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## New universality class in three dimensions?: the antiferromagnetic $RP^2$ model

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### Abstract

We present the results of a Monte Carlo simulation of the  $RP^2$  model in three dimensions with negative coupling. We observe a second order phase transition between the disordered phase and an antiferromagnetic, unfrustrated, ordered one. We measure, with a finite size scaling analysis, the thermal exponent, obtaining  $\nu = 0.784(8)$ . We have found two magnetic-type relevant operators whose related  $\eta$  exponents are  $0.038(2)$  and  $1.338(8)$  respectively.

*Keywords:*  $RP^2$ ; Non-linear sigma model; Antiferromagnetism; Critical exponents; Phase transitions; Universality; Finite size scaling

The theory of critical phenomena offers a common framework to study problems in Condensed Matter Physics (CMP) and in High Energy Physics (HEP). In both areas, the concepts of Spontaneous Symmetry Breaking (SSB) and of Universality allow to relate problems in principle very different.

The usual Heisenberg model, associated with the standard ferromagnetic Non-Linear  $\sigma$  Model (NL $\sigma$ M), has a SSB pattern of type  $SO(3)/SO(2)$ . With the introduction of nontrivial Antiferromagnetic (AF) interactions the SSB pattern normally changes completely and, usually, frustration is generated. In particular, a SSB pattern shared by several AF models is  $SO(3) \times SO(2)/SO(2)$ . For instance, some frustrated quantum AF Heisenberg models [1], or the helimagnets and canted spin systems [2,3] are exam-

ples of this behavior. Frustrated quantum spin models are specially interesting because of their possible relation with High Temperature Superconductivity [4].

As a general consequence of the Weinberg theorem the low energy physics of a system is completely determined by its SSB pattern, the effective Lagrangian for the system being the corresponding NL $\sigma$ M. In this framework a study has been carried out for  $SO(3) \times SO(2)/SO(2)$  in perturbation theory [3], where the main conclusion reached is that the only possible nontrivial critical point in three dimensions is that of  $O(4)$ . In spite of that, we have found in a nonperturbative lattice formulation of a model with similar symmetry properties, a critical point with exponents that seem to be different from those of  $O(4)$  computed in Ref. [5].

From the HEP point of view, it is of great interest to understand whether AF interactions can generate new universality classes. One could even hope that nontrivial antiferromagnetism would be the ingredient

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needed in order to nonperturbatively formulate interacting theories in four dimensions [6,7].

In a previous work [8] we found that, on a three dimensional AF O(3) model, the only new phase transitions generated were first order. We will consider in this letter the  $\text{RP}^2 \equiv S^2/Z_2$  (real projective space) spin model in three dimensions. We place the spins on a cubic lattice with a nearest neighbors interaction:

$$S = \beta \sum_{\langle ij \rangle} (\mathbf{v}_i \cdot \mathbf{v}_j)^2, \quad (1)$$

where  $\{\mathbf{v}_i\}$  are normalized real three-components vectors. The local  $Z_2$  symmetry  $\mathbf{v}_i \rightarrow -\mathbf{v}_i$  is preserved even after the SSB (Elitzur's theorem), and so, the sense of a spin is irrelevant, it is only its direction that matters. In the AF case, (1) is related [9] with a lattice discretization of the action  $\int \text{tr}[P(R^{-1}\partial_\mu R u)^2]$ , where  $R \in SO(3)$ , and  $P$  is the diagonal matrix  $\{g, g, -g\}$ , with  $g$  being the coupling. This is just a particular case of the NL $\sigma$ M considered in Ref. [3].

For  $\beta$  positive this model presents a weak first order phase transition which has been used to describe liquid crystals [10]. The ordered phase corresponds to states where all spins are aligned.

For  $\beta$  negative there is also an AF ordered phase with a more complex structure [11]. There is a second order phase transition between the disordered phase and an ordered AF one. Let us call a site even (odd) when the sum of its coordinates  $x + y + z$  is even (odd). A state where, for instance, all spins on even sites are aligned in a given direction, and those on odd sites lie randomly in the orthogonal plane, has zero energy. So, at  $T = 0$  the ground state is highly degenerate with a global O(2) symmetry. However, when fluctuations are taken into account, the scenario is much more complex. We will show, with Monte Carlo simulations, that the O(2) symmetry is broken in the critical region. In a forthcoming paper [9] we shall discuss the very low temperature regime as well as further details of the vacuum structure on the critical region.

