Zeno inhibition of polarization rotation in an optically active medium

Isabel Gonzalo[1], **Miguel A. Porras**[2] **and Alfredo Luis**[1]

[1]Departamento de Óptica, Facultad de Ciencias Fsicas, Universidad Complutense de Madrid, 28040 Madrid, Spain

[2]Grupo de Sistemas Complejos, Universidad Politécnica de Madrid, Rios Rosas 21, 28003 Madrid, Spain

Abstract. We describe an experiment in which the rotation of the polarization of light propagating in an optically active water solution of D-fructose tends to be inhibited by frequent monitoring whether the polarization remains unchanged. This is an example of Zeno effect of remarkable pedagogical interest because of its conceptual simplicity, easy implementation, low cost, and because the same Zeno effect holds at classical and quantum levels. An added value is the demonstration of Zeno effect beyond typical idealized assumptions in a practical setting with real polarizers.

Keywords: Zeno effect, Polarization, Optical activity

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1. Introduction

The quantum Zeno effect is the inhibition of the evolution of the state of a quantum system by frequent measurement. Misra and Sudarshan [1] were the first to call the effect by that name since it recalls the arrow paradox of the Greek philosopher Zeno of Elea. However this effect was much earlier described by Turing in 1954 [2], closely related work was published [3, 4], and a general derivation was provided in 1974 [5]. The original Zeno paradox argues that a flying arrow is at every instant of time in a portion of space equal to its own length. According to Zeno, this is equivalent to be at rest at every instant, and the "sum" of these motionless arrows cannot constitute a motion [6].

Maybe unexpectedly, quantum physics provides a confirmation of the intuition of the Greek philosopher. This is usually illustrated by a particle that tends to abandon its initial quantum state following a typical Hamiltonian evolution. When observing, close to the initial instant, whether the system has already abandoned the initial state, the most probable outcome is "not". In such a case, quantum state reduction forces the particle to go back to the initial state, in a kind of Sisyphus-like punishment. With a frequent enough observation rate, the probability that the particle remains arrested in the initial state is as close to one as desired. The paradox is that the mere presence of a detector counteracts the system dynamics even if the particle never "touches" the detector. This is dramatically illustrated by the optical detection of bombs that explode when illuminated. The Zeno effect, implemented as a mild but continuous interrogation, allows to safely detect the bomb without being touched by any photon [7, 8]. The quantum Zeno effect has been demonstrated with ions [9], polarized photons [10], cold atoms [11] and Bose-Einstein condensates [12], an continues attracting interest for its fundamental implications in quantum measurement [13, 14] as well as its applications to preserve coherence in quantum information [15]. For a review, see Ref. [16].

The Zeno effect has also been described in classical physics, although it loses most of its paradoxical flavor. In classical optics, we are used, for example, to perfect absorbers that are perfect reflectors, since perfect sensing of field penetration impedes field penetration. Most of classical Zeno effects occur in wave systems, many of them light waves with a quantum wave analog, whose natural propagation is inhibited or altered by observation. Several demonstrations of Zeno-like effects in classical optics can be found in Refs. [17–22].

In this paper we describe the observation of the Zeno effect in the polarization rotation of linearly polarized light caused by an optically active medium. This Zeno effect was suggested in Ref. [23], but not implemented in practice. A given initial polarization direction of light is the initial state of the system that the active medium tends to modify. The active medium is a solution of fructose in water. Polarizers with their axis parallel to the initial polarization direction and immersed in the solution, monitor whether the polarization remains the initial one. The most frequent these measurements, the most likely the field remains vibrating in the initial direction. This is conceptually similar to the Zeno effect in the optical rotation by Faraday effect [18] (not implemented neither), but our setting is clearly much more simple.

We believe that the experimental verification of this Zeno effect has a three-fold added value to its pure scientific research interest. The first one is its conceptual simplicity and ease of implementation, with a very low cost. This makes the experiment attractive for teachers and for undergraduate students to learn the essentials of the Zeno effect. The second point is that we are observing a Zeno effect that is the same at the quantum and classical levels. With a single photon light source, for instance, the experiment should be repeated a large number of times, and the probability that the polarization state remains the initial one would be given by the fraction of the ensemble of photons detected in the initial sate. With a classical source with many photons, as a cheap laser pointer, this probability is given by the fraction of light intensity that remains polarized in the initial polarization state, since intensity and number of photons are proportional. The third item is the observation of how the Zeno effect holds in a practical setting with real polarizers. This is beyond the typical presentations of the Zeno effect plenty of idealized assumptions, and gives room to an insightful physical analysis of the effect of many practical contributions, in the line of recent analysis of Zeno effect under imperfect detection [24].

