OBSERVABLE STRUCTURE FUNCTIONS

J. Montero
Dept. of Statistics and O.R.
Faculty of Mathematics
Complutense University
28040 Madrid, Spain

Abstract. This paper deals with the concept of structure function in Reliability Theory. Complete lattices are considered in order to model the space of performance levels for both system and components, leading to a general concept of structure function. Measurability with respect to the associate order topology is also assumed. On one hand, some basic concepts in classical Reliability Theory are translated into this context, where a particular measure has been defined over the space of components. On the other hand, the idea of duality is analyzed in this context.
Keywords: Coherent Systems, Dual Structure Functions, Reliability Theory.

1. INTRODUCTION

Classical Reliability Theory develops probabilistic properties of binary systems with binary components, where only two states are allowed: perfect functioning and complete failure. Such a systems are usually represented by their structure functions, which assign the performance level of the system -"0" if it is failed, "1" if it is functioning- to each profile of states for its components -"0_i" if component i is failed, "1_i" if component i is functioning-. In this way, the relationship between the performance levels of components and performance level of the system itself can be explained by a mapping $\phi:\{0,1\}^n\to\{0,1\}$, n being the number of components of the system (see, e.g., Barlow & Proschan[1]). But as pointed out by many reliability practitioners, components and systems can in fact take intermediate state levels. Performance of many real systems can not be explained just by considering perfect functioning and absolute failure.

Some mathematical models for non-binary systems have been proposed in the past. A first generalization tries to develop a theory of finite multistate systems, where a finite and linearly ordered set of performance levels is assumed (see, e.g., El-Neweihi *et al.*[2] and Griffith[3]). A second generalization has been introduced by some authors (see, e.g., Baxter[4,5], and Block & Savits[6]) by considering a continuum of different states between both extreme performance levels, that is, by assuming a real interval as the space of states for the system and each component (for example, the unit interval [0,1]). Their associate "continuum" structure function will be in this case given by a mapping $\phi:[0,1]^n\to[0,1]$ (see also Montero[7]).

In this paper we consider a general complete lattice as the basic structure of states, and an associate measure defined on the Borel σ-field generated by the order topology is also assumed. The concept of dual structure function in then discussed within this general framework.

2.- GENERAL STRUCTURE FUNCTIONS

It can be assumed that the space of performance levels for the system and its components must be in any case a partially ordered set, that is, a pair $(L,\geq)$ given by a non-empty set L and a binary relation $\geq$ over L verifying reflexivity ($a\geq a$ $\forall a\in L$), anti symmetry ($a=b$ if $a\geq b$ and $b\geq a$ hold simultaneously) and transitivity ($a\geq c$ if $a\geq b$ and $b\geq c$ hold for some $b\in L$). As usual, $a>b$ means that $a\geq b$ holds but not $b\geq a$, and then it is understood that the level of performance "a" is strictly higher than level "b". But few properties can be developed if such a lattice is not supposed to be at least complete. We shall therefore assume that for each non-empty subset $M\subset L$, its *sup*M element (*sup*$M\geq b$ $\forall b\in M$ and *sup*$M\leq a$ if $a\geq b$ $\forall b\in M$) and its *inf*M element (*inf*$M\leq b$ $\forall b\in M$ and *inf*$M\geq a$ if $a\leq b$ $\forall b\in M$) exist in L. Being L complete, it is then assured that L is also compact and it has a lowest level $0=$*inf*L (complete failure) and a highest level $1=$*sup*L (perfect functioning). A general approach to the idea of structure function must be based upon such a basic structure of performace levels. Any complete lattice $(L,\geq)$ can be considered as a "general space" of performance levels.

Following Montero *et al.*[8], any mapping $\phi:L^n \to L_0$ defined over a general space of performance levels $(L,\geq)$ for each component and another general space of performance levels $(L_0,\geq)$ for the system itself will be called "general structure function" (GSF). If $L_0=L$, such a general structure function is then called L-structure function (LSF for short). The usual product lattice is then associated to L^n, in such a way that $x \leq y$ holds for two $x,y \in L^n$ if and only if $x_i \leq y_i$ $\forall i$ ($x<y$ holds for two $x,y \in L^n$ if and only if $x \leq y$ but not $y \leq x$). Moreover, since L is compact, its product space L^n is assured to be -by Tychonoff theorem- also compact (see, e.g., Willard[9]).

