Bayesian Analysis of Realized Matrix-Exponential GARCH Models

Manabu Asai

Faculty of Economics Soka University, Japan

Michael McAleer

Department of Finance Asia University, Taiwan and
Discipline of Business Analytics University of Sydney Business School,
Australia and Econometric Institute Erasmus School of Economics
Erasmus University Rotterdam, The Netherlands and
Department of Quantitative Economics
Complutense University of Madrid, Spain and
Institute of Advanced Sciences
Yokohama National University, Japan

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Keywords   Multivariate GARCH; Realized Measure; Matrix-Exponential; Bayesian Markov chain Monte Carlo method; Asymmetry.

JEL Classification   C11, C32.

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Manabu Asai

Faculty of Economics
Soka University, Japan

Michael McAleer

Department of Finance
Asia University, Taiwan
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1 Introduction

Estimation and forecasting time-varying co-volatilities between assets plays an important role in asset pricing, portfolio selections, and risk management. A popular approach is to estimate conditional covariance matrices via the multivariate models of the class of generalized autoregressive conditional heteroskedasticity (GARCH) (see the survey paper by McAleer (2005) as an example).

Over the past two decades, realized measures of volatility have received unprecedented attention in the academic literature on modeling and forecasting of stock market returns volatility. In the traditional literature on GARCH models, Engle and Gallo (2006) and Shephard and Sheppard (2010), among others, incorporated realized measures for modeling and forecasting volatility. In addition, Hansen, Huang, and Shek (2012) suggested the realized GARCH framework, which provides a structure for the joint modeling of returns and realized measures of volatility. By extending the work of Hansen, Huang, and Shek (2012), Hansen and Huang (2016) developed the realized exponential GARCH (EGARCH) model, which is an extension of the EGARCH model of Nelson (1991).

In this paper, we consider a multivariate extension of the realized EGARCH model of Hansen and Huang (2016). For the model specification, two features of the EGARCH model of Nelson (1991) are to accommodate asymmetric effects and to guarantee the positive value of conditional volatility via the exponential transformation. For asymmetric effects from asset returns to future volatility, Hansen and Huang (2016) consider the second-order approximation of Hermite polynomials, rather than the absolute value function of standardized returns, as used in Nelson (1991). We consider a multivariate version of this type of specification, and develops news impact curves, as in Engle and Ng (1993). Turning to the positive definiteness of conditional co-volatility, we use the matrix-exponential transformation as in Chiu, Leonard, and Tsui (1996) and Kawakatsu

(2005). We also incorporate measurement errors of realized volatility and co-volatility measures in the specification of the new realized matrix-exponential GARCH model.

For estimating the multivariate GARCH models, Vrontos et al. (2003) estimated several bivariate ARCH and GARCH models, and found that maximum likelihood (ML) estimates of the parameters were different from their Bayesian MCMC estimates. As the differences can be caused by the non-normality of the parameters, Vrontos et al. (2003) suggest careful interpretation of the ML estimation. For this reason, the Bayesian Markov chain Monte Carlo (MCMC) technique is considered in the paper, so that non-normal posterior distributions are obtained.

The remainder of the paper is organized as follows. Section 2 introduces the new realized MEGARCH model and its news impact curve. Section 3 explains the MCMC algorithm for the Bayesian estimation in detail. Section 4 provides an empirical example for three stocks traded on the New York Stock Exchange. Section 4 compares five kinds of symmetric and asymmetric models, using the deviance information criterion of Spiegelhalter et al. (2002) and the Frovenius norm of forecast error. Section 4 also examines the MCMC estimates of the parameters, and presents the news impact curves for describing effects from stock returns to future (co-)volatility of its own and other assets.. Finally, Section 5 gives some concluding remarks.

2   Realized Matrix-Exponential GARCH Model

We develop a multivariate extension of the realized EGARCH model of Hansen and Huang (2016). Let y_t and x_t be a financial asset return at day t and the log of its realized volatility measure,

respectively. The Realized EGARCH of Hansen and Huang (2016) is given by:

$$y_t = \varepsilon_t \exp\left(h_t/2\right), \quad \varepsilon_t \sim N(0,1)$$

$$x_t = \xi + h_t + \delta(\varepsilon_t) + u_t, \quad u_t \sim N(0, \sigma_u^2) \tag{1}$$

$$h_{t+1} = \omega + \phi(h_t - \omega) + \tau(\varepsilon_t) + \psi u_t,$$

where $\delta(\varepsilon) = \delta_1\varepsilon + \delta_2(\varepsilon^2 - 1)$ and $\tau(\varepsilon) = \tau_1\varepsilon + \tau_2(\varepsilon^2 - 1)$, and ε_t and u_t follow independent normal distributions. The conditional log-volatility, h_t, is determined by past information of innovation terms, ε_t and u_t. The asymmetric function, $\delta(\varepsilon)$ and $\tau(\varepsilon)$, can be interpreted as the second-order approximation of the Hermite polynomials. Hansen and Huang (2016) consider a more general framework, but their empirical results indicate that the above specification is sufficient for practical purposes.

We extend the univariate model (1) to the multivariate case by considering matrix-exponential transformation, as in Chiu et al. (1996). Chiu et al. (1996) suggested using the matrix-exponential transformation in order to guarantee the positive definiteness of the covariance matrix. Kawakatsu (2006) developed matrix-exponential GARCH models. For any square matrix, A, the matrix-exponential transformation is defined by $\mathrm{Exp}(X) = \sum_{i=0}^{\infty}(1/i!)A^i$, with $A^0 = I$. The same result is obtained by working with the spectral decomposition, as we have $\mathrm{Exp}(A)$ by replacing the eigenvalues by their exponential transformation. Note that $\mathrm{Exp}(A)$ is positive definite, whenever A is symmetric. In the same manner, $\mathrm{Log}(B)$ is defined by its spectral decomposition of a positive definite matrix, B, with replacement of the logarithmic transformation of eigenvalues.

Let $\boldsymbol{y}_t$ be an m-dimensional vector of financial asset returns. Define the matrix-logarithmic transformation of an $m \times m$ matrix of realized volatility measures as $\boldsymbol{X}_t$. We consider the realized

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matrix-exponential GARCH (MEGARCH) model defined by:

$$\boldsymbol{y}_t = \text{Exp}\left(\frac{1}{2}\boldsymbol{H}_t\right)\boldsymbol{\varepsilon}_t, \quad \boldsymbol{\varepsilon}_t \sim N(\boldsymbol{0}, \boldsymbol{I}_m) \tag{2}$$

$$\boldsymbol{X}_t = \boldsymbol{\Xi} + \boldsymbol{H}_t + \boldsymbol{\Delta}(\boldsymbol{\varepsilon}_t) + \boldsymbol{U}_t, \quad \boldsymbol{u}_t = \text{vech}(\boldsymbol{U}_t) \sim N(\boldsymbol{0}, \boldsymbol{\Sigma}_u) \tag{3}$$

$$\boldsymbol{H}_{t+1} = \boldsymbol{\Omega} + \boldsymbol{\Phi} \odot (\boldsymbol{H}_t - \boldsymbol{\Omega}) + \boldsymbol{G}(\boldsymbol{\varepsilon}_t) + \boldsymbol{\Psi} \odot \boldsymbol{U}_t, \tag{4}$$

where $\boldsymbol{\Xi}$, $\boldsymbol{\Omega}$, $\boldsymbol{\Phi}$, $\boldsymbol{\Psi}$ are $m \times m$ symmetric matrices of parameters, $\boldsymbol{\Sigma}_u$ is an m^*-dimensional positive definite matrix with $m^* = m(m+1)/2$, $\odot$ denotes the operator of element-wise multiplication (Hadamard product), $\boldsymbol{\varepsilon}_t$ and $\boldsymbol{u}_t$ follow independent multivariate normal distributions, and $\boldsymbol{\Delta}(\boldsymbol{\varepsilon})$ and $\boldsymbol{G}(\boldsymbol{\varepsilon})$ are matrices of asymmetric functions, as defined below. By the specification (2)-(4), the conditional volatility matrix, $\text{Exp}(\boldsymbol{H}_t)$, is determined by past information of innovation terms, $\boldsymbol{\varepsilon}_t$ and $\boldsymbol{u}_t$.

