

A DECOMPOSITION THEOREM FOR POLYMEASURES

FERNANDO BOMBAL, DAVID PÉREZ-GARCÍA, AND IGNACIO VILLANUEVA

ABSTRACT. We prove that every countably additive polymeasure can be decomposed in a unique way as the sum of a “discrete” polymeasure plus a “continuous” polymeasure. This generalizes a previous result of Saeki.

1. INTRODUCTION

Since the work of Riesz and [1], the interplay between measure theory and linear operators defined on spaces of continuous functions has proven to be a very fruitful tool in mathematical analysis [4]. As in the linear case, multilinear operators (or tensor products) on $C(K)$ spaces play a prominent role in the multilinear and holomorphy theory of Banach spaces [2] and have applications in Harmonic Analysis [3], Complex Analysis [5] and even Quantum Information Theory [6]. Thanks to the so-called polymeasures [2, 6, 15], one has at hand a multilinear measure theory that, as in the linear case, can help to get a deeper insight in the theory. Hence, the understanding of polymeasures can (and indeed does) improve our knowledge about multilinear operators on $C(K)$ spaces (as examples of that see [7]).

In this paper we work in this direction by providing a decomposition of every countably additive polymeasure (in a unique way) as the sum of a “discrete” polymeasure plus a “continuous” one. This simplifies the problem of studying general polymeasures to the the study of the discrete and continuous case which, in many cases, is simpler [8].

Our result provides (with a simple proof based on measure theory) a generalization of a result in [14], where the author proves a decomposition theorem for the elements of $(C(K_1) \widehat{\otimes}_\pi \cdots \widehat{\otimes}_\pi C(K_n), X)^*$, where K_i are compact Hausdorff spaces, X is a Banach space such that X^* verifies certain geometrical decomposition (which turns out to be equivalent to X^* not containing isomorphic copies of c_0), and $E \widehat{\otimes}_\pi F$ stands for the projective tensor product of E and F .

The notations and terminology used along the paper will be the standard in Banach Space Theory and Measure Theory, as for instance in [4]. However, before going any further we shall explain some terminology and state some known facts we shall use: X will always be a Banach space, and X^* its dual. K, K_i will always be compact Hausdorff spaces, and $C(K)$ will denote the Banach space of the continuous scalar valued measures with the supremum norm. Given a compact space K and its Borel σ -algebra Σ , we write $B(K)$ for the completion under the supremum norm of the

2000 *Mathematics Subject Classification.* 46G25.

All three authors were partially supported by DGICYT grant BMF2001-1284.

space $S(K)$ of the Σ -simple scalar valued functions. It is well known that $C(K) \xrightarrow{1} B(K) \xrightarrow{1} C(K)^{**}$, where $\xrightarrow{1}$ denotes isometric embedding.

Let $T : C(K_1) \times \cdots \times C(K_n) \longrightarrow X$ be a multilinear operator. Then, we know that T can be extended to a multilinear operator $\bar{T} : B(K_1) \times \cdots \times B(K_n) \longrightarrow X^{**}$, in a unique way if we ask \bar{T} to be separately weak*-weak* continuous, when the weak* topology we consider in $B(K_i)$ is given by the isometric inclusion $B(K_i) \hookrightarrow C(K_i)^{**}$ (see [2]). In fact, this extension is nothing but the restriction of the so called Aron-Berner extension $AB(T) : C(K_1)^{**} \times \cdots \times C(K_n)^{**} \longrightarrow X^{**}$ to the product $B(K_1) \times \cdots \times B(K_n)$ (see, for instance [5]).

Once we have defined \bar{T} , we can define the set function

$$\gamma : \Sigma_1 \times \cdots \times \Sigma_n \longrightarrow X^{**}$$

given by

$$\gamma(A_1, \dots, A_n) = \bar{T}(\chi_{A_1}, \dots, \chi_{A_n}).$$

Thus defined, γ is a weak* countably additive *polymeasure*, that is, a separately additive set function such that, for every $x^* \in X^*$, $x^* \circ \gamma$ is separately countably additive.

