EDICIONES
COMPLUTENSE

Iberian Journal of the History of Economic Thought
e-ISSN: 2386-5768

ARTÍCULOS

The Classical Theory of Relative Prices: Evidence from the Spanish Economy

Víctor Magariño Lafalla
Departamento de Economía de la Universidad Carlos III de Madrid. Universidad Carlos III

https://dx.doi.org/10.5209/ijhe.99633 Recibido: 13/12/2024 • Revisado: 02/01/25 • Aceptado: 22/09/25

EN **Abstract.** This paper examines the development and current standing of the theory of value and distribution dominant in the classical and Marxian traditions of economic thought, while exploring its empirical adequacy in Spanish input-output data for the years 2010, 2015 and 2016. Estimates of direct prices and prices of production are used to measure their respective proximity, as well as the distance from each to market prices. The classical/Marxian system of prices of production is also subjected to testing for the possibility of pronounced price curvature. The results suggest that all three sets of prices are significantly close to each other, thereby strengthening the empirical support for the cross-sectional hypothesis derived from such account of relative prices. In addition, the evidence gathered from the Spanish economy does not support the case for twisted price curves as an empirically relevant phenomenon.
Keywords. Price/value deviations; classical economics; Marxian economics; relative prices; prices of production.

JEL Codes: B14; E11; C67.

ES La teoría clásica de los precios relativos: evidencias de la economía española

ES **Resumen.** Este artículo examina el desarrollo y la posición actual de la teoría del valor y la distribución predominante en las tradiciones clásica y marxista del pensamiento económico, explorando a su vez su adecuación empírica en datos de insumo-producto de España para los años 2010, 2015 y 2016. Se utilizan estimaciones de precios directos y precios de producción para medir su proximidad, así como la distancia de cada uno de ellos a los precios de mercado. El sistema clásico/marxista de precios de producción también se somete a pruebas para evaluar la posibilidad de una curvatura pronunciada de precios. Los resultados sugieren que los tres conjuntos de precios son significativamente cercanos entre sí, lo que refuerza el respaldo empírico a la hipótesis transversal derivada de esta concepción de los precios relativos. Además, la evidencia recopilada de la economía española no respalda la existencia de curvas de precios distorsionadas como fenómeno empíricamente relevante.
Palabras clave. Desviaciones precio/valor; economía clásica; economía marxista; precios relativos; precios de producción.

Códigos JEL: B14; E11; C67.

PT A teoria clássica dos preços relativos: evidências da economia espanhola

PT **Resumo.** Este artigo examina o desenvolvimento e a posição atual da teoria do valor e da distribuição, predominante nas tradições clássica e marxista do pensamento econômico, explorando sua adequação empírica por meio de dados de insumo-produto da Espanha para os anos de 2010, 2015 e 2016. Estimativas de preços diretos e preços ao produtor são utilizadas para medir sua proximidade e distância em relação aos preços de mercado. O sistema clássico/marxista de preços ao produtor também é testado para avaliar a possibilidade de uma curvatura de preços acentuada. Os resultados sugerem que os três conjuntos de preços são significativamente próximos entre si, fortalecendo o suporte empírico para a hipótese transversal derivada dessa concepção de preços relativos. Além disso, as evidências coletadas da economia espanhola não corroboram a existência de curvas de preços distorcidas como um fenômeno empiricamente relevante.

Palavras-chave. Desvios de preço/valor; economia clássica; economia marxista; preços relativos; preços ao produtor.

JEL classificação: B14; E11; C67.

Cómo citar: Magariño Lafalla, V. (2026): The Classical Theory of Relative Prices: Evidence from the Spanish Economy. *Iberian Journal of the History of Economic Thought*, 13(1), 49-65. https://dx.doi.org/10.5209/ijhe.99633.

1. Introduction

The problem of relative price formation has occupied a central place in the development of political economy since its earliest formulations. Long before the emergence of marginalist theory, classical political economists sought to explain the observable regularities of market prices by appealing not to individual preferences, but to objective conditions of production and reproduction, mediated by competition. A common thread running through these early contributions is the distinction between market prices, which fluctuate continuously under the pressure of changing conditions, and underlying regulating magnitudes (variously referred to as natural values, intrinsic values or fundamental prices) which act as centres of gravity for such movements[1].

Within this classical tradition, competition is not conceived as a state of equilibrium, nor as the outcome of rational optimisation by representative agents, but rather as a turbulent social process through which persistent regularities are enforced behind the backs of individual producers. Market prices are therefore not treated as arbitrary outcomes of subjective valuation, but as magnitudes that are constrained and regulated by the conditions under which commodities are reproducibly produced. The central theoretical question that follows from this perspective is straightforward: if market prices tend to gravitate around determinate centres of gravity, what are the underlying determinants of those centres, and how do they relate to the observable structure of production in each economy?