According to classical binary approaches (see, e.g., Barlow and Proschan[1]), we shall restrict our study to "coherent" structure functions ϕ, that is, those structure functions being monotonic ($\phi(x_1,x_2,\ldots,x_n) \geq \phi(y_1,y_2,\ldots,y_n)$ holds whenever $x_i \geq y_i$ $\forall i$) and with no irrelevant components. By an irrelevant component we mean here a component i such that $\phi(y_i,x)=\phi(z_i,x)$ $\forall x \in L^n$ $\forall y_i,z_i \in L$, where $(v_i,x)=(x_1,\ldots,x_{i-1},v_i,x_{i+1},\ldots,x_n)$. Component i will be irrelevant in a given monotonic system ϕ if and only if $\phi(0_i,x)=\phi(1_i,x)$ $\forall x \in L^n$. Any system which can be represented by a GSF being monotonic and with no irrelevant alternatives will be called "general coherent system".

The key point for this paper is to notice that structure functions modeling real systems assume in fact the existence of an appropriate measure on the initial space of components. This associate measure reveals which states in L^n are really observable, or how the structure function is really recognized. Hence, two GSF ϕ_1 and ϕ_1 defined on the same measure space are indistinguishable if they are identical almost everywhere with respect to such a measure. If we consider the Borel σ-fields $\mathcal{B}$ and $\mathcal{B}_0$ generated by the natural order topology in lattices L and L_0, both being obtained by considering the family of sets of the form $\{y/y<x\}$ together with those of the form $\{y/y>x\}$ as a sub base (see, e. g., Willard[8]), then we can assume without loss of practical generality that our GSF $\phi:L^n\to L_0$ is measurable with respect the product σ-field ($\phi^{-1}(B)\in\sigma(\mathcal{B}^n)$ $\forall B\in\mathcal{B}_0$, where $\sigma(\mathcal{B}^n)$ is the minimum σ-field containing the product topology $\mathcal{B}^n$). Hence, the system must be always analyzed with respect to some fixed measure μ defined on $\sigma(\mathcal{B}^n)$. Moreover, it is well known that "being identical almost everywhere" defines an equivalence relation on the set of measurable functions.

Definition 1.- Let $(L^n,\sigma(\mathcal{B}^n),\mu)$ be a fixed measure space, L being a general space of performance levels. An observable structure function (OSF) is an equivalence class of measurable GSF being indistinguishable between them.

Each OSF Φ can be represented by any of its elements $\phi\in\Phi$, although we will usually try to define the most useful element as its canonical element (perhaps the most simple analytical mapping). Concepts and results relative to GSF must be properly translated to the OSF context, analyzing the whole equivalence class. For example, a "coherent" OSF must be an equivalence class containing some coherent GSF, leading to the following definition.

Definition 2.- An OSF Φ is said to be coherent if

i) there exists $\phi\in\Phi$ being monotonic, and

ii) there is no irrelevant component for any given GSF $\phi\in\Phi$.

The following result represents a useful necessary condition:

Proposition 1.- If ϕ is a coherent observable structure function, then

$$\mu\{x \;/\; \phi(0_i,x)<\phi(1_i,x)\} > 0 \qquad \forall i$$

holds for any given monotonic $\phi\in\Phi$.