We say that a polymeasure $\gamma : \Sigma_1 \times \cdots \times \Sigma_n \longrightarrow X$ is countably additive (resp. regular) if it is separately countably additive (resp. regular).

Given a polymeasure $\gamma : \Sigma_1 \times \cdots \times \Sigma_n \longrightarrow X$, as in the case $n = 1$, its *semivariation* is defined as the set function

$$\|\gamma\| : \Sigma_1 \times \cdots \times \Sigma_n \longrightarrow [0, +\infty]$$

given by

$$\|\gamma\|(A_1, \dots, A_n) = \sup \left\{ \left\| \sum_{m_1=1}^{r_1} \cdots \sum_{m_n=1}^{r_n} a_{m_1}^1 \cdots a_{m_n}^n \gamma(A_{m_1}^1, \dots, A_{m_n}^n) \right\| \right\}$$

where the supremum is taken over all the finite Σ_i -partitions $(A_{m_i}^i)_{m_i=1}^{r_i}$ of A_i ($1 \leq i \leq n$) and all the collections $(a_{m_i}^i)_{m_i=1}^{r_i}$ in the unit ball of the scalar field.

It is easy to see that, if for some $1 \leq i \leq n$ we have $A_i = B_i \cup C_i$ (disjointly), then

$$(1) \quad \|\gamma\|(A_1, \dots, A_n) \leq \|\gamma\|(A_1, \dots, B_i, \dots, A_n) + \|\gamma\|(A_1, \dots, C_i, \dots, A_n)$$

We can also define the *supremation* of a polymeasure as the set function

$$\bar{\gamma} : \Sigma_1 \times \cdots \times \Sigma_n \longrightarrow [0, +\infty]$$

given by

$$\bar{\gamma}(A_1, \dots, A_n) = \sup_{B_i \subset A_i, B_i \in \Sigma_i} \{\|\gamma(B_1, \dots, B_n)\|\}$$

In [6] it is proved that $\bar{\gamma} \leq \|\gamma\| \leq 4^n \bar{\gamma}$.

Regarding this, we will also need the following lemma. Its simple proof, which uses just induction and the separated regularity of the polymeasure, can be seen in [12].

Lemma 1.1. *Let $\gamma : \Sigma_1 \times \cdots \times \Sigma_n \longrightarrow X$ be a regular polymeasure. Then, given $(A_1, \dots, A_n) \in \Sigma_1 \times \cdots \times \Sigma_n$, for every $\epsilon > 0$ there exist compact sets $(C_1, \dots, C_n) \in \Sigma_1 \times \cdots \times \Sigma_n$ with $C_i \subset A_i$ ($1 \leq i \leq n$) such that*

$$\|\gamma(A_1, \dots, A_n) - \gamma(C_1, \dots, C_n)\| < \epsilon.$$

For the theory of polymeasures and its applications to the study of multilinear operators on $C(K)$ spaces see [2], [12], [15] and the references therein.

If E_1, \dots, E_n, X are Banach spaces, a multilinear operator $T : E_1 \times \cdots \times E_n \longrightarrow X$ is said to be *completely continuous* if, given weakly Cauchy sequences $(x_i^m)_{m \in \mathbb{N}} \subset E_i$, ($1 \leq i \leq n$), the sequence $(T(x_1^m, \dots, x_n^m))$ is norm convergent, equivalently, if given weakly null sequences $(x_i^m)_{m \in \mathbb{N}} \subset E_i$, ($1 \leq i \leq n$), the sequence $(T(x_1^m, \dots, x_n^m))$ is norm null. These operators are studied, among other places, in [10, 11, 13].

In this note it will be specially relevant [15, Theorem 5], which states that, with the above notation, T is completely continuous if and only if γ is countably additive (equivalently if and only if γ is regular). It follows from [9, Corollary 4.8] that in this case T is also *unconditionally converging*, in the sense defined in [7] (see also [9]).

