Supplementary for:

Blessing of Dimensionality in Spiking Neural Networks: The by-chance functional learning*

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Code availability

The Matlab code used for simulations is available at https://blogs.mat.ucm.es/vmakarov/downloads/

Appendices

A Dynamics of single synapses

Let $\boldsymbol{x}$ be the input pattern exciting a neuron. Without loss of generality, we assume that it arrives at $t \in [0, \Delta)$. The spike train in the j-th channel $\{t_{ji}\}$ is a stationary Poisson point process with the rate $f_j = f_{\max} x_j$. Then, $N_j = f_j \Delta$ is the expected number of spikes the neuron receives from this channel.

By integrating Eq. (6) from the main text, we get the dynamics of the synaptic random variable:

$$y_j(t) = \sum_i e^{-\frac{t-t_{ji}}{\tau_s}} H(t - t_{ji}), \tag{A.1}$$

where $H(\cdot)$ is the Heaviside function. Then, the expectation (over the ensemble) of y_j is given by

$$\begin{aligned} \mathrm{E}\left[y_j\right](t) &= \frac{N_j}{\Delta} \int_0^\Delta e^{-\frac{t-t_{ji}}{\tau_s}} H(t - t_{ji})\, \mathrm{d}t_{ji} = \\ &= f_j \tau_s \left(1 - e^{-\frac{t}{\tau_s}}\right). \end{aligned} \tag{A.2}$$

Then, Eq. (10) from the main text follows. For $t \gg \tau_s$, we can approximate:

$$\mathrm{E}\left[\boldsymbol{y}\right](t) \approx f_{\max} \tau_s \boldsymbol{x}. \tag{A.3}$$

We can write Eq. (8) from the main text in the form:

$$\dot{w}_j = -A_j(t)(w_j - B_j(t)), \quad j = 1, \ldots, n, \tag{A.4}$$

where

$$A_j(t) = \mu y_{\mathrm{ps}}(t)(1 + \eta y_j(t)), \quad B_j(t) = \frac{\eta y_j(t)}{1 + \eta y_j(t)}. \tag{A.5}$$

Approximating $\boldsymbol{y}$ by its expectation (A.3), we obtain

$$\dot{\boldsymbol{w}} = -\boldsymbol{a}(t) \circ (\boldsymbol{w} - \boldsymbol{w}^*), \tag{A.6}$$

where $\boldsymbol{a}(t) = \mu y_{\mathrm{ps}}(t)(1 + \eta f_{\max} \tau_s \boldsymbol{x})$ and

$$\boldsymbol{w}^* = \boldsymbol{x} \oslash (\alpha + \boldsymbol{x}), \quad \alpha = \frac{1}{\eta f_{\max} \tau_s}. \tag{A.7}$$

Finally,

$$\boldsymbol{w}(t) - \boldsymbol{w}^* = (\boldsymbol{w}(0) - \boldsymbol{w}^*) \circ e^{-\int_0^t \boldsymbol{a}(s) ds}. \tag{A.8}$$

Thus, each postsynaptic spike leads to an exponential convergence of $\boldsymbol{w}(t)$ to $\boldsymbol{w}^*$ (see Fig. 2C in the main text).

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B Neuronal firing to frequency patterns

B.1 Response to a novel stimulus

We assume that a neuron has random synaptic weights $\boldsymbol{w}(0) \sim U^n[0,1]$ and it receives a random pattern $\boldsymbol{x} \sim U^n[0,1]$. Then, the probability of firing to the pattern is:

$$P_{\text{fr}} := \Pr\left(\mathrm{E}\left[I\right] > \theta\right), \quad \mathrm{E}\left[I\right] = \frac{\beta}{n}\langle \boldsymbol{w}, \mathrm{E}\left[\boldsymbol{y}\right]\rangle - \gamma. \tag{B.1}$$

Using the expectation (Eq. (29) in the main text), we get:

$$P_{\text{fr}} = 1 - \Pr\left(\rho \leq \frac{\theta + \gamma}{\beta \tau_s f_{\max}}\right) = 1 - \Pr\left(\rho \leq \frac{\lambda}{4}\right), \tag{B.2}$$

where ρ is the pattern-synaptic match (Eq. (13) in the main text) and λ is the inhibitory-excitatory ratio (Eq. (15) in the main text). Since ρ implies a sum over $n \gg 1$ i.i.d. terms, we can use the Central Limit Theorem [1] and approximate the distribution $\rho \sim \mathcal{N}(\mu, \sigma/\sqrt{n})$. The mean and the standard deviation of the product of two independent random variables uniformly distributed on $[0,1]$ are: $\mu = \frac{1}{4}$ and $\sigma = \frac{\sqrt{7}}{12}$. Then, Eq. (B.2) yields Eq. (14) in the main text.

B.2 Response to the learned stimulus

After learning stimulus $\boldsymbol{x}$, the synaptic weights are given by Eq. (11) from the main text. Thus, the pattern-synaptic match is:

$$\rho_{\text{lrn}} = \frac{1}{n}\sum_{j=1}^{n} \frac{x_j^2}{\alpha + x_j}. \tag{B.3}$$

Then, the first two moments are:

$$\mu_{\text{lrn}} = \mathrm{E}\left[\frac{x^2}{\alpha + x}\right] = f_1(\alpha), \quad \sigma_{\text{lrn}}^2 = \mathrm{E}\left[\frac{x^4}{(\alpha + x)^2}\right] - \mu_{\text{lrn}}^2 = f_2(\alpha), \tag{B.4}$$

where, after simple but tedious calculations, we get the expressions for $f_{1,2}(\alpha)$ provided by Eq. (18) from the main text. Then, similar arguments as in Sect. B.1 give the firing probability:

$$P_{\text{fr}} = \Phi\left(\sqrt{n}(\mu_{\text{lrn}} - \lambda/4)/\sigma_{\text{lrn}}\right), \tag{B.5}$$

which yields Eq. (17) in the main text.

B.3 Response to arbitrary stimulus after learning

After learning stimulus $\boldsymbol{x}$, the response to another stimulus $\boldsymbol{\xi} \neq \boldsymbol{x}$ is defined by the pattern-synaptic match:

$$\rho_{\text{oth}} = \frac{1}{n}\sum_{j=1}^{n} \frac{x_j \xi_j}{\alpha + x_j}. \tag{B.6}$$

Then, we get the moments:

$$\mu_{\text{oth}} = \mathrm{E}\left[\frac{x\xi}{\alpha + x}\right] = f_3(\alpha), \quad \sigma_{\text{oth}}^2 = \mathrm{E}\left[\frac{x^2\xi^2}{(\alpha + x)^2}\right] - \mu_{\text{oth}}^2 = f_4(\alpha), \tag{B.7}$$

where the expressions for $f_{3,4}(\alpha)$ are provided by Eq. (20) from the main text. Then, we get the firing probability to an arbitrary stimulus after learning $\boldsymbol{x}$:

$$P_{\text{fr}} = \Phi\left(\sqrt{n}(\mu_{\text{oth}} - \lambda/4)/\sigma_{\text{oth}}\right), \tag{B.8}$$

which yields Eq. (19) in the main text.

References

[1] Montgomery D.C., Runger G.C. *Applied statistics and probability for Engineers.* 6th ed., Wiley, Hoboken, NJ, 2014.