Proof: Obviously, this condition means that if we can not find a monotonic $\phi\in\Phi$, or it verifies

$$\mu\{x \;/\; \phi(0_i,x)<\phi(1_i,x)\} = 0$$

for some component i, then Φ is not coherent. In fact: if we suppose the existence of such a monotonic GSF $\phi\in\Phi$ and such component i, then the mapping φ defined by $\varphi(x)=\phi(0_i,x)$ $\forall x\in L^n$ would be a monotonic GSF identical almost everywhere to ϕ, since $\varphi(x)=\phi(0_i,x)\le\phi(0_i,y)=\varphi(y)$ holds whenever $x\le y$. But then component i is an irrelevant component of the GSF φ:

$$\varphi(0_1,x)=\phi(0_1,x)=\varphi(1_1,x) \quad \forall x$$

3.- DUAL OBSERVABLE L-STRUCTURE FUNCTIONS

A certain duality appears in any lattice $(L,\geq)$, since lattice $(L,\leq)$ will always be another lattice. However, $(L,\geq)$ is not always isomorphic to $(L,\leq)$. But this isomorphism appears in practice very often, since the set of performance levels are usually defined according to a high internal duality which assures the existence of a one-to-one mapping $\delta:(L,\geq)\rightarrow(L,\leq)$ being order-preserving ($\delta(a)\leq\delta(b)$ if and only if $a\geq b$). In this section we shall translate the idea of duality from binary reliability theory to the context of our observable structure functions, extending some results obtained in Montero *et al.*[8]

In general, $\delta(0)=1$, $\delta(1)=0$ and $\delta(a)>\delta(b)$ if and only if $a<b$ hold for any order-preserving isomorphism δ. But it must be pointed out that in general such an isomorphism needs not to be unique and may even not exist. In any case, the idea of dual structure is behind such a dual isomorphism.

Example 1.- Let us consider the real unit interval $[0,1]$. Obviously, δ_1 such that $\delta_1(x)=1-x$ $\forall x\in[0,1]$ and δ_2 such that $\delta_2(x)=1-x^2$ $\forall x\in[0,1]$, are both one-to-one mappings being order preserving.

The following definition was proposed in Montero *et al.*[8]

Definition 3.- Let us consider a LSF ϕ. Supposed a one-to-one mapping $\delta:L\to L$ being order preserving, its δ-dual structure function is given by

$$\phi^{\delta}(x_1,x_2,\ldots,x_n) = \delta^{-1}[\phi(\Delta(x))]$$

where $\Delta:(L^n,\sigma(\mathcal{B}^n))\to(L^n,\sigma(\mathcal{B}^n))$ is the associated one-to-one mapping such that

$$\Delta(x_1,\ldots,x_n)=(\delta(x_1),\ldots,\delta(x_n)) \quad \forall x\in L^n$$

Though order-preserving isomorphisms are not unique in general and they may even not exist, it can be noticed that dual structure can be unique in some particular cases. For example, if L is a linearly ordered set and a "series" system

$$\phi_{\sigma}(x_1,x_2,\ldots,x_n)=min(x_1,x_2,\ldots,x_n)$$

is given, its usual dual structure function is in all cases the "parallel" structure

$$\phi_{\pi}(x_1,x_2,\ldots,x_n)=max(x_1,x_2,\ldots,x_n)$$

Indeed, parallel structure function is the δ-dual structure function of series structure for any given isomorphism δ defined in the linearly ordered set L of performance levels: since

$$y=max(x_1,x_2,\ldots,x_n) \Leftrightarrow \delta(y)=min(\delta(x_1),\delta(x_2),\ldots,\delta(x_n))$$

then we have that $\phi^{\delta}(x)=\delta^{-1}[min(\delta(x_1),\ldots,\delta(x_n))]=max(x_1,\ldots,x_n)$ $\forall x\in[0,1]$ holds for any order-preserving isomorphism δ in L.

However, the property $(\phi^{\delta})^{\delta}=\phi$ can not be assured in general. But as shown in Montero *et al.*[8], given δ a one-to-one and order preserving mapping, the mapping Δ is also a measurable one-to-one mapping. Hence, if ϕ is a measurable LSF, any δ-dual LSF ϕ^{δ} will be also measurable. Furthermore, every δ-dual LSF ϕ^{δ} of a coherent LSF ϕ is assured to be coherent (see Montero *et al.*[8]). When the concept of duality and such duality closure theorem is going to be translated into the context of observable L-structure functions, we find out that the δ-dual set $\Phi^{\delta}=\{\phi^{\delta}/\phi\in\Phi\}$ of a coherent observable L-structure function Φ is not assured to be also coherent, neither an equivalence class of measurable and indistinguishable L-structure functions, as it is shown in the following example.

