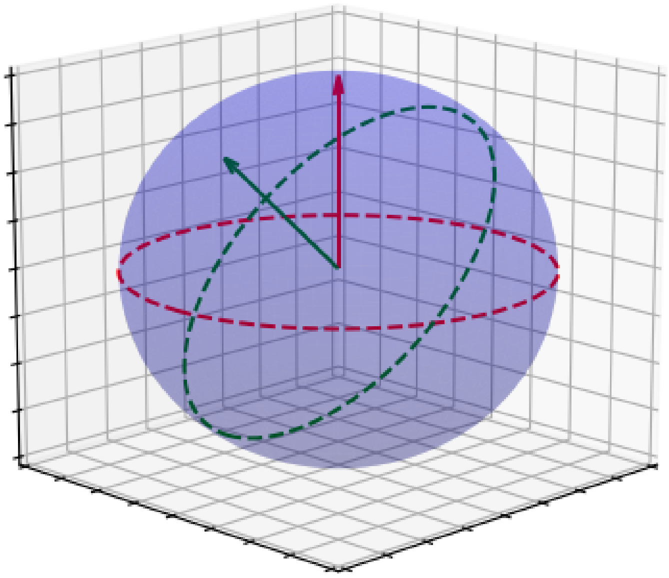

Figure 2: Vacuum states for the real triplet representation of SU(2).

where each of the arrows represent a vacuum choice and the associated circumference represents the unbroken generator.

If we diagonalize the M_A^2 matrix, once again we obtain the same mass eigenvalues, proving that the vacuum state choice is completely arbitrary and does not affect the underlying physics. Analogously to the previous cases, we have a symmetric matrix which we can make diagonal with orthogonal rotations.

$$D_A = P^t M_A^2 P = v^2 g^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{46}$$

As predicted, we get 2 massive bosons and a massless one. This is the first time we are left with a massless gauge boson which is a candidate to be called what we know as the photon, this will also have some additional implication which will be discussed during section 3.3. If we have a look at the eigenvectors associated with these eigenvalues, we can see that they are in complete relation to the eigenvectors associated to the scalar bosons and therefore to the vacuum state choice.

$$\text{Eigenvalue } 0 \rightarrow \vec{u}_0 = \vec{n}, \tag{47}$$

$$\text{Eigenvalue } v^2 g^2 \rightarrow \vec{u}_{v^2 g^2} = \vec{w}_0. \tag{48}$$

But we have not found the correct model yet, as we still need 1 more massive boson while keeping the massless one to build our EW theory according to the known experimental data.

To sum up briefly what we saw with this 3 simpler examples, the Higgs mechanism does not only give an explanation to how gauge bosons can acquire masses without breaking the great properties that symmetries give to the theory, but it also allows for gauge bosons to receive different masses or even remain massless. This fact makes this mechanism much more interesting as that is precisely what we will want to achieve in our final model to reproduce the experimental data. As it is well known, the massive EW gauge bosons, do not have the same masses [13].

3 The SM of electroweak interactions $SU(2)_L \times U(1)_Y$

After going over some simpler symmetry groups to get a catch on the systematic approach we are following, we are ready to dive deep into the symmetry model which gives us the correct predictions matching the experiments. This model was originally proposed by Glashow, Weinberg and Salam, which is why it is often found by the name of the GWS theory [8, 9, 10]. We start by thinking again about the complex doublet representation of SU(2). As we know from our previous study, this theory gives us 3 massive gauge bosons while we need to consider a photon gauge boson as part of our theory, therefore we add an extra U(1) symmetry to build our full SU(2) x U(1) model. But a complete theory requires to also know what are the symmetry groups acting on, that is why we added sub-indexes to the symmetry groups. We will call the charge under this U(1) group, the hypercharge (Y), and we are assigning the scalar field ϕ a hypercharge equal to 1/2. This election is, as many others, arbitrary, a different choice would affect what we call electric charge though as we will see later. With this assignment we can give the transformation of the scalar field under our symmetry group,

