The Fit of Dynamic Equilibrium Models of Exchange Rate[#]

Juan Ángel Jiménez Martín[a (*)] Rafael Flores de Frutos[b]

[a] Dpto. de Fundamentos de Análisis Económico II, Universidad Complutense, Somosaguas, Madrid, 28223, Spain

[b] Dpto. de Fundamentos de Análisis Económico II, Universidad Complutense, Somosaguas, Madrid, 28223, Madrid, Spain

---

Abstract

The two-country monetary model has become a fundamental tool for explaining the behavior of the exchange rate. However, the popularity of this approach is not justified by its empirical support. One of the reasons for the empirical "failure" of exchange rate models could be the econometric approach applied. In this paper, an alternative procedure for evaluating the fit of dynamic equilibrium models of exchange rate is suggested. This approach is applied to three theoretical models: Lucas (1982), Svensson (1985), and Grilli and Roubini (1992).

Keywords: exchange rate, equilibrium model, seasonality

JEL classification: F31, F37, G15

---

* Corresponding author. Tel.: +34 91 394 23 55. Fax: +34 91 394 2613;

e_mail: juanangel@ccee.ucm.es

[#] We are grateful to A. Novales for helpful comments. We would like to acknowledge the financial support of the Ministerio de Educación, Spain, trough project BEC2003-03965

I. Introduction

The model of the representative agent has become a fundamental tool for explaining the behavior of the exchange rate. Lucas (1978, 1982) Helpman and Razin (1979, 1982) Stockman (1980, 1983, 1987) Svensson (1985), Hodrick (1989) or Grilli and Roubini (1992) are classical references.

This literature has also contributed to the financial theory development. Lucas (1982) or Svensson (1985) are core references in the research on models for assets valuation in foreign currencies [see Baskhi and Chen, 1997 or Cao, 2001] or on determinants of risk premiums in the foreign exchange market [see Hodrick, 1989, Singleton, 1990, Kaminsky and Peruga, 1990, Engel, 1992a and 1992b, Dutton, 1993, Bekaert, 1994 and Hu, 1997].

However, the popularity of this approach is not justified by its empirical support. Meese and Rogoff (1983a , 1983b), Chinn and Meese (1995), Hodrick (1989), Roubini and Grilli (1995), Hu (1997) or Kim and Roubini (2000) show the difficulties in forecasting or explaining monthly exchange rate behaviour, using fundamental variables.

One of the reasons for the empirical validation "failure" of exchange rate models could be the econometric approach applied. Some assumptions made in order to obtain feasible econometric expressions from non-linear economic models, might be too restrictive. Also, reduced-form estimation equations of dynamic equilibrium models of exchange rate include domestic (foreign) money supply and real income among their relevant variables. Seasonality is inherent in such sets of explanatory variables. Econometricians, rather than explicitly investigating the economic underlying seasonal variation, typically remove its effects by using seasonally adjusted data. Wallis (1974) shows that the seasonal adjustment may distort the relations between variables.

In this paper, another procedure for evaluating fitness of dynamic equilibrium models, different from the standard econometric approach, is proposed. Following the works of Watson (1993), Kydland and Prescott (1982) and Prescott (1986), the economic model is viewed as an approximation to the stochastic processes generating the actual data. Therefore, if the economic model is correctly specified, then it could generate data with similar characteristics to those of observed data

In this approach, econometrics plays a neutral role in determining the ability of models to explain the exchange rate behaviour. The question of interest is whether "an economy derived from some simple exchange rate equilibrium model, could generate monthly time series of the exchange rate with similar statistical properties to those present in the observed exchange rate”. The paper develops a general framework to evaluate approximations between real and theoretical exchange rate.

Using standard assumptions on preferences, expressions for the exchange rate of the following theoretical models, are computed: (1) Lucas (1982), (2) Svensson (1985), and (3) Grilli and Roubini (1992). Then, approximating theoretical variables by their observed counterparts and giving values, on a wide range, to model parameters, time series of the British Pound / US Dollar (GBP/USD) exchange rate are generated. Finally, actual and simulated time series are compared.

The paper remains as follows. Section II summarizes the most relevant aspects of dynamic equilibrium models for the exchange rate. Section III presents the results of the simulation procedure, and Section IV shows concluding remarks.

II. Dynamic Equilibrium Models of the Exchange Rate

The main features of the most simple dynamic equilibrium model for the exchange rate are: (1) The world consists of two countries which produce one good each, (2) markets are in equilibrium due price flexibility, (3) agents have identical, homothetic and correctly defined preferences, (4) production and money stock are exogenous. Early contributions to this approach are: Lucas (1982), based on Lucas (1978), Helpman and Razin (1979, 1982), and Stockman (1980, 1983,1987). For these authors, the equilibrium exchange rate is a consequence of the individuals optimising behaviour, who invest in financial claims and face an intertemporal budget constraint.

For Stockman (1980) (ST) and Lucas (1982) (LU), the exchange rate is an endogenous variable. They provide a closed-form solution where the exchange rate is a non linear function of variables and preferences. Svensson (1985) (SV) and Hodrick (1989) (HO) extend the ST and LU original models in order to study the real effects of monetary policy. While in ST or LU models, new information arrives (i.e.: the state is observed) after closing the goods market and before opening the asset market, in SV and HO new information arrives before opening the goods market and after closing the asset market. Currencies are held to provide future liquidity services, they are stored because their value are priced endogenously as the shadow price of liquidity constraints. Expectations on future states have effects on the current value of money. All this leads to a new expression for the exchange rate, where expectations on future purchasing power of both currencies play a crucial role. For SV and HO the exchange rate is a forward looking price with agents accumulating money due to uncertainty about future.

Lucas (1990) incorporates liquidity constrains on assets markets and studies their effects on prices[1]. Former theoretical models were unable to induce enough volatility in the interest rate yield.

Grilli and Roubini (1992) (GR) present an open economy model with two countries. It is an extension of Lucas (1990) work that studies the effects on financial asset prices of liquidity constraints in asset markets. GR introduce cash in advance constraints in the asset market and shows that: (1) The equilibrium exchange rate depends on money demand in asset markets and the share of money used in asset transactions; (2) interest rates differentials determine the exchange rate and (3) the excess volatility of interest rates spills over to the exchange rate market and leads to excess volatility of exchange rates too. Table 1 shows the closed forms solutions the exchange rate derived from the ST, LU, HO, SV and GR models respectively.

1 "If cash is required for trading in securities, then the quantity of cash -of liquidity- available for this purpose at any time will in general influence the prices of securities traded at that time. That is, the price of a security will in general depend not only on the properties of the income stream to which it is a claim Bits fundamental- but also on the liquidity in the market at the time is traded", Lucas (1990, p. 237)

Table 1. Closed-form solutions for exchange rate

ST / LU	SV / HO[2]	GR
$S_t^{ST} = \frac{M_t^D}{M_t^F}\frac{Y_t^F}{Y_t^D}\frac{U_F(c_{it}^D, c_{it}^F)}{U_D(c_{it}^D, c_{it}^F)}$	$S_t^{SV/HO} = \frac{E_t\left[U_F\left(c_{it+1}^D, c_{it+1}^F\right)*\frac{Y_{t+1}^F}{M_{t+1}^F}\right]}{E_t\left[U_D\left(c_{it+1}^D, c_{it+1}^F\right)*\frac{Y_{t+1}^D}{M_{t+1}^D}\right]}$	$S_t^{GR} = \frac{\left(1 - Z_t^D\right)}{\left(1 - Z_t^F\right)}\frac{M_t^D}{M_t^F}\frac{Y_t^F}{Y_t^D}\frac{U_F\left(c_{it}^D, c_{it}^F\right)}{U_D\left(c_{it}^D, c_{it}^F\right)}\frac{q_t^D}{q_t^F}$

Notation:

Subscripts and superscripts	
t	Index variable for time.
D	Domestic country.
F	Foreign country.
$X^j_{i\,t}$	Superscripts generally refer to a type of good (Domestic or Foreign). Subscripts refer to the agent who holds or consumes the good. Therefore.
M^F_{Dt}	Is foreign holdings of home money on date *t*
U_D	Equilibrium marginal utility of domestic consumption.
U_F	Equilibrium marginal utility of foreign consumption

Major variables and parameter definition	
$C^j_{\ i}$	Consumption of goods of country *j* by the agent of country *i*
M^j_t	Total amount of money of country *j*
N^j_i	Amount of money of country *j* held by agent of country *i* for transaction in the good market.
P^j_t	The local currency prices of good *j* in country *j*
$q^j_{\ t}$	The local currency prices of one period discount bond j in country *j*
S_t	Nominal Spot exchange rate expressed as the domestic price of foreign currency.
S_t^{GR}	Equilibrium exchange rate from Grilli and Roubini (1992)
S_t^{LU}	Equilibrium exchange rate from Hodrick (1989)
S_t^{LU}	Equilibrium exchange rate from Lucas (1982)
S_t^{ST}	Equilibrium exchange rate from Stockman (1980)
S_t^{SV}	Equilibrium exchange rate from Svensson (1985)
Y^j_t	Goods Endowment of country j
Z^j_t	Amount of money of country j held by the agent for transactions in the asset market

III. Simulation Procedure

The existence of closed form solutions for the exchange rate is the key of our proposal. In all cases, the equilibrium exchange rate is a non linear function of preferences or expected preferences, outputs and money. If preferences, expectation generating process, outputs and money stocks are adequately approximated, it is possible to generate time series of the corresponding theoretical exchange rate. The ability of theoretical models for replicating the actual exchange rate behaviour will be judged by comparing actual and simulated time series.

Monthly time series of the exchange rate are simulated from LU, SV and GR theoretical models, the steps of the simulation procedure are:

(1) Choice of a particular functional form for preferences. Two types of utility

[2] Hodrick (1989) analyses a version of the cash in advance model presented in Svensson (1985a, b) and adds a discussion of exogenous fiscal policy and examine time-varying conditional variances of the exogenous processes.

functions are considered: Separable (S) utility function and CES utility function (Table 2). Simulation exercises have been carried out for both types of functions.

Table 2. Utility functions: CES and Separable[3]

CES	SEPARABLE
$U\left[c_{it}^{D},\ c_{it}^{F}\right]=\frac{1}{1-\gamma}[(c_{it}^{D})^{\varepsilon}+(c_{it}^{F})^{\varepsilon}]^{\frac{1-\gamma}{\varepsilon}}$	$U\left[c_{it}^{D},\ c_{it}^{F}\right]=\frac{1}{1-\gamma_D}\left(c_{it}^{D}\right)^{1-\gamma_D}+\frac{1}{1-\gamma_F}\left(c_{it}^{F}\right)^{1-\gamma_F}$

(2) Derivation of the corresponding solution for the exchange rate. This is done by introducing the particular functional form for preferences into the equilibrium solution. In general, the resulting equilibrium exchange rate is a non linear function of outputs, monetary aggregates, interest rates (only in GR), risk aversion and the intertemporal substitution elasticity, see Table 3.

(3) Approximation of the theoretical variables by their observed counterparts. Outputs are approximated by monthly industrial production indexes and monetary aggregates are approximated by M2. The chosen interest rates are: the Three Months Interbank UK interest rate and the Deposits Certificates US interest rate.

(4) Computation of expectations. They have been carried out by using univariate Autoregressive Integrated Moving Average (ARIMA) models [Box-Jenkins, 1970].

(5) Values assignation to the risk aversion and elasticity of substitution parameters. Although they are fundamental elements in asset valuation, experimental research has provided little guidance about their true values[4]. In this paper, the risk aversion takes the values [0, 1, 2, 3, 4], and the elasticity of substitution is allowed to vary from 0 to 1, by steps of 0.10 units. In both

3 The constant-elasticity-of-substitution (CES) function implies that the elasticity of substitution between domestic and foreign is constant ($\sigma = (1/(1-\varepsilon))$). As σ approaches 1, CES function becomes the Cobb-Douglas function. *Separable preferences* are the extreme case of infinite elasticity of substitution, or perfect substitutability between goods. γ is the coefficient of relative risk aversion that is the inverse of intertemporal elasticity of substitution. γ determines the household's willingness to shift consumption between different periods: The smaller is γ, the more slowly marginal utility falls as consumption rises, and so the more willing the household is to allow its consumption to vary over time. If γ is close zero, for example, utility is almost linear, and so the household is willing to accept large swings in its consumption to take advantage of small differences between its discount rate and the rate of return it gets on its saving. In the special case of $\gamma \rightarrow 1$ the utility function simplifies to logarithmic.

4 Arrow (1971) summarizes a number of studies and concludes that relative risk aversion with respect to wealth is almost constant. He further argues on theoretical grounds that should be approximately one. Friend and Blume (1975) present evidence based upon the portfolio holdings of individuals that the relative risk aversion is larger, with their estimates being in the range of two. Kydland and Prescott (1982), in their study of aggregate fluctuations, found that they needed a value between one and two to mimic the observed relative variability of consumption and investment. Altug (1983) estimates the parameter to near cero. Kehoe (1983), studying the response of small countries balance of trade to terms of trade shocks, obtained estimates near one. Hildreth and Knowles (1986), in their study of the behaviour of farmers also obtain estimates between one and two. Mehra and Prescott (1985) present evidence for restricting the value of relative risk aversion to be a

cases, the grid has been set to cover the interval of two standard deviation around the estimated parameters (Table 4) which have been obtained by the General Method of Moments (GMM).[5]

(6) Simulation of exchange rate time series. For each model, Table 3 provides the expressions for calculating theoretical exchange rate.