The classical answer to this question is grounded in the analysis of production costs understood in a specific sense. In the simplest analytical setting (namely, a system of simple commodity production characterised by reproducible techniques, monetary exchange, and competition among independent producers, but without a capitalist–labourer class division) relative prices are regulated by the labour requirements necessary to reproduce commodities under prevailing technical conditions. In such a setting, competition among producers tends to equalise incomes per unit of labour time, with the result that relative prices gravitate around magnitudes proportional to vertically integrated labour requirements. These magnitudes, which may be formally expressed as the sum of the direct and indirect labour time embodied in each commodity, provide a technical anchor for relative prices and establish the basic intuition behind the classical theory of value.

Once capitalist relations of production are introduced, however, this initial result must be further specified. The separation between workers and the means of production, together with the competitive mobility of capital across sectors, gives rise to a systematic tendency toward the equalisation of profit rates. Under these conditions, the centres of gravity of market prices are no longer directly proportional to embodied labour time, except in the limiting case where capital intensities are uniform across industries. Instead, relative prices gravitate around a transformed system which incorporates both the technical conditions of production and the social conditions governing the distribution of surplus value. The resulting deviations between labour values and prices of production are not arbitrary but are themselves law-governed and reflect differences in capital intensity across sectors.

A crucial implication of this two-stage determination is that the regulation of market prices cannot be understood independently of the competitive process through which it is enforced. Intrasectorial competition gives rise to correlated prices and persistent dispersion of profit rates across firms operating under heterogeneous technical conditions, while intersectoral competition tends to equalise profit rates across sectors at the level of the regulating capitals (that is, those capitals operating under the most reproducible technical conditions). Prices of production may therefore be interpreted as the long-run centres of gravity of market prices corresponding to these regulating conditions, rather than as simple averages of observed prices or costs. This conception of real competition, developed most systematically in recent classical-Marxian literature, provides the theoretical foundation upon which the present analysis is built.

[1] This distinction can already be found in the writings of Petty (1662) and Cantillon (1755), is developed further by the Physiocrats (Quesnay 1760), and is subsequently systematised by Smith (2007) and Ricardo (2001), before being reformulated and generalised within Marx's (1999) critique of political economy.

From an empirical standpoint, the classical theory of relative prices gives rise to two closely related hypotheses. The first is a cross-sectional hypothesis, according to which, at any given point in time, market prices should be found to lie at relatively short distances from both direct prices (prices proportional to embodied labour requirements) and prices of production. The second is a dynamic hypothesis, according to which movements in market prices over time should be closely correlated with movements in their underlying centres of gravity. While the latter requires time-series or panel data and lies beyond the scope of the present paper, the former can be directly tested using input-output data, which allow for the construction of vertically integrated labour coefficients and the formal estimation of prices of production.

The purpose of this paper is to examine the empirical strength of the classical theory of relative prices in the Spanish economy using input-output tables for the years 2010, 2015, and 2016. By computing vectors of direct prices, prices of production, and observed market prices for sixty-three industries, and by measuring the distances between these vectors using both weighted and unweighted metrics, the paper evaluates the degree to which market prices conform to their theoretically predicted centres of gravity. In addition, the paper investigates the behaviour of prices of production as functions of the rate of profit, with the aim of assessing the empirical relevance of claims concerning pronounced price curvature, technique reswitching, and capital reversal.

The contribution of the paper is threefold. First, it provides new empirical evidence on the classical theory of relative prices for the Spanish economy, complementing and extending an existing body of cross-sectional studies conducted on other countries. Second, it offers a transparent formalisation of the relationship between labour values, direct prices, and prices of production within a pure circulating-capital framework, thereby clarifying the nature and magnitude of price-value deviations. Third, by examining the linearity of price paths and the stability of price-value rankings, it contributes to the ongoing debate on the empirical significance of Sraffian critiques of the classical-Marxian approach.

The remainder of the paper is organised as follows. Section 2 reviews the empirical literature on the relationship between labour values, prices of production, and market prices. Section 3 develops the theoretical framework, proceeding from simple commodity production to capitalist competition and formalising the determination of prices of production under real competition. Section 4 presents the methodology and empirical results obtained from Spanish input-output data. Section 5 examines the behaviour of prices of production as the rate of profit varies and evaluates the empirical relevance of price curvature and reswitching phenomena. Section 6 concludes.

2. Empirical Literature Review

Although the data, methods, and computational tools available during the time of Smith, Ricardo, and Marx were not sufficiently developed to allow for a systematic empirical evaluation of the hypotheses derived from their theories of relative prices, the construction and widespread use of input-output tables (first popularised by Nobel Prize-winning economist Wassily Leontief) made such analyses increasingly feasible in the second half of the twentieth century. Since then, a large empirical literature has emerged, covering a wide range of countries and historical periods, which has overwhelmingly pointed toward the empirical strength of the classical account of relative price formation.