As multivariate extensions of asymmetric functions, $\delta(\varepsilon)$ and $\tau(\varepsilon)$, we define:

$$\boldsymbol{\Delta}(\boldsymbol{\varepsilon}) = \boldsymbol{\varepsilon}\boldsymbol{\delta}' + \boldsymbol{\delta}\boldsymbol{\varepsilon}' + \boldsymbol{\Upsilon} \odot (\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}' - \boldsymbol{I}_m),$$
$$\boldsymbol{G}(\boldsymbol{\varepsilon}) = \boldsymbol{\varepsilon}\boldsymbol{\gamma}' + \boldsymbol{\gamma}\boldsymbol{\varepsilon}' + \Lambda \odot (\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}' - \boldsymbol{I}_m), \tag{5}$$

respectively, where $\boldsymbol{\delta}$ and $\boldsymbol{\gamma}$ are m-vectors of parameters, and $\boldsymbol{\Upsilon}$ and Λ are $m \times m$ symmetric matrices. By the specification (5), we obtain, $E[\boldsymbol{\Delta}(\boldsymbol{\varepsilon}_t)] = \boldsymbol{O}$ and $E[\boldsymbol{\Delta}(\boldsymbol{G}_t)] = \boldsymbol{O}$ with finite covariance matrices.

By the structure of the realized MEGARCH model (2)-(5), $\boldsymbol{y}_t$ and $\boldsymbol{X}_t$ are stationary, if $\{\boldsymbol{H}_t\}$ is stationary. Since the process, $\{\boldsymbol{G}(\boldsymbol{\varepsilon}_t) + \boldsymbol{\Psi} \odot \boldsymbol{U}_t\}$, has an independent and identical distribution with mean *zero*, it can be considered as an innovation term. Hence the stationary condition is $|\phi_{ij}| < 1$, where ϕ_{ij} is the (i, j)-element of $\boldsymbol{\Phi}$.

We may consider an alternative specification of the asymmetric function based on the MEGARCH

model of Kawakatasu (2006), as:

$$\boldsymbol{\Delta}(\boldsymbol{\varepsilon}) = \mathrm{unvech}\left\{\boldsymbol{B}_\delta\boldsymbol{\varepsilon} + \boldsymbol{F}_\delta(\boldsymbol{\varepsilon}^* - E(\boldsymbol{\varepsilon}^*))\right\},$$

$$\boldsymbol{G}(\boldsymbol{\varepsilon}) = \mathrm{unvech}\left\{\boldsymbol{B}\boldsymbol{\varepsilon} + \boldsymbol{F}(\boldsymbol{\varepsilon}^* - E(\boldsymbol{\varepsilon}^*))\right\},$$

$$(6)$$

where $\boldsymbol{\varepsilon}^*$ is an m-vector with ith element $|\varepsilon_i|$, unvech $(\cdot)$ is the inverse operator of vech, and $\boldsymbol{B}_\delta$, $\boldsymbol{F}_\delta$, $\boldsymbol{B}$, and $\boldsymbol{F}$ are the $m^* \times m$ matrices with $m^* = m(m+1)/2$. These two realized MEGACRH models reduce to the MEGARCH model by omitting (3), and setting $\boldsymbol{\Psi} = \boldsymbol{O}$ in (4). In our empirical analysis for model comparisons, we use the two MEGARCH and two realized MEGARCH models, namely the MEGARCH model with (5) (quadratic function), the MEGARCH model with (6) (Kawakatsu, 2006), the Realized MEGARCH model with (2)-(5), and the realized MEGARCH model with (2)-(4) and (6) (Kawaktsu-type).

As an extension of Engle and Ng (1993), we can plot the news impact curve (NIC) via the function

$$\boldsymbol{V} = \mathrm{Exp}\left(\boldsymbol{\Omega} + G\left(\underline{\boldsymbol{V}}^{-1/2}\boldsymbol{y}\right)\right),$$

$$(7)$$

whih is obtained by setting $\boldsymbol{H}_t = \boldsymbol{\Omega}$, $\boldsymbol{U}_t = \boldsymbol{O}$, and $\mathrm{Exp}(\boldsymbol{H}_t) = \underline{\boldsymbol{V}}$ in the realized MEGARCH model, where $\underline{\boldsymbol{V}} = E(\boldsymbol{y}_t\boldsymbol{y}_t')$.

3 Bayesian Estimation

3.1 MCMC Algorithm

In this subsection, we describe the Markov chain Monte Carlo (MCMC) technique for estimating the realized MEGARCH model. For this purpose, we review briefly the development of MCMC method in the GARCH literature. In the univariate class, Geweke (1989) suggested the importance sampling technique for estimating ARCH models, while Bauwens and Lubrano (1998), Kim, Shephard and Chib (1998), Nakatsuma (2000), Vrontos, Dellaportas and Politis (2000), and Mitsui and Watanabe (2003) proposed several MCMC methods. Asai (2006) compared the above

MCMC methods, and found that the best method is the so-called 'tailored' approach based on the acceptance-rejection Metropolis-Hastings (ARMH) algorithm, with respect to the mixing, efficiency and computational requirement. For multivariate GARCH models, Vrontos, Dellaportas and Politis (2003) used the Metropolis-Hastings algorithm of Vrontos, Dellaportas and Politis (2000), while Ishihara, Omori and Asai (2015) worked with the tailored ARMH algorithm.

Based on recent developments of the MCMC technique, we can use a fast and efficient approach, namely, the Delayed Rejection & Adaptive Metropolis (DRAM) algorithm of Haario et al. (2006). The DRAM algorithm combines two ideas in the MCMC literature: adaptive Metropolis samplers (Haario et al. 1999, 2001) and delayed rejection (Tierney and Mira, 1999; Green and Mira, 2001; Mira 2002). The adaptive Metropolis sampler is based on the idea to create a Gaussian proposal distribution with a covariance matrix calibrated using the sample path of the MCMC chain. The basic idea of the delayed rejection is that, upon rejection in a Metropolis-Hastings algorithm, instead of advancing time and retaining the same position, a higher stage move is proposed to improve efficiency of the resulting MCMC estimators. We can apply the DRAM algorithm to various univariate and multivariate conditional volatility models.