$$\phi(x) \to e^{i\Theta_a(x)T^a} e^{i\Theta_Y(x)Y} \phi(x). \tag{49}$$

For the SM Higgs doublet we have,

$$T^a = \frac{\sigma^a}{2}, \quad Y = \frac{1}{2}\mathbf{1}. \tag{50}$$

One last thing we have to address before going through the usual mass calculations we are already used to is the meaning of the L sub-index we wrote on our SU(2) group. Experiments have shown that left-handed particles do not behave the same way right-handed particles do. It has been shown that left-handed particles form doublets under the SU(2) group while right-handed particles transform as singlets. Meaning that they do not interact with this group. The $SU(2)_L$ and $U(1)_Y$ groups commute between each other. For this reason, the elements of the same $SU(2)_L$ doublet will have to share the same hypercharge. Some examples of doublets are,

$$\begin{pmatrix} q_u \\ q_d \end{pmatrix}_L, \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L, \tag{51}$$

on the other hand, examples of singlets are,

$$q_{uR}, q_{dR}, e^-_R, \tag{52}$$

and ν_{eR} if we extend the SM with right-handed neutrinos. Any other generation of quarks or leptons would behave in the exact same way both for doublets and singlets.

We can characterize the general vacuum state as we did with the doublet representation of the simple SU(2) group,

$$\phi_0 = \frac{ve^{i\xi}}{\sqrt{2}} \begin{pmatrix} e^{i\gamma} sin(\theta) \\ cos(\theta) \end{pmatrix} \equiv \frac{ve^i\xi}{\sqrt{2}} \vec{\mathfrak{n}}, \tag{53}$$

where $\mathfrak{n}$ has been used to emphasize that we are talking about a complex unitary vector. As it is no surprise, we now look for the gauge boson masses, so we need the covariant derivative. In this case, as we have a group which is not a simple Lie group but the direct product of 2 different groups, we will use different names for the three SU(2) gauge fields and for the U(1) one, to avoid unnecessary confusion.

$$D_\mu\phi = \left(\partial_\mu - \frac{ig}{2}W_\mu^a\sigma^a - \frac{ig'}{2}B_\mu\right)\phi, \tag{54}$$

with W_μ and B_μ the SU(2) and U(1) gauge fields, respectively. Notice that the coupling constants are not necessarily the same for both groups so we name them g and g' to distinguish them. We can expand our fields around this vacuum state as we did in eq.(20). In fact, the calculations involving the mass eigenstates of the scalar particles are not affected by adding the U(1) group to our symmetry, so the results in this part are identical to the ones obtained in sec.2.2.1 that we can see in eqs. (26) and (27) by just renaming our parameters as follows: $\xi = \beta$ and $\gamma = \alpha - \beta$. A remainder of what results we got in that section, we found 3 Goldstone bosons and a massive scalar particle, this is a good start as now we have a group with 4 different generators. We know that we will get 3 massive gauge bosons and a massless one. Now we just need to perform the explicit calculations to discover if the masses that arise from the Higgs mechanism explain the experimental observations. From the kinetic term, $(D^\mu\phi)^\dagger(D_\mu\phi)$, we have a look at the relevant term for our mass calculation,

$$\Delta\mathcal{L} = \frac{v^2}{8}\left(e^{-i\gamma}\sin\theta, \cos\theta\right)\left(gW^{a\mu}\sigma^a + g'B^\mu\right)\left(gW_\mu^b\sigma^b + g'B_\mu\right)\begin{pmatrix} e^{i\gamma}\sin\theta \\ \cos\theta \end{pmatrix}. \tag{55}$$

If we operate explicitly this matrix product, remembering the properties of the Pauli matrices, and then reordered the information as we have been doing with all the different models by building the Hessian matrix, we obtain,

$$M^2_{GWS} = \frac{g^2v^2}{4}\begin{pmatrix} 1 & 0 & 0 & \frac{g'}{g}\sin(\zeta)\cos(\gamma) \\ 0 & 1 & 0 & -\frac{g'}{g}\sin(\zeta)\sin(\gamma) \\ 0 & 0 & 1 & -\frac{g'}{g}\cos(\zeta) \\ \frac{g'}{g}\sin(\zeta)\cos(\gamma) & -\frac{g'}{g}\sin(\zeta)\sin(\gamma) & -\frac{g'}{g}\cos(\zeta) & \frac{g'^2}{g^2} \end{pmatrix}. \tag{56}$$

As we can see, we have an interaction between the U(1) gauge boson and the SU(2) gauge bosons, but not between SU(2) bosons themselves. It is also worth noting that the angle θ we had been using to describe the vacuum state in eq.(53) is no longer visible, and instead a new parameter, $\zeta = 2\theta$ emerged while the global phase described by ξ has completely vanished from the analysis. This indicates that not only does the choice of the vacuum not affect the mass eigenstates, but that a global phase introduced into the vacuum will not even affect the eigenvectors. The relative phase between the lower and upper component of our doublet is the only appearing phase. This fact also indicates that there was a "better" way to express our general vacuum state in terms the angle 2θ. As we have been discussing through out all this work, "better" is just an aesthetic property and any vacuum choice is physically equivalent. Just for once, and because this is the physical model that will give the correct mass predictions, we will give an in depth analysis of the particular and usual vacuum state before diagonalizing the general matrix.