A path-breaking contribution to this literature is Shaikh's (1984) study, which employed Italian input-output data from Marzi and Varri, as well as Leontief's input-output tables for the United States, to test the degree of cross-sectional and intertemporal association between relative prices and relative labour values. In addition to log-linear regression analysis, Shaikh examined the mean absolute deviation (MAD) of sectoral direct prices (that is, prices proportional to labour values) and prices of production from observed market prices. Working within a pure circulating-capital framework, he found that both direct prices and prices of production lie remarkably close to market prices, with deviations ranging from 17% to 19% in the Italian economy and from 20% to 25% in the United States. These results provided the first systematic empirical support for the classical hypothesis that market prices gravitate around objectively determined centres of gravity.

Shaikh's methodology and findings subsequently opened the door to many empirical studies that sought both to replicate his results and to refine his procedures. Among the most influential of these contributions are those by Ochoa (1984, 1989), who used more detailed U.S. data covering a subset of years within the period 1947-1972 and extended the analysis to include fixed capital. Ochoa found that the average MAD between prices of production and market prices was approximately 13.6%, while the corresponding deviation for direct prices was around 12.2%; the average MAD between prices of production and direct prices amounted to 16.9%. In addition, Ochoa introduced the mean absolute weighted deviation (MAWD), which accounts for differences in industry size, as well as the normalised vector distance (NVD) as a scale-invariant measure. Both metrics yielded similarly small deviations, thereby reinforcing the robustness of the original findings.

From these initial works onward, a substantial number of studies have been conducted on different economies and across varying time spans, consistently corroborating the empirical regularities identified by Shaikh and Ochoa. This includes further analyses of the U.S. economy (Bienenfeld 1988; Shaikh 1998, 2016), as well as studies based on data from the United Kingdom (Cockshott, Cottrell and Michaelson 1995; Cockshott and Cottrell 1997, 2005), Yugoslavia (Petrovic 1987), Greece (Tsoulfidis and Maniatis 2002, 2007), Sweden (Zachariah 2004), several Asian economies (Tsoulfidis and Rieu 2006; Tsoulfidis 2008; Montibeler and Sánchez 2014; Tsoulfidis and Paitaridis 2016), Germany (Fröhlich 2012), Canada (Tsoulfidis and Paitaridis 2009), and a subset of world countries (Zachariah 2006; Isikara & Mokre 2022). a remarkably consistent picture: labour values and prices of production are both found to be closely aligned with observed market prices, with deviations that are moderate in magnitude and stable across countries and time periods.

Within this broad empirical tradition, the study by Sánchez and Nieto (2010) is of relevance for the present paper. Using Spanish input-output data, the authors compute labour values, prices of production, and market prices, and assess their relationships using a combination of regression techniques, distance measures, and angular comparisons. Their results confirm that both labour values and prices of production bear a close relationship to market prices in the Spanish economy, with relatively small deviations in magnitude. Moreover, Sánchez and Nieto observe that direct prices often appear closer to market prices than prices of production, a finding they interpret as reflecting the relatively compressed dispersion of capital intensities across sectors and the empirical proximity between labour values and prices of production themselves. Their analysis thus provides an important empirical benchmark for Spain.

Alongside these supportive findings, the empirical literature has also generated a sustained methodological debate concerning the interpretation of observed price-value correspondences. A central contribution in this respect is Kliman (2002), who argues that the high cross-sectional correlations typically obtained between aggregate sectoral prices and labour values are driven by industry size effects rather than by any underlying regulatory relationship. His claim is not simply that correlations may be spurious in principle, but that both regression coefficients and commonly used distance measures may be misleading insofar as they reflect the low dispersion of sectoral aggregates rather than genuine value regulation. To address this, Kliman proposes deflating both prices and values by a proxy for industry scale (specifically, aggregate costs) and reports that the resulting size-adjusted variables exhibit weak and unstable relationships in U.S. data.

Related, though conceptually distinct, criticisms are advanced by Nitzan and Bichler (2009), who challenge the empirical labour theory of value on two main grounds. First, they argue that empirical studies purportedly correlating prices and labour values in fact correlate monetary magnitudes with other monetary magnitudes, thereby engaging in a form of circular reasoning. Second, they maintain that observed price-value correlations are spurious artefacts of industry scale, arising from the multiplication of unit prices and unit values by heterogeneous output quantities.