Example 2.- Let us consider binary systems with two components (that is, $\phi:\{0,1\}^2\to\{0,1\}$), and the probability measure $\mathbb{P}$ such that $\mathbb{P}\{(0,0)\}=\mathbb{P}\{(0,1)\}=\mathbb{P}\{(1,1)\}=1/3$. If we define $\delta(a)=1-a$ $\forall a\in\{0,1\}$, it is clear that any two LSF ϕ and ϕ such that $\phi_1(x)=\phi_2(x)$ $\forall x\neq(1,0)$ are identical almost everywhere. But if $\phi_1(1,0)\neq\phi_2(1,0)$, then ϕ_1^{δ} is not identical almost everywhere to ϕ_2^{δ}, since

$$\phi_1^{\delta}(0,1)=\delta^{-1}[\phi_1(1,0)]\neq\delta^{-1}[\phi_2(1,0)]=\phi_2^{\delta}(0,1)$$

with $\mathbb{P}\{(0,1)\}>0$.

The desired closure theorem will appear when a "natural" isomorphism δ can be defined preserving the measure μ associated to $(L^n,\sigma(\mathcal{B}^n))$, in the sense that the measure of any Borel set is equal to the measure of its image, that is:

$$\mu(B) = \mu\{y\in L^n \ / \ \exists x\in B,\ y=\Delta(x)\} \qquad \forall B\in\sigma(\mathcal{B}^n) \qquad (1)$$

Clearly, this condition on δ is equivalent to impose

$$\mu[x,y] = \mu(\Delta[x,y]) \qquad \forall x,y\in L^n$$

on Δ. Moreover, it is then clear that such a measure μ must be a product measure (a more general concept of duality could have been proposed by considering different one-to-one mappings for each component). In any case, our concept of dual observable structure function can be properly considered only when such a condition (1) is verified; otherwise, the existence of a measure-preserving isomorphism is not assured. Uniqueness of measure-preserving isomorphisms is neither true in general, but some general results can be obtained.

Proposition 2.- Let us consider a measure μ defined on $(L^n,\sigma(\mathcal{B}^n))$ such that $\mu(\{0,0,\dots,0\})\neq\mu(\{1,1,\dots,1\})$. Since $\delta(0)=1$ for any one-to-one mapping being order-preserving, it is clear that it can not be simultaneously measure-preserving.

Example 3.- Let us consider systems with one component, with $L=\{0,1\}^2$ as the space of performance levels, and the probability measure $\mathbb{P}$ on $(L,\mathcal{B})$ such that $\mathbb{P}(\{(i,j)\})=1/4$ $\forall(i,j)\in\{0,1\}^2$. Then the mapping δ_1 such that $\delta_1(i,j)=(1-i,1-j)$ $\forall(i,j)$, and the mapping δ_2 such that $\delta_2(i,j)=(1-j,1-i)$ $\forall(i,j)$, are both one-to-one, order-preserving and measure preserving.

Proposition 3.- Let us suppose that $\mu(z,1]=0$ for some $z\neq 1$, and that for any given $x\in L^n$, $x\neq 0$, there exists $y\in L^n$, $y\leq x$, such that $\mu[0,y)>0$. Then there is no measure-preserving isomorphism in L.

Proof: Trivial since $\mu[0,\delta(z))=0$ in order to be measure preserving, and therefore $\mu[0,y)=0$ $\forall y\leq\delta(z)$ must hold.

Theorem 1.- Let us consider a complete linearly ordered set L, and a σ-finite product measure μ associated to $(L^n, \sigma(\mathcal{B}^n))$. If there exists a measure preserving isomorphism δ in L, its associated Δ is unique almost everywhere.