(7) Comparison between simulated (*SimExR)* and observed (*ObsExR*) time series. This is done by comparing the univariate ARIMA models for both series. Order of integration type and degree of autocorrelation between *SimExR* and *ObsExR* should be very similar, provided the theoretical model is an adequate approximation for the true data generation process.

Table 3. Closed-form solutions for exchange rate under CES and Separable preferences [(a)]

	CES	SEPARABLE
LU	$S_t^L = \frac{M_t^D}{M_t^F}\left(\frac{Y_t^F}{Y_t^D}\right)^{\varepsilon}$	$S_t^L = 2^{(\gamma_F-\gamma_D)}\frac{M_t^D}{M_t^F}\frac{\left(Y_t^F\right)^{(1-\gamma_F)}}{\left(Y_t^D\right)^{(1-\gamma_D)}}$
SV	$S_t^{SV} = \frac{E_t\left\{\left[\left(Y_{t+1}^D\right)^{\varepsilon}+\left(Y_{t+1}^F\right)^{\varepsilon}\right]^{\frac{1-\gamma}{\varepsilon}-1}\frac{\left(Y_{t+1}^F\right)^{\varepsilon}}{N_{t+1}^F}\right\}}{E_t\left\{\left[\left(Y_{t+1}^D\right)^{\varepsilon}+\left(Y_{t+1}^F\right)^{\varepsilon}\right]^{\frac{1-\gamma}{\varepsilon}-1}\frac{\left(Y_{t+1}^D\right)^{\varepsilon}}{N_{t+1}^D}\right\}}$	$S_t^{SV} = \frac{E_t\left\{\left(\frac{1}{2}\right)^{-\gamma_F}\frac{\left(Y_{t+1}^F\right)^{1-\gamma_F}}{M_{t+1}^F}\right\}}{E_t\left\{\left(\frac{1}{2}\right)^{-\gamma_D}\frac{\left(Y_{t+1}^D\right)^{1-\gamma_D}}{M_{t+1}^D}\right\}}$
GR[(b)]	$S_t^{GR} = \frac{E_t\left\{\left[\left(Y_{t+1}^D\right)^{\varepsilon}+\left(Y_{t+1}^F\right)^{\varepsilon}\right]^{\frac{1-\gamma}{\varepsilon}-1}\frac{\left(Y_{t+1}^F\right)^{\varepsilon}}{N_{t+1}^F}\frac{1}{q_t^F}\right\}}{E_t\left\{\left[\left(Y_{t+1}^D\right)^{\varepsilon}+\left(Y_{t+1}^F\right)^{\varepsilon}\right]^{\frac{1-\gamma}{\varepsilon}-1}\frac{\left(Y_{t+1}^D\right)^{\varepsilon}}{N_{t+1}^D}\frac{1}{q_t^D}\right\}}$	$S_t^{GR} = \frac{E_t\left\{\left(\frac{1}{2}\right)^{-\gamma_F}\frac{\left(Y_{t+1}^F\right)^{1-\gamma_F}}{M_{t+1}^F}\frac{1}{q_t^F}\right\}}{E_t\left\{\left(\frac{1}{2}\right)^{-\gamma_D}\frac{\left(Y_{t+1}^D\right)^{1-\gamma_D}}{M_{t+1}^D}\frac{1}{q_t^D}\right\}}$

Notes: (a) Following Lucas (1982), we assume *pooling equilibria* ($c^D_{it}=Y^D_t / 2,\ c^F_{it}=Y^F_t / 2$)

(b) Following Lucas (1982) and Svensson (1985), production and money stock are stochastic.

maximum of ten, though without specifying a concrete value.

5 Hansen and Singleton (1982) show how to apply GMM to a Consumption-Based Capital Asset Pricing model. Following Lucas (1978), they suppose that a representative consumer chooses stochastic consumption and investment plans to maximize your utility. The dynamic optimization problem implies a set of stochastic Euler equations that must be satisfied in equilibrium. They estimate the parameters of preferences directly from Stochastic Euler equations. We use GMM to estimate the parameters when the consumption are approximated by the UK and US industrial production indexes and the stocks returns are approximated by the returns on the FT-100 (FT) of London Stock Exchange and the returns on the Dow-Jones (DJ) of New York Stock Exchange. Table 4 shows the estimation results.

Table 4. GMM estimation of utility function parameters[(a)]

β - SEPA[(b)]	β – CES	γ_D	γ_F	γ- CES	ε - CES	J_Sta - SEPA[(d) (e)]	J_Sta - CES
0.989 [(c)] (0.01)	0.993 ($6*10^{-6}$)	1.13 (0.08)	1.33 (0.16)	1.08 (0.00)	0.037 ($9*10^{-5}$)	15.84 (0.26)	13.05 (0.44)

Notes:

(a) Instruments are: a constant term, lagged production growth rates, lagged monetary aggregates growth rates, and lagged rates of return

(b) β is the time discount factor.

(c) Estimated standard errors in parentheses

(d) J-statistic, for testing the validity of overidentifying restrictions. Under the null hypothesis, the overidentifying restrictions are satisfied, the J-statistic (i.e., the minimized value of the objective function) times the number of observations is asymptotically χ_q^2, with degrees of freedom equal to the number of overidentifying restrictions

(e) P- values represented in parentheses

Results

Tables 6-8 and 10-12 report a variety of descriptive statistics of *SimExR*, over the sample period 1990:01-1998:04: Mean (M), standard deviation (Std), skewness (Skw), kurtosis (Kt) and the order of integration (d). In addition, the tables show the maximum likelihood estimation of the parameters of ARIMA models. Diagnostic checks are developed to detect model inadequacy. Descriptive statistics of the residuals from estimated models are reported: mean ($\bar{a}_t$) and estimated mean standard error ($\hat{\sigma}_{\bar{a}}$), estimated standard errors ($\hat{\sigma}_a$) and Ljung-Box Q-statistics at lag 12 to test for serial correlation (Q(12))

Tables 6 to 8 show the ARIMA models for *SimExR* when agents are represented by separable utility functions (*SimExR**S***). For each model, all combinations with five possible values (0, 1, 2, 3, 4) for the risk aversion parameters are considered, i.e. 25 cases.

Table 6 shows the results for the LU model. All simulated time series, except *SimExRS11*[6] (Figure 5), show a strong seasonal behaviour. Either they need a seasonal difference or show a seasonal AR(1) factor with parameter close to 1. The case of *SimExRS11* is different, it does not show seasonality because it corresponds to the case of a logarithmic utility function. In this type of function, production (the variable with the strongest seasonal component)disappears from the data generating process, leaving the ratio between money stocks as the only relevant variable in determining exchange rate. Although *SimExRS11* is an I(1) variable (i.e., integrated of order 1) as the actual exchange rate, it exhibits an increasing trend (Pound depreciation) during the sample interval (1990-1998). The opposite feature is showed by the actual GBP/USD. As the risk aversion

6 *SimExRS* γ_F γ_D identifies the preferences. *SimExRS01:* the first number, "*0*",denotes risk aversion parameter in foreign goods, the last number, "1", denotes risk aversion parameter in domestic goods

parameter approaches to 4, industrial production becomes more important as well as the seasonal component of the *SimExRS*.

Table 7 shows the case of SV model. Although SV shows important differences with respect to LU, the results are quite similar. In all cases, except for the logarithmic utility function, the seasonal component is very strong.

Table 8 shows the GR model simulations. In this case, the short term interest rates enter in the data generation process with important effects. All simulated time series are I(1) with SimExRS11 and SimExRS21 following random walks as the actual GBP/USD exchange rate. In these special cases, it is possible to test for cointegration between *ObsExR* and *SimExR*. Cointegration with the actual GBP/USD exchange rate is found when the risk aversion parameter for the foreign good is lower than 3. However, the slope of the *SimExR* time series has the wrong sign, the cointegration coefficient is always negative, see Table 9.

Figures 2-4 present 3 interesting cases for the times series of simulated exchange rate for the LU, SV and GR models: *SimExRS11* (logarithmic utility function), *SimExRS14*, in this case simulated exchange rate for the LU and SV models replicates appreciation observed time series, nonetheless the seasonal component is particularly large, *SimExRS41* provides striking evidence against these risk aversion parameters, because the seasonal fluctuations are large and regular.

Tables 10 to 12 show ARIMA models for *SimExR* when agents are represented by a CES type utility functions (*SimExR**CES***). For SV and GR models, 55 cases are considered that arise from combining 5 values for the risk aversion parameter (γ) with 11 possible values for the parameter which determines the substitution elasticity (ε) between goods (0, .1, .2, .3, .4, .5, .6, .7, .8, .9, 1). Only 15 cases are presented in this paper, corresponding to 5 values of the risk aversion parameter (0, 1, 2, 3, 4) combined with three possible values for the substitution elasticity (.1, .5, 1). In the case of LU, where γ is not relevant, 11 time series have been generated, one for each value of ε.

Table 10 shows the results for LU model. In this case ε weighs the relative production between countries, also it is the factor that makes *SimExR* to differ. All time series show a strong seasonal component which is evident by the presence of a seasonal unit root.

Table 11 shows the case of SV model. Here, agents' expectations vary according to the degree of risk aversion. Again, all time series show a clear seasonal behaviour. Nevertheless, the seasonal unit root is not always present, in particular when ε is small.

A common feature in both models is that all *SimExR* time series show Pound depreciation.

Table 12 shows the simulations corresponding to GR model. In many time series the seasonal structure does not appear. In addition, in many cases is possible to identify a random walk process as the univariate data generation process. The problem, as in the case of separable utility function, the cointegration coefficient is negative, see Table 13.

Short term interest rates play an important role for both generated and actual time series to have similar statistical properties. Thus, is possible to find some theoretical models able to produce time series following random walk processes. However, short term interest rates are not enough for replicating the actual exchange rate slope. *SimExR*, under GR, seem to be very similar to the ratio of interest rates. Last column of Table 13 shows the correlation coefficient between the ratio of interest rates and the *SimExR* time series, it varies between 0.94 and 0.98.

Some simulated time series are shown in Figures 5-8. Four cases are considered that arise from combining the lowest value for ε (the substitution elasticity = 0.1), with the 5 possible values for ν (the risk aversion parameter:0, 1, 2, 3, 4. For ε = 0.1, the weights of relative production between countries becomes less important as well as the seasonal components. In these cases, the GR model reproduces the stochastic process of *ObsExR* series, and, in four of fifty cases it is possible to identify a random walk process as the univariate data generation process.

IV. Conclusions

In this paper, we ask whether Equilibrium models of exchange rate can generate monthly time series of the exchange rate with similar statistical properties to those present in the actual data. This work judges the fit of exchange rate equilibrium models using a methodology based on the works of Watson (1993), Kydland and Prescott (1982) and Prescott (1986). In this approach, it is asked whether data from a real economy share certain characteristics with data generated by the artificial economy described by an economic model. For evaluating the fit of the dynamic structural economic model, we simulate monthly time series for the GBP/USD exchange rate based on GR, LU and SV models.

The results of this paper suggest that LU and SV models cannot match the stochastic process generating the actual data. All time series show a strong seasonal component and fails to capture the pound appreciation. Seasonal movements in times series simulated from the GR model are noticeably smaller than for the simulated under LU and SV models. In many cases is possible to identify a random walk process as the univariate data generation process, but, direct examination shows that the ratio of interest rates seems to drive the nominal exchange rate, i.e. none fundamental variables, except interest rate, may have no explanatory power for exchange rate. It seems implausible that the ratio of interest rates would be important enough to drive alone the nominal exchange rate.

The evidence presented here offers a rejection of the equilibrium model of exchange rate. One possible explanation Output and money time series have a strong seasonal component, and it is obvious that economic agents must react to that. The seasonal variation in economic time series must be an integrated part of the optimizing behaviour of economic agents.

Acknowledgements

We would like to acknowledge the financial support of the Ministerio de Educación, Spain, trough project BEC2003-03965. We are grateful to A. Aznar, M. Gracia, J. M. González-Páramo, J. Del Hoyo and A. Novales for helpful comments.

References

Altug, S., 1983. "Gestion lags and the business cycle: an empirical analysis", *Carnegie-Mellon Working Paper* , presented in The Econometric Society meeting, Stanford University, Carnegie-Mellon University, Pittsburgh, PA.

Arrow, K. J., 1971. *Essays in the Theory of Risk-Bearing*, North-Holland, Amsterdam.

Bakshi, G., Chen, Z., 1997. "Equilibrium valuation of Foreign Exchange Claims", *The Journal of Finance*, 52, 799-826.

Bekaert, G., 1994. "Exchange rate volatility and deviations from unbiasedness in cash-in-advance model", *Journal of International Economic,* 36, 29-52.

Box, G.E.P., Jenkins, G. M., 1970. *Time Series Analysis, Forecasting and Control*, ed. Holden-Day, San Francisco.

Box, G. E. P., Tiao, G. C., 1975. Intervention Analysis with applications to economic and environmental problems", *Journal of American Statistical Association,* 70, 70-79.

Cao, M., 2001. "Systematic jump risk in a small open economy: simultaneous equilibrium valuation of options on the market portfolio and exchange rate", *Journal of International Money and Finance,* 20, 191-218.

Chinn, M. D., Meese, R. A., 1995. "Banking on currency forecasts: How predictable is change in money", *Journal of International Economics,* 38, 161-178.

Dutton, J., 1993. "Real and Monetary Shocks and risk premia in forward markets for foreign exchange", *Journal of Money, Credit, and Banking*, 25, 731-754.