These critiques have elicited detailed responses in the literature. Cockshott and Cottrell (2005) show that Kliman's deflation procedure effectively removes the informational content of the data by "dividing through by the signal", since empirical estimates of labour values are themselves derived from cost data, so that cost-deflated prices and values are two random perturbations removed from the underlying magnitudes. More recently, Cockshott, Cottrell, and Valle Baeza (2014) provide a systematic rebuttal to Nitzan and Bichler's arguments, demonstrating that the calculation of vertically integrated labour values from monetary input-output tables is invariant to the price vector used, up to a scalar, and that the proposed correlation between unit prices and unit values is not well-defined on dimensional grounds. From this perspective, the relevant empirical question is not whether aggregate price-value correlations are "spurious" in a statistical sense, but whether labour-time magnitudes provide a systematically better account of the structure of relative prices than alternative value bases (a question that the accumulated empirical evidence continues to answer in the affirmative).

Finally, empirical attention has also been devoted to assessing the relevance of Sraffian phenomena such as twisted price curves, capital reversal, and the reswitching of techniques. The accumulated evidence suggests that, while such phenomena are theoretically possible, they should be regarded as anomalies rather than as empirically pervasive features of actual economies (Krelle 1977; Ochoa 1984; Shaikh 1998, 2012, 2016). The observed near-linearity of price curves, capital-output ratios, and wage-profit relations is broadly consistent with the close empirical alignment between market prices, labour values, and prices of production documented in the literature.

3. The Competitively Enforced Objectivity of Long-Run Price Formation

This section develops a reconstruction of the classical theory of relative prices as a sequence of determinations proceeding from more abstract to more concrete levels of analysis. The aim is not to provide a historical narrative of the evolution of classical political economy, but rather to clarify the logical structure underlying its account of price formation and to identify the precise role played by labour-time magnitudes, prices of production, and market prices within that structure. The analysis begins by examining the competitive regulation of prices in a simple commodity-producing economy characterised by reproducible techniques and monetary exchange and then proceeds to incorporate the specifically capitalist mechanisms of wage labour, surplus value, and intersectoral capital mobility. This methodological approach allows for a clear distinction between the technical conditions of reproduction and the social conditions governing distribution, a distinction that proves crucial for understanding both the transformation of labour values into prices of production and the theoretical status of the so-called transformation problem.

At each stage of the analysis, competition is treated not as a state of equilibrium, but as a real social process through which persistent regularities are enforced behind the backs of individual producers. Prices are therefore analysed as centres of gravity around which market prices fluctuate, rather than as equilibrium magnitudes realised instantaneously. The purpose of the section is to make explicit the mechanisms through which such centres of gravity are formed, to formalise their determination within a multi-sector framework, and to clarify the relationship between labour-time accounting and the observable structure of relative prices in capitalist economies.

3.1. Simple Commodity Production and the

Labour-Time Regulation of Relative Prices

To identify the technical determinants of relative prices prior to the introduction of specifically capitalist relations of production, it is useful to begin with a simple commodity-producing economy. This analytical abstraction is characterised by reproducible techniques of production, monetary exchange, and competition among independent producers, but abstracts from the separation between workers and the means of production and from the generalised mobility of capital as capital. The purpose of this abstraction is not to describe an actual historical stage of development, but to isolate the mechanisms through which relative prices are regulated solely by the technical requirements of social reproduction.[2]

Within such a system, competition operates as a regulating process among producers producing similar commodities under heterogeneous technical conditions. Producers employing inferior techniques are systematically undercut by those operating closer to the prevailing social norm, while producers employing superior techniques are drawn toward the dominant price structure through competitive pressures. As a result, market prices are not arbitrary outcomes of individual valuation but tend to gravitate around determinate centres of gravity reflecting socially necessary conditions of production. At this level of abstraction, what competition tends to equalise is not a rate of profit, but net income per unit of labour time. This distinction is crucial: profit-rate equalisation presupposes capitalist relations of production and will be introduced only at a later analytical stage.

To make this mechanism precise, consider an economy with N industries. Let $\vec{\mathrm{l}} \equiv (1 \times \mathrm{N})$ denote the row vector of direct labour coefficients, $\mathbf{A} \equiv (\mathrm{N} \times \mathrm{N})$ the matrix of technical input coefficients, and $\mathbf{N}$ the corresponding identity matrix. Let $\vec{\mathrm{p}} \equiv (1 \times \mathrm{N})$ denote the row vector of natural (centre-of-gravity) prices.

For any sector j, the monetary cost of intermediate inputs required to produce one unit of output is given by $(\vec{\mathrm{p}}\mathbf{A})_j$, while the unit price received upon sale is p_j. The difference, $p_j - (\vec{\mathrm{p}}\mathbf{A})_j$, represents the net income generated per unit of output, that is, revenue net of replacement costs. Dividing this magnitude by the direct labour time required per unit of output, l_j, yields the net income per unit of labour time:

$$\frac{\mathrm{p_j}-(\vec{\mathrm{p}}\mathbf{A})_\mathrm{j}}{\mathrm{l_j}} \quad (1)$$