3.1 Standard vacuum state

If we rewrite eq.(56) with a particularly easy choice of the vacuum state, $\phi_0 = \left(0, v/\sqrt{2}\right)$ which corresponds to $\theta = \xi = 0$ we can still see there is a mix between B_μ and W_μ^3 gauge fields. Obtaining the eigenvectors and eigenvalues of this matrix is almost immediate as we only have to work on the

2×2 block relating the mixed gauge bosons. This way we obtain the mass squared matrix,

$$D_{\text{GWS}} = \begin{pmatrix} \frac{v^2 g^2}{4} & 0 & 0 & 0 \\ 0 & \frac{v^2 g^2}{4} & 0 & 0 \\ 0 & 0 & \frac{v^2\left(g^2+g'^2\right)}{4} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \tag{57}$$

giving us 3 massive gauge bosons, 2 of which have the same mass and a massless boson. The P basis change matrix gives us the transformation between the group generators and the mass eigenstates,

$$P_{\text{GWS}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{g}{\sqrt{g^2+g'^2}} & \frac{g'}{\sqrt{g^2+g'^2}} \\ 0 & 0 & \frac{-g'}{\sqrt{g^2+g'^2}} & \frac{g}{\sqrt{g^2+g'^2}} \end{pmatrix}. \tag{58}$$

Now we can directly read the eigenvectors from the base transformation matrix and finally give name to the EW gauge bosons.

$$W_\mu^+ = \frac{1}{\sqrt{2}}\left(W_\mu^1 - iW_\mu^2\right), \tag{59}$$

$$W_\mu^- = \frac{1}{\sqrt{2}}\left(W_\mu^1 + iW_\mu^2\right), \tag{60}$$

$$Z_\mu^0 = \frac{1}{\sqrt{g^2+g'^2}}\left(gW_\mu^3 - g'B_\mu\right), \tag{61}$$

$$A_\mu = \frac{1}{\sqrt{g^2+g'^2}}\left(g'W_\mu^3 + gB_\mu\right). \tag{62}$$

The definitions of the $W^\pm$ bosons are completely arbitrary since we have a 2 dimensional eigenspace, and therefore we have the freedom to redefine the 2 basis vectors of that space. The usual and very useful way to do it, is to choose them such that the 2 gauge bosons are also electric charge eigenstates, same way the Z_μ^0 is neutrally charged. This affirmation is by no means trivial and should not be taken for granted. Despite the undeniable interest that diving into this topic has, it is not within this work to rigorously prove that fact. Nevertheless, you might be able to convince yourself of the validity of the claim by looking at the way the charged currents associated to this bosons couple to fermions of which we do know the electric charge, as can be seen in [11]. This might well be considered an unsatisfactory solution by some people so we will give another alternative path to prove this claim at sec.3.3 of this work. Finally, the masses squared of this 4 EW mediator particles are,

$$m_W^2 = \frac{v^2 g^2}{4}, \tag{63}$$

$$m_Z^2 = \frac{v^2\left(g^2+g'^2\right)}{4}, \tag{64}$$

$$m_A^2 = 0. \tag{65}$$

The mix between the SU(2) third generator and the U(1) generator, gave the corresponding boson, Z_μ^0 an extra mass in comparison to its otherwise indistinguishable partners, the $W^\pm$ bosons, as we had seen in the pure SU(2) example, eq. (32). With this analysis in mind we can go back to the general vacuum state study.