We explain the delayed rejection and the adaptive Metropolis-Hastings algorithm of the DRAM algorithm of Haario et al. (2006). For the delayed rejection part, suppose the current position of the Markov chain is $\boldsymbol{\theta}_n = \theta$. As in a regular Metropolis-Hastings, a candidate move, $\boldsymbol{\theta}_c^{(1)}$, is generated from a candidate generating density, $q_1(\theta, \cdot)$ and accepted with the usual probability $\alpha_1(\theta, \theta_c^{(1)}) = \min\left(1, \frac{N_1}{D_1}\right)$, where $N_1 = \pi(\theta_c^{(1)})q_1(\theta_c^{(1)}, \theta)$ and $D_1 = \pi(\theta)q_1(\theta, \theta_c^{(1)})$ When it is rejected, instead of retaining the same position, $\boldsymbol{\theta}_{n+1} = \theta$, as we would do in a standard Metropolis-Hastings, a second stage move, $\boldsymbol{\theta}_c^{(1)}$, is proposed. The second stage proposal is allowed to depend not only on the current position of the chain but also on the rejected proposal: $q_2(\theta, \theta_c^{(1)}, \cdot)$.

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The second stage proposal is accepted with probability $\alpha_2(\theta, \theta_c^{(1)}, \theta_c^{(2)}) = \min\left(1, \frac{N_2}{D_2}\right)$, where $N_2 = \pi(\theta_c^{(2)})q_1(\theta_c^{(2)}, \theta_c^{(1)})q_2(\theta_c^{(2)}, \theta_c^{(1)}, \theta)[1-\alpha_1(\theta_c^{(2)}, \theta_c^{(1)})]$ and $D_2 = \pi(\theta)q_1(\theta, \theta_c^{(1)})q_2(\theta, \theta_c^{(1)}, \theta_c^{(2)})[1-\alpha_1(\theta, \theta_c^{(1)})]$. This process of delaying rejection can be iterated. If q_i denotes the proposal density at the i-th stage, the acceptance probability at that stage is given by $\alpha_i(\theta, \theta_c^{(1)}, \ldots, \theta_c^{(i)}) = \min\left(1, \frac{N_i}{D_i}\right)$, where

$$D_i = q_i(\theta, \ldots, \theta_c^{(i)})[q_{i-1}(\theta, \ldots, \theta_c^{(i-1)})[q_{i-2}(\theta, \ldots, \theta_c^{(i-2)}) \cdots$$

$$[q_2(\theta, \theta_c^{(1)}, \theta_c^{(2)})[q_1(\theta, \theta_c^{(1)})\pi(\theta) - N_1] - N_2] - N_3] \cdots - N_{i-1}]$$

$$N_i = \pi(\theta_c^{(i)})q_1(\theta_c^{(i)}, \theta_c^{(i-1)})q_2((\theta_c^{(i)}, \theta_c^{(i-1)}, \theta_c^{(i-2)}) \cdots q_i(\theta_c^{(i)}, \theta_c^{(i-1)}, \ldots, \theta)$$

$$\times [1 - \alpha_1(\theta_c^{(i)}, \theta_c^{(i-1)})][1 - \alpha_2((\theta_c^{(i)}, \theta_c^{(i-1)}, \theta_c^{(i-2)})] \cdots [1 - \alpha_{i-1}(\theta_c^{(i)}, \theta_c^{(i-1)}, \ldots, \theta_c^{(1)})].$$

As shown in Mira (2001), the process of delaying rejection can be interrupted at any stage. In our analysis, we try at most 5 times to move away from the current position, otherwise we let the chain stay where it is.

Now we turn to the explanation of the adaptive Metropolis-Hastings algorithm. The basic idea is to create a Gaussian proposal distribution with a covariance matrix calibrated using the sample path of the MCMC chain. Following Haario et al. (2006), when the current position is $\boldsymbol{\theta}_n = \theta_n$, we consider the proposal distribution with the multivariate normal distribution with mean θ and covariance matrix defined by:

$$S_n = \begin{cases} S_0, & n \leq n_0 \\ s_p \text{Cov}(\theta_0, \ldots, \theta_n), & n > n_0 \end{cases}$$

where $s_p = 2.4^2/p$, p is the dimension of $\boldsymbol{\theta}$, and Cov creates the sample covariance matrix. In our analysis, we set $n_0 = 2,000$. Combining the two approaches, Haario et al. (2006) showed the ergodicity of the DRAM algorithm.

3.2 Bayesian Estimation of Realized MEGARCH Models

Let $\boldsymbol{\theta} = (\text{vech}(\boldsymbol{\Omega})', \text{vech}(\boldsymbol{\Phi})', \boldsymbol{\gamma}', \text{vech}(\boldsymbol{\Lambda})', \text{vech}(\boldsymbol{\Xi})', \boldsymbol{\delta}', \text{vech}(\boldsymbol{\Upsilon})', \text{vech}(\boldsymbol{\Psi})', \text{vech}(\boldsymbol{L})')'$, where $\boldsymbol{L}$ is the lower triangular matrix calculated by the Cholesky decomposition of $\boldsymbol{\Sigma}_u$. Rather than sampling $\boldsymbol{\Sigma}_u$, we consider sampling $\boldsymbol{L}$ as this parametrization always guarantees the positive definiteness of $\boldsymbol{\Sigma}_u$ without imposing any restrictions. For $\boldsymbol{Y} = (\boldsymbol{y}_1, \ldots, \boldsymbol{y}_T)$ and $\boldsymbol{X} = (\boldsymbol{X}_1, \ldots, \boldsymbol{X}_T)$, the joint density function is given by:

$$f(\boldsymbol{Y}, \boldsymbol{X}|\boldsymbol{\theta}) = \prod_{t=1}^{T} f(\boldsymbol{y}_t|\boldsymbol{H}_t) f(\boldsymbol{X}_t|\boldsymbol{y}_t, \boldsymbol{H}_t, \boldsymbol{\theta}) \tag{8}$$

with

$$f(\boldsymbol{y}_t|\boldsymbol{H}_t) \propto |\text{Exp}(\boldsymbol{H}_t)|^{-1/2} \exp\left(-\frac{1}{2}\boldsymbol{y}_t'\text{Exp}(-\boldsymbol{H}_t)\boldsymbol{y}_t\right),$$

$$f(\boldsymbol{X}_t|\boldsymbol{y}_t, \boldsymbol{H}_t, \boldsymbol{\theta}) \propto |\boldsymbol{\Sigma}_u|^{-1/2} \exp\left(-\frac{1}{2}\text{vech}\left(\boldsymbol{U}_t\right)' \boldsymbol{\Sigma}_u^{-1}\text{vech}\left(\boldsymbol{U}_t\right)\right),$$

where $\boldsymbol{U}_t = \boldsymbol{X}_t - \boldsymbol{\Xi} - \boldsymbol{H}_t - \boldsymbol{\Delta}\left(\text{Exp}(-0.5\boldsymbol{H}_t)\boldsymbol{y}_t\right)$ and $\boldsymbol{H}_1 = \boldsymbol{\Omega}$.