Proof: since μ is σ-finite, it is assured the existence of a partition $\{A_n\}_{n\in\mathbb{N}}\subset\sigma(\mathcal{B}^n)$ of L^n such that $\mu(A_n)<\infty$ $\forall n\in\mathbb{N}$. Let A_n be fixed, and let us define

$$B = \{x\in L^n \;/\; \exists C\in\sigma(\mathcal{B}^n),\ x\in C,\ C \text{ open with } \mu(C)=0\}$$

In any case,

$$\mu(A_n-B) = \inf\ \{\mu(A_n\cap(\bigcup_{x\notin B} D_x)),\ \{D_x\ni x\}_{x\notin B}\subset\sigma(\mathcal{B}^n) \text{ open sets}\}$$

because it is clear that for any given $y\in B$ and any given $x\notin B$ there exists an open set $D_x\in\sigma(\mathcal{B}^n)$, $D_x\ni x$, such that $y\notin D_x$ (for example, if $x<y$ then $x_j<y_j$ for some j, and $\{z/z_j<y_j\}$ is an open set containing x but not y). If we consider now an open cover $\{C_x\}_{x\in L^n}\subset\sigma(\mathcal{B}^n)$ with $x\in C_x$ for each $x\in L^n$ and $\mu(C_x)=0$ if $x\in B$, a finite open sub cover can be picked in such a way that

$$\mu(A_n) = \mu((\bigcup_{k=1}^{m} C_{x_k})\cap A_n) \leq \mu(\bigcup_{x_k\notin B}(C_{x_k}\cap A_n)) + \sum_{x_k\in B}\mu(C_{x_k}\cap A_n) =$$

$$= \mu(\bigcup_{x_k \notin B} (C_{x_k} \cap A_n)) \leq \mu((\bigcup_{x \notin B} C_x) \cap A_n)$$

Therefore $\mu(A_n)=\mu(A_n-B)$ $\forall n$ holds, $\mu(B\cap A_n)=0$ $\forall n$ also holds, and

$$\mu(B)=\sum_n \mu(B\cap A_n)=0$$

But the value $\Delta(x)$ is unique for any given $x\notin B$: if δ_1 and δ_2 are two measure-preserving isomorphisms in L such that $\delta_1(x_j)<\delta_2(x_j)$ for some j, then it is clear that for any fixed set A_n

$$\mu\{y\in A_n/y_j>\delta_1(x_j)\} =$$

$$= \mu\{y\in A_n/y_j>\delta_2(x_j)\} - \mu\{y\in A_n/\delta_1(x_j)<y_j\leq\delta_2(x_j)\}$$

Analogously,

$$\mu\{y\in A_n/y_j<\delta_2(x_j)\} =$$

$$= \mu\{y\in A_n/y_j<\delta_1(x_j)\} - \mu\{y\in A_n/\delta_1(x_j)\leq y_j<\delta_2(x_j)\}$$

in such a way that

$$\mu\{y\in A_n/\delta_1(x_j)<y_j\leq\delta_2(x_j)\} = \mu\{y\in A_n/\delta_1(x_j)\leq y_j<\delta_2(x_j)\} = 0$$

Therefore,

$$\mu\{y \;/\; \delta_1(x_j)<y_j<\delta_2(x_j)\} =$$

$$= \sum_{n\in\mathbb{N}} \mu\{y\in A_n \;/\; \delta_1(x_j)<y_j<\delta_2(x_j)\} = 0$$

and hence

$$\mu\{y \;/\; x_j>y_j>\delta_1^{-1}(\delta_2(x_j))\} = 0$$

$$\mu\{y \;/\; \delta_2^{-1}(\delta_1(x_j))>y_j>x_j\} = 0$$

in contradiction with the fact that $x\notin B$: since

$$\mu\{y/y_j=x_j\} = \mu\{y/y_j=\delta_1(x_j)\} = \mu\{y/y_j=\delta_2(x_j)\} = 0$$

it is clear that

$$\{y \;/\; \delta_1^{-1}(\delta_2(x_j))<y_j<\delta_2^{-1}(\delta_1(x_j))\}$$

would be an open set with zero measure containing the element $x\notin B$.