The notation $\cdots^{[i]}$ means that the i^{th} coordinate is not involved.

Definition 1.2. *We say that a polymeasure $\gamma : \Sigma_1 \times \cdots \times \Sigma_n \longrightarrow X$, representing an operator $T : C(K_1) \times \cdots \times C(K_n) \longrightarrow X$, is continuous if, given $(s_1, \dots, s_n) \in K_1 \times \cdots \times K_n$, for any $\epsilon > 0$ there exist neighborhoods A_i of s_i ($1 \leq i \leq n$) such that*

$$\|\gamma\|(A_1, \dots, A_n) < \epsilon.$$

Definition 1.3. *We say that a polymeasure $\gamma : \Sigma_1 \times \cdots \times \Sigma_n \longrightarrow X$, representing an operator $T : C(K_1) \times \cdots \times C(K_n) \longrightarrow X$, is discrete if there exist sequences $(s_i^{m_i})_{m_i \in \mathbb{N}} \subset K_i$ ($1 \leq i \leq n$) and a sequence of scalars $(\alpha_{m_1 \dots m_n})_{m_i \in \mathbb{N}}$ such that*

$$\gamma(A_1, \dots, A_n) = \sum_{m_1} \cdots \sum_{m_n} \alpha_{m_1 \dots m_n} \delta_{s_1^{m_1}}(A_1) \cdots \delta_{s_n^{m_n}}(A_n),$$

where

$$\delta_{s_i^{m_i}}(A_i) = \begin{cases} 1 & \text{if } s_i^{m_i} \in A_i \\ 0 & \text{if } s_i^{m_i} \notin A_i \end{cases}$$

Remark 1.4. It is important to notice that, in this definition, the convergence of the series need *not* be unconditional. In fact, in the case of scalar measures, the convergence is unconditional if and only if $v(\gamma) < \infty$, equivalently if and only if γ can be extended to a product measure [3].

2. THE RESULT

The following two lemmas contain most of the technical parts of the proof of the main result.

Lemma 2.1. *Let $T : C(K_1) \times \cdots \times C(K_n) \longrightarrow X$ be a multilinear operator such that its associated polymeasure $\gamma : \Sigma_1 \times \cdots \times \Sigma_n \longrightarrow X$ is countably*

additive. Given any $(s_1, \dots, s_n) \in K_1 \times \dots \times K_n$ such that $\gamma(\{s_1\}, \dots, \{s_n\}) = 0$, for every $\epsilon > 0$ there exist neighborhoods A_i of s_i ($1 \leq i \leq n$) such that

$$\|\gamma\|(A_1, \dots, A_n) < \epsilon.$$

Proof. First we will prove that given an operator $T : C(K_1) \times \dots \times C(K_n) \rightarrow X$ with a countably additive associated polymasure $\gamma : \Sigma_1 \times \dots \times \Sigma_n \rightarrow X$, given $\mathbf{s} := (s_1, \dots, s_n) \in K_1 \times \dots \times K_n$ and given $\epsilon > 0$ there exist neighborhoods A_i of s_i ($1 \leq i \leq n$) such that

$$(2) \quad \|\gamma\|(A_1 \setminus \{s_1\}, \dots, A_n \setminus \{s_n\}) < \epsilon.$$

Assume this is not true. Then there exists $\epsilon_0 > 0$ such that for every neighborhood A_i of s_i ($1 \leq i \leq n$)

$$(3) \quad \|\gamma\|(A_1 \setminus \{s_1\}, \dots, A_n \setminus \{s_n\}) > \epsilon_0.$$