Competition among independent producers tends to equalise this magnitude across sectors, since producers operating under techniques that yield below-average income per labour hour are progressively displaced, while those operating under superior conditions face pressure to conform to the prevailing price structure. Denoting the uniform net income per unit of labour time by $\mathrm{w} > 0$, the competitive equalisation condition may therefore be written, for all $\mathrm{j} = 1, \ldots, \mathrm{N}$, as:

$$\frac{\mathrm{p_j}-(\vec{\mathrm{p}}\mathbf{A})_\mathrm{j}}{\mathrm{l_j}} = \mathrm{w} \leftrightarrow \mathrm{p_j} - (\vec{\mathrm{p}}\mathbf{A})_\mathrm{j} = \mathrm{w}\,\mathrm{l_j} \quad (2)$$

In vector notation, this system can be expressed compactly:

$$\vec{\mathrm{p}} - \vec{\mathrm{p}}\mathbf{A} = \mathrm{w}\,\vec{\mathrm{l}} \leftrightarrow \vec{\mathrm{p}}(\mathbf{I} - \mathbf{A}) = \mathrm{w}\,\vec{\mathrm{l}} \quad (3)$$

Provided that the spectral radius of $\mathbf{A}$ satisfies $\rho(\mathbf{A}) < 1$, the matrix $(\mathbf{I}-\mathbf{A})$ is invertible and the system admits a unique solution. Multiplying both sides on the right by $(\mathbf{I}-\mathbf{A})^{-1}$ yields:

$$\vec{\mathrm{p}} = \mathrm{w}\,\vec{\mathrm{l}}(\mathbf{I} - \mathbf{A})^{-1} \quad (4)$$

Defining the row vector of vertically integrated labour requirements as

$$\vec{\mathrm{v}} \equiv \vec{\mathrm{l}}(\mathbf{I} - \mathbf{A})^{-1} \quad (5)$$

it follows that $\vec{\mathrm{p}} = \mathrm{w}\,\vec{\mathrm{v}}$.

Since wis a scalar common to all sectors, relative prices are therefore given by

$$\frac{\mathrm{p_i}}{\mathrm{p_j}} = \frac{\mathrm{v_i}}{\mathrm{v_j}} \;\forall\, \mathrm{i, j} \quad (6)$$

This result establishes that, in a system of simple commodity production, relative prices are regulated by relative labour-time requirements. Importantly, the magnitudes $\vec{\mathrm{v}}$ should not be interpreted as labour costs, nor as sums of wage payments, but as expressions of the technical conditions of social reproduction, capturing the total direct and indirect labour time required to reproduce each commodity. The proportionality between natural prices and vertically integrated labour requirements thus reflects the equalisation of net income per unit of labour time enforced by competition at this level of abstraction.

The determination obtained here is necessarily provisional. Once capitalist relations of production are introduced (specifically, wage labour, surplus value, and the competitive mobility of capital) the technical regulation of relative prices must be supplemented by an analysis of distribution and profit-rate equalisation. It is to this more concrete determination of relative prices under capitalist competition that the analysis now turns.

3.2. Capitalist Production, Prices of Production, and the Transformation Problem

The result obtained in the previous section establishes that, in a system of simple commodity production, relative prices are regulated by relative labour-time requirements, as competition among independent producers tends to equalise net income per unit of labour time. Once capitalist relations of production are introduced, however, this determination must be further specified. The separation between workers and the means of production, together with the generalised mobility of capital across sectors, gives rise to new competitive regularities that systematically modify the technical regulation of prices derived at the previous level of abstraction.

Under capitalist production, commodities are produced not merely to recover their technical re-

[2] This methodological use of simple commodity production as an analytical abstraction is standard within classical political economy and should be understood as a logical device rather than as a historical claim. See, among others, Ricardo (1821, chs. 1 & 4-6) and Shaikh (2016, ch. 9).

production costs, but to realise a surplus in monetary form. Competition among capitals therefore does not operate on the equalisation of income per labour hour, but on the equalisation of profit rates across sectors. At the same time, profit-rate equalisation does not eliminate heterogeneity at the firm level: within each sector, firms operating under different technical conditions continue to earn different rates of profit. What is equalised through intersectoral competition is the profit rate of the regulating capitals, that is, those capitals operating under the most reproducible technical conditions. The long-run centres of gravity of market prices under capitalism are therefore prices of production, defined as those prices that allow regulating capitals to recover their costs of production and realise the average rate of profit.