We assume the multivariate (truncated) normal distribution for the prior distribution:

$$\text{vech}(\boldsymbol{\Omega}) \sim N(\boldsymbol{\mu}_\omega, \boldsymbol{\Sigma}_\omega), \quad \text{vech}(\boldsymbol{\Phi}) \sim N(\boldsymbol{\mu}_\phi, \boldsymbol{\Sigma}_\phi) \times \boldsymbol{S}(\boldsymbol{\Phi}), \quad \boldsymbol{\gamma} \sim N(\boldsymbol{\mu}_\gamma, \boldsymbol{\Sigma}_\gamma),$$

$$\text{vech}(\boldsymbol{\Lambda}) \sim N(\boldsymbol{\mu}_\lambda, \boldsymbol{\Sigma}_\lambda), \quad \text{vech}(\boldsymbol{\Xi}) \sim N(\boldsymbol{\mu}_\xi, \boldsymbol{\Sigma}_\xi), \quad \boldsymbol{\delta} \sim N(\boldsymbol{\mu}_\delta, \boldsymbol{\Sigma}_\delta),$$

$$\text{vech}(\boldsymbol{\Upsilon}) \sim N(\boldsymbol{\mu}_\upsilon, \boldsymbol{\Sigma}_\upsilon), \quad \text{vech}(\boldsymbol{\Psi}) \sim N(\boldsymbol{\mu}_\psi, \boldsymbol{\Sigma}_\psi), \quad \text{vech}(\boldsymbol{L}) \sim N(\boldsymbol{\mu}_l, \boldsymbol{\Sigma}_l),$$

where $\boldsymbol{S}(\boldsymbol{\Phi})$ is the region in which $\boldsymbol{\Phi}$ satisfies the stationary condition, $\mu_\omega = -4\ \mathbf{1}_{m^*}$, $\boldsymbol{\Sigma}_\omega = 5I_{m^*}$, $\mu_\phi = \text{vech}(0.15I_m + 0.8\ \mathbf{1}_m\mathbf{1}_m')$, $\boldsymbol{\Sigma}_\phi = 5I_{m^*}$, $\mu_\gamma = \mathbf{0}$, $\boldsymbol{\Sigma}_\gamma = 5I_m$, $\mu_\lambda = 0.01\ \mathbf{1}_{m^*}$, $\boldsymbol{\Sigma}_\lambda = 5I_{m^*}$, $\mu_\xi = \mathbf{0}$, $\boldsymbol{\Sigma}_\xi = 5I_{m^*}$, $\boldsymbol{\mu}_\delta = \mathbf{0}$, $\boldsymbol{\Sigma}_\delta = 5I_m$, $\mu_\upsilon = 0.01\ \mathbf{1}_{m^*}$, $\boldsymbol{\Sigma}_\upsilon = 5I_{m^*}$, $\mu_\psi = 0.01\ \mathbf{1}_{m^*}$, $\boldsymbol{\Sigma}_\psi = 5I_{m^*}$, and $\mathbf{1}_m$ is the m-dimensional vector of ones.

With the prior distribution $p(\boldsymbol{\theta})$, we consider the joint posterior distribution:

$$\pi(\boldsymbol{\theta}|\boldsymbol{Y}, \boldsymbol{X}) \propto p(\boldsymbol{\theta})f(\boldsymbol{Y}, \boldsymbol{X}|\boldsymbol{\theta}),$$

which is proportional to the prior density times the joint density. To obtain the posterior quantities of the parameters from the posterior distribution, we implement the MCMC algorithm:

1. Initialize $\boldsymbol{\theta}$.

2. Generate $\boldsymbol{\theta}|\boldsymbol{Y}, \boldsymbol{X}$ by the DRAM algorithm.

3. Go to Step 2.

For the implementation issues, we discard the first 5,000 samples as the burn-in and use the subsequent 10,000 samples for estimation. We also set S_0 as the asymptotic covariance matrix of the ML estimates of the realized MEGARCH model, and draw an initial value of $\boldsymbol{\theta}$ from a multivariate normal distribution with the mean of their corresponding values of prior distribution and the covariance matrix, S_0.

3.3   Illustration with Simulated Data

We generated $T = 1,000$ observations for the bivariate $(m = 2)$ realized MEGARCH model (2)-(5), with parameter values listed in Table 1, which also shows the ML and Bayesian MCMC results. For the Bayesian MCMC results, the posterior means are computed by averaging the simulated draws; The 95% credible intervals are calculated using the 2.5th and 97.5th percentiles of the simulated draws. P-value is for the convergence diagnostics suggested by Geweke (1992), in which the test statistic approaches to the standard normal distribution if the corresponding Markov chain is stationary for the retained 10,000 samples. P-values indicate that all series converge after discarding the first 5,000 samples. Compared wth the ML estimates, the posterior means are close to the ML estimates. The 95% credible intervals are wider than the 95% confidence interval for the data set. The acceptance rate for the DRAM algorithm is 0.9435, which is close to 1 due to the delayed rejection part of the DRAM.

Figure 1 shows the estimated posterior densities for $(\omega_{11}, \omega_{21}, \omega_{22}, \phi_{11}, \phi_{21}, \phi_{22})$. Except for ϕ_{11}, the estimated desnities are far from the normal distribution with the same mean and variance. As shown by Vrontos et al. (2003) and Asai (2006), the posterior densities for the GARCH class model are generally non-normal.

Figure 2 illustrates the news impact curve, which shows effects from ith return to the (i, j)th element of the one-step ahead conditional covariance matrix $(i, j = 1, 2)$, given by equation (7). As a feature of the MCMC technique, we obtain its posterior mean and 95% interval simultaneously. For the specification with the data set, the effects from the ith return $(i = 1, 2)$ on its own conditional variance draws asymmetric curves, while the effects on other conditional variance give (nearly) flat lines. The effects on conditional co-volatility give asymmetric curves, but they are negligible when the impacts are positive.

4 Empirical Analysis

Three stocks traded on the New York Stock Exchange are selected, namely: Bank of America (BAC), General Electric (GE), and International Business Machines (IBM). Based on the vector of returns for the $m = 3$ stocks computed for a 1-minute interval of trading day at t between 9:30 am and 4:00 pm, we calculated the daily values of the estimates of integrated co-volatility matrix, via the approach of Koike (2016). The estimator of Koike (2016) is robust to jumps and microstructure noise, and has an ability to handle asynchronicity of the times at which transactions are recorded. As in Asai and McAleer (2017), we modify the estimator of Koike (2016) to guarantee the positive definiteness of the covariance matrix, by the threshold method of Bickel and Levina (2008). We denote the estimate of integrated co-volatility matrix at dat t as $\boldsymbol{C}_t$, giving $\boldsymbol{X}_t = \text{Log}(\boldsymbol{C}_t)$. The sample period starts at August 31, 2006, and ends on October 26,

2012, giving 1500 observations. We also calculated the corresponding return vector, $\boldsymbol{y}_t$.

The sample is divided into two periods. The first 1000 observations cover the bankruptcy of Lehmann Brothers (September 15, 2008) and the Greek government requesting an initial loan from the EU (April 23, 2010). The former is the start of the global financial crisis, while the latter is the beginning of the European debt crisis. The last 500 observations are used for forecasting.