Engel, C., 1992b. "The risk premium and the liquidity premium in foreign exchange markets", *International Economic Review*, 33, 871-879.

Engel, C., 1992a. "On the foreign exchange risk premium in a general equilibrium model", *Journal of International Economics,* 32, 305-319.

Friend, I., Blume, M. E., 1975. "The demand for risky assets", *American Economic Review*, 65, 900-922.

Grilli, V., Roubini, N., 1992. "Liquidity and exchange rates", *Journal of International Economics,* 33, 339-352.

Hansen, L. P., Singleton, K. J., 1982. "Generalized instrumental variables estimation of nonlinear rational expectations models", *Econometrica*, 50 (5), 1269-1286.

Helpman, E., Razin, A., 1982. "A comparison of exchange rate regimes in the presence of imperfect capital markets", *International Economic Review,* 23, 365-88.

Helpman, E., Razin, A., 1979. "Towards a consistent comparison of alternative exchange rate systems", *Canadian Journal of Economics,* 12, 394-409.

Hildreth, C., Knowles, G. J., 1986. "Farmers' Utility Functions", en *Bayesian Inference and Decision Techniques: Essays in Honor of Bruno de Finetti. Studies in Bayesian Econometrics and Statistics Series*, 6, Amsterdam and Oxford: North-Holland; distributed in the U.S. and Canada by Elsevier Science, New York, in Goel, P. and Zellner, A. (ed.), 291-317.

Hodrick, R. J., 1989. “Risk uncertainty, and exchange rates”, *Journal of Monetary Economics,* 23, 433-459.

Hu, X., 1997. “Macroeconomic Uncertainty and risk premium in the foreign exchange market”, *Journal of International Money and Finance,* 16, 699-718.

Kaminsky, G., Peruga, R., 1990. “Can a time-varying risk premium explain excess returns in the forward market for foreign exchange?”, *Journal of International Economics,* 28, 47-70.

Kehoe, T., 1983. “Dynamics of the current account: theoretical and empirical analysis”, *Working Paper*, Harvard University, Cambridge, MA.

Kim, S., Roubini, N., 2000. “Exchange anomalies in the industrial countries: a solution with a sturctural VAR approach”, *Journal of Monetary Economics*, 45, 561-586.

Kydland, F. E., Prescott, E. C., 1982. “Time to Build and Aggregate Fluctuations”, *Econometrica*, 50, 1345-1370.

Lucas, R. E. JR, 1978. “Asset Prices in an Exchange Economy”, *Econometrica*, 46, 1429-45.

Lucas, R. E. JR, 1982. “Interest rates and currency prices in a two-country world”, *Journal of Monetary Economics*, 10, 335-359.

Lucas, R. E. JR, 1990. “Liquidity and interest rates”, *Journal of Economic Theory,* 50, 237-264.

Meese, R. A., Rogoff, K., 1983b. “The out-of-sample failure of empirical exchange rate models: sampling error or misspecification?”, en *Exchange Rates and International Macroeconomics,* in Frenkel, J. (ed.), Chicago, University of Chicago Press.

Meese, R. A., Rogoff, K., 1983a. “Empirical exchange rate models of the seventies: Do they fit out of sample”, *Journal of International Economics,* 14, 3-24.

Mehra, R., Prescott, E. C., 1985. “The Equity premium: a puzzle”, *Journal of Monetary Economics,* 15, 145-161.

Prescott, E. C., 1986. “Theory ahead of business-cycle measurement”, *Carnegie-Rochester Conference Ser. Public Policy,* 25, 11-44.

Roubini, N., Grilli, R., 1995. “Liquidity models in open economies: theory and empirical evidence”, *NBER,* October, wp. 5313.

Singleton, K. J., 1990. “Specification and estimation of intertemporal asset pricing models”, *Handbook of Monetary Economics,* Volume I, in Friedman B. M. and Hahn F. H. (eds.).

Stockman, A. C., 1987.“The Equilibrium Approach to exchange rates”, *Federal Reserve Bank of Richmond Economic Review*; 73(2), 12-30.

Stockman, A. C., 1983.“Real exchange rates under alternative nominal exchange rate systems”, *Journal of International Money and Finance,* 2(2), 147-166.

Stockman, A. C., 1980. “A theory of exchange rate determination”, *Journal of Political Economy,* 88, 673-698.

Svensson, L. E. O., 1985. “Currency prices, terms of trade, and interest rates. A general equilibrium asset-pricing cash in advance approach”, *Journal of International Economics*, 18, 17-41.

Watson, M. W., 1993. "Measures of fit for calibrated models", *Journal of Political Economy*, 101, 1011-1041.

Wallis, K. F., 1974. "Seasonal Adjustment and the relation between variables", *Journal of the American Statistical Association,* 69, 13-32.

Apéndice

Sección 1: Descripción

En la simulación se utilizan datos mensuales sin desestacionalizar desde 1986:01 hasta 1998:04. El tipo de cambio nominal se define como el precio de la moneda extranjera (US dollar-USD) in términos de la monedad doméstica (British Pound-GBP). La oferta monetaria se mide por M2 y el Índice Producción Industrial (IPI) se utiliza para aproximar la renta. Para aproximar los tipos de interés a corto plazo (R), en el caso del Reino Unido se utilizan el tipo interbancario a tres meses, para Estados Unidos se consideran los tipos de los certificados de depósitos a tres meses. Ambos son medias de los datos diarios. Los índices bursátiles provienen del *Financial Times.* Para el Reino Unido se utiliza el FT-100 (FT) y para Estados Unidos el Dow-Jones (DJ), la base para ambos índices es Diciembre de 1994.

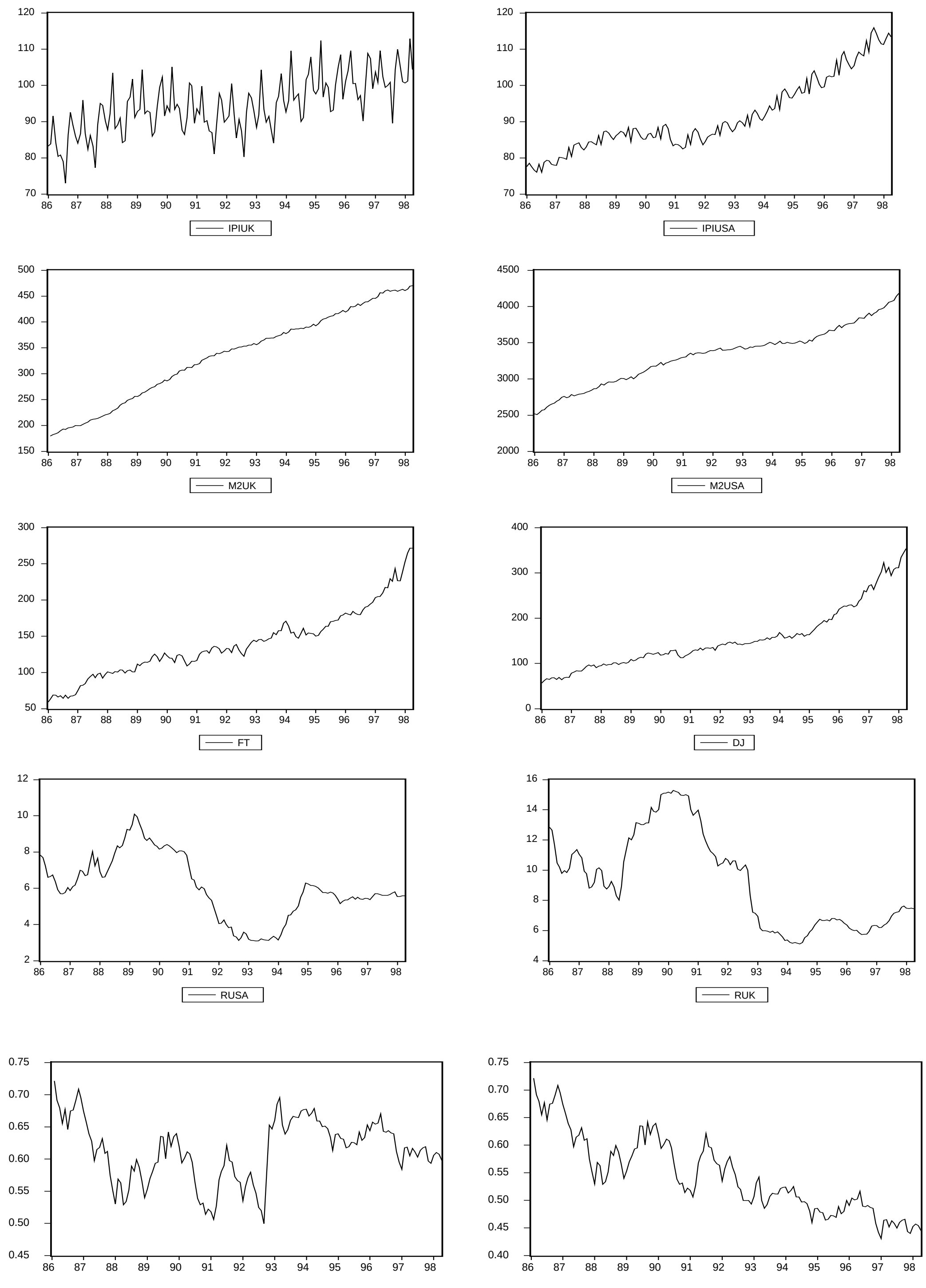
El tipo de cambio y los datos de producción proceden de la OCDE y M2 se obtiene de las Estadísticas Financieras Internacionales del FMI.

Como paso preliminar para la simulación se contrasta la presencia de valores extremos. Se desarrolla análisis de intervención [Box and Tiao, 1975] en septiembre de 1992 in el tipo de cambio GBP/USD y en el tipo de interés a corto plazo del Reino Unido (el gobierno británico decide sacar la libra temporalmente del sistema Monetario Europeo). El DJ, FT y el tipo de interés a corto plazo de Estados Unidos presentan valores extremos en octubre de 1987 (*Crash* del mercado de acciones). M2UK presenta un valor extremo en diciembre de 1992. El IPI presenta efecto Semana Santa. En las simulaciones se utilizan las series temporales corregidas de estas intervenciones.

Como se puede ver en la Figura 1 y en la Tabla 5, el análisis de los datos indica que las series nos son estacionarias. Las series temporales del IPI y M2 presentan un fuerte componente estacional. El tipo de cambio GBP/USD se aprecio durante el período de estudio, excepto en la devaluación de la libra en la crisis de septiembre de 1992. El proceso estocástico de paseo aleatorio es consistente con el proceso generador de datos para el tipo de cambio GBP/USD y para las series de rendimientos bursátiles, DJ y FT. La Figura 1 muestra todas las series observadas.

Section 2: Real Data

Figure 1. Real Data[7]



GBP/USD

GBP/USD (I)

[7] GBP/USD (I) does not include the September 1992 devaluation, see Table 5.

Table .5. Summary of ARIMA models fitted to real data [(a)]