Formally, consider again an economy with N industries, characterised by the technical coefficients matrix $\mathbf{A}$ and the direct labour coefficients row vector $\vec{l}$. Let $\vec{p_\pi}$ denote the row vector of prices of production, w the uniform real wage rate, and π the uniform rate of profit. In a circulating-capital framework, prices of production satisfy the following system:

$$\overrightarrow{p_\pi} = (\overrightarrow{p_\pi}\mathbf{A} + w\vec{l})(1+\pi) \quad (7)$$

Analogously to our simple production argument, this equation states that unit prices equal unit costs marked up by the uniform rate of profit. Rearranging yields:

$$\overrightarrow{p_\pi}(\mathbf{I} - (1+\pi)\mathbf{A}) = w(1+\pi)\vec{l} \quad (8)$$

Provided that the spectral radius of $(1+\pi)\mathbf{A}$ is less than unity, this system admits a unique solution,

$$\overrightarrow{p_\pi} = w(1+\pi)\,\vec{l}(\mathbf{I} - (1+\pi)\mathbf{A})^{-1} \quad (9)$$

Equation (9) makes explicit why the simple-production result derived in Section 3.1 does not carry over unchanged to capitalist economies. When $\pi = 0$, prices of production reduce to the prior $\vec{p} = w\vec{l}\,(\mathbf{I} - \mathbf{A})^{-1} = w\vec{v}$ expression, reproducing the proportionality between prices and vertically integrated labour requirements. For $\pi > 0$, however, prices depend on the modified inverse $(\mathbf{I} - (1+\pi)\,\mathbf{A})^{-1}$, which reweights successive rounds of indirect inputs by powers of $(1+\pi)$. This reweighting reflects the fact that, under capitalist production, intermediate inputs must be replaced at prices that themselves include an average profit, so that surplus value enters the reproduction process at each round of inter-industry replacement.

The same point can be seen by expanding (9) as a Neumann series:

$$\overrightarrow{p_\pi} = w\sum_{k=0}^{\infty}(1+\pi)^{k+1}\,\vec{l}\mathbf{A}^k \quad (10)$$

By contrast, vertically integrated labour requirements are given by the expression:

$$\vec{v} = \sum_{k=0}^{\infty}\vec{l}\mathbf{A}^k \quad (11)$$

The deviation between prices of production and labour values thus arises from the systematic reweighting of indirect labour layers by the profit rate. This deviation is neither accidental nor arbitrary, but a necessary consequence of capitalist competition and profit-rate equalisation.

At this point, it is useful to adopt the vertical decomposition of prices associated with Smith and formalised by Shaikh[3]. Any commodity price can be written as the sum of vertically integrated labour costs and vertically integrated profits,

$$p_i = \mu_i^* + \phi_i^*, \quad (12)$$

where μ_i^* denotes the total direct and indirect wage bill embodied in commodity i, and ϕ_i^* the total direct and indirect profits embodied in it. Under conditions of uniform wages and competitive profit-rate equalisation, $\mu_i^* = w\,v_i$, so that prices of production may be expressed as

$$p_{\pi,i} = w\,v_i(1+\omega_i^*), \quad \omega_i^* \equiv \frac{\phi_i^*}{\mu_i^*} \quad (13)$$

Relative prices of production can therefore be written as

$$\frac{p_{\pi,i}}{p_{\pi,j}} = \frac{v_i}{v_j}\,\gamma_{i,j}, \quad \gamma_{i,j} \equiv \frac{1+\omega_i^*}{1+\omega_j^*} \quad (14)$$

Equation (14) makes explicit the sense in which the labour-time regulation of relative prices derived in Section 3.1 becomes perturbed under capitalist production. Relative prices of production are anchored in relative labour values but are modified by a distributional/technical disturbance factor reflecting differences in vertically integrated profit-wage ratios (ω_i^*) across sectors. Importantly, this formulation also clarifies why such deviations are typically theorised to be moderate. If the dispersion of (ω_i^*) across sectors is limited, which is likely the case given the fact that vertically integrated profit-wage ratios are weighted combinations of all the economy's individual ones (Shaikh 1984, 67-68) , then the disturbance factor $\gamma_{i,j}$ will be close to unity for most pairs of commodities, implying bounded and systematic deviations from labour-value proportionality.

This perspective is reinforced by Shaikh's analysis of Marx's transformation procedure. Marx's original transformation, which applies the average rate of profit to costs expressed in labour-value terms, may be understood as the first step of a logically coherent adjustment process: outputs are revalued at prices of production, while inputs initially remain valued at labour values. Once this inconsistency is recognised, the natural completion consists in feeding the transformed prices back into the valuation of the input bundle and reiterating the equalisation of the profit rate. This successive-approximation procedure, developed explicitly by Shaikh (1977) and anticipated in related form by Bródy (1970) and Mor-

[3] The theoretical foundations of such procedure were initially laid out by Smith (2007, 41-46), and were subsequently expanded upon by Shaikh (1984 and 2016) precisely to discuss, among related things, the conditions that lead prices of production to deviate from direct prices and the empirical question of how large such deviations may be on the existing input-output data.

ishima (1973), generates a sequence of price vectors converging to a fixed point corresponding to the fully transformed system of prices of production described in Equation (9).