3.2 General vacuum state

If we diagonalize the mass squared matrix in eq. (56) we will of course obtain the same mass eigenvalues as in the particular vacuum choice. As we have seen with all the previous simple models, the eigenvalues are independent of the vacuum choice, but the eigenstates in eqs. (59)–(62) will change depending on the vacuum state. To sum up, the expected result of the calculations we will perform is to obtain the same gauge bosons masses but the concrete combinations of fields we will name photon, $W^{\pm}$ and Z boson will change. For aesthetic and historical reasons, we will make a small variable change, as we give the following definition,

$$\tan\theta_W = \frac{g'}{g}, \tag{66}$$

where θ_W is the Weinberg angle. There is very precise experimental determination of this parameter [12]. , This angle also gives a very straight forward idea of how strong the coupling of the gauge bosons is, as for a limit value, $\theta_W = 0$ the U(1) gauge boson is not mixed at all with the SU(2) bosons. With this definition we proceed to the diagonalization of matrix (56), we obtain the same mass eigenvalues which have already been discussed so we will not use more time for a known discussion. We are centering our attention in the matrix eigenvectors. These will give our new gauge bosons in terms of the group gauge fields.

$$W^I_\mu = \mathcal{N}\left(\cos\zeta W^1_\mu + \sin\zeta\cos\gamma W^3_\mu\right), \tag{67}$$

$$W^{II}_\mu = \sin\gamma W^1_\mu + \cos\gamma W^2_\mu, \tag{68}$$

$$Z^0_\mu = \begin{pmatrix} -\cos\theta_W\sin\zeta\cos\gamma \\ \cos\theta_W\sin\zeta\sin\gamma \\ \cos\theta_W\cos\zeta \\ -\sin\theta_W \end{pmatrix}\cdot\begin{pmatrix} W^1_\mu \\ W^2_\mu \\ W^3_\mu \\ B_\mu \end{pmatrix}, \tag{69}$$

$$A_\mu = \begin{pmatrix} -\sin\theta_W\sin\zeta\cos\gamma \\ \sin\theta_W\sin\zeta\sin\gamma \\ \sin\theta_W\cos\zeta \\ \cos\theta_W \end{pmatrix}\cdot\begin{pmatrix} W^1_\mu \\ W^2_\mu \\ W^3_\mu \\ B_\mu \end{pmatrix}. \tag{70}$$

As we anticipated, the concrete field combinations that represent our physical bosons depend on the vacuum state choice. As we did for the standard vacuum state, we could combine the W^I and W^{II} bosons in an appropriate way to form electric charge eigenstates. This task was already of considerable difficulty in the simplest choice of vacuum and as it is easy to imagine, it is much harder now. For this reason we will keep working with the $W^{I,II}$ bosons while keeping in mind that there is a lineal combination of them that would represent the $W^{\pm}$ bosons. In eq.(67) $\mathcal{N}$ is a real constant that ensures that the eigenstate is correctly normalized.

Lets sum up what we found with the $SU(2)\times U(1)_Y$ model so far. From the scalar sector of our Lagrangian, eq.(1) we found a massive scalar particle which we called the Higgs boson, eq.(26) and 3 massless Goldstone bosons, which as we expected from the Goldstone theorem, combined with the gauge sector of the Lagrangian to give us 3 massive gauge bosons; the $W^{I,II}_\mu$ and the Z^0_μ, as well as a massless boson, the photon A_μ. The masses squared we found for this bosons are,

$$m^2_{W^{I,II}} = \frac{v^2 g^2}{4}, \tag{71}$$

$$m^2_{Z^0} = \frac{v^2 g^2}{4}\frac{1}{\cos^2\theta_W}, \tag{72}$$

$$m^2_A = 0. \tag{73}$$

With this result we could already be satisfied as we have found a symmetry model that gives rise to the correct gauge boson masses that we were looking for to build the EW theory, but masses are not the only relevant parameter to build the SM. This is why we are going to make a bit of a change and, for once, focus on a different physical property, electric charge. The definition of this operator and the following discussion will help us understand the difference between left and right-handed particles. This will not only give answer to this question but will also enable us to have a more conclusive answer to some of the topics we have left open throughout the previous discussion.