Table 2 presents the descriptive statistics of the returns, volatility ad co-volatility. The empirical distribution of the returns is heavily skewed to the left and is highly leptokurtic. Regarding volatility and co-volatility, they are skewed to the right, with evidence of heavy-tails in all series. Figure 3 shows the volatility and co-volatility of the three stocks, indicating that volatilities and co-volatilities are high in the period of turbulence caused by the global and European financial crises.

Using the return, volatility and co-volatility for three stocks, we estimated two kinds of realized MEGARCH models: one is based on the asymmetric function (5), while the other is based on (6). To compare these models, we also estimate two MEGARCH models (2) and (4), with $\boldsymbol{U}_t = O$ and $\boldsymbol{G}(\boldsymbol{\varepsilon})$ defined by (5) or (6), respectively. As a benchmark model, we use the BEKK model of Baba et al. (1987) and Engle and Kroner (1995):

$$\boldsymbol{y}_t = \underline{\boldsymbol{H}}_t^{1/2} \boldsymbol{\varepsilon}_t,$$

$$\underline{\boldsymbol{H}}_{t+1} = \underline{\boldsymbol{\Omega}} + \underline{\boldsymbol{A}}(\boldsymbol{y}_t \boldsymbol{y}_t' - \underline{\boldsymbol{\Omega}})\underline{\boldsymbol{A}}' + \underline{\boldsymbol{B}}(\underline{\boldsymbol{H}}_t - \underline{\boldsymbol{\Omega}})\underline{\boldsymbol{B}}',$$

(9)

where $\underline{\boldsymbol{H}}_t$ is the conditional covariance matrix, $\underline{\boldsymbol{A}}$ and $\underline{\boldsymbol{B}}$ are $m \times m$ square matrices, $\underline{\boldsymbol{\Omega}}$ is an m-dimensional positive definite matrix. The specification is known as the 'variance targeting' BEKK model (see Laurent, Rombouts, and Violante (2012)).

Table 3 presents diagnostic results for the MCMC methods for estimating the five models. The acceptance rate is close to one for the delayed rejection of the DRAM algorithm. The minimum of

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thw P-values diagnostics suggested by Geweke (1992) indicates that all Markov chains converged.

To compare the in-sample performance of the five models, we use the DIC (deviance informa-tion criterion) of Spiegelhalter et al. (2002), defined by:

$$DIC = E_{\boldsymbol{\theta}|\boldsymbol{Y},\boldsymbol{X}}[D(\boldsymbol{\theta})] + p_D,$$

$$p_D = E_{\boldsymbol{\theta}|\boldsymbol{Y},\boldsymbol{X}}[D(\boldsymbol{\theta})] - D(E_{\boldsymbol{\theta}|\boldsymbol{Y},\boldsymbol{X}}[\boldsymbol{\theta}]), \qquad D(\boldsymbol{\theta}) = -2\log f(\boldsymbol{Y},\boldsymbol{X}|\boldsymbol{\theta}) + C_{data}$$

where C_{data} is a constant term that depends only on the data set $\{\boldsymbol{Y},\boldsymbol{X}\}$. Table 4 shows the estimates of DIC for the five models. The realized MEGARCH model (quadratic function) has the smallest DIC estimate, while the BEKK model has the largest DIC. Taking account of the minimum and maximum values in the fluctuations of DIC, there are clear differences between the realized MEGARCH models and the MEGACH models, and differences between the MEGARCH models and BEKK model. On the other hand, there is an overlap of the distributions of DICs of the two realized MEGACRH models. Compared with the differences in the models, the differences based on the asymmetric function are minor.

Figure 4 shows the box plots of the posterior densities for the parameters of the realized MEGARCH model with the quadratic function. Figure 4 implies that the posterior densities have skewed and heavy-tails, which are non-normal. The densities of ϕ_{ij} have a mass larger than 0.9, indicating high persistence for the process of $\boldsymbol{H}_t$. To examine the relationship between returns and future (co-)volatilities, we need a careful consideration since the (i,j)-element of $\boldsymbol{H}_t$ does not necessarily correspond to the (i,j)-element of $\text{Exp}(\boldsymbol{H}_t)$. Figure 5 presents the news impact curves for the realized MEGARCH models with the quadratic function. Figure 5(a)(j)(r) shows the asymmetric news impacts for their own future volatilities. Figure 5(g)(m) indicates that the

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news impacts from GE/IBM to future volatility of BAC are relatively large, while Figure 5(h)(l) illustrates the asymmetric effects from GE/IBM to future co-volatility between BAC and itself. The remaining news effects are relatively small. The results are reasonable as the financial sector is considered to be sensitive to the fluctuations of stocks in other sectors for the period which covers the GFC and EDC.

Fixing the sample size as $T = 1,000$, we re-estimated all the models to obtain one-step-ahead forecasts of co-volatility matrices based on updated posterior distributions. We use $\boldsymbol{C}_h$ as a proxy for the true co-volatility matrix, and define the forecast error matrix as:

$$\boldsymbol{E}_h = \boldsymbol{C}_h - \mathrm{Exp}\left(\boldsymbol{H}_h^f\right),$$

where $\boldsymbol{H}_h^f$ is the one-step-ahead forecast at time h. We compare the five models' out-of-sample forecast root-mean-squared error (RMSE) based on the Frobenius norm of the forecast error, which is defined by

$$FN = \frac{1}{500} \sum_h ||\boldsymbol{E}_{T+h}|| = \frac{1}{500} \sum_t \left[\sum_{i,j} e_{ij,T+h}^2 \right]^{1/2}, \tag{10}$$

for the last 500 observations.

Table 5 presents the estimates of FN. The realized MEGACRH model (quadratic function) has the smallest posterior mean, which is close to the results of the alternative realized MEGARCH model. The upper bounds of the 95% credible interval of the FN of the realized MEGARCH models are smaller than the lower bounds for the BEKK and the two MEGARCH models, implying the superiority of the structure of the realized MEGARCH model. Figure 6 shows the forecasts of the dynamic correlations and their corresponding realized measures calculated via $c_{ij,t}/\sqrt{x_{ii,t}c_{jj,t}}$, where $c_{ij,t}$ is the (i,j) element of $\boldsymbol{C}_t$. Figure 6 indicates that the forecasts of the correlation dynamics capture the realized correlation well.

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For the period which covers the GFC and EDC, the empirical results show that the new realized MEGARCH model gives the best fit to the data, and the best forecasts. Between the two realized MEGARCH models, the quadratic asymmetric function provides better results, although the difference is minor.

5   Conclusion

The paper developed a new realized matrix-exponential GARCH (MEGARCH) model, which is an extension of univariate realized exponential GARCH model of Hansen and Huang (2016). We considered the Bayesian MCMC estimation technique, which gives non-normal posterior distributions. Using returns and realized measures of the co-volatility matrix for three stocks traded on NYSE, we found that the realized MEGARCH models outperformed the BEKK and MEGARCH models for in-sample and out-of-sample performance. The news impact curves based on the posterior densities presented reasonable results.

The new realized MEGARCH model and Bayesian MCMC estimation open the possibility for many interesting research directions. We may develop a realized version of the BEKK model, the dynamic conditional correlation model of Engle (2002), and the Cholesky GARCH model of Dellaportas and Pourahmadi (2012). For these issues, we need to wait for further research on modeling the appropriate specification of the measurement equation of the realized measure of co-volatility matrix, so that its conditional and realized covariance matrices are positive definite.