In order to discuss the observables measured, let us construct the (traceless) tensorial field  $\mathbf{T}$  with components

$$T_i^{\alpha\beta} = v_i^\alpha v_i^\beta - \frac{1}{3} \delta^{\alpha\beta}, \quad (2)$$

and its Fourier transform  $\hat{\mathbf{T}}$  in a  $L \times L \times L$  lattice with periodic boundary conditions.

The intensive staggered (nonstaggered) magnetization can be defined in terms of the tensorial field as the sum of the spins on even sites minus (plus) those on odd sites, or equivalently

$$\mathbf{M}_s = \frac{1}{V} \hat{\mathbf{T}}_{(\pi, \pi, \pi)} \quad (\mathbf{M} = \frac{1}{V} \hat{\mathbf{T}}_{(0,0,0)}), \quad (3)$$

where  $V$  is the lattice volume. We have observed a phase transition at  $\beta \sim -2.41$  for which  $\mathbf{M}_s$  is an order parameter (zero value in the disordered phase and a clear nonzero value in the  $L \rightarrow \infty$  limit in the ordered one). The magnetization  $\mathbf{M}$  is also an order parameter. As these operators correspond to different irreducible representations of the translations group, we will study the scaling properties of each observable independently.

To measure in a Monte Carlo simulation on a finite lattice we have constructed scalars under the O(3) group. For the magnetization and the susceptibility we compute respectively

$$M = \langle \sqrt{\text{tr} \mathbf{M}^2} \rangle, \quad \chi = V \langle \text{tr} \mathbf{M}^2 \rangle, \quad (4)$$

and analogously for the staggered observables.

We have also measured the *second momentum* correlation length, which is expected to have the same scaling behavior at the critical point as the exponential (physical) one, but it is much easier to measure [12]

$$\xi = \left( \frac{\chi/F - 1}{4 \sin^2(\pi/L)} \right)^{1/2}, \quad (5)$$

where  $F$  is the mean value of the trace of  $\hat{\mathbf{T}}$  squared at minimal momentum ( $2\pi/L$  in direction  $x$ ,  $y$  or  $z$ ). To define  $\xi_s$  we use  $\chi_s$  and compute  $F_s$  from  $\hat{\mathbf{T}}$  at momentum  $(2\pi/L + \pi, \pi, \pi)$  and permutations.

The action (1) is suitable for cluster update methods by using Wolff's embedding algorithm [13]. We have checked the performance of both the Swendsen-Wang [14] and the single cluster methods [13]. Unfortunately, due to the AF character of the interaction, the critical slowing down is not reduced in any case, as there is always a large cluster that contains most of the lattice sites. In fact, we have measured a dynamic exponent  $z \approx 2$  for both methods.

We have also developed a Metropolis algorithm. Near the transition the spin fluctuations are large, and a

Table 1

Number of Monte Carlo sweeps performed for different lattice sizes. Measures have been taken every 10 sweeps. The integrated autocorrelation times (in sweeps) for both susceptibilities are also displayed. They have been computed using a selfconsistent 6 $\tau$  window. The statistical errors are below the 5% level. We have discarded in each case about  $200\tau_{\chi_s}$  iterations for thermalization.

$L$	MC sweeps ( $\times 10^6$ )	$\tau_{\chi_s}$	$\tau_{\chi}$
6	6.71	7.4	5.8
8	17.07	11.4	7.4
12	6.51	24.8	12.8
16	22.14	44.1	21.4
24	8.77	107	48
32	10.13	179	87
48	3.93	430	205

spin proposal uniformly distributed over the sphere is accepted with nearly 30% probability. So we have used a 3 hit algorithm, reaching a mean 70% acceptance.

Regarding the performance of the three methods mentioned for a given lattice size, the differences are very small in terms of the CPU time. We have selected the Metropolis method which is slightly faster.