3.3 Electric charge operator in the SM for a general vacuum

As we have deeply discussed, the Higgs mechanism breaks the symmetry of the theory to give rise to masses for the particles that we include into the theory, but as we have also seen, not all symmetry generators are always broken by the vacuum. When this is the case, the vacuum itself is still invariant under some subgroup of the original symmetry group. Hence, we can find a transformation (g_Q) that leaves ϕ_0 invariant. The infinitesimal generator of this transformation is what we will call electric charge (Q). We can then express the transformation around the identity as an expansion in terms of the generators T^A of the full group and an equal number of real infinitesimal parameters Θ_A,

$$g_Q = \mathbf{1} + Q\epsilon + \mathcal{O}(\epsilon^2) = \mathbf{1} + i\Theta_A T^A + \mathcal{O}(\Theta_A^2). \tag{74}$$

The condition for g_Q to leave the vacuum invariant is completely equivalent to,

$$(Q\epsilon)\phi_0 = \left(\Theta_A T^A\right)\phi_0 = 0, \tag{75}$$

with $\epsilon \sim \Theta_A$. It is important to note that the existence of such charge operator that satisfies eq.(75) is not guaranteed. In fact, we can extract which symmetry models are susceptible of having a charge operator based on the study we have already performed in the scalar and gauge sectors of our Lagrangian. As we already discussed, the Higgs mechanism does not always break all the group generators. When one generator is left unbroken, this symmetry is preserved by the vacuum. We can then only search for the charge operator in the models that have left a massless gauge boson. As we know, this is a direct consequence of not breaking all the group generators. It is also worth discussing that if we find one solution to eq.(75) then, we can find infinitely many as all its multiples will also solve it. This normalization ambiguity can be solved by fixing the value of one of the real parameters Θ_A.

3.3.1 The Higgs doublet

As a practical example, we will calculate the explicit form of the electric charge operator for a fundamental $SU(2)_L$ representation and a hypercharge of 1/2. This election did not happen by chance as this are the exact properties we gave our scalar field ϕ for the general vacuum state previously discussed. To do that, we particularize eq.(75) for the $SU(2)_L \times U(1)_Y$ group as follows,

$$\left(\Theta_A T^A\right)\phi_0 = \left(\Theta_a T^a + \Theta_Y Y\right)\phi_0 = 0. \tag{76}$$

And for the Higgs doublet, the group generators take the form shown in eq.(50). As discussed, this equation has an ambiguity in its normalization, therefore we can choose a fixed value for one of real constants Θ to make the solution unique, following this reasoning, we choose,

$$\Theta_Y = 1. \tag{77}$$

After all this considerations, we are ready to explicitly find the combination of generators that "kills" the vacuum. For that we use the known sigma matrices expressions as well as the general vacuum state represented by eq.(53). Substituting all this information in eq.(76) we get a 2×2 lineal equation system with complex coefficients which we can easily convert into a 4×4 real system which does not offer many complications to be solved. We therefore find the following values for the real parameters Θ_a

$$\Theta_1 = -\sin\zeta\cos\gamma, \tag{78}$$

$$\Theta_2 = \sin\zeta\sin\gamma, \tag{79}$$

$$\Theta_3 = \cos\zeta. \tag{80}$$

We can then see that for the standard vacuum choice, $\phi_0 = \left(0, v/\sqrt{2}\right)^t$, the electric charge operator reduces to the well known relation,

$$Q = T^3 + Y. \tag{81}$$

This formal relation also holds true for different representations and hypercharge values. This gives us an easy path to build this operator for any other particles in the SM. If we build the explicit matrix for the fundamental representation and $Y = 1/2$ as our goal was, we get the charge operator for the Higgs doublet,

$$Q = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}. \tag{82}$$

This means that both the upper and lower component of the scalar field ϕ are charge eigenstates but have a different eigenvalue. We can therefore rewrite,

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}. \tag{83}$$

If we have a look at the standard vacuum scalar eigenstates for this group, which can be found in eq.(23), we can see that the Higgs bosons is part of the lower component. For this reason we can conclude that the Higgs bosons has the following properties,

$$h \quad \longrightarrow \quad S_h = 0\ ,\ M_h^2 = 2\mu^2\ ,\ Q_h = 0. \tag{84}$$