For simplicity, we first describe the ideal theory that predicts that the rotation of the polarization direction in optically active media tends to be inhibited by increasing number of measurements, being completely inhibited in the limit of infinitely frequent measurements. The Zeno effect is indeed observed in the experiment as an increase of the intensity in the initial polarization state as the number of measurements increases, but the use of non-ideal, imperfect measurement devices prevents from reaching the limit of complete inhibition. The experimental data are correctly interpreted by taking into account the losses in the polarizers.

2. Ideal Zeno effect in optically active media

An optically active medium rotates the polarization direction of light traversing it. Optically active media are characterized by having a chiral structure. If, as in our experiment, the medium is a solution of chiral molecules, an excess of either lefthanded or righthanded enantiomers of the chiral molecule is needed for the medium to show optical activity. We recall that lefthanded and righthanded enantiomers of a chiral molecule are mirror or specular images of each other, their molecular conformations being not superimposable.

Suppose that linearly polarized light propagates in an optically active solution from $z = 0$ to $z = L$. We express the initial state of light as $|\Psi_0\rangle = E_0\,|V\rangle$, where E_0 is the electric field amplitude, $I_0 = E_0^2$ is proportional to the intensity, and $|V\rangle$ denotes the polarization direction, which is taken to be the vertical direction with respect to the lab bench. At a distance z within the solution, the polarization direction is rotated by an angle α, so that the light state at z within the solution will be

$$|\Psi\rangle = E_0(\cos\alpha|V\rangle + \sin\alpha|H\rangle), \tag{1}$$

where $|H\rangle$ denotes the horizontal direction. For a homogeneous solution, the rotated angle is proportional to z i. e., $\alpha = Az$, where A is a constant.

Let us wonder how much light remains vertically polarized at the final plane $z = L$. For this, we place a vertical polarizer at $z = L$ that transmits the projection of the incident light onto the vertical direction. The light state after the polarizer is then $|\Psi_1\rangle = E_0\cos\alpha|V\rangle = \cos\alpha|\Psi_0\rangle$, where $\alpha = AL$, whose intensity is $I_1 = E_0^2\cos^2\alpha = I_0\cos^2\alpha$. The fraction of light intensity that remains vertically polarized at $z = L$ is then given by $i_1 \equiv I_1/I_0 = \cos^2\alpha$.

We may wish to repeat the above measurement twice by placing two equispaced vertical polarizers at $z = L/2$ and $z = L$. The optical rotation from one to another polarizer is then one-half, i. e., $\alpha = AL/2$. The light state after the first polarizer will be $E_0\cos\alpha|V\rangle$, and after the second polarizer $|\Psi_2\rangle = E_0\cos^2\alpha|V\rangle = \cos^2\alpha|\Psi_0\rangle$, giving an intensity $I_2 = E_0^2\cos^4\alpha$ after the last polarizer. The fraction of light intensity that remains vertically polarized at $z = L$ is now $i_2 \equiv I_2/I_0 = \cos^4\alpha$.

In general, for N equispaced vertical polarizers, the rotation from one the the next polarizer is $\alpha = AL/N$, the light state after the last polarizer will be

$$|\Psi_N\rangle = \cos^N\left(\frac{AL}{N}\right)|\Psi_0\rangle, \tag{2}$$

with an intensity $I_N = E_0^2\cos^{2N}(AL/N)$, giving the fraction

$$i_N = \cos^{2N}\left(\frac{AL}{N}\right) \tag{3}$$

of light intensity that remains vertically polarized at $z = L$.

As N increases, the factors $\cos^N(AL/N)$ and $\cos^{2N}(AL/N)$ in Eqs. (2) and (3) increase monotonically, and reach the value unity in the limit $N \to \infty$. This means that the final polarization state $|\Psi_N\rangle$ approaches the initial state $|\Psi_0\rangle$ and, in particular, the light intensity transmitted by the last polarizer approaches the incident intensity. Figure 1 (closed circles) illustrate this behavior for a few values of the angle AL rotated in the length L of the solution.