In Table 1 we display the number of Monte Carlo sweeps performed for the different lattice sizes as well as the integrated autocorrelation times for the observables  $\chi$ ,  $\chi_s$ . The total CPU time has been the equivalent of 12 months of DEC Alpha AXP3000 distributed over several workstations.

Every 10 Monte Carlo sweeps we store individual measures of the energy and of the Fourier transform of the tensorial magnetization at suitable momentum values. We have used the spectral density method [15] to extrapolate in a neighborhood of the critical point. The data presented here correspond to simulations at two  $\beta$  values ( $-2.41$  and  $-2.4$ ). We compute the quantities referred above as well as their  $\beta$ -derivatives through the connected correlations with the energy.

We have firstly analyzed several quantities that present a peak near the transition point. As we have found that the specific heat does not diverge, we have to limit ourselves to study quantities related with magnetization operators. The advantage of measuring a peak height is that its position also defines an apparent critical point allowing for a very simple and accurate measure. Unfortunately, quantities like the  $\beta$ -derivatives of the magnetizations or the connected susceptibilities ( $\chi_{\text{con}} \equiv \chi - VM^2$ ) present their peaks

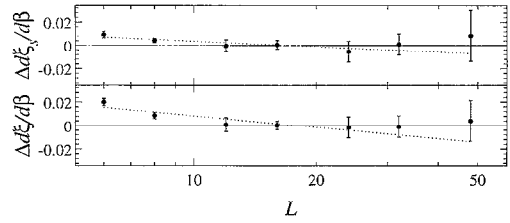


Fig. 1. Deviations from a power law fit of the  $\beta$ -derivative of the staggered (upper side) and nonstaggered (lower side) correlation lengths at the critical point, using data from lattice sizes with  $L \geq 16$ . The dotted lines correspond to a fit using all data sizes.

far away from the critical point suffering from large corrections to scaling. For example,  $\chi_{\text{con}}$  in the  $L = 16$  lattice peaks at  $\beta = -2.29$  where the correlation length  $\xi$  is one half of its value at the critical point. This is due to the weakness of the tensorial ordering: the staggered magnetization, for instance, does not reach one-half of its maximum until  $\beta < -3.5$ .

Another possibility is to obtain the infinite volume critical point by other means and, then, to measure the different quantities at this point. By studying the matching of the scaling function  $\langle (\text{tr} \mathbf{M}_s^2)^2 \rangle / \langle (\text{tr} \mathbf{M}_s^2)^2 \rangle$ , as well as that of  $\xi_s(L, \beta)/L$  and the corresponding nonstaggered quantities, we conclude that

$$\beta_c \in [-2.415, -2.405]. \quad (6)$$

To improve the above determination it is necessary a careful consideration of the corrections to scaling. This subject will be discussed elsewhere [9].

In the case of quantities that change rapidly at the critical point as the magnetizations do, the errors in the determination of the critical point affect very much the results, and this method is not accurate. Nevertheless, we have found important quantities, like the  $\beta$ -derivatives of the correlation lengths, which are very stable. Both  $d\xi_s/d\beta$  and  $d\xi/d\beta$  should scale as  $L^{1+1/\nu}$ ; fitting the data from all lattice sizes we obtain an acceptable fit:  $\nu = 0.793(2)$  with  $\chi^2/\text{dof} = 2.0/5$  and  $\nu = 0.787(2)$  with  $\chi^2/\text{dof} = 5.5/5$  respectively. However, if we discard the  $L = 6$  data the fitted parameters change significantly and change again after discarding the  $L = 8$  ones. The fits for  $L \geq 12$  and  $L \geq 16$  agree within errors. We, thus, choose the  $L \geq 12$  data for computing  $\nu$  but take the statistical error from the fit with  $L \geq 16$  (see Fig. 1):

Table 2

Critical exponents obtained from a Finite Size Scaling analysis using data from lattices of sizes  $L$  and  $2L$ . In the second row we show the operator used for each column.