Variables	$\nabla^d \nabla_s$	ARIMA (R) (p, d, q)	ARIMA (S) (P, D, Q)	$\overline{a}$ $(\hat{\sigma}_{\overline{a}})$	$\hat{\sigma}_a$	Q(12)	Outliers[(b)]	ARIMA MODELS [(c) (d)]
M2USA	$\nabla\nabla_{12}$	(1,1,0)	(0,1,1)	0.012 (0.773)	8.98	10.85	5/93, 5/94, 6/95, 3/96, 8/97.	$(1\text{-}0.561B)\nabla\nabla_{12}\, Y_t = (1\text{-}0.480B^{12})\, a_t$ (0.072) (0.080)
	$\nabla\nabla_{12}$	(1,1,0)	(-1,1,0)	0.109 (0.779)	9.05	8.78	5/87, 2/93,5/93,5/94 6/95,3/96, 4/96,8/97	$(1\text{-}0.558B)\,(1\text{-}0.480B^{12}\nabla\nabla_{12}Y_t = a_t$ (0.072) (0.078)
M2UK	$\nabla\nabla_{12}$	(1,1,0)	(1,1,0)	-0.017 (0.156)	1.81	9.38	6/89*, 7/96, 6/97, 9/97*	$Y_t = -21.452\ \xi_t^{S12/92} + N_t$ (1.729) $(1+0.210\,B)\ \nabla\nabla_{12}N_t = (1\text{-}0.805\,B^{12})\, a_t$ (0.084) (0.046)
	$\nabla\nabla_{12}$	(1,1,0)	(3,1,0)	-0.010 (0.157)	1.83	11.63	6/89*, 6/90, 9/92, 7/96, 6/97*, 9/97*	$Y_t = -21.238\ \xi_t^{S12/92} + N_t$ (1.622) $(1+0.154\,B)\,(1+0.665\,B^{12} +0.436\,B^{24} +0.375\,B^{36})\ \nabla\nabla_{12}N_t = a_t$ (0.087) (0.094) (0.108) (0.091)
	$\nabla\nabla_{12}$	(-3,1,0)	(1,1,0)	-0.026 (0.113)	1.32	12.44	11/90, 9/92, 6/93,5/94, 4/95, 3/96,7/96,8/96	$Y_t = -7.598\ \xi_t^{S6/89} - 22.457\ \xi_t^{S12/92} + 6.157\ \xi_t^{S6/97} - 6.860\ \xi_t^{I9/97} + N_t$ (1.100) (1.106) (1.265) (0.971) $(1+0.174\,B +0.229\,B^2 +0.374\,B^3)\ \nabla\nabla_{12}N_t = (1\text{-}\ 0.648\,B^{12})\, a_t$ (0.082) (0.079) (0.081) (0.067)
	$\nabla\nabla_{12}$	(3,1,0)	(3,1,0)	-0.025 (0.113)	1.31	10.68	11/90, 9/92, 3/96, 7/96, 4/97.	$Y_t = -7.671\ \xi_t^{S6/89} - 22.414\ \xi_t^{S12/92} + 6.676\ \xi_t^{S6/97} - 6.701\ \xi_t^{I9/97} + N_t;$ (1.085) (1.077) (1.255) (0.960) $(1+0.156\,B +0.254\,B^2 +0.383\,B^3)(1+0.636\,B^{12} +0.380\,B^{24} +0.280\,B^{36})\ \nabla\nabla_{12}N_t = a_t$ (0.082) (0.078) (0.084) (0.091) (0.107) (0.095)
IPIUK	∇_{12}	(3,0,0)	(3,1,1)	-0.006 (0.128)	1.46	19.05	9/88,9/90,4/91,5/92,1/93, 10/93, 2/96	$Y_t = -2.193\ \xi_t^{SS} + N_t\,;$ (0.472) $(1+0.002\,B - 0.296\,B^2 - 0.583\,B^3)\,(1\text{-}\ 0.245\,B^{12} + 0.343\,B^{24} + 0.184\,B^{36})\,[\nabla_{12}\, N_t - 1.684] = (1\text{-}0.852\,B^{12})\, a_t$ (0.072) (0.067) (0.071) (0.091) (0.078) (0.076) (0.172) (0.029)
	$\nabla\nabla_{12}$	(2,1,0)	(3,1,1)	0.162 (0.131)	1.60	19.36	9/88,9/90,4/91,5/92, 2/96 11/97.	$Y_t = -2.105\ \xi_t^{SS} + N_t\,;$ (0.481) $(1+0.962\,B +0.623\,B^2)\,(1\text{-}\ 0.221\,B^{12} + 0.303\,B^{24} + 0.187\,B^{36})\ \nabla\nabla_{12}\, N_t = (1\text{-}0.826\,B^{12})\, a_t$ (0.068) (0.068) (0.094) (0.078) (0.076) (0.039)
IPIUSA	$\nabla\nabla_{12}$	(3,1,0)	(1,1,0)	0.045 (0.050)	0.58	9.82	7/89, 11/90*, 12/90, 4/95, 2/96.	$Y_t = -0.675\ \xi_t^{SS} + N_t;$ (0.134) $(1+0.045\,B - 0.131\,B^2 - 0.239\,B^3)\ \nabla\nabla_{12}\, N_t = (1\text{-}0.532\,B^{12})\, a_t$ (0.083) (0.084) (0.084) (0.073)
	$\nabla\nabla_{12}$	(3,1,0)	(3,1,0)	0.047 (0.049)	0.57	5.83	2/87, 7/89, 11/90*, 12/90 2/96.	$Y_t = -0.711\ \xi_t^{SS} + N_t;$ (0.116) $(1+0.051\,B - 0.182\,B^2 - 0.217\,B^3)(1\text{-}\ 0.386\,B^{12} + 0.360\,B^{24} + 0.257\,B^{36})\ \nabla\nabla_{12}N_t = a_t$ (0.084) (0.084) (0.085) (0.084) (0.084) (0.087)
RCUK	∇	(3,11)	(0,0,0)	$-2.2*10^{-2}$ $(2.6*10^{-2})$	0.32	11.6	3/86, 4/86, 10/86, 5/87 12/88, 6/89, 10/89, 10/90	$Y_t = (-1.49 - 1.11\,B)\ \xi_t^{S10/92} + N_t;$ (0.32) (0.32) $(1 + 0.11B - 0.25\,B^2 + 0.22\,B^3)\ \nabla\, N_t = (1 - 0.57\,B)\ a_t$ (0.24) (0.11) (0.08) (0.24)
RCUSA	∇	(3,1,0)	(0,0,0)	$-1.0*10^{-3}$ $(1.7*10^{-2})$	0.20	6.9	9/87,1/91,7/92	$Y_t = 0.57\ \xi_t^{I1087} + 0.55\ \xi_t^{I1287} + N_t;$ (0.11) (0.11) $(1\text{-}\ 0.51\,B + 0.18\ B^2 - 0.19\,B^3)\ \nabla\, N_t = a_t$ (0.08) (0.09) (0.08)
FT ($\lambda = 0$)[(e)]	∇	(0,1,0)	(0,0,0)	$1.6*10^{-18}$ $(3.5*10^{-3})$	0.04	8.5	01/89,10/89,05/90,10/97	$Y_t = (-0.31 - 0.11\,B)\ \xi_t^{S1087} + N_t;$ (0.02) (0.02) $(\nabla N_t - 0.012) = a_t$ (0.003)
DJ ($\lambda = 0$)	∇	(1,1,0)	(0,0,0)	$-1.5*10^{-13}$ $(3.1*10^{-3})$	0.04	14.4	01/87,11/87,8/90, 9/90	$Y_t = -0.31\ \xi_t^{S1087} + N_t;$ (0.04) $(1 + 0.21\,B)\,(\nabla N_t - 0.014) = a_t$ (0.08) (0.003)
GBP/USD	∇	(0,1,0)	(0,0,0)	$-1.9*10^{-5}$ $(1.4*10^{-3})$	0.02	7.19	07/86,02/88,07/88,06/89,0 8/89,09/89,04/93	$Y_t = (0.08 + 0.08\,B)\ \xi_t^{S1092} + N_t;$ (0.02) (0.02) $\nabla\, N_t = a_t$

Notes:

(a) ∇: *Difference Operator*; **B**: *backward shift operator;* $\nabla_s = (1\text{-}B^s)$. Descriptive statistics of the residuals from estimated models are reported: mean and estimated mean standard error $\overline{a}\ /\left(\hat{\sigma}_{\overline{a}}\right)$, estimated standard errors ($\hat{\sigma}_a$) and Ljung-Box Q-statistics at lag 12 to test for serial correlation (Q(12)).

(b) Residuals over two standard errors (* Residuals over three standard errors)

(c) Estimated standard errors in parentheses

(d) $\xi_t^{I"T"} = \begin{cases} 1 & t = T \\ 0 & t \neq T \end{cases}; \quad \xi_t^{S"T"} = \begin{cases} 1 & t \geq T \\ 0 & t < T \end{cases}; \quad \xi_t^{SS} = \begin{cases} 1 & t = \text{month within Easter holidays} \\ 0 & \text{Re}\ st \end{cases}$

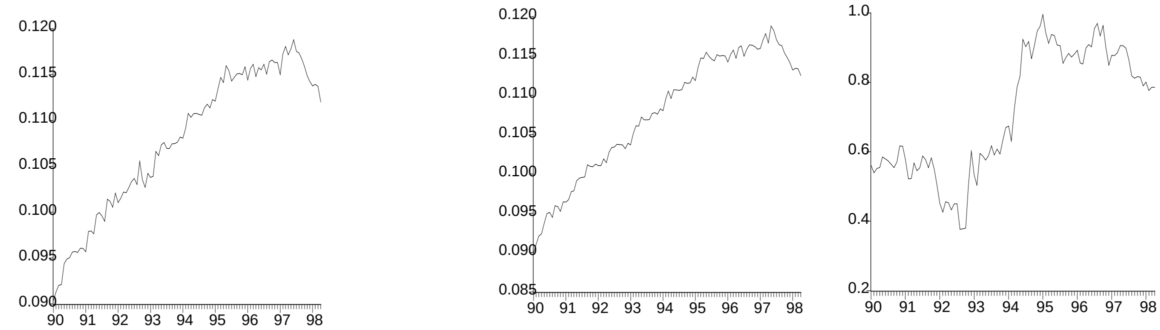
(e) Box-Cox transformation

Section 3: *SimExR*: Separable preferences[8]

Figure 2

EQUILIBRIUM EXCHANGE RATE FOR $\gamma_D = 1$, $\gamma_F = 1$

LU SV GR



LU: $(1-0.44B^{12}-0.41B^{24})\nabla Y_t = a_t$
(0.10) (0.10)

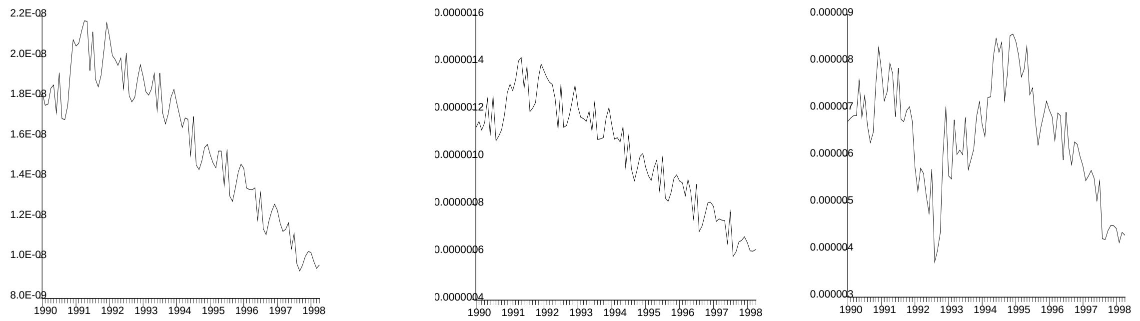
SV: $(1+0.23B+0.02B^2-0.28B^3)(1-0.50B^{12})\nabla Y_t = a_t$
(0.10) (0.10) (0.10) (0.09)

GR: $\nabla Y_t = a_t$

Figure 3

EQUILIBRIUM EXCHANGE RATE FOR $\gamma_D = 1$, $\gamma_F = 4$

LU SV GR



LU: $(1+0.37B^{12}+0.24B^{24})\nabla\nabla_{12} Y_t = a_t$
(0.09) (0.09)

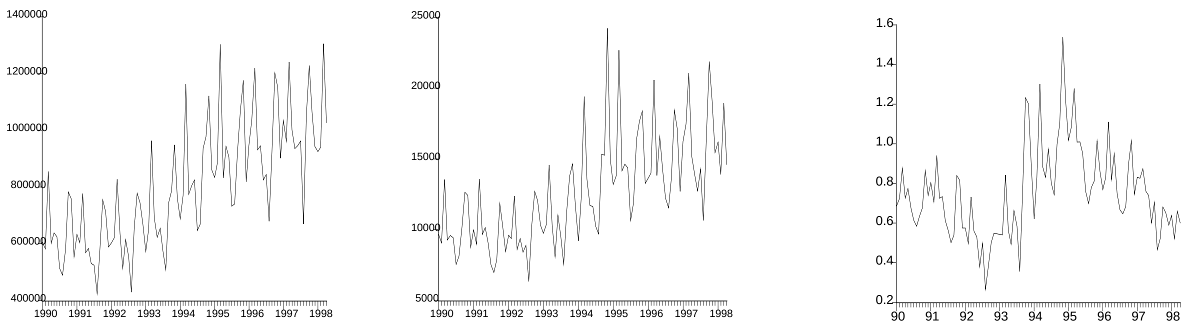
SV: $(1+0.64B^{12}+0.53B^{24}+0.39B^{36})\nabla\nabla_{12} Y_t = a_t$
(0.09) (0.10) (0.09)

GR: $(1+0.23B)(1-0.41B^{12}-0.16B^{24})\nabla Y_t = a_t$
(0.10) (0.09) (0.08)

Figure 4

EQUILIBRIUM EXCHANGE RATE FOR $\gamma_D = 4$, $\gamma_F = 1$

LU SV GR



LU: $(1+0.96B+0.66B^2)\nabla\nabla_{12} Y_t = a_t$
(0.07) (0.08)

SV: $(1-0.33B)(1+0.33B^{12}+0.39B^{24}+0.63B^{36})\nabla\nabla_{12} Y_t = (1-0.97B)a_t$
(0.11) (0.10) (0.09) (0.12) (0.02)

GR: $(1+0.42B+0.34B^2)(1-0.47B^{12}-0.29B^{24})\nabla Y_t = a_t$
(0.10) (0.10) (0.10) (0.10)

---

[8] ∇: *Difference Operator*; **B**: *backward shift operator;* $\nabla_s = (1-B^s)$

Table 6. Summary of ARIMA models fitted to the *SimExR* (LU model / Separable Preferences)