It must be noticed that the set B defined in the proof of last theorem is obviously an open set: if $x\in B$, there exists C, $x\in C$, such that $\mu(C)=0$, and then for any given element $y\in C$ we have that $\mu(C)=0$, in such a way that $C\subset B$ holds; hence, the complement L^n-B is closed. Moreover, it is clear that our linearly ordered set L is Hausdorff (there exist two disjoint open sets $U_x\ni x$ and $V_y\ni y$ for any pair of elements $x\neq y$) and, being L compact, L is also a T_4-space (if A and B are two disjoint closed sets, there exist two disjoint open sets $U\supset A$ and $V\supset B$). And it is also known that every product of compact and Hausdorff spaces (L^n in our case) is a T_4-space (see e.g. Willard[8]).

The following result justifies the uniqueness of a "natural" duality when dealing with random continuous systems of independent components:

Theorem 2.- Let us consider a closed interval L of the compact real line $\mathbb{R}\cup\{-\infty,\infty\}$, and let us assume that a proper product probability measure is defined in the L^n, being absolutely continuous with density function $f(x_1).f(x_2)...f(x_n)>0$ $\forall x\in L^n$. Then it is defined a unique measure-preserving isomorphism.

Proof: trivial since the distribution function associated to f is continuous and strictly increasing in L.

Example 4.- Let us consider the real unit hypercube $[0,1]^n$ with its associate Borel σ-field and the Uniform distribution $\mathbb{P}$ on $[0,1]^n$, characterized by

$$\mathbb{P}[y,z] = \prod_{i=1}^{n} (z_i - y_i) \qquad \forall y,z \in L^n,\ z \geq y$$

then such a "natural" isomorphism in $L=[0,1]$ is unique and it is given by $\delta(a)=1-a$ $\forall a$. Indeed, since $\mathbb{P}[0,x]=\mathbb{P}[\Delta(x),\Delta(1)]$ must be verified due to condition (3), then

$$\prod_{i=1}^{n} x_i = \prod_{i=1}^{n} (1-\delta(x_i)) \qquad \forall x \in L^n$$

Therefore, just taking $\delta(a)=1-a$ $\forall a \in [0,1]$ we have the unique order-preserving and measure-preserving isomorphism. This example shows how the standard dual continuum structure function in most papers

$$\phi^{\delta}(x_1,x_2,\ldots,x_n) = 1-\phi(1-x_1,1-x_2,\ldots,1-x_n) \qquad \forall x \in [0,1]^n$$

was really the unique dual structure which can be defined when the space of states is observed according the Lebesgue measure (ϕ has been defined from the real unit hypercube into the real unit interval, and distance between points in the real line is usually measured according such a Lebesgue measure).

In order to get a general duality closure theorem for coherent OSF we need the following lemma.

Lemma 1.- Let us assume the existence of an isomorphism δ in the general space of states L being measure-preserving. Then the dual set of any OSF is also an OSF.

Proof: Let ϕ be a fixed LSF. Then it is enough to prove that the set of LSF being identical almost everywhere to ϕ^{δ} is just the dual set of all LSF being identical almost everywhere to ϕ. Indeed, since

$$\mu\{x \ / \ \varphi(x)=\phi(x)\} = \mu\{x \ / \ \delta^{-1}[\varphi(x)]=\delta^{-1}[\phi(x)]\} =$$
$$= \mu\{\Delta(y) \ / \ \delta^{-1}[\varphi(\Delta(y))]=\delta^{-1}[\phi(\Delta(y))]\} =$$
$$= \mu(\Delta\{y \ / \ \varphi^{\delta}(y)=\phi^{\delta}(y)\}) = \mu\{y \ / \ \varphi^{\delta}(y)=\phi^{\delta}(y)\}$$

and

$$\mu\{x \ / \ \varphi(x)=\phi^{\delta}(x)\} = \mu\{y \ / \ \varphi^{-\delta}(y)=\phi(y)\}$$

where $\varphi^{-\delta}(y)=\delta[\varphi(\Delta^{-1}(y))]$, then the result is immediate. Hence, dual L-structure functions of any two identical almost everywhere L-structure functions are also identical almost everywhere, and if ϕ^{δ} is identical almost everywhere to φ^{δ}, then ϕ and φ are also identical almost everywhere.