$L$	$\nu$		$\eta_s$		$\eta$	
	$d\xi_s/d\beta$	$d\xi/d\beta$	$\chi_s$	$M_s$	$\chi$	$M$
6	0.786(6)	0.790(6)	0.0431(10)	0.0474(9)	1.442(2)	1.447(2)
8	0.785(4)	0.781(4)	0.0375(7)	0.0409(8)	1.413(2)	1.416(2)
12	0.789(8)	0.782(9)	0.0357(17)	0.0382(18)	1.391(3)	1.393(3)
16	0.787(9)	0.781(8)	0.0371(19)	0.0390(19)	1.379(4)	1.381(4)
24	0.77(2)	0.77(2)	0.038(5)	0.038(5)	1.362(8)	1.365(9)

$$\begin{aligned} \xi_s : \quad \nu &= 0.788(7) \\ \xi : \quad \nu &= 0.779(6). \end{aligned} \quad (7)$$

To estimate the errors, not considered in (7), associated with the uncertainty in the determination of the critical point we repeat the fits with  $\beta$  at the limits of the interval (6). We observe that  $\nu$  changes by an amount of a 1%.

To avoid the problems reported above, we have also used a method directly based on the Finite Size Scaling ansatz, that allows to write the mean value of any operator  $O$  as

$$\langle O(L, \beta) \rangle = L^x f_O(\xi(L, \beta)/L) + \dots, \quad (8)$$

where  $\xi(L, \beta)$  is the correlation length measured at coupling  $\beta$  in a size  $L$  lattice,  $f_O$  is a smooth operator-dependent function and  $x$  depends also on  $O$ . The dots stand for corrections to scaling.

Measuring  $\langle O \rangle$  at the same coupling in lattices  $2L$  and  $L$  and using (8) we can write for their quotient

$$Q_O = 2^x \frac{f_O\left(\frac{\xi(2L, \beta)}{2L}\right)}{f_O\left(\frac{\xi(L, \beta)}{L}\right)} + \dots \quad (9)$$

Considering the dependence of  $Q_O$  on  $\rho \equiv \xi(2L, \beta)/\xi(L, \beta)$ , we just have to measure at the point where  $\rho = 2$  to obtain  $Q_O = 2^x$  up to corrections to scaling.  $x$  can be written in terms of the critical exponents:  $x = \gamma/\nu$  for  $\chi$ ,  $x = -\beta/\nu$  for  $M$ , etc.. To compute  $\nu$  we can use  $O = d\xi/d\beta$  for which  $x = 1 + 1/\nu$ . As the quantity  $\rho$  is an observable and not an external parameter, like  $\beta$ , this procedure does not require a previous determination of  $\beta_c$ . Another advantage is that the result depends only on measures in just two lattices which allows a better error estimation. In columns 2

and 3 of Table 2 we report the results for the thermal exponent  $\nu$  obtained from data in lattices  $L$  (first column) and  $2L$ , using as operators the  $\beta$ -derivatives of the correlation lengths  $\xi_s$  and  $\xi$  respectively. Even in the smaller lattices we do not observe any corrections to scaling. The data from columns 2 and 3 are very correlated statistically and so, by taking the mean value, the errors are only slightly reduced. We select as our best estimation the mean of the results for the lattices 16–32:

$$\nu = 0.784(8). \quad (10)$$

To be compared with the values  $\nu(O(3)) = 0.704(6)$  [16] and  $\nu(O(4)) = 0.748(9)$  [5].

In the case of the magnetic exponents, due to the large slope of  $Q$  as a function of  $\rho$  (see Fig. 2), a correct determination of the errors requires to take into account the statistical correlations of the whole data. We obtain for  $\gamma_s$  and  $\beta_s$ , in the most favorable cases, errors as small as a 0.1%. From these we compute, using the scaling relations  $\gamma_s/\nu = 2 - \eta_s$  and  $2\beta_s/\nu = D - 2 + \eta_s$ , the values of  $\eta_s$  with acceptable accuracy (see columns 4 and 5 of Table 2). In this case, the corrections to scaling are only significant for the  $L = 6$  lattice. We quote as our preferred value

$$\eta_s = 0.038(2). \quad (11)$$

For the nonstaggered sector, we observe that the usual susceptibility diverges much more slowly than the staggered one ( $\gamma_s - \gamma \sim 1$ ). The results for  $\eta$ , using the corresponding scaling relations, are reported in the last two columns of Table 2. In this case the corrections to scaling are nonnegligible for all lattice sizes. As the data fit very well to a linear function of  $1/L$  we take as the  $L \rightarrow \infty$  value