From this standpoint, the divergence between labour values and prices of production is not arbitrary but reflects the cumulative effect of applying profit-rate equalisation to the valuation of intermediate inputs at each round of reproduction. While the existence and uniqueness of the limiting price vector follow from the properties of the underlying reproduction system, the empirical magnitude of the resulting deviations depends on the dispersion of capital intensities across sectors. When such dispersion is limited, the successive corrections remain moderate, providing a theoretical basis for the near-linearity of price paths and the relatively small price–value deviations observed in practice.

The iterative completion of Marx's transformation procedure thus resolves the internal inconsistency of the original transformation and yields a uniquely defined system of prices of production. By successively feeding transformed output prices back into the valuation of the input bundle and reapplying profit-rate equalisation, the procedure converges to the fully transformed price system described in Equation (9). However, this resolution proceeds by taking classical labour values as the underlying cost magnitudes and examining how capitalist competition systematically modifies them through the equalisation of profit rates. The apparent inconsistency traditionally associated with the transformation problem therefore arises from comparing two magnitudes defined at different levels of abstraction: labour values capture the technical conditions of reproduction independently of distribution, while prices of production reflect the social conditions of reproduction under capitalist competition. Once this distinction is made explicit, the problem ceases to be one of logical inconsistency and becomes instead a question of how labour-time accounting should be extended to remain consistent with capitalist relations of production and distribution.

Recent work by Ian Wright (2014, 2017) provides a compelling resolution to this issue. Wright observes that classical labour values integrate labour costs over the technical reproduction of means of production, but abstract from the labour required to reproduce capitalist consumption. In a steady-state capitalist economy, however, profit income is systematically spent on consumption goods, whose production itself requires labour. Wright therefore defines an augmented technique that incorporates capitalist consumption into the reproduction process and introduces the corresponding super-integrated labour coefficients,

$$\tilde{\vec{\mathrm{v}}} \equiv \vec{\mathrm{l}}(\mathbf{I} - \mathbf{A} - \mathbf{C})^{-1}, \quad (15)$$

where $\mathbf{C}$ denotes the matrix of capitalist consumption coefficients. Solving the production-price system under this augmented technique yields:

$$\overrightarrow{\mathrm{p}_{\pi}} = \mathrm{w}\,\tilde{\vec{\mathrm{v}}} \quad (16)$$

Equation (16) restores a direct proportionality between labour-time magnitudes and prices of production, once labour costs are defined at the appropriate level of social reproduction. From this perspective, the transformation problem does not reflect a failure of the labour theory of value, but rather the use of an incomplete measure of labour cost under capitalist conditions. When labour-time accounting is extended to include the reproduction of capitalist consumption, prices of production and labour-time costs are once again dual representations of the same underlying social process.

4. Measurement and Cross-Sectional Evidence

The analysis developed in Section 3 establishes the theoretical structure of the classical theory of relative prices and clarifies the mechanisms through which labour-time magnitudes regulate market prices under both simple and capitalist production. Crucially, the relevant theoretical categories (labour values, direct prices, and prices of production) are defined in objective and operational terms. This makes it possible to move beyond purely conceptual analysis and to confront the theory with empirical evidence.

Input-output tables provide the appropriate empirical basis for such an exercise. By recording, in a consistent and economy-wide framework, the technical relations between industries, the use of intermediate inputs, and the direct labour requirements of production, they allow for the construction of vertically integrated labour coefficients and for the estimation of the price systems implied by the classical theory. Once these price vectors have been constructed, their empirical relevance can be assessed by examining their proximity to observed market prices.

This section operationalises the theoretical categories introduced in Section 3 and evaluates their empirical performance using Spanish input-output data for the years 2010, 2015, and 2016. Labour values, direct prices, and prices of production are defined in matrix terms, and the distances between the corresponding price vectors and observed market prices are measured using a set of scale-invariant and size-sensitive metrics. The aim is not to estimate behavioural parameters or to fit reduced-form relationships, but to test the central cross-sectional implication of the classical theory: that market prices lie at relatively short distances from their theoretically determined centres of gravity.

4.1 Labour Values, Direct Prices and Prices of Production

While the formal definition of labour values was introduced at an abstract level in Section 3.1, the present subsection specifies how these magnitudes are operationalised using empirical input-output data. In particular, the labour coefficients employed here correspond to observed, homogenised labour inputs, constructed from sectoral employment and hours-worked data as described in Appendix A. The technical coefficients matrix likewise reflects observed inter-industry input requirements. The purpose of restating the labour-value equations is therefore not to introduce new theoretical content, but to make explicit the procedure by which the relevant price vectors are computed from data.