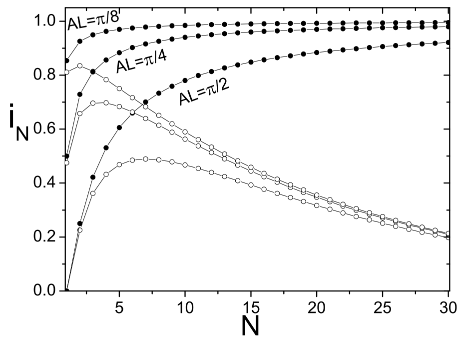

Figure 1. Fractional intensity $i_N = \cos^{2N}(AL/N)$ after N vertical ideal polarizers (closed circles), and fractional intensity $i_N = T^N \cos^{2N}(AL/N)$ after N vertical polarizers of transmittance, e. g., $T = 0.95$ (open circles). From the lowest to the highest curves, the angle rotated by the medium without any polarizer is $AL = \pi/2, \pi/4$ and $\pi/8$.

A similar Zeno effect takes place for a single photon. If the initial state of a photon is $|\Psi_0\rangle = |V\rangle$, optical activity turns it into the superposition state $|\Psi\rangle = \cos\alpha|V\rangle + \sin\alpha|H\rangle$. A polarizer monitors whether the polarizer remains vertically polarized, projecting into $|\Psi_1\rangle = \cos\alpha|V\rangle$, i. e., into the state $|V\rangle$ with probability $i_1 = \cos^2\alpha$. Similarly as above, the result of the projections of N polarizers will be given by Eq. (2), meaning that the state is $|V\rangle$ with a probability given by Eq. (3). The photon thus remains in a the initial vertical polarization state with a probability approaching unity. A similar quantum Zeno effect takes place for a spin-1/2 particle precessing in a magnetic field and suffering frequent measurements of the spin.

3. Experiment

Figure 2 shows a sketch of our experimental setup for the observation of the Zeno effect. We point out that the above ideal Zeno effect, particularly the limit $i_N \to \infty$ for $N \to \infty$, is unattainable in practice because measurement devices are never perfect. For example, polarizer transmittances T parallel to the transmission axis smaller than unity, lead to the opposite situation that i_N always approaches 0 for sufficiently large number of polarizers, as seen in Fig. 1 (open circles). Thus we can only expect to observe a real Zeno effect consisting on an increase of i_N when the first few polarizers are introduced.

We used a water solution of D-fructose (also called levulose) because of its strong optical rotatory power, low cost and availability. The molecular formula of levulose is $C_6H_{12}O_6$. The prefix "D" stands for the hydroxy group attached to the the right side of the asymmetric carbon furthest from the carbonyl. The optical activity is usually specified by the specific rotatory power $[\alpha]_D^{20} = -92$ degrees, which means that a water solution of levulose with concentration of 1 gr/ml at 20°C rotates linearly polarized light 92 degrees lefthanded (from the viewpoint of receiver) when the propagation length of light at 589 nm (the sodium D line) is 1 dm. Once the required concentration is fixed for the experiment (see below), the solution is heated until it boils for a few seconds to obtain a quite transparent solution without bubbles.

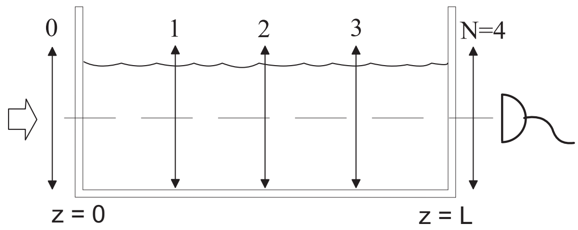

Figure 2. Light coming from the left and vertically polarized by the first polarizer, propagates in the optically active medium contained in a cuvette of length L. A detector measures the light power at the exit of the last vertical polarizer situated outside the cuvette. The vertical polarizers inside the cuvette delimit equally light paths inside the active medium.

As a light source we used a diode laser emitting almost unpolarized light at 657 nm wavelength with power 2.8 mW. Other similar sources, as common red laser pointers, can be used as well. The solution is contained in a $L = 14.4$ cm long cuvette with entrance and exit facets made of glass. At the end of the cuvette, a commercial digital photometer (IF PM Industrial Fiber Optics) collecting the emerging light measures its power. The polarizer labeled as 0 in Fig. 2 placed before the entrance facet prepares the incident light in a vertically polarized state. All polarizers used in the experiment are cut to proper size from a large, 0.7 mm thick, polarizer sheet with crossed transmission of about 0.002 per cent for visible light.

Since the powers measured in the experiment are proportional to the intensities, the fraction of light intensity i_1 that remains vertically polarized at $z = L$ without intermediate polarizers ($N = 1$) is evaluated as the quotient $i_1 = P_1/P_0$ of the readouts of the power meter P_0 with only the preparing polarizer 0 and the power P_1 when a vertical polarizer after the exit facet of the cuvette is also placed. The theoretical curves in Fig. 1 suggests that the increase of intensity due to rotation inhibition will be more easily observable when the rotation angle in the length of the cuvette is $\alpha = AL = \pi/2$. To obtain the concentration providing this rotation, we place the two vertical polarizers before and after the entrance and exit facets of the cuvette and solve slowly small amounts of levulose until no light is detected. This choice sets the value of $i_1 = P_1/P_0$ to zero.