Theorem 3.- Let δ be a measure-preserving isomorphism defined on the general space of states $(L^n, \sigma(\mathcal{B}^n), \mu)$, and let Φ be a coherent OSF. Then Φ^{δ} is also a coherent OSF.

Proof: From the previous lemma, it is assured that Φ^{δ} is an observable structure function. The result follows from the fact that any δ-dual LSF ϕ^{δ} of a coherent LSF ϕ is always a coherent LSF (see Montero *et al.*[8]).

Therefore, only in case of dealing with a one-to-one mapping being order-preserving and measure-preserving it makes sense to consider the dual system.

4.- FINAL COMMENTS

This paper on observable structure functions has been devoted mainly to the problem of duality. As in classical reliability theory, the concept of dual structure function will be useful in analyzing systems where components are subject to two kinds of failure. This is the case in most safety systems. For example, if we consider a control flood of two independent sluices in a river, we find that each sluice can respond at a level x_1 to a command to close and at a level y_1 to a command to open (we can assume that both $x_1, y_1 \in L$, being L a linearly ordered set with a natural order-preserving isomorphism δ between the degrees of response to each command). If we want to close the flood, it is clear that the system can be represented by a series structure function depending on the values x_1 and x_2; but if we want to open the flood, the system will be represented by a parallel structure function depending on the values y_1 and y_2. Then it will be very useful that both failure to close and failure to open can be analyzed using the same structure function (one is the dual structure of the other, no matter the particular isomorphism δ on L).

In any case, it is clear that the concept of structure function proposed in this paper is too general for most practical situations. It seems a must to develop in deepness intermediate generalized models between binary systems and our GSF. Discrete systems and continuum systems have been studied in the past, but we must also develop concepts and properties for "usual" structure functions (USF) $\phi:L^n \to L_0$ with $L_0=L$, L being an arbitrary linearly ordered set. Another important case is that one in which the system can be modeled according to "multivalued" usual structure functions (MUSF), that is, those systems that can be described by a mapping $\phi:L^n \to L_0$, where $L_0=L^k$ $(k<n)$ and L is a linearly ordered set. If our measure μ defined on the space of states is a probability measure $\mathbb{P}$, a reliability function R, $R(a)=\mathbb{P}\{x \in L^n / \phi(x) \geq a\}$ $\forall a \in L_0$, can be defined in order to know the probability distribution of the performance of these USF or MUSF systems. First results (some general reliability bounds for USF and MUSF) can be found in Montero *et al.*[8]

ACKNOWLEDGEMENTS

This research has been supported by Dirección General de Investigación Científica y Técnica (National Grant number PB88-0137).

REFERENCES

[1] R.E. Barlow & F. Proschan (1975) *Statistical Theory of Reliability and Life Testing*. Holt-Rinehart-Winston, New York.

[2] E. El-Neweihi; F. Proschan & J. Sethuraman (1978) *Multistate coherent systems*. Journal of Applied Probability 15, 675-688.

[3] W.S. Griffith (1980) *Multistate reliability models*. Journal of Applied Probability 17, 735-744.

[4] L.A. Baxter (1984) *Continuum structures* I. Journal of Applied Probability 19, 391-402.

[5] L.A. Baxter (1986) *Continuum structures* II. Mathematical Proceedings of the Cambridge Philosophical Society 99, 331-338.

[6] H.W. Block & T.H. Savits (1984) *Continuous multistate structure functions*. Operations Research 32, 703-714.

[7] J. Montero (1988) *Coherent fuzzy systems*. Kybernetes 17, 28-33.

[8] J. Montero; J. Tejada & J. Yáñez *General structure functions*. To appear in Kybernetes.

[9] S. Willard (1970) *General Topology*. Addison-Wesley, Reading.