S_h is the Higgs bosons spin, which is of course 0 as it is an scalar particle. For any other $SU(2)_L$ doublet like the ones in eq.(51), the upper component will always have 1 more electric charge unit than the lower component, as Y might change from doublet to doublet, but they all share the same $T^3 = \frac{\sigma^3}{2}$. Nevertheless, we just saw that this is not trivially true for a general vacuum state, as you would get the corresponding combination of generators according to the parameters Θ_a we have found. This combination does not make Q a diagonal matrix so we can find its eigenvalues to see how it acts on doublets,

$$Q = \frac{1}{2}\begin{pmatrix} 2\cos^2\theta & -e^{i\gamma}\sin 2\theta \\ -e^{-i\gamma}\sin 2\theta & 2\sin^2\theta \end{pmatrix}, \tag{85}$$

$$D_Q = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}. \tag{86}$$

We have reintroduced the parameter θ so that the relation with the vacuum is seen more easily. Note that except for the complex phases and the normalization factor, this matrix is identical to one we have already seen in eq.(8) so the eigenvectors will also be closely related. The matrix eigenvectors will again tell us what are the exact combinations that represent the physical charge eigenstates,

$$\text{Eigenvalue } 0 \quad \longrightarrow \quad \vec{w}_0 = \begin{pmatrix} e^{i\gamma}\cos\theta \\ \sin\theta \end{pmatrix} = \mathfrak{n}, \tag{87}$$

$$\text{Eigenvalue } 1 \quad \longrightarrow \quad \vec{w}_1 = \begin{pmatrix} -e^{i\gamma}\sin\theta \\ \cos\theta \end{pmatrix} = \mathfrak{n}_\perp. \tag{88}$$

As we can see, the eigenstate corresponding to the eigenvalue 0 is always the one defined by the vacuum state as seen in eq.(53). The perpendicular space is the eigenspace with eigenvalue 1. This means that the Higgs boson which is also found in the direction defined by the vacuum, maintains the exact same physical properties we described in (84). We can now see that the charge eigenvalues do not change and the observations on how this operator acts on doublets stands as discussed. This is a new proof that the vacuum state choice affects the definition of the operators that describe the physical reality, but it cannot affect the true physical properties, not only with masses as we have seen plenty of times but also with any other observable. On the other hand, for $SU(2)_L$ singlets in eq.(52), the T^a generators are 0, so even for a the general vacuum choice, the charge is trivially just the same as the hypercharge. Now we have precise knowledge on how the electric charge operator is defined. This gives us an alternative and more rigorous way of finding the charge eigenstates $W^{\pm}$. As we discussed in 3.2, there is a lineal combination, analogous to equations (59) and (60) that gives us two charge eigenstates. Building the corresponding charge operator would be a way to look for this particular combination, but this is something that we will not look into.

4 Conclusions

Through out this work, we have studied the Higgs mechanism and the consequences of choosing a completely general vacuum state. Firstly and most importantly, we have concluded that the physical properties, represented by the eigenvalues of each of the operators we have studied (mass and electric charge) are not affected by this arbitrary election. It is therefore impossible to infer from the charges and masses what vacuum election was made to carry out the calculations. This arbitrary election leaves its mark in the form of the different eigenstates. However, we have also seen through the different examples that there is a difference between the scalar and gauge sectors. For the scalar mass eigenstates, as it is easiest to see in the U(1) example studied in section 2.1, a global phase in the vacuum ϕ_0 does affect the scalar eigenvectors. Conversely, for the gauge bosons, this global phase in the vacuum is completely irrelevant, even for the eigenvectors. This pattern repeats itself during the rest of the examples. The reason for this is that in the scalar sector, this global phase in the vacuum state also provides a relative phase between the vacuum and the φ_i expansion fields. This does not happen in the expansion of the kinetic term that gives place to the gauge boson mass terms as these always just include a $|\phi_0|^2$ factor. This automatically kills any global phase introduced into the vacuum.

Going more into the detailed conclusions of global symmetry SSB. We have seen that in all the studied examples we found one massive scalar particle, the Higgs boson and one or more massless bosons. The Higgs boson has always been found as the combination of scalar fields pointing in the radial direction defined by the vacuum, and for the SM we found its main properties as shown in eq.(84). On the contrary, all the perpendicular directions remain as massless Goldstone bosons. Theories with more than one Higgs boson are mathematically possible but such models are beyond the SM and the scope of this work [14].

Concerning the electric charge operator, we can conclude that it behaves very similarly to the mass squared operators. Its eigenvalues do not change with the vacuum and only the eigenvectors which define the physical eigenstates are affected. We have also explicitly studied the difference between various particle representations. The $SU(2)_L$ fundamental representation transforms as doublets and will always have a component with one extra electric charge unit. On the other hand, $SU(2)_L$ singlets always have an electric charge equal to their hypercharge.

To finish, I would like to highlight once again the main conclusion one can extract from this work: the physical quantities are never affected by the choice researchers might take regarding the vacuum state.

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