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Table 1: Comparison of ML and Bayesian MCMC Results for Simulated Data

Parameter Value	ML Estimate	95% Confidence Interval	Posterior Mean	95% Credible Interval	P-value (Conv.)
$\omega_{11} = 1$	1.1521	$(0.7930, 1.5112)$	1.2743	$(0.3371, 2.6156)$	0.9185
$\omega_{21} = 0$	0.1850	$(0.0289, 0.3411)$	0.1915	$(-0.1830, 0.5547)$	0.8065
$\omega_{22} = 1$	1.6565	$(1.3510, 1.9621)$	1.8296	$(0.9419, 2.6976)$	0.9612
$\phi_{11} = 0.98$	0.9579	$(0.9522, 0.9636)$	0.9610	$(0.9371, 0.9830)$	0.9972
$\phi_{21} = 0.80$	0.8098	$(0.7963, 0.8232)$	0.8068	$(0.7563, 0.8464)$	0.9980
$\phi_{22} = 0.95$	0.9443	$(0.9355, 0.9532)$	0.9503	$(0.9227, 0.9805)$	0.9996
$\gamma_1 = -0.4$	-0.2482	$(-0.2620, -0.2344)$	-0.2550	$(-0.2978, -0.2083)$	0.9830
$\gamma_2 = -0.2$	-0.2135	$(-0.2235, -0.2035)$	-0.2142	$(-0.2447, -0.1817)$	0.9566
$\lambda_{11} = 0.4$	0.3406	$(0.3174, 0.3638)$	0.3706	$(0.2934, 0.4377)$	0.9624
$\lambda_{21} = 0.3$	0.1703	$(0.1554, 0.1851)$	0.1963	$(0.1434, 0.2495)$	0.9236
$\lambda_{22} = 0.2$	0.0889	$(0.0827, 0.0952)$	0.0934	$(0.0740, 0.1156)$	0.9018
$\xi_{11} = -1.0$	-1.4393	$(-1.5255, -1.3531)$	-1.5320	$(-1.7445, -1.3128)$	0.9789
$\xi_{21} = -0.5$	-0.6166	$(-0.6459, -0.5872)$	-0.6234	$(-0.6965, -0.5408)$	0.9878
$\xi_{22} = -1.5$	-2.1303	$(-2.1945, -2.0662)$	-2.1929	$(-2.4124, -1.9637)$	0.9567
$\delta_1 = -0.1$	-0.0856	$(-0.0978, -0.0735)$	-0.0947	$(-0.1303, -0.0555)$	0.9865
$\delta_1 = -0.2$	-0.2106	$(-0.2211, -0.2001)$	-0.2138	$(-0.2543, -0.1763)$	0.9695
$\upsilon_{11} = 0.20$	0.1700	$(0.1480, 0.1920)$	0.1930	$(0.1136, 0.2524)$	0.9234
$\upsilon_{21} = 0.15$	0.0971	$(0.0852, 0.1090)$	0.1194	$(0.0728, 0.1650)$	0.9028
$\upsilon_{22} = 0.10$	0.0371	$(0.0317, 0.0426)$	0.0352	$(0.0066, 0.0659)$	0.8096
$\psi_{11} = 0.1$	0.4361	$(0.4188, 0.4534)$	0.4421	$(0.3471, 0.5010)$	0.9631
$\psi_{21} = 0.3$	1.0591	$(1.0446, 1.0735)$	1.0598	$(1.0129, 1.1046)$	0.9990
$\psi_{22} = 0.2$	0.4798	$(0.4523, 0.5073)$	0.4827	$(0.3891, 0.5837)$	0.9839
$l_{11} = 0.500$	0.9144	$(0.8811, 0.9478)$	0.9404	$(0.8262, 1.0736)$	0.9985
$l_{21} = 0.075$	0.0666	$(0.0367, 0.0965)$	0.0919	$(0.0018, 0.1845)$	0.7750
$l_{31} = -0.060$	-0.0734	$(-0.1055, -0.0412)$	-0.0594	$(-0.1590, -0.0326)$	0.7154
$l_{22} = 0.250$	0.4268	$(0.4078, 0.4457)$	0.4307	$(0.3900, 0.4748)$	0.9686
$l_{32} = 0.120$	0.1558	$(0.1302, 0.1814)$	0.1640	$(0.0852, 0.2535)$	0.8600
$l_{33} = 0.200$	0.4097	$(0.3950, 0.4243)$	0.4286	$(0.3810, 0.4753)$	0.9960
Acceptance Rate			0.9435		

Note: P-value is for the convergence diagnostic statistics suggested by Geweke (1992).

Table 2: Descriptive Statistics of Returns, Volatilities and Co-Volatilities

Stock	Mean	Std.Dev.	Skew.	Kurt.
Return				
BAC	−0.2423	3.6214	−0.2525	13.054
GE	0.1230	1.2211	−0.3447	7.1232
IBM	−0.0511	1.8591	−0.1674	11.200
Volatility				
BAC	4.1286	10.338	8.8001	134.34
GE	0.7827	1.7139	8.4851	107.38
IBM	1.7100	4.0020	7.8978	92.847
Co-Volatility				
(BAC,GE)	0.3694	1.2924	9.0682	124.53
(BAC,IBM)	0.6565	2.3697	7.9425	89.802
(IBM,GE)	0.4781	1.1807	7.5428	80.038

Table 3: MCMC Diagnostic Results

Model	AR	Min. P-value	Median P-value	Max. P-value
Real MEGARCH (qd)	1.0000	0.1629	09345	0.9996
Real MEGARCH (abs)	1.0000	0.1624	0.8681	1.0000
MEGARCH (qd)	1.0000	0.1767	0.8600	0.9996
MEGARCH (abs)	0.9697	0.1765	0.7521	0.9920
BEKK	1.0000	0.1710	0.8821	0.9965

Note: 'AR' denotes the acceptance rate of the DRAM algorithm. P-value is for the convergence diagnostic statistic of Geweke (1992).

Table 4: DIC Estimates

Model	Ranking	DIC	S.E.	$\text{DIC}_{\max}$	$\text{DIC}_{\min}$
Real MEGARCH (qd)	1	25776.56	(155.84)	26278.34	25293.42
Real MEGARCH (abs)	2	26246.57	(334.48)	27648.75	25148.74
MEGARCH (qd)	3	32414.06	(53.14)	32677.01	32292.61
MEGARCH (abs)	4	32706.78	(150.76)	33252.62	32287.65
BEKK	5	36373.84	(67.64)	36679.49	36217.02

Table 5: Estimates of Frovenious Norm for Out-of-Sample Forecast Errors

Model	Ranking	Posterior Mean	95% Credible Interval
Real MEGARCH (qd)	1	5.9444	$(5.8446, 6.1054)$
Real MEGARCH (abs)	2	6.1601	$(6.0535, 6.3466)$
MEGARCH (qd)	3	7.3730	$(7.2344, 7.6333)$
MEGARCH (abs)	4	9.8408	$(9.6167, 10.6414)$
BEKK	5	10.0824	$(9.8822, 10.4021)$