$SimExRS\gamma_F\gamma_D$[(a)]	**M.**	**Std.**	**Skw**	**Kt**	$\nabla^d \nabla_{s}$.[(b)]	**ARMA (R) (p, q)**	**ARMA (S) (P, Q)**	$\bar{a}$ $(\hat{\sigma}_{\bar{a}})$	$\hat{\sigma}_a$	**Q(12)**	**ARIMA MODELS**
SimExRS00	$1.1*10^{-1}$	$1.5*10^{-2}$	0.15	2.5	$\nabla\nabla_{12}$	(9, 0)	(1, 0)	$-9.3*10^{-5}$ $(2.3*10^{-4})$	$2.3*10^{-3}$	10.3	$(1+0.71B+0.22B^2-0.23B^3-0.35B^9)\ (1+0.26B^{12})\ \nabla\nabla_{12}Y_t = a_t$ (0.09) (0.12) (0.09) (0.07) (0.11)
SimExRS01	$0.5*10$	$8.6*10^{-1}$	0.14	1.6	$\nabla\nabla_{12}$	(0, 0)	(1, 0)	$-1.7*10^{-3}$ $(4.4*10^{-3})$	$4.4*10^{-2}$	13.8	$(1+0.31B^{12})\ \nabla\nabla_{12}Y_t = a_t$ (0.10)
SimExRS02	$2.5*10^{2}$	$0.6*10$	0.25	1.7	$\nabla\nabla_{12}$	(2, 1)	(0, 0)	$-5.9*10^{-2}$ $(64*10^{-2})$	$64*10^{-1}$	10.1	$(1+1.06B+0.72B^2)\ \nabla\nabla_{12}Y_t = (1+0.29B)a_t$ (0.11) (0.08) (0.16)
SimExRS03	$1.2*10^{4}$	$3.4*10^{3}$	0.40	2.0	$\nabla\nabla_{12}$	(2, 0)	(0, 0)	$0.3*10$ $(5.8*10)$	$5.8\ 10^{2}$	8.7	$(1+0.95B+0.66B^2)\ \nabla\nabla_{12}Y_t = a_t$ (0.07) (0.08)
SimExRS04	$6.1*10^{5}$	$2.1*10^{5}$	0.53	2.4	$\nabla\nabla_{12}$	(2, 0)	(0, 0)	$6.6*10^{2}$ $(4.3*10^{3})$	$4.3*10^{4}$	10.3	$(1+0.95B+0.67B^2)\ \nabla\nabla_{12}Y_t = a_t$ (0.07) (0.07)
SimExRS10	$2.2*10^{-3}$	$1.6*10^{-4}$	-0.22	3.3	$\nabla\nabla_{12}$	(3, 0)	(1, 0)	$-5.1*10^{-6}$ $(4.6*10^{-6})$	$4.6*10^{-5}$	9.7	$(1+0.92B+0.51B^2-0.33B^9)(1+0.30B^{12})\ \nabla\nabla_{12}Y_t = a_t$ (0.07) (0.07) (0.06) (0.10)
SimExRS11	$1.1*10^{-1}$	$7.8*10^{-3}$	-0.49	2.0	∇	(0, 0)	(2, 0)	$-7.0*10^{-5}$ $(4.8*10^{-5})$	$4.8*10^{-4}$	9.7	$(1-0.44B^{12}-0.41B^{24})\ \nabla Y_t = a_t$ (0.10) (0.10)
SimExRS12	$0.5*10$	$6.6*10^{-1}$	0.30	1.8	$\nabla\nabla_{12}$	(2, 0)	(0, 0)	$-1.1*10^{-2}$ $(1.2*10^{-2})$	$1.2*10^{-1}$	20.0	$(1+0.91B+0.61B^2)\ \nabla\nabla_{12}Y_t = a_t$ (0.08) (0.08)
SimExRS13	$2.5*10^{2}$	$4.9*10$	0.26	2.2	$\nabla\nabla_{12}$	(2, 0)	(0, 0)	$-4.2*10^{-1}$ $(0.1*10)$	$0.1*10^{2}$	19.2	$(1+0.96B+0.66B^2)\ \nabla\nabla_{12}Y_t = a_t$ (0.07) (0.08)
SimExRS14	$1.3*10^{4}$	$3.3*10^{3}$	0.44	2.4	$\nabla\nabla_{12}$	(2, 0)	(0, 0)	$-1.4*10$ $(8.4*10)$	$8.4*10^{2}$	18.3	$(1+0.96B+0.66B^2)\ \nabla\nabla_{12}Y_t = a_t$ (0.07) (0.08)
SimExRS20[(c)]	$4.7*10^{-5}$	$4.8*10^{-6}$	-0.05	2.5	$\nabla\nabla_{12}$	(10, 0)	(3, 0)	$-2.5*10^{-7}$ $(1.0*10^{-7})$	$1.0*10^{-6}$	9.1	$(1+0.67B+0.32B^2+0.23B^{10})(1+0.23B^{12}+0.34B^{24}+0.49B^{36})\ \nabla\nabla_{12}Y_t = a_t$ (0.10) (0.10) (0.09) (0.10) (0.09) (0.10)
SimExRS21[(c)]	$2.2*10^{-3}$	$1.1*10^{-4}$	-0.80	2.7	$\nabla\nabla_{12}$	(1, 0)	(3, 0)	$-3.2*10^{-6}$ $(1.6*10^{-6})$	$1.6*10^{-5}$	12.7	$(1-0.27B)(1+0.43B^{12}+0.38B^{24}+0.48B^{36})\ \nabla\nabla_{12}Y_t = a_t$ (0.10) (0.09) (0.10) (0.09)
SimExRS22	$1.1*10^{-1}$	$7.6*10^{-3}$	0.07	3.2	$\nabla\nabla_{12}$	(2, 2)	(0, 0)	$2.0*10^{-2}$ $(2.6*10^{-2})$	$2.6*10^{-1}$	26.4	$(1+1.1B+0.84B^2)\ \nabla\nabla_{12}Y_t = (1+0.48B+0.43B^2)\ a_t$ (0.09) (0.07) (0.13) (0.13)
SimExRS23	$0.5*10$	$7.4*10^{-1}$	0.29	3.1	$\nabla\nabla_{12}$	(2, 2)	(0, 0)	$-9.9*10^{-3}$ $(2.3*10^{-2})$	$2.3*10^{-1}$	21.0	$(1+1.1B+0.80B^2)\ \nabla\nabla_{12}Y_t = (1+0.31B+0.29B^2)\ a_t$ (0.10) (0.08) (0.14) (0.13)
SimExRS24	$2.6*10^{2}$	$5.2*10$	0.44	3.0	$\nabla\nabla_{12}$	(2, 0)	(0, 0)	$-7.4*10^{-1}$ $(0.2*10)$	$0.2*10^{2}$	29.0	$(1+0.95B+0.64B^2)\ \nabla\nabla_{12}Y_t = a_t$ (0.07) (0.08)

Table.6. (continued). Summary of ARIMA models fitted to the *SimExR* (LU model / Separable Preferences)

***SimExRS*$\gamma_F\gamma_D$**	**M.**	**Std.**	**Skw**	**Kt**	$\nabla^d\,\nabla_s$.	**ARMA (R) (p, q)**	**ARMA (S) (P, Q)**	$\overline{a}$ $(\hat{\sigma}_{\overline{a}})$	$\hat{\sigma}_a$	**Q(12)**	**ARIMA MODELS**
***SimExRS30*[(c)]**	$1.0*10^{-6}$	$1.8*10^{-7}$	-0.15	2.0	$\nabla\nabla_{12}$	(10, 0)	(2, 0)	$-2.1*10^{-9}$ $(3.0*10^{-9})$	$3.0*10^{-8}$	11.8	$(1+0.38B-0.05B^2-0.34B^3+0.27B^{10})(1-0.02B^{12}+0.34B^{24})\,\nabla\nabla_{12}Y_t = a_t$ (0.09) (0.11) (0.09) (0.10) (0.10) (0.09)
***SimExRS31*[(c)]**	$4.8*10^{-5}$	$6.3*10^{-6}$	-0.49	2.3	$\nabla\nabla_{12}$	(0, 0)	(3, 0)	$-1.2*10^{-7}$ $(6.6*10^{-8})$	$6.6*10^{-7}$	14.6	$(1+0.45B^{12}+0.36B^{24}+0.34B^{36})\,\nabla\nabla_{12t} = a_t$ (0.10) (0.10) (0.09)
SimExRS32	$2.3*10^{-3}$	$2.5*10^{-4}$	-0.45	2.7	∇	(4, 1)	(1, 0)	$-9.9*10^{-6}$ $(6.2*10^{-6})$	$6.2*10^{-5}$	16.3	$(1-0.14B-0.12B^2-0.23B^3+0.44B^4)(1-0.94B^{12})\,\nabla Y_t =(1-0.64B)\,a_t$ (0.18) (0.13) (0.10) (0.09) (0.03) (0.19)
SimExRS33	$1.1*10^{-1}$	$1.4*10^{-2}$	0.16	3.3	∇	(4, 1)	(1, 0)	$-7.6*10^{-4}$ $(5.0*10^{-4})$	$5.0*10^{-3}$	16.9	$(1+0.05B-0.03B^2-0.25B^3+0.35B^4)(1-0.94B^{12})\,\nabla Y_t =(1-0.73B)\,a_t$ (0.19) (0.19) (0.16) (0.10) (0.03) (0.10)
SimExRS34	$5.4*10$	$9.6*10^{-1}$	0.49	3.6	∇	(4, 1)	(1, 0)	$-3.8*10^{-2}$ $(3.5*10^{-2})$	$3.5*10^{-1}$	16.3	$(1+0.15B+0.03B^2-0.25B^3+0.29B^4)(1-0.94B^{12})\,\nabla Y_t =(1-0.73B)\,a_t$ (0.19) (0.19) (0.16) (0.10) (0.03) (0.18)
***SimExRS40*[(c)]**	$2.1*10^{-8}$	$5.7*10^{-9}$	-0.09	1.8	$\nabla\nabla_{12}$	(3, 0)	(2, 0)	$-3.4*10^{-11}$ $(8.6*10^{-11})$	$8.6*10^{-10}$	15.7	$(1+0.33B-0.08B^2-0.36B^3)(1-0.00B^{12}+0.32B^{24})\,\nabla\nabla_{12}Y_t = a_t$ (0.09) (0.11) (0.10) (0.10) (0.09)
***SimExRS41*[(c)]**	$1.0*10^{-6}$	$2.2*10^{-7}$	-0.28	2.0	$\nabla\nabla_{12}$	(0, 0)	(2, 0)	$-2.1*10^{-9}$ $(2.4*10^{-8})$	$2.4*10^{-8}$	18.8	$(1+0.37B^{12}+0.24B^{24})\,\nabla\nabla_{12}Y_t = a_t$ (0.09) (0.09)
***SimExRS42*[(c)]**	$4.9*10^{-5}$	$9.2*10^{-6}$	-0.33	2.3	$\nabla\nabla_{12}$	(4, 0)	(0, 0)	$-1.2*10^{-7}$ $(1.7*10^{-7})$	$1.7*10^{-6}$	26.6	$(1+0.28B-0.00B^2-0.20B^3+0.25B^4)\,\nabla\nabla_{12}Y_t =a_t$ (0.10) (0.10) (0.10) (0.10)
***SimExRS43*[(c)]**	$2.4*10^{-3}$	$4.3*10^{-4}$	-0.07	2.6	$\nabla\nabla_{12}$	(5, 1)	(0, 0)	$-3.2*10^{-6}$ $(2.9*10^{-6})$	$2.9*10^{-5}$	12.6	$(1+0.66B+0.21B^2-0.04B^3+0.36B^4+0.41B^5)\,\nabla\nabla_{12}Y_t =a_t$ (0.11) (0.11) (0.11) (0.11) (0.09)
SimExRS44	$1.1*10^{-1}$	$0.2*10^{-1}$	0.34	3.2	$\nabla\nabla_{12}$	(5, 0)	(0, 0)	$-1.3*10^{-5}$ $(1.2*10^{-5})$	$1.2*10^{-4}$	14.6	$(1+0.80B-0.40B^2+0.10B^3+0.38B^4+0.41B^5)\,\nabla\nabla_{12}Y_t =a_t$ (0.09) (0.12) (0.13) (0.12) (0.10)
ObsExR	$5.1*10^{-1}$	$4.6*10^{-2}$	0.64	2.7	∇	(0, 0)	(0, 0)	$-1.9*10^{-3}$ $(1.5*10^{-3})$	$1.5*10^{-2}$	6.6	$\nabla Y_t = a_t$

Notes:

(a) Variable definitions: *SimExRS*$\gamma_F\,\gamma_D$ identifies preferences. *SimExRS01:* the first number (*0*) denotes risk aversion parameter in foreign goods, the last number (1) denotes risk aversion parameter in domestic good.