Let $\vec{l} \equiv (1 \times N)$ empirical homogenized direct labour coefficients row vector, $\vec{v} \equiv (1 \times N)$ empirical vertically integrated labour time requirements row vector, $\mathbf{A} \equiv (N \times N)$ empirical technical coefficients matrix, and $\mathbf{I} \equiv (N \times N)$ identity matrix, for an economy with N industries. Given the definition of labour values as the sum of the direct and indirect labour times incurred in the production of a certain commodity, these can be formally expressed as in Equation (5):

$$\vec{v} = \vec{l} + (\vec{v}\mathbf{A}) \leftrightarrow \vec{v} = \vec{l}(\mathbf{I} - \mathbf{A})^{-1} \quad (17)$$

To operationalise direct prices, we first establish the condition that the sum of the outputs produced by all N industries valued at direct prices be equal to the sum of these outputs valued at market prices. This condition defines the money unit that characterizes these prices. If we let $\vec{p_\delta} \equiv (1 \times N)$ row vector of direct prices, $\vec{Q} \equiv (N \times 1)$ sectoral outputs column vector, and $\vec{m} \equiv (1 \times N)$ row vector of market prices, then such condition is given by:

$$\overrightarrow{p_\delta}\vec{Q} = \vec{m}\vec{Q} \quad (18)$$

Now, for direct prices to be directly proportional to labour values, there must exist a proportionality constant (β) such that $\vec{p_\delta} \equiv \beta\vec{v}$. Given the condition expressed in equation (12) we can determine this proportionality constant in the following manner:

$$\overrightarrow{p_\delta}\vec{Q} = \vec{m}\vec{Q} \rightarrow (\beta\vec{v})\vec{Q} = \vec{m}\vec{Q} \rightarrow \beta = \frac{\vec{m}\vec{Q}}{\vec{v}\vec{Q}} \quad (19)$$

where the division between $\vec{m}\vec{Q}$ and $\vec{p_\delta}\vec{Q}$ results in a scalar because both cross products give (1×1) matrices. Therefore, direct prices can be expressed as:

$$\overrightarrow{p_\delta} = \beta\vec{v} = \frac{\vec{m}\vec{Q}}{\vec{v}\vec{Q}}\vec{v} \quad (20)$$

As far as prices of production are concerned, in Section 3.2 they were defined as the centres of gravity around which market prices oscillate, reflecting homogeneous profit and wage rates for the entire economy, and being determined primarily by the direct and indirect labour cost of producing one unit of each industry's output. If we do not abstract from turnover times and include fixed capital into the total capital advanced, then the classical/Marxian system of prices of production is given by:

$$\overrightarrow{p_\pi} = w\vec{l} + \overrightarrow{p_\pi}(\mathbf{A} + \mathbf{D}) + \pi\overrightarrow{p_\pi}[\mathbf{K} + \left(\mathbf{A} + (\vec{b}\vec{l})\right)] < \mathbf{t} >, \quad (21)$$

where we let $\vec{p_\pi} \equiv (1 \times N)$ row vector of unnormalized prices of production, $\pi \equiv$ homogeneous profit rate, $\vec{b} \equiv (N \times 1)$ column vector of the basket of goods consumed by workers with their real wage, $< \mathbf{t} > \equiv (N \times N)$ diagonal matrix of turnover times, $\mathbf{K} \equiv (N \times N)$ fixed capital coefficients matrix, and $\mathbf{D} \equiv (N \times N)$ depreciation coefficients matrix.

However, due to data limitations on fixed capital and depreciation coefficients, the model employed in this study is a pure circulating capital model in which we abstract from such factors (including turnover times) by setting **D = 0, K =0** and $< \mathbf{t} > = \mathbf{I}$. The pure circulating capital model, while not being the most complete representation of prices of production, is nevertheless still highly useful and indicative of the potential results that a fixed capital model may yield. The classical/Marxian pure circulating capital price system is then operationalised by the expression:

$$\overrightarrow{p_\pi} = w\vec{l} + \overrightarrow{p_\pi}\mathbf{A} + \pi\overrightarrow{p_\pi}\left(\mathbf{A} + (\vec{b}\vec{l})\right) \quad (22)$$

We can now rearrange this equation to calculate prices of production by first noting that the uniform wage rate is equivalent to the cross product between the vector of prices of production and the vector of the goods basket representing workers' consumption, that is, $w = \vec{p_\pi}\vec{b}$. If we also let , $\mathbf{A}^*=[\mathbf{A}+(\vec{b}\vec{l})]$, and $\mathbf{H} = \mathbf{A}^* (\mathbf{I} - \mathbf{A}^*)^{-1}$, then Equation (22) gives:

$$\overrightarrow{p_\pi} = (\overrightarrow{p_\pi}\mathbf{A}^*) + (\pi\overrightarrow{p_\pi}\mathbf{A}^*) \rightarrow \frac{1}{\pi}\overrightarrow{p_\pi} = \overrightarrow{p_\pi}\mathbf{H} \quad (23)$$

The informed reader will recognize in equation (17) the expression of an eigenvalue problem for the matrix $\mathbf{H}$, a real square matrix containing only positive entries, which, by application of the Perron-Frobenius