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Figure 1: Estimated Posterior Density for Simulated Data
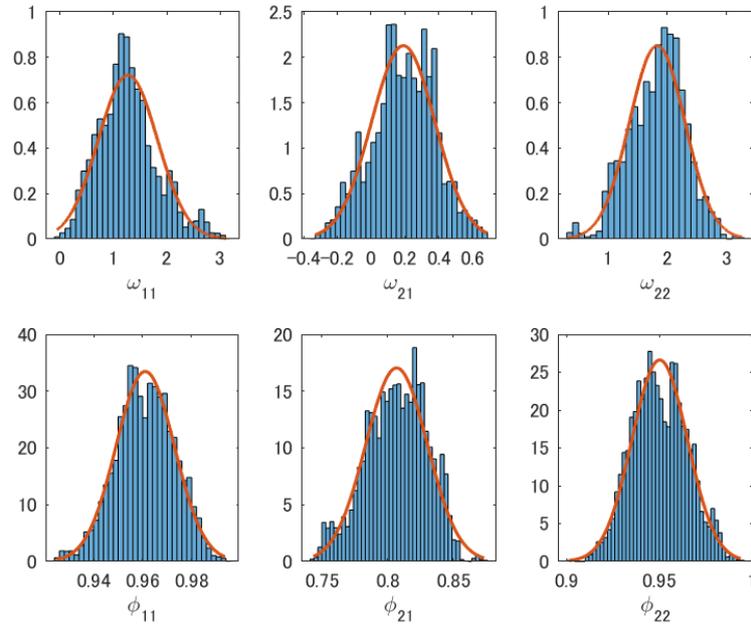
Note: Histograms are normalized to estimate densities. The red line shows the corresponding normal density with the same mean and variance.

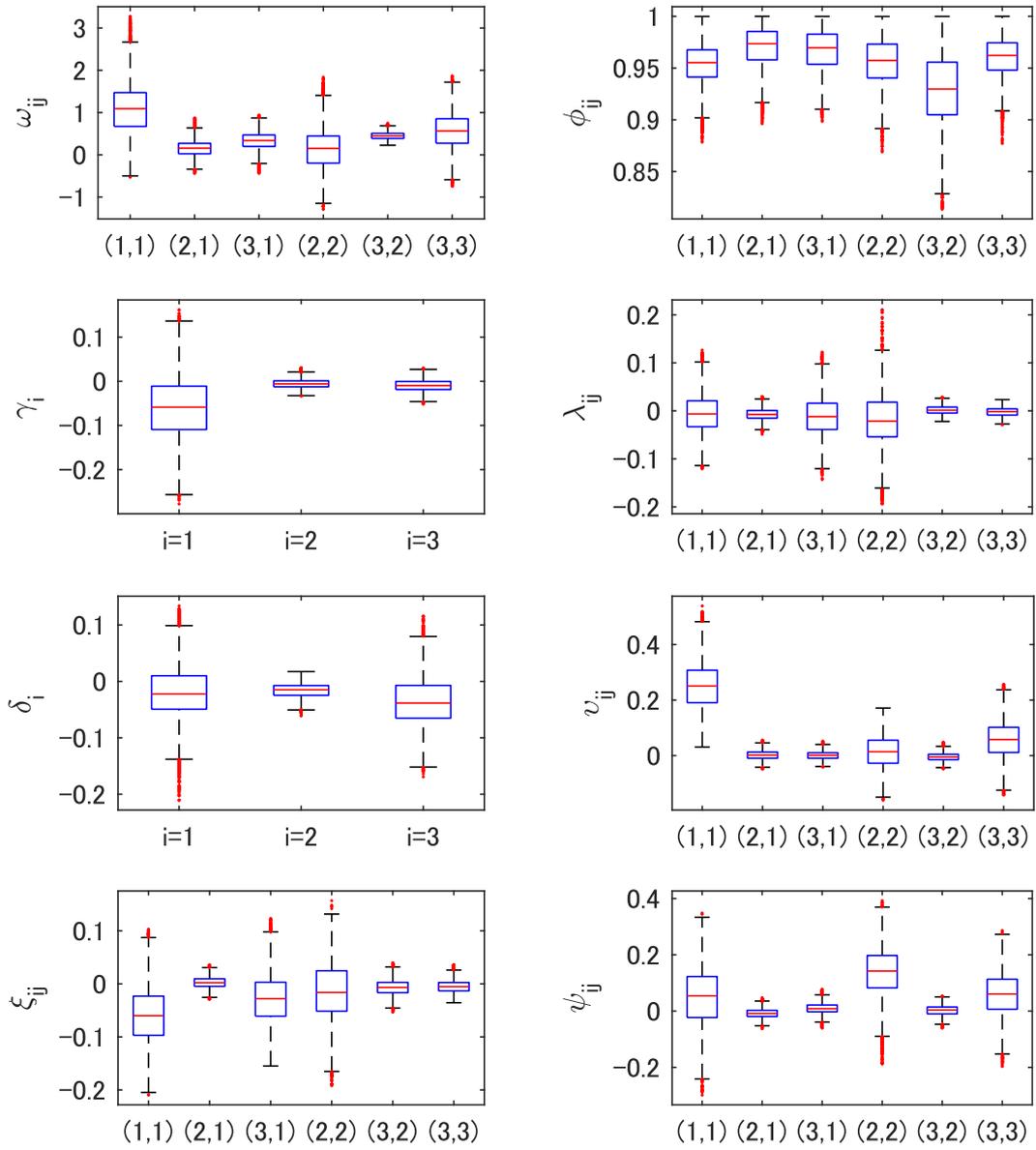Figure 2: Estimated News Impact Curves for Simulated Data
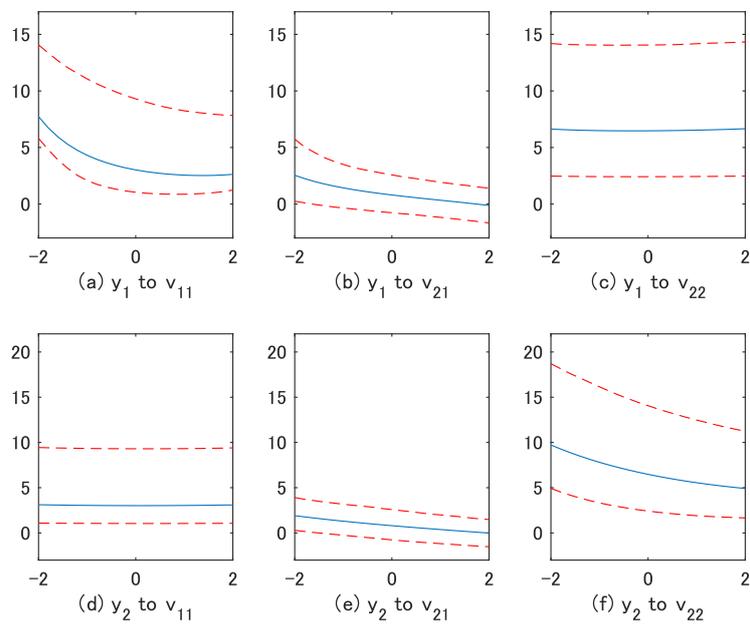
Note: Posterior mean (solid line) and 95% interval (dotted lines) of the news impact curve via equation (7).