We pick first arbitrary neighborhoods A_i^1 of s_i ($1 \leq i \leq n$). Since (3) holds, we can apply Lemma 1.1 together with the comments above it to obtain that for every $1 \leq i \leq n$ there exists a compact set $C_i^1 \subset A_i^1 \setminus \{s_i\}$ such that

$$\|\gamma(C_1^1, \dots, C_n^1)\| > \frac{\epsilon_0}{2 \cdot 4^n}$$

For every $1 \leq i \leq n$ we define now $A_i^2 := A_i^1 \setminus C_i^1$. This is again a neighborhood of s_i , so we can repeat the reasoning to find compact sets C_i^2 ($1 \leq i \leq n$) such that

$$\|\gamma(C_1^2, \dots, C_n^2)\| > \frac{\epsilon_0}{2 \cdot 4^n}$$

Proceeding inductively for every $1 \leq i \leq n$ we construct a sequence $(C_i^m)_{m \in \mathbb{N}}$ of compact mutually disjoint sets such that, for every $m \in \mathbb{N}$,

$$(4) \quad \|\gamma(C_1^m, \dots, C_n^m)\| > \frac{\epsilon_0}{2 \cdot 4^n}$$

But this is not possible: first let us note that the operator $\varphi : c_0 \rightarrow B(K_i)$ defined by

$$\varphi(e_m) = \chi_{C_i^m}$$

is clearly continuous, therefore the sequence $(\chi_{C_i^m})_{m \in \mathbb{N}} \subset B(\Sigma_i)$ weakly converges to 0. On the other hand, according to [9, Corollary 4.8] or [15, Proposition 8] the multilinear operator \bar{T} defined in the introduction is completely continuous, therefore the sequence $(\bar{T}(\chi_{C_1^m}, \dots, \chi_{C_n^m}))_m = (\gamma(C_1^m, \dots, C_n^m))_m$ must converge to 0 in norm, a contradiction with (4). This proves (2).

To finish the proof of the lemma, we use induction. The result is known to be true for $n = 1$; we suppose it true for $n - 1$ and we prove the case n . We first observe that for every $1 \leq i \leq n$, the restricted polymasure $\gamma_{s_i} : \Sigma_1 \times \dots \times \Sigma_n \rightarrow X$ defined by

$$\gamma_{s_i}(A_1, \dots, A_n) = \gamma(A_1, \dots, A_{i-1}, \{s_i\}, A_{i+1}, \dots, A_n)$$

are all of them as in the hypothesis of the lemma. Therefore, given $\epsilon > 0$, by taking finite intersections if necessary, we can consider, for every $1 \leq i \leq n$

a neighborhood A_i of s_i verifying (2) and the result of the lemma for all of the restricted polymeasures above mentioned.

Finally, using (1), we have that

$$\|\gamma\|(A_1, \dots, A_n) \leq \|\gamma_{s_1}\|(A_2, \dots, A_n) + \|\gamma\|(A_1 \setminus \{s_1\}, A_2, \dots, A_n),$$

and we can use again (1) in $\|\gamma\|(A_1 \setminus \{s_1\}, A_2, \dots, A_n)$ to obtain that

$$\begin{aligned} \|\gamma\|(A_1, \dots, A_n) &\leq \|\gamma_{s_1}\|(A_2, \dots, A_n) + \|\gamma_{s_2}\|(A_1 \setminus \{s_1\}, A_3, \dots, A_n) + \\ &\quad + \|\gamma\|(A_1 \setminus \{s_1\}, A_2 \setminus \{s_2\}, A_3, \dots, A_n). \end{aligned}$$

Continuing this procedure we can conclude that

$$\begin{aligned} \|\gamma\|(A_1, \dots, A_n) &\leq \|\gamma_{s_1}\|(A_2, \dots, A_n) + \|\gamma_{s_2}\|(A_1 \setminus \{s_1\}, A_3, \dots, A_n) + \\ &\quad \dots + \|\gamma_{s_n}\|(A_1 \setminus \{s_1\}, \dots, A_{n-1} \setminus \{s_{n-1}\}) + \\ &\quad + \|\gamma\|(A_1 \setminus \{s_1\}, \dots, A_n \setminus \{s_n\}) \\ &\leq (n+1)\epsilon. \end{aligned}$$

□

Remark 2.2. What this lemma essentially says is that a polymeasure γ is continuous if and only if $\gamma(\{s_1\}, \dots, \{s_n\}) = 0$ for any point $(s_1, \dots, s_n) \in K_1 \times \dots \times K_n$.