(b) **∇**: *Difference Operator*; **B**: *backward shift operator;* $\nabla_s = (1-B^s)$

(c) *SimExR* shows appreciation as *ObsExR*

Table 7. Summary of ARIMA models fitted to the *SimExR* (SV model / Separable Preferences)

SimExRS$\gamma_F\gamma_D$[(a)]	**M.**	**Std.**	**Skw**	**Kt**	$\nabla^d \nabla_s$ [(b)]	**ARMA (R) (p, q)**	**ARMA (S) (P, Q)**	$\overline{a}$ $(\hat{\sigma}_{\overline{a}})$	$\hat{\sigma}_a$	**Q(12)**	**ARIMA MODELS**
SimExRS00	$1.1*10^{-1}$	$1.5*10^{-2}$	0.15	2.6	$\nabla\nabla_{12}$	(0, 1)	(2, 0)	$4.9*10^{-5}$ $(3.4*10^{-4})$	$3.4*10^{-3}$	28.2	$(1+0.18B^{12}+0.46B^{24})\nabla\nabla_{12}Y_t=(1-0.86B^{12})a_t$ (0.10) (0.11) (0.05)
SimExRS01	$0.5*10$	$8.7*10^{-1}$	0.16	1.7	$\nabla\nabla_{12}$	(1, 0)	(2, 0)	$-3.3*10^{-3}$ $(6.4*10^{-6})$	$6.4*10^{-2}$	14.5	$(1+0.21B)(1+0.54B^{12}+0.28B^{24})\nabla\nabla_{12}Y_t=a_t$ (0.10) (0.11) (0.12)
SimExRS02	$2.5*10^{2}$	$5.6*10$	0.30	1.8	$\nabla\nabla_{12}$	(0, 1)	(2, 0)	$3.5\ 10^{-2}$ $(9.2*10^{-1})$	$0.9*10$	31.4	$(1+0.18B^{12}+0.26B^{24})\nabla\nabla_{12}Y_t=(1-0.57B)a_t$ (0.11) (0.12) (0.09)
SimExRS03	$1.3*10^{4}$	$3.6*10^{3}$	0.48	2.2	$\nabla\nabla_{12}$	(0, 1)	(2, 0)	$0.5*10$ $(1.0*10^{2})$	$1.0*10^{3}$	22.5	$(1+0.29B^{12}+0.27B^{24})\nabla\nabla_{12}Y_t=(1-0.68B)a_t$ (0.11) (0.12) (0.07)
SimExRS04	$6.2*10^{5}$	$2.3*10^{5}$	0.71	2.7	$\nabla\nabla_{12}$	(0, 1)	(2, 0)	$7.2*10^{2}$ $(8.9*10^{3})$	$8.9*10^{4}$	15.9	$(1+0.34B^{12}+0.28B^{24})\nabla\nabla_{12}Y_t=(1-0.74B)a_t$ (0.11) (0.11) (0.07)
SimExRS10	$2.2*10^{-3}$	$1.7*10^{-4}$	-0.28	3.2	$\nabla\nabla_{12}$	(1, 1)	(2, 0)	$-6.1*10^{-6}$ $(7.0*10^{-6})$	$7.5*10^{-5}$	21.9	$(1+0.17B)(1+0.25B^{12}+0.36B^{24})\nabla\nabla_{12}Y_t=(1-0.59B)a_t$ (0.15) (0.10) (0.11) (0.12)
SimExRS11	$1.1*10^{-1}$	$7.8*10^{-3}$	-0.49	2.0	∇	(3, 0)	(1, 0)	$4.3*10^{-5}$ $(7.6*10^{-5})$	$7.6*10^{-4}$	13.0	$(1+0.23B+0.02B^{2}-0.28B^{3})(1-0.50B^{12})\nabla Y_t=a_t$ (0.10) (0.10) (0.10) (0.09)
SimExRS12	$0.53*10$	$6.8*10^{-1}$	0.14	2.0	$\nabla\nabla_{12}$	(2, 0)	(2, 0)	$-1.5*10^{-4}$ $(1.7*10^{-2})$	$1.7*10^{-1}$	25.8	$(1+0.79B+0.39B^{2})(1+0.20B^{12}+0.34B^{24})\nabla\nabla_{12}Y_t=a_t$ (0.10) (0.10) (0.11) (0.11)
SimExRS13	$2.6*10^{2}$	$5.3*10$	0.46	2.5	$\nabla\nabla_{12}$	(1, 1)	(3, 0)	$0.1*10$ $(0.2*10)$	$0.2*10^{2}$	29.2	$(1-0.36B)(1+0.27B^{12}+0.35B^{24}+0.64B^{36})\nabla\nabla_{12}Y_t=(1-0.97B)a_t$ (0.10) (0.09) (0.09) (0.11) (0.02)
SimExRS14	$1.3*10^{4}$	$3.7*10^{3}$	0.78	3.3	$\nabla\nabla_{12}$	(1, 1)	(3, 0)	$1.1*10^{2}$ $(1.6*10^{2})$	$1.6*10^{3}$	22.9	$(1-0.33B)(1+0.33B^{12}+0.39B^{24}+0.63B^{36})\nabla\nabla_{12}Y_t=(1-0.97B)a_t$ (0.11) (0.10) (0.09) (0.12) (0.02)
SimExRS20[(c)]	$4.7*10^{-5}$	$5.1*10^{-6}$	0.09	2.4	$\nabla\nabla_{12}$	(0, 1)	(3, 0)	$-1.8*10^{-7}$ $(1.5*10^{-7})$	$1.5*10^{-6}$	15.9	$(1+0.28B^{12}+0.36B^{24}+0.45B^{36})\nabla\nabla_{12}Y_t=(1-0.53B)a_t$ (0.09) (0.09) (0.10) (0.09)
SimExRS21[(c)]	$2.3*10^{-3}$	$1.2*10^{-4}$	-0.72	2.7	$\nabla\nabla_{12}$	(0, 0)	(3, 0)	$5.9*10^{-11}$ (2.610^{-6})	$2.6*10^{-5}$	17.0	$(1+0.51B^{12}+0.48B^{24}+0.50B^{36})\nabla\nabla_{12}Y_t=a_t$ (0.09) (0.10) (0.10)
SimExRS22	$1.1*10^{-1}$	$8.2*10^{-3}$	0.30	3.2	$\nabla\nabla_{12}$	(4, 1)	(3, 0)	$-5.8*10^{-4}$ $(3.1*10^{-4})$	$3.1*10^{-3}$	12.6	$(1+0.25B^{4}-0.42B^{9})(1+0.27B^{12}+0.30B^{24}+0.36B^{36})\nabla\nabla_{12}Y_t=(1-0.62B)a_t$ (0.09) (0.10) (0.10) (0.10) (0.10) (0.09)
SimExRS23	$0.5*10$	$8.1*10^{-1}$	0.63	3.7	$\nabla\nabla_{12}$	(9, 1)	(3, 0)	$3.7*10^{-2}$ $(3.4*10^{-2})$	$3.4*10^{-1}$	12.7	$(1-0.46B^{9})(1+0.36B^{12}+0.35B^{24}+0.50B^{36})\nabla\nabla_{12}Y_t=(1-0.77B)a_t$ (0.11) (0.10) (0.10) (0.10) (0.08)
SimExRS24[(c)]	$2.6*10^{2}$	$6.3*10$	1.10	4.7	∇_{12}	(9, 0)	(3, 0)	$0.2*10$ $(0.2*10)$	$0.2*10^{2}$	16.8	$(1-0.29B-0.54B^{9})(1+0.24B^{12}+0.27B^{24}+0.52B^{36})\nabla_{12}Y_t=a_t$ (0.09) (0.10) (0.09) (0.08) (0.09)

Table.7. (continued). Summary of ARIMA models fitted to the *SimExR* (SV model / Separable Preferences)

SimExRS $\gamma_F\gamma_D$	**M.**	**Std.**	**Skw**	**Kt**	$\nabla^d\,\nabla_s$.[(b)]	**ARMA (R) (p, q)**	**ARMA (S) (P, Q)**	$\bar{a}$ $(\hat{\sigma}_{\bar{a}})$	$\hat{\sigma}_a$	**Q(12)**	**ARIMA MODELS**
SimExRS30[(c)]	$1.0*10^{-6}$	$1.9*10^{-7}$	-0.04	2.0	$\nabla\nabla_{12}$	(1, 0)	(2, 0)	$-2.1*10^{-9}$ $(4.2*10^{-9})$	$4.2*10^{-8}$	15.2	$(1+0.54B)(1+0.25B^{12}+0.26B^{24})\ \nabla\nabla_{12}\ Y_t = a_t$ (0.08) (0.10) (0.10)
SimExRS31[(c)]	$4.8*10^{-5}$	$6.5*10^{-6}$	-0.46	2.3	$\nabla\nabla_{12}$	(0, 0)	(3, 0)	$-1.5*10^{-7}$ $(1.1*10^{-7})$	$1.1*10^{-6}$	19.2	$(1+0.58B^{12}+0.51B^{24}+0.43B^{36})\ \nabla\nabla_{12}Y_t = a_t$ (0.09) (0.10) (0.10)
SimExRS32	$2.3*10^{-3}$	$2.7*10^{-4}$	-0.34	2.8	$\nabla\nabla_{12}$	(4, 1)	(3, 0)	$-16*10^{-6}$ $(8.2*10^{-6})$	$8.2*10^{-5}$	22.7	$(1+0.33B^4)(1+0.29B^{12}+0.34B^{24}+0.40B^{36})\ \nabla\nabla_{12}Y_t =(1-0.48B)a_t$ (0.09) (0.09) (0.10) (0.10) (0.10)
SimExRS33	$1.1*10^{-1}$	$0.2*10^{-1}$	0.44	3.7	∇_{12}	(9, 0)	(3, 0)	$-1.1*10^{-4}$ $(7.6*10^{-4})$	$7.6*10^{-3}$	18.7	$(1-0.40B-0.44B^9)(1+0.30B^{12}+0.31B^{24}+0.51^{36})\nabla\nabla_{12}Y_t = a_t$ (0.09) (0.09) (0.10) (0.09) (0.09)
SimExRS34	0.6*10	0.1*10	0.11	5.6	∇_{12}	(9, 0)	(3, 0)	$1.5*10^{-2}$ $(6.3*10^{-2})$	$6.3*10^{-1}$	16.5	$(1-0.27B-0.41B^9)(1+0.36B^{12}+0.35B^{24}+0.57B^{36})\nabla_{12}\ Y_t = a_t$ (0.10) (0.10) (0.09) (0.09) (0.09)
SimExRS40[(c)]	$2.2*10^{-8}$	$5.9*10^{-9}$	-0.01	1.9	$\nabla\nabla_{12}$	(1, 0)	(3, 0)	$-6.0*10^{-11}$ $(1.1*10^{-10})$	$1.1*10^{-9}$	7.0	$(1+0.38B^4)(1+0.32B^{12}+0.28B^{24}+0.28B^{36})\ \nabla\nabla_{12}\ Y_t =a_t$ (0.09) (0.09) (0.10) (0.10)
SimExRS41[(c)]	$1.0*10^{-6}$	$2.3*10^{-7}$	-0.26	2.0	$\nabla\nabla_{12}$	(0, 0)	(3, 0)	$-3.7*10^{-9}$ $(3.5*10^{-9})$	$3.5*10^{-8}$	17.4	$(1+0.64B^{12}+0.53B^{24}+0.39B^{36})\ \nabla\nabla_{12}Y_t =a_t$ (0.09) (0.10) (0.09)
SimExRS42[(c)]	$4.9*10^{-5}$	$9.6*10^{-6}$	-0.28	2.3	$\nabla\nabla_{12}$	(4, 1)	(3, 0)	$-3.5*10^{-7}$ $(2.2*10^{-7})$	$2.2*10^{-6}$	16.0	$(1+0.31B^4)(1+0.37B^{12}+0.35B^{24}+0.34B^{36})\ \nabla\nabla_{12}Y_t =(1-0.39B)a_t$ (0.10) (0.10) (0.10) (0.09) (0.10)
SimExRS43[(c)]	$2.4*10^{-3}$	$4.7*10^{-4}$	0.05	2.8	$\nabla\nabla_{12}$	(4, 1)	(3, 0)	$-2.9*10^{-5}$ $(1.8*10^{-5})$	$1.8*10^{-5}$	19.1	$(1+0.28B^4)(1+0.34B^{12}+0.37B^{24}+0.42B^{36})\ \nabla\nabla_{12}Y_t =(1-0.64B)a_t$ (0.10) (0.09) (0.09) (0.09) (0.08)
SimExRS44	$1.2*10^{-1}$	$2..7*10^{-2}$	0.76	4.5	∇_{12}	(9, 0)	(3, 0)	$-2.1*10^{-4}$ $(14*10^{-4})$	$1.4*10^{-2}$	17.1	$(1-0.36B-0.39B^9)(1+0.34B^{12}+0.34B^{24}+0.50B^{36})\ \nabla_{12}Y_t = a_t$ (0.10) (0.10) (0.10) (0.09) (0.10)
ObsExR	$5.1*10^{-1}$	$4.6*10^{-2}$	0.64	2.7	∇	(0, 0)	(0, 0)	$-1.9*10^{-3}$ $(1.5*10^{-3})$	$1.5*10^{-2}$	6.6	$\nabla Y_t = a_t$

Notes:

(a) Variable definitions: *SimExRS* $\gamma_F\ \gamma_D$ identifies preferences. *SimExRS01:* the first number (*0*) denotes risk aversion parameter in foreign goods, the last number (1) denotes risk aversion parameter in domestic goods .

(b) ∇: *Difference Operator*; **B**: *backward shift operator;* $\nabla_s = (1-B^s)$

(c) *SimExR* shows appreciation as *ObsExR*.