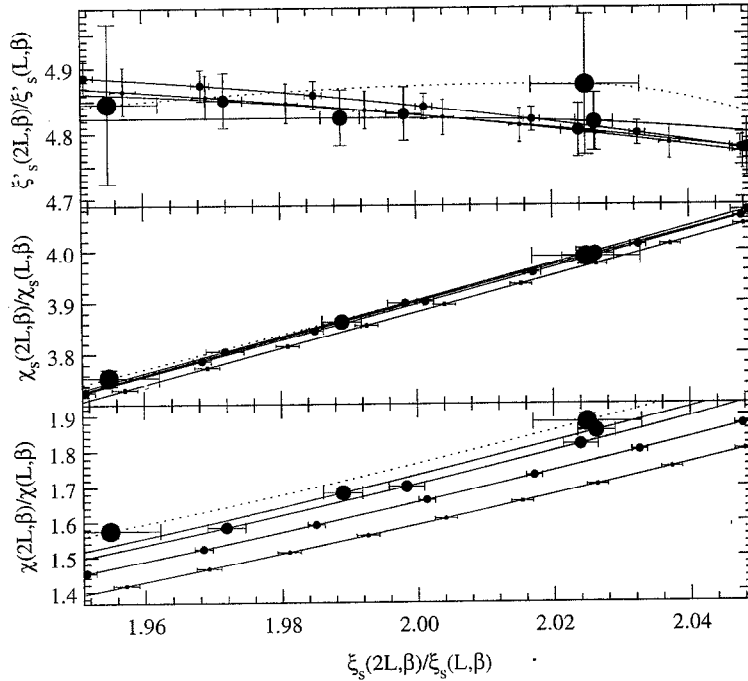


Fig. 2. Quotients of several observables as a function of the quotient of the staggered correlation lengths. The sizes of the symbols are a growing function of the lattice size.  $\xi'_s$  is the  $\beta$ -derivative of the staggered correlation length.

$$\eta = 1.338(8), \quad (12)$$

where the error is half statistical and half due to the possible deviations from linearity. Notice that the large value of  $\eta$  means that, at the critical point, the spatial correlation function ( $G(r)|_{\beta=\beta_c} \sim |r|^{-1-\eta}$ ), in the nonstaggered sector, decreases much faster than in the staggered case.

Our results show that the O(3) symmetry is fully broken in the ordered phase near the critical point. If we discard order  $t^\beta$  terms ( $t$  being the reduced temperature),  $\mathbf{M}$  is zero, so the magnetization tensors of the even and odd sublattices are opposite, and the eigenvectors corresponding to the maximum eigenvalues are orthogonal. Studying the finite size scaling of the operator  $[\mathbf{M}_s, \mathbf{M}] = \frac{1}{2}[\mathbf{M}_{\text{even}}, \mathbf{M}_{\text{odd}}]$  we observe a behavior of type  $L^{-(\beta+\beta_s)/\nu}$ . We thus conclude that considering the order  $t^\beta$  terms the orthogonality will only hold approximately what implies that the O(2) symmetry of the ground state does not remain.

We have studied a spin model in three dimensions with the symmetries of the O(3) group but with very interesting new properties. It presents an ordered vac-

uum where the O(3) symmetry is fully broken. The model has two odd (magnetic type) relevant operators with different associated  $\eta$  exponents. Finally, the values obtained for the exponents  $\nu$  (Eq. (10)) and  $\eta$  for the staggered channel (Eq. (11)) are very different from those of the O(3) model, and they are 3 standard deviations apart from those computed in Ref. [5] for the O(4) model. From the analysis using different lattice sizes we are confident that the systematic errors due to finite size effects are small and it seems difficult that they could make our results compatible with those obtained in Ref. [5].

We think that in addition to the interest of the model by itself, the results suggest further studies of related models, like the addition of vector interactions, or four dimensional systems.

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