Lemma 2.3. *Let $T : C(K_1) \times \dots \times C(K_n) \longrightarrow X$ be a multilinear operator such that its associated polymeasure $\gamma : \Sigma_1 \times \dots \times \Sigma_n \longrightarrow X$ is countably additive. Then there exist at most countably many $s = (s_1, \dots, s_n) \in K_1 \times \dots \times K_n$ such that*

$$\gamma(\{s_1\}, \dots, \{s_n\}) \neq 0.$$

Proof. We reason by induction on n . For $n = 1$ (in that case γ is just a regular countably additive measure) the result is well known. We suppose the result true for $n - 1$ and we set to prove it for n . Suppose the result is not true in this case. Then there exists $\epsilon > 0$ such that the set

$$C := \{(s_1, \dots, s_n) \text{ such that } \|\gamma(\{s_1\}, \dots, \{s_n\})\| > \epsilon\}$$

is not countable. We will inductively choose a sequence $(x_m)_{m \in \mathbb{N}} \subset C$. We start by choosing $x_1 := (s_1^1, \dots, s_n^1)$ any element of C . We show now how to choose x_m once $x_1 = (s_1^1, \dots, s_n^1), \dots, x_{m-1} = (s_1^{m-1}, \dots, s_n^{m-1})$ have been chosen. For every $1 \leq j \leq m - 1$ and for every $1 \leq i \leq n$ we consider the (regular, countably additive) $(n - 1)$ -polymeasure

$$\gamma_{s_i^j} : \Sigma_1 \times \overset{[i]}{\dots} \times \Sigma_n \longrightarrow X$$

given by

$$\gamma_{s_i^j}(A_1, \overset{[i]}{\dots}, A_n) = \gamma(A_1, \dots, A_{i-1}, \{s_i^j\}, A_{i+1}, \dots, A_n)$$

for every $(A_1, \overset{[i]}{\dots}, A_n) \in \Sigma_1 \times \overset{[i]}{\dots} \times \Sigma_n$

By the inductive step (on n) we know that the set

$$C_i^j := \left\{ (s_1, \dots, s_{i-1}, s_i^j, s_{i+1}, \dots, s_n) \text{ such that} \right.$$

$$\left. (s_1, \overset{[i]}{\dots}, s_n) \in K_1 \times \overset{[i]}{\dots} \times K_n \text{ and } \gamma(s_1, \dots, s_{i-1}, s_i^j, s_{i+1}, \dots, s_n) = \right.$$

$$= \gamma_{s_i^j}(s_1, \dots, s_n) \neq 0\}$$

is at most countable.

Hence the set

$$C' := \bigcup_{i=1}^n \bigcup_{j=1}^{m-1} C_i^j$$

is at most countable, and therefore $C \setminus C'$ is not empty. So, we choose x_m any element in $C \setminus C'$.

Once we have chosen $(x_m = (s_1^m, \dots, s_n^m))_{m \in \mathbb{N}} \subset C$ as above, for every $1 \leq i \leq n$ we consider the set

$$S_i := \bigcup_{m \in \mathbb{N}} \{s_i^m\}.$$

Since $S_i \in \Sigma_i$ and since γ is countably additive, we have that $\gamma(S_1, \dots, S_n)$ is well defined and

$$\gamma(S_1, \dots, S_n) = \sum_{m_1} \cdots \sum_{m_n} \gamma(\{s_1^{m_1}\}, \dots, \{s_n^{m_n}\}).$$

But the construction of the sequence (x_m) implies that whenever there exist $i, k \in \{1, \dots, n\}$ such that $m_i \neq m_k$, we have that $\gamma(\{s_1^{m_1}\}, \dots, \{s_n^{m_n}\}) = 0$, hence

$$\gamma(S_1, \dots, S_n) = \sum_m \gamma(\{s_1^m\}, \dots, \{s_n^m\})$$

but this series can not converge because $\|\gamma(\{s_1^m\}, \dots, \{s_n^m\})\| > \epsilon$, thus we reached a contradiction. \square

Theorem 2.4. *Let $T : C(K_1) \times \cdots \times C(K_n) \longrightarrow X$ be a multilinear operator with a countably additive associated polymasure $\gamma : \Sigma_1 \times \cdots \times \Sigma_n \longrightarrow X$. Then γ can be written in a unique way as*

$$\gamma = \gamma_c + \gamma_d$$

where γ_c is continuous and γ_d is discrete.