Table 8 Summary of ARIMA models fitted to the *SimExR* (GR model / Separable Preferences)

***SimExRS*$\gamma_F\gamma_D$**(a)	**M.**	**Std.**	**Skw**	**Kt**	$\nabla^d \nabla_{s.}$(b)	**ARMA (R) (p, q)**	**ARMA (S) (P, Q)**	$\overline{a}$ $(\hat{\sigma}_{\overline{a}})$	$\hat{\sigma}_a$	**Q(12)**	**ARIMA MODELS**
SimExRS00	$7.1*10^{-1}$	$2.1*10^{-1}$	0.00	1.5	∇	(4, 0)	(0, 0)	$4.3*10^{-3}$ $(5.3*10^{-3})$	$5.3*10^{-2}$	11.8	$(1+0.14B+0.10B^2-0.20B^3+0.25B^4)\ \nabla Y_t = a_t$ (0.10) (0.10) (0.10) (0.10)
SimExRS01	$3.5*10$	$1.2*10$	-0.03	1.3	∇	(0, 0)	(1, 0)	$1.7*10^{-1}$ $(1.9*10^{-1})$	$1.9*10^{0}$	8.8	$(1-0.20B^{12})\ \nabla Y_t = a_t$ (0.10)
SimExRS02	$1.7*10^{3}$	$6.4*10^{2}$	0.01	1.3	∇	(1, 0)	(2, 0)	$0.8*10$ $(1.4*10)$	$1.4*10^{2}$	29.8	$(1-0.26B)\ (1-0.25B^{12}-0.38B^{24})\ \nabla Y_t = a_t$ (0.10) (0.09) (0.09)
SimExRS03	$8.5*10^{4}$	$3.6*10^{4}$	0.10	1.5	∇	(2, 0)	(2, 0)	$3.7*10^{2}$ $(9.7*10^{2})$	$9.7*10^{3}$	19.3	$(1+0.50B+0.36B^2)\ (1-0.37B^{12}-0.41B^{24})\ \nabla Y_t = a_t$ (0.10) (0.10) (0.10) (0.10)
SimExRS04	$4.2*10^{6}$	$2.0*10^{6}$	0.19	1.6	∇	(2, 0)	(2, 0)	$1.6*10^{4}$ $(6.3*10^{4})$	$6.3*10^{5}$	15.9	$(1+0.44B+0.33B^2)\ (1-0.49B^{12}-0.32B^{24})\ \nabla Y_t = a_t$ (0.10) (0.10) (0.10) (0.11)
SimExRS10	$1.5*10^{-2}$	$0.3*10^{2}$	-0.01	1.9	∇	(2, 0)	(1, 0)	$3.5*10^{-5}$ $(1.2*10^{-4})$	$1.2*10^{-3}$	11.8	$(1+0.21B+0.19B^2)\ (1-0.19B^{12})\ \nabla Y_t = a_t$ (0.10) (0.10) (0.10)
SimExRS11	$7.1*10^{-1}$	$1.8*10^{-1}$	-0.10	1.5	∇	(0, 0)	(0, 0)	$-2.0*10^{-3}$ $(3.7*10^{-3})$	$3.7*10^{-2}$	15.5	$\nabla Y_t = a_t$
SimExRS12	$3.5*10$	$1.0*10$	-0.06	1.5	∇	(1, 0)	(2, 0)	$1.2*10^{-1}$ $(2.7*10^{-1})$	$2.7*10^{0}$	38.9	$(1+0.27B)\ (1-0.20B^{12}-0.31B^{24})\ \nabla Y_t = a_t$ (0.10) (0.09) (0.09)
SimExRS13	$1.7*10^{3}$	$6.0*10^{2}$	0.06	1.7	∇	(2, 0)	(2, 0)	$0.5*10$ $(1.9*10)$	$1.9*10^{2}$	21.4	$(1+0.50B+0.38B^2)\ (1-0.34B^{12}-0.38B^{24})\ \nabla Y_t = a_t$ (0.10) (0.10) (0.10) (0.10)
SimExRS14	$8.6*10^{4}$	$3.5*10^{4}$	0.20	1.9	∇	(2, 0)	(2, 0)	$2.1*10^{2}$ $(1.3*10^{3})$	$1.3*10^{4}$	18.2	$(1+0.42B+0.34B^2)\ (1-0.47B^{12}-0.29B^{24})\ \nabla Y_t = a_t$ (0.10) (0.10) (0.10) (0.10)
SimExRS20	$3.0*10^{-4}$	$5.3*10^{-5}$	0.14	2.9	∇	(2, 0)	(2, 0)	$-1.3*10^{-8}$ $(2.6*10^{-6})$	$2.6*10^{-5}$	12.7	$(1+0.35B+0.22B^2)\ (1-0.16B^{12}-0.19B^{24})\ \nabla Y_t = a_t$ (0.10) (0.10) (0.10) (0.08)
SimExRS21	$1.5*10^{-2}$	$2.9*10^{-3}$	-0.04	2.3	∇	(0, 0)	(0, 0)	$5.5*10^{-6}$ $(8.5*10^{-5})$	$8.5*10^{-4}$	16.8	$\nabla Y_t = a_t$
SimExRS22	$7.2*10^{-1}$	$1.7*10^{-1}$	-0.02	2.1	∇	(1, 0)	(2, 0)	$1.2*10^{-3}$ $(5.6*10^{-3})$	$5.6*10^{-2}$	31.3	$(1+0.24B)\ (1+0.18B^{12}+0.30B^{24})\ \nabla Y_t = a_t$ (0.10) (0.09) (0.09)
SimExRS23	$3.6*10$	$1.0*10$	0.13	2.2	∇	(2, 0)	(2, 0)	$3.5*10^{-2}$ $(4.0*10^{-1})$	$4.0*10^{0}$	20.6	$(1+0.44B+0.36B^2)\ (1+0.18B^{12}+0.30B^{24})\ \nabla Y_t = a_t$ (0.10) (0.09) (0.09)
SimExRS24	$1.7*10^{3}$	$6.1*10^{2}$	0.31	2.4	∇	(2, 0)	(2, 0)	$0.2.*10$ $(2.6*10)$	$2.6*10^{2}$	18.3	$(1+0.36B+0.33B^2)\ (1-0.45B^{12}-0.28B^{24})\ \nabla Y_t = a_t$ (0.10) (0.10) (0.10) (0.10)

Table 8 (continued). Summary of ARIMA models fitted to the *SimExR* (GR model / Separable Preferences)

SimExRS$\gamma_F\gamma_D$	**M.**	**Std.**	**Skw**	**Kt**	$\nabla^d\ \nabla_s.$ [(b)]	**ARMA (R)** **(p, q)**	**ARMA (S)** **(P, Q)**	$\bar{a}$ $(\hat{\sigma}_{\bar{a}})$	$\hat{\sigma}_a$	**Q(12)**	**ARIMA MODELS**
SimExRS30	$6.4*10^{-6}$	$1.05*10^{-6}$	0.10	3.2	∇	(2, 0)	(2, 0)	$-1.7*10^{-8}$ $(6.1*10^{-8})$	$6.1*10^{-7}$	10.4	$(1+0.50B+0.27B^2)\ (1-0.18B^{12}-0.17B^{24})\ \nabla Y_t = a_t$ (0.10) (0.10) (0.10) (0.08)
SimExRS31	$3.1*10^{-4}$	$5.1*10^{-5}$	-0.03	2.9	∇	(0, 0)	(1, 0)	$-4.8*10^{-7}$ $(2.3*10^{-6})$	$2.3*10^{-5}$	16.3	$(1-0.34B^{12})\ \nabla Y_t = a_t$ (0.09)
SimExRS32	$1.5*10^{-2}$	$2.9*10^{-3}$	0.09	2.9	∇	(2, 0)	(2, 0)	$-8.9*10^{-6}$ $(1.3*10^{-4})$	$1.3*10^{-3}$	21.5	$(1+0.34B+0.22B^2)\ (1-0.22B^{12}-0.32B^{24})\ \nabla Y_t = a_t$ (0.10) (0.10) (0.10) (0.09)
SimExRS33	$7.3*10^{-1}$	$1.8*10^{-1}$	0.31	3.0	∇	(2, 0)	(2, 0)	$5.0*10^{-4}$ $(8.6*10^{-3})$	$8.6*10^{-2}$	20.8	$(1+0.43B+0.38B^2)\ (1-0.33B^{12}-0.38B^{24})\ \nabla Y_t = a_t$ (0.10) (0.10) (0.10) (0.10)
SimExRS34	3.6*10	1.1*10	0.57	3.3	∇	(2, 0)	(2, 0)	$-3.9*10^{-3}$ $(5.6*10^{-1})$	$5.6*10^{0}$	19.8	$(1+0.35B+0.35B^2)\ (1-0.47B^{12}-0.27B^{24})\ \nabla Y_t = a_t$ (0.10) (0.10) (0.10) (0.10)
SimExRS40	$1.4*10^{-7}$	$2.6*10^{-8}$	-0.21	2.7	∇	(2, 0)	(2, 0)	$-5.0*10^{-10}$ $(1.3*10^{-9})$	$1.3*10^{-8}$	92.9	$(1+0.49B+0.25B^2)\ (1-0.24B^{12}-0.21B^{24})\ \nabla Y_t = a_t$ (0.10) (0.10) (0.09) (0.08)
SimExRS41	$6.5*10^{-6}$	$1.2*10^{-6}$	-0.32	2.5	∇	(1, 0)	(2, 0)	$-1.1*10^{-8}$ $(5.2*10^{-8})$	$5.2*10^{-7}$	16.9	$(1+0.23B)(1-0.41B^{12}-0.16B^{24})\ \nabla Y_t = a_t$ (0.10) (0.09) (0.08)
SimExRS42	$3.1*10^{-4}$	$6.1*10^{-5}$	-0.08	2.8	∇	(2, 0)	2, 0)	$-5.31*10^{-7}$ $(2.8*10^{-6})$	$2.8*10^{-5}$	22.9	$(1+0.36B+0.25B^2)\ (1-0.26B^{12}-0.34B^{24})\ \nabla Y_t = a_t$ (0.10) (0.10) (0.09) (0.08)
SimExRS43	$1.5*10^{-2}$	$3.6*10^{-3}$	0.34	3.2	∇	(2, 0)	(2, 0)	$-2.6*10^{-5}$ $(1.9*10^{-4})$	$1.9*10^{-3}$	21.3	$(1+0.44B+0.39B^2)\ (1-0.34B^{12}-0.37B^{24})\ \nabla Y_t = a_t$ (0.10) (0.10) (0.09) (0.08)
SimExRS44	$7.5*10^{-1}$	$2.2*10^{-1}$	0.77	4.0	∇	(2, 0)	(2, 0)	$-6.4*10^{-4}$ $(1.2*10^{-1})$	$1.2*10^{0}$	20.3	$(1+0.34B+0.35B^2)\ (1-0.47B^{12}-0.25B^{24})\ \nabla Y_t = a_t$ (0.10) (0.10) (0.10) (0.09)
ObsExR	$5.1*10^{-1}$	$4.6*10^{-2}$	0.64	2.7	∇	(0, 0)	(0, 0)	$-1.9*10^{-3}$ $(1.5*10^{-3})$	$1.5*10^{-2}$	6.6	$\nabla Y_t = a_t$

Notes:

(a) Variable definitions: SimExRS $\gamma_F\ \gamma_D$ identifies the preferences. SimExRS*01:* the first number (*0*) denotes risk aversion parameter in foreign goods, the last number (1) denotes risk aversion parameter in domestic goods.

(b) **∇**: *Difference Operator*; **B**: *backward shift operator;* $\nabla_s = (1-B^s)$

Table 9. Testing for cointegrating among *SimExRa* and *ObsExR* under GR model. Separable preferences

SimExRS $\gamma_F v_D$	ß$_0$	ß$_1$	R^2	D-F[(a)(b)(c)] 1lag	D-F 2lag	corr.[(d)]
SimExRS00	0.62 [(e)] (0.01)	$-1.6*10^{-1}$ $(0.2*10^{-1})$	0.51	-3.40	-3.16	0.95
SimExRS01	0.61 (0.01)	$-0.3*10^{-2}$ $(0.1*10^{-2})$	0.55	-3.40	-3.32	0.96
SimExRS02	0.61 (0.01)	$-5.5*10^{-5}$ $(4.8*10^{-6})$	0.57	-3.48	-3.45	0.95
SimExRS03	0.60 (0.01)	$-9.6*10^{-7}$ $(8.8*10^{-7})$	0.55	-3.73	-3.52	0.91
SimExRS04	0.58 (0.01)	$-1.7*10^{-8}$ $(1.6*10^{-9})$	0.52	-3.94	-3.62	0.88
SimExRS10	0.64 (0.02)	$-8.6*10^{0}$ $(1.1*10^{0})$	0.37	-2.80	-2.72	0.94
SimExRS11	0.63 (0.01)	$-1.7*10^{-1}$ $(0.2*10^{-1})$	0.46	-2.92	-2.95	0.99
SimExRS12	0.62 (0.01)	$-0.3*10^{-2}$ $(0.1*10^{-2})$	0.55	-3.40	-3.32	0.97
SimExRS13	0.60 (0.01)	$-5.4*10^{-5}$ $(5.7*10^{-6})$	0.48	-3.40	-3.17	0.93
SimExRS14	0.60 (0.01)	$-9.6*10^{-7}$ $(8.8*10^{-7})$	0.45	-3.63	-3.31	0.88
SimExRS20	0.62 (0.02)	$-3.0*10^{2}$ $(0.8*10^{2})$	0.12	-2.13	-2.18	0.81
SimExRS21	0.63 (0.02)	$-0.8*10$ $(0.1*10)$	0.25	-2.41	-2.39	0.94
SimExRS22	0.62 (0.02)	$-1.6*10^{-1}$ $(0.2*10^{-1})$	0.32	-2.64	-2.61	0.94
SimExRS23	0.60 (0.01)	$-2.7*10^{-3}$ $(0.4*10^{-3})$	0.34	-2.97	-2.78	0.90
SimExRS24	0.59 (0.01)	$-4.3*10^{-5}$ $(6.2*10^{-6})$	0.33	-3.20	-2.93	0.85
SimExRS30	0.49 (0.03)	$3.5*10^{3}$ $(4.5*10^{3})$	0.01	-2.05	-2.16	0.44
SimExRS31	0.55 (0.03)	$-1.3*10^{2}$ $(0.9*10^{2})$	0.02	-1.97	-2.06	0.68
SimExRS32	0.58 (0.02)	$-4.\ 8*10^{0}$ $(1.5*10^{0})$	0.09	-2.12	-2.15	0.77
SimExRS33	0.58 (0.02)	$-9.9*10^{-2}$ $(2.4*10^{-2})$	0.15	-2.40	-2.33	0.77
SimExRS34	0.57 (0.01)	$-1.7*10^{-3}$ $(0.4*10^{-3})$	0.17	-2.64	-2.49	0.74
SimExRS40	0.41 (0.02)	$7.5*10^{-5}$ $(1.6*10^{-5})$	0.19	-2.60	-2.59	0.01
SimExRS41	0.44 (0.02)	$1.0*10^{4}$ $(0.4*10^{4})$	0.07	-2.25	-2.34	0.24
SimExRS42	0.49 (0.02)	$4.9*10$ $(7.6*10)$	0.00	-2.03	-2.14	0.42
SimExRS43	0.53 (0.02)	$-1.3*10^{0}$ $(1.3*10^{0})$	0.01	-1.99	-2.10	0.52
SimExRS44	0.54 (0.02)	$-0.4*10^{-1}$ $(0.2*10^{-1})$	0.04	-2.11	-2.17	0.56

Notes:

a) To test for cointegration we estimate by OLS the following model for the exchange rate data, from 1990:01 to 1998:04:

$$ObsExR_t = \beta_0 + \beta_1 SimExR_t + u_t^{ExR}$$

Where, $ObsExR_t$ and $SimExR_t$ are observed and simulated exchange rate, respectively. Then the augmented Dickey-Fuller t test, on the estimated residuals, is used. If u_t^{ExR} is I(0) then the variables $ObsExR_t$ and $SimExR_t$ will be cointegrated with cointegrating vector (1, -ß$_1$).

b) MacKinnon critical values for Augmented Dickey-Fuller test (significance level) : -3.50 (1%), -2.89 (5%), -2.58 (10%)

c) ADF test regression include a constant and i lag (i =1, 2).

d) Correlation between *SimExR* and interest rates ratio.