The red circles circles in Fig. 3 represent the experimentally obtained values i_N for increasing number of equispaced, intermediate polarizers up to $N = 7$. The value of i_N for given N is obtained as the quotient $i_N = P_N/P_0$ between the detected power P_N with N polarizers (including the last one after the exit facet of the cuvette) and P_0 with only the preparing polarizer 0 before the entrance facet. The error bars account for the inaccuracy in the values of i_N due to the limited precision of the power measurements and their random fluctuations. As seen, i_N increases significantly with N, but it does at a significantly lower rate than in the theory of the ideal Zeno effect (gray circles in Fig. 3), as expected for real polarizers used, and starts to decrease with $N = 6$ polarizers.

The experimental values of i_N can be explained by a simple modification to the ideal Zeno effect described in Sec. 2. The polarizers transmittance in the direction parallel to the transmission axis is not unity, but was measured to be $T_a \simeq 0.82$ in air and $T_s \simeq 0.90$ in the solution for the laser wavelength. The effect of the transmittances is then taken into account if we evaluate the fractional intensity after the N polarizers,

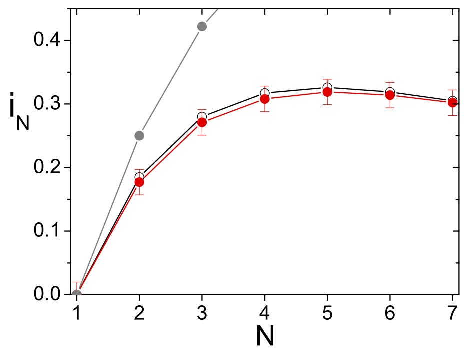

Figure 3. Fraction of intensity i_N that remains linearly polarized in the vertical direction as a function of the number of polarizers N. Gray circles: ideal Zeno effect from Eq. (3). Red circles: average values of three series of experimental values. Open circles: modification of the ideal Zeno effect as evaluated from Eq. (4).

$N-1$ in the solution and the last one in air, with the expression

$$i_N = T_a T_s^{N-1} \cos^{2N}\left(\frac{AL}{N}\right), \tag{4}$$

where $AL = \pi/2$ for the solution used in the experiment. The values of i_N obtained from Eq. (4) (open circles in Fig. 3) are seen to yield an adequate description of the observed behavior. The slightly lower experimental values can be understood from inaccuracies in the polarizers alignment.

The above experimental results hold in the classical-optics domain but are easily translated to the quantum domain under a simple assumption. This assumption is that in relation to the mean intensity, a single mode field state in a linear regime produces the same results as an ensemble of independent photons. This is in fact the contents of the popular saying by Dirac that every photon interferes with itself, i. e., independently of the others [26]. The conditions of linearity, e. g., $E_N = tE_0$ and of having a single mode field, hold in a good degree approximation in the above experiment. This leads to the relation between classical intensities, or powers, $P_N = i_N P_0$, with $i_N = |t|^2$, and hence to the relation $\bar{n}_N = i_N \bar{n}_0$ between mean number of photons. When dealing with a single photon, the only possible outcomes are those of a Hamlet-like question: photon or no photon. After the above relation between mean numbers, we get that the probability of having an output photon is i_N.

Even beyond the condition of linearity, the equivalence of a single mode with an ensemble of independent photons holds for laser beams well above the laser threshold. In this case the field tends to be a Glauber coherent state where photons follow Poisson statistics, i. e., they are independent. The field state is then an eigenstate of the annihilation operator and the removal of any photon does not alter the field state, i. e., no photon cares about the life or death of the others [25].

4. Conclusion

We believe that the experiment described in this paper constitutes the simplest possible arrangement for the observation of the Zeno effect, which makes it particularly suited as an undergraduate experiment. The same effect of inhibition of polarization rotation in optically active media by frequent observation would also be observed with an ensemble of individual photons. We can then say that we have observed the Zeno effect in the polarization state of the photons of a classical source of light. Also, much emphasis is usually made in the fact that the evolution of a quantum state can be inhibited by more and more frequent measurements. The present experiment goes a step beyond typical idealized analyses by showing the limitations of the Zeno effect in a practical setting with real measurement devices.

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