Proof. Let γ be as in the hypothesis. Let $(s_m)_{m \in \mathbb{N}} := (s_1^m, \dots, s_n^m)_{m \in \mathbb{N}} \subset K_1 \times \cdots \times K_n$ be the sequence (possibly finite or even empty) of elements in $K_1 \times \cdots \times K_n$ such that $\gamma(\{s_1\}, \dots, \{s_n\}) \neq 0$ (see Lemma 2.3). For every $1 \leq i \leq n$ we define the set $S_i \subset K_i$ by

$$S_i = \bigcup_{m \in \mathbb{N}} \{s_i^m\}.$$

S_i , being the countable union of closed sets, is a Borel set of K_i . Hence we can define γ_d by

$$\gamma_d(A_1, \dots, A_n) = \gamma(A_1 \cap S_1, \dots, A_n \cap S_n)$$

for every $(A_1, \dots, A_n) \in \Sigma_1 \times \cdots \times \Sigma_n$. Since γ is countably additive, so is γ_d , therefore

$$\begin{aligned} \gamma_d(A_1, \dots, A_n) &= \gamma(A_1 \cap S_1, \dots, A_n \cap S_n) = \\ &= \sum_{m_1} \cdots \sum_{m_n} \gamma(\{s_1^{m_1}\}, \dots, \{s_n^{m_n}\}) \delta_{s_1^{m_1}}(A_1) \cdots \delta_{s_n^{m_n}}(A_n) \end{aligned}$$

and γ_d is discrete. As remarked above, the series is known to be convergent in the given order, but in general it is not unconditionally convergent.

We define now $\gamma_c = \gamma - \gamma_d$. Then γ_c is regular and countably additive and clearly

$$\gamma_c(\{s_1\}, \dots, \{s_n\}) = 0$$

for every $(s_1, \dots, s_n) \in K_1 \times \dots \times K_n$. By Lemma 2.1, this makes γ_c continuous.

The uniqueness of the decomposition is clear. \square

To see the relation with Saeki's result [14, Theorem 1], note that condition \mathcal{P} in [14, p. 34] is equivalent to X^* not containing isomorphic copies of c_0 . It follows now from [1, Proposition 11] that in that case every multilinear operator $T : C(K_1) \times \dots \times C(K_n) \rightarrow X^*$ (with representing polymeasure $\gamma : \Sigma_1 \times \dots \times \Sigma_n \rightarrow X^*$) is unconditionally converging (as for the definition of [7]), and now it follows from [9, Theorem Corollary 4.8] and [15, Theorem 5] that γ is countably additive.

However, the converse is not at all as easy, that is, it is not trivial (if possible) to derive our result directly from Saeki's one. The reason is that this would be essentially the same as giving a positive answer to the following question:

Let X be a Banach space, let T be an operator $T : C(K_1) \times \dots \times C(K_n) \rightarrow X$ with representing polymeasure γ . Suppose that T is unconditionally converging (equivalently, suppose that γ is countably additive). Does T factor through a Banach space not containing c_0 ?