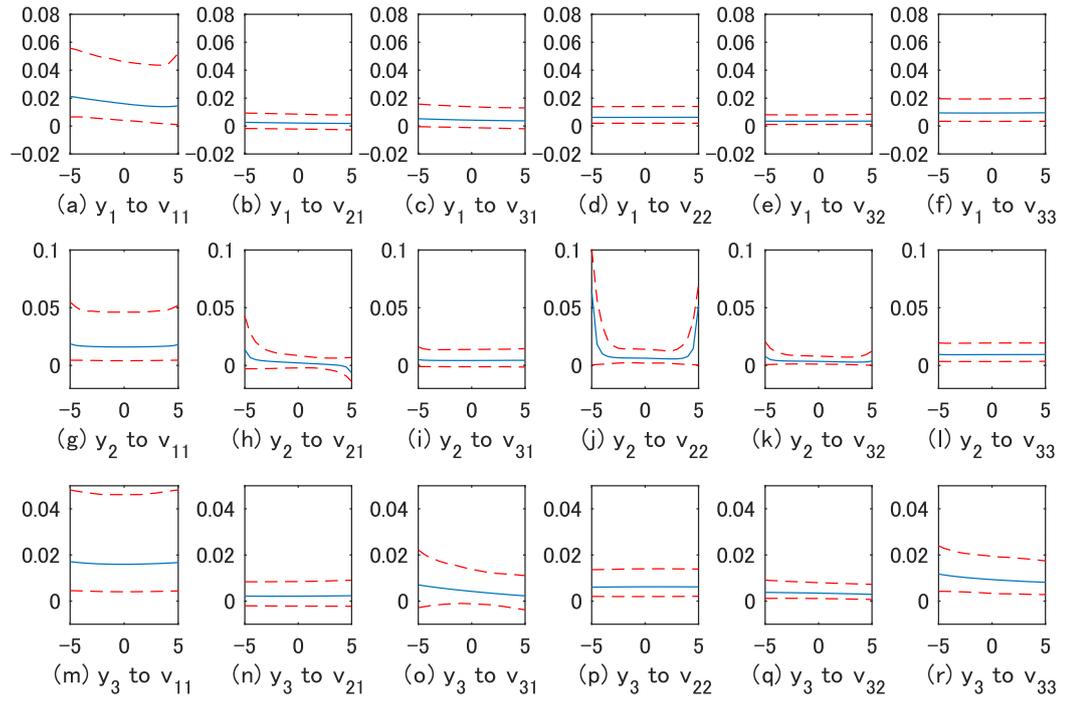Figure 3: Estimated Co-Volatility Matrix for Stock Returns
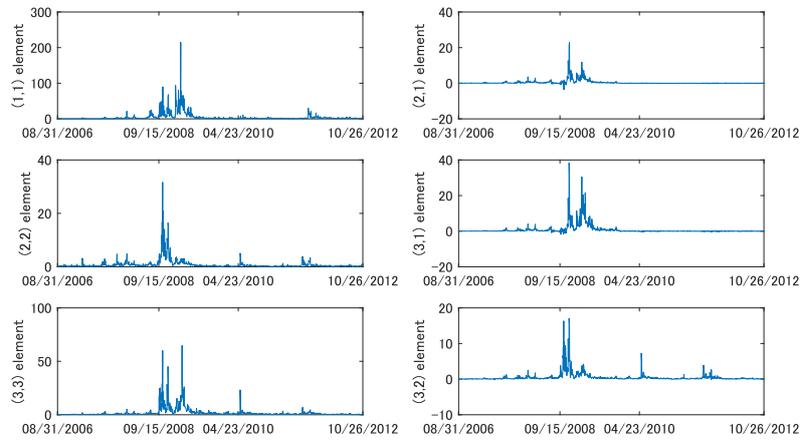
Note: Time series plots of (i, j)th element of covariance matrix; BAC $(i, j = 1)$, GE $(i, j = 2)$ and IBM $(i, j = 3)$.

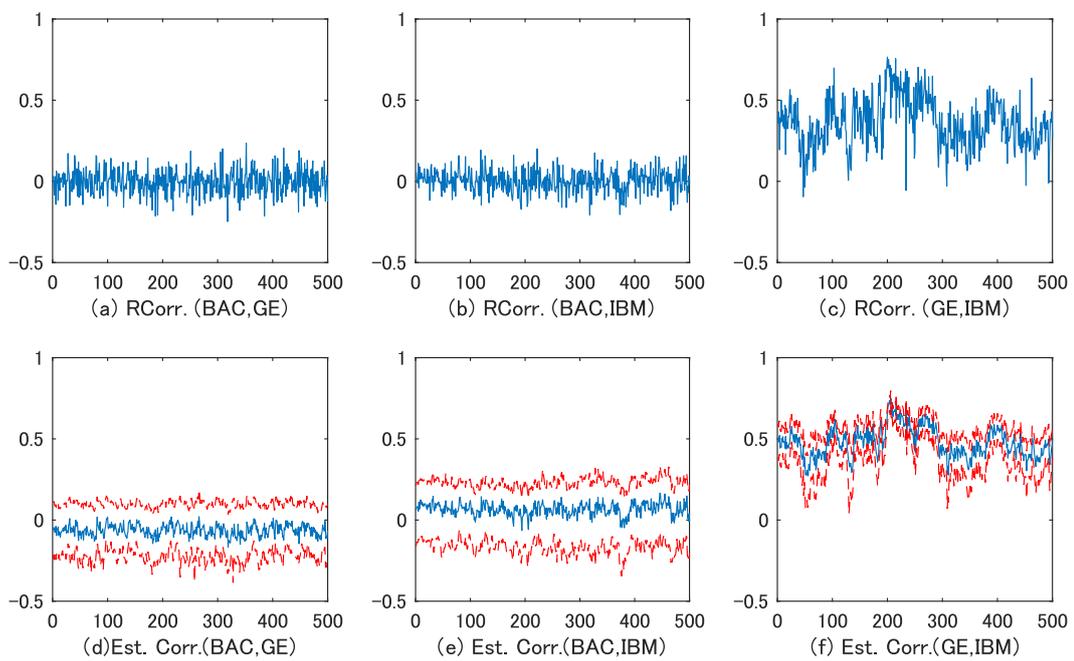Figure 4: Boxplots of Posterior Samples for the Realized MEGARCH Model

Note: The numbers for i and j indicate one of three stocks, namely, (1) BAC, (2) GE, and (3) IBM.

Figure 5: Estimated News Impact Curves for Stock Market Data

Note: Posterior mean (solid line) and 95% interval (dotted lines) of the news impact curve via equation (7), which measures the effect on (i, j)th element of co-volatility matrix by the impact of kth return, where the number indicates three stocks, namely, (1) BAC, (2) GE, and (3) IBM.

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Figure 6: Realized Measures and Forecasts of Dynamic Correlations

(a) RCorr. (BAC,GE)   (b) RCorr. (BAC,IBM)   (c) RCorr. (GE,IBM)

(d)Est. Corr.(BAC,GE)   (e) Est. Corr.(BAC,IBM)   (f) Est. Corr.(GE,IBM)

Note: Figures 6(a)-(c) show the realized correlations, while Figures 6(d)-(f) illustrate the estimated posterior means (solid line) and 95% intervals (dotted lines) of the one-step-ahead forecasts of correlation dynamics.