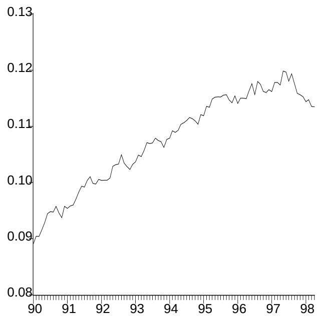
e) Estimated standard errors in parentheses.

Section 4: *SimExR*: CES preferences[9]

Figure 5

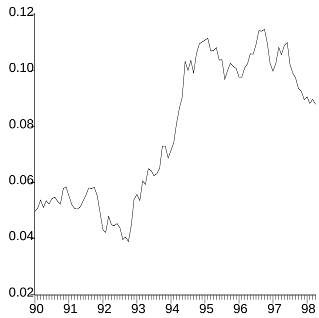
EQUILIBRIUM EXCHANGE RATE FOR $\gamma = 0$ $\varepsilon = 0.1$

LU



SV

GR



$\underset{(0.10)}{(1-0.44B^{12}}\underset{(0.10)}{-0.41B^{24})}\nabla Y_t = \underset{(0.09)}{(1-0.44B^{12})}$ at

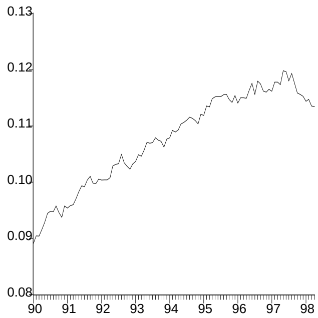
$\underset{(0.09)}{(1-0.38\,B)}\,\underset{(0.09)}{(1-0.44\,B^{12})}\,\nabla Y_t = a_t$

$\nabla Y_t = a_t$

Figure 6

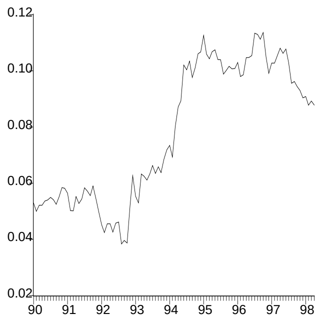
EQUILIBRIUM EXCHANGE RATE FOR $\gamma = 1$, $\varepsilon = 0.1$

LU



SV

G R



$\underset{(0.10)}{(1-0.44B^{12}}\underset{(0.10)}{-0.41B^{24})}\nabla Y_t = \underset{(0.09)}{(1-0.44\,B^{12})}\,a_t$

$\underset{(0.09)}{(1+0.45\,B)}\,\underset{(0.11)}{(1-0.47\,B^{12}}\underset{(0.11)}{-0.22B^{24})}\,\nabla Y_t = \underset{(0.08)}{(1-0.78\,B)}\,a_t$

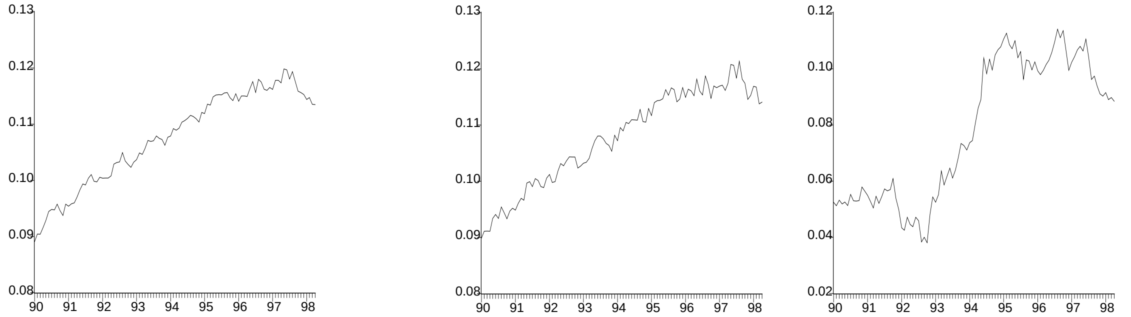
$\nabla Y_t = a_t$

[9] ∇: *Difference Operator*; **B**: *backward shift operator;* $\nabla_s = (1-B^s)$

Figura 7

EQUILIBRIUM EXCHANGE RATE FOR $\gamma = 2$, $\varepsilon = 0.1$

LU | SV | GR



$(1-0.44B^{12}-0.41B^{24})\nabla Y_t = (1-0.44B^{12})a_t$
(0.10) (0.10) (0.09)

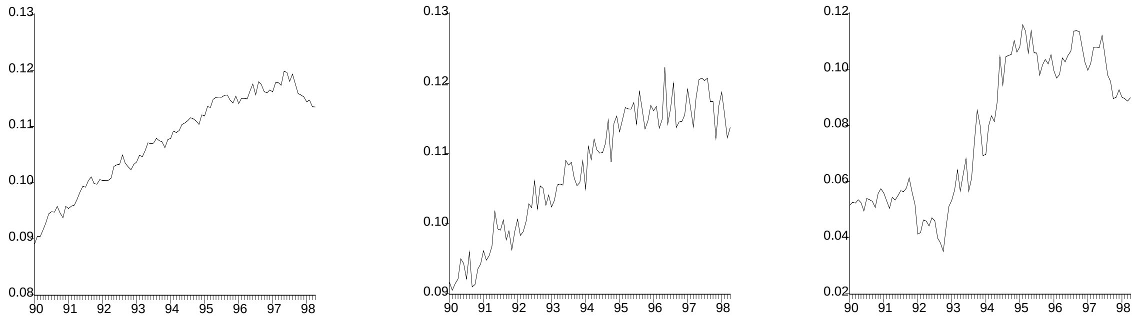
$(1+0.42B+0.23B^2)(1-0.51B^{12})\nabla Y_t = a_t$
(0.10) (0.10) (0.09)

$(1+0.21B^{12})\nabla Y_t = a_t$
(0.10)

Figure 8

EQUILIBRIUM EXCHANGE RATE FOR $\gamma = 3$, $\varepsilon = 0.1$

LU | SV | GR



$(1-0.44B^{12}-0.41B^{24})\nabla Y_t = (1-0.44B^{12})$ at
(0.10) (0.10) (0.09)

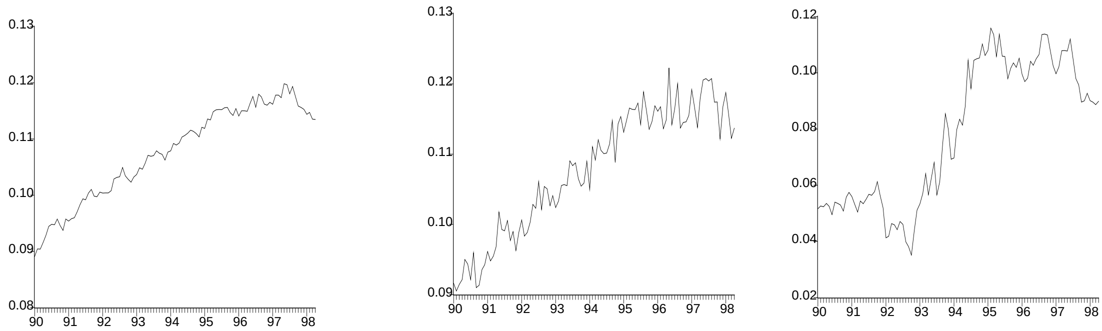
$(1+0.59B+0.29B^2)(1-0.36B^{12})\nabla Y_t = a_t$
(0.10) (0.10) (0.09)

$\nabla Y_t = a_t$

Figure 13

EQUILIBRIUM EXCHANGE RATE FOR $\gamma = 4$, $\varepsilon = 0.1$

LU | SV | GR



$(1-0.44B^{12}-0.41B^{24})\nabla Y_t = (1-0.44B^{12})a_t$
(0.10) (0.10) (0.09)

$(1+0.62B+0.29B^2)(1-0.18B^{12})\nabla Y_t = a_t$
(0.10) (0.10) (0.08)

$\nabla Y_t = a_t$

Table 10. Summary of ARIMA models fitted to the *SimExR* (LU model / CES Preferences)

SimExRCES $\mathcal{E}*10^{(a)}$	**M.**	**Std.**	**Skw**	**Kt**	$\nabla^{d}\,\nabla_{s}$.$^{(b)}$	**ARMA (R) (p, q)**	**ARMA(S) (P, Q)**	$\bar{a}$ $(\hat{\sigma}_{\bar{a}})$	$\hat{\sigma}_a$	**Q(12)**	**ARIMA MODELS**
SimExRCES0	$1.1*10^{-1}$	$7.8*10^{-3}$	-0.49	2.0	∇	(0, 0)	(2, 0)	$-7*10^{-5}$ $(4.8*10^{-5})$	$4.8*10^{-4}$	14.0	$(1-0.44B^{12}-0.41B^{24})\ \nabla\ Y_t = (1-0.44\ B^{12})\ a_t$ (0.10) (0.10) (0.09)
SimExRCES1	$1.1*10^{-1}$	$8.3*10^{-3}$	-0.44	2.0	∇	(3, 0)	(2, 0)	$-5.6*10^{-5}$ $(6.0*10^{-5})$	$6.0*10^{-4}$	13.2	$(1+0.33B-0.09B^{2}-0.41B^{3})\ (1-0.53B^{12}-0.32B^{24})\ \nabla Y_t = a_t$ (0.09) (0.10) (0.10) (0.10 (0.11)
SimExRCES2	$1.1*10^{-1}$	$8.9*10^{-2}$	-0.37	2.0	∇	(3, 0)	(1, 0)	$-6.1*10^{-5}$ $(8.0*10^{-5})$	$8.0*10^{-4}$	19.1	$(1+0.41B-0.01B^{2}-0.45B^{3})\ (1-0.90B^{12})\ \nabla Y_t = a_t$ (0.09) (0.10) (0.09) (0.05)
SimExRCES3	$1.1*10^{-1}$	$9.5*10^{-3}$	-0.31	2.0	$\nabla\nabla_{12}$	(9, 0)	(1, 0)	$-9.9*10^{-5}$ $(8.8*10^{-5})$	$8.8*10^{-4}$	5.8	$(1+0.53B-0.01B^{2}-0.39B^{3}-0.37B^{9})\ (1+0.37B^{12})\ \nabla\nabla_{12} Y_t = a_t$ (0.09) (0.11) (0.09) (0.07) (0.10)
SimExRCES4	$1.1*10^{-1}$	$1.0*10^{-2}$	-0.24	2.1	$\nabla\nabla_{12}$	(9, 0)	(1, 0)	$-1.0*10^{-4}$ $(1.1*10^{-4})$	$1.1*10^{-3}$	6.7	$(1+0.59B-0.01B^{5}-0.35B^{3}-0.36B^{9})\ (1+0.34B^{12})\ \nabla\nabla_{12} Y_t = a_t$ (0.09) (0.12) (0.09) (0.07) (0.10)
SimExRCES5	$1.1*10^{-1}$	$1.1*10^{-2}$	-0.17	2.1	$\nabla\nabla_{12}$	(9, 0)	(1, 0)	$-1.1*10^{-4}$ $(1.3*10^{-4})$	$1.3*10^{-3}$	7.4	$(1+0.63B-0.11B^{2}-0.31B^{3}-0.36B^{9})\ (1+0.31B^{12})\ \nabla\nabla_{12} Y_t = a_t$ (0.09) (0.11) (0.09) (0.07) (0.10)
SimExRCES6	$1.1*10^{-1}$	$1.2*10^{-2}$	-0.10	2.2	$\nabla\nabla_{12}$	(9, 0)	(1, 0)	$-1.1*10^{-4}$ $(1.5*10^{-4})$	$1.5*10^{-3}$	8.1	$(1+0.66B+0.15B^{2}-0.29B^{3}-0.36B^{9})\ (1+0.30B^{12})\ \nabla\nabla_{12} Y_t = a_t$ (0.09) (0.12) (0.09) (0.07) (0.10)