If this were the case, there would be no complemented copies of c_0 in $\ell_\infty \widehat{\otimes}_\pi \ell_\infty$, a question long time (and still) open. The reasoning for this is the following: Every bilinear operator $T : \ell_\infty \times \ell_\infty \rightarrow c_0$ is unconditionally converging (this follows from [9]). Suppose that c_0 is complemented in $\ell_\infty \widehat{\otimes}_\pi \ell_\infty$, let $\hat{P} : \ell_\infty \widehat{\otimes}_\pi \ell_\infty \rightarrow c_0$ be the projection and let $P : \ell_\infty \times \ell_\infty \rightarrow c_0$ be the bilinear operator associated to \hat{P} . If P factors through a space not containing c_0 , so does \hat{P} , and this forces \hat{P} to be unconditionally converging, which is clearly impossible.

We note that in [8], the authors show an example of a Banach lattice X and an unconditionally converging operator $T : X \rightarrow c_0$ such that T does not factor through any space not containing c_0 .

REFERENCES

- [1] F. Bombal, M. Fernández and I. Villanueva, Unconditionally converging multilinear operators, *Math. Nach.*, **226** (2001), 5-15.
- [2] F. Bombal and I. Villanueva, Multilinear operators on spaces of continuous functions, *Funct. Approx. Comment. Math.*, **26** (1998), 117-126.
- [3] F. Bombal and I. Villanueva, Integral operators on the product of $C(K)$ spaces, *J. Math. Anal. Appl.*, **264** (2001), 107-121.
- [4] J. Diestel and J. J. Uhl, *Vector Measures*, Mathematical Surveys, No. 15. American Math. Soc., Providence, R.I., 1977.
- [5] S. Dineen, *Complex Analysis on Infinite Dimensional Spaces*, Springer 1999.
- [6] I. Dobrakov, On integration in Banach spaces, VIII (polymeasures). *Czech. Math. J.*, **37** (112) (1987), 487-506.

- [7] M. Fernández Unzueta, Unconditionally convergent polynomials in Banach spaces and related properties, *Extracta Math.* **12** (1997), 305–307.
- [8] N. Ghoussoub and W. B. Johnson, Counterexamples to several problems on the factorization of bounded linear operators, *Proc. Amer. Math. Soc.* **92** (1984) 233–238.
- [9] J. Gutiérrez and I. Villanueva, Extensions of multilinear operators and Banach space properties, *Proc. Roy. Soc. Edimb. Series A* **133 A** (2003) 1–18.
- [10] A. Pełczyński, On weakly compact polynomial operators on B-spaces with Dunford-Pettis property. *Bull. Acad. Polon. Sci. Ser. Sci. Math. Astr. Phys.*, **11** (1963), 371–378.
- [11] A. Pełczyński, A theorem of Dunford-Pettis type for polynomial operators. *Bull. Acad. Polon. Sci. Ser. Sci. Math. Astr. Phys.*, **11** (1963), 379–386.
- [12] D. Pérez-García and I. Villanueva, Orthogonally additive polynomials on spaces of continuous functions. Preprint.
- [13] R. A. Ryan, Dunford-Pettis properties *Bull. Acad. Polon. Sci. Ser. Sci. Math.*, **27** (1979), 373–379.
- [14] S. Saeki, Tensor products of $C(X)$ -spaces and their conjugate spaces, *J. Math. Soc. Japan.*, **28** (1976), 33–47.
- [15] I. Villanueva, Completely continuous multilinear operators on $C(K)$ spaces, *Proc. Amer. Math. Soc.* **128** (1999), 793–801.

E-mail address: bombal@mat.ucm.es

DEPARTAMENTO DE ANÁLISIS MATEMÁTICO, FACULTAD DE MATEMÁTICAS, UNIVERSIDAD COMPLUTENSE DE MADRID, MADRID 28040

E-mail address: dperezg@escet.urjc.es

ÁREA DE MATEMÁTICA APLICADA, ESCET, UNIVERSIDAD REY JUAN CARLOS, DEPARTAMENTAL II, 28933 MÓSTOLES (MADRID)

E-mail address: ignaciov@mat.ucm.es

DEPARTAMENTO DE ANÁLISIS MATEMÁTICO, FACULTAD DE MATEMÁTICAS, UNIVERSIDAD COMPLUTENSE DE MADRID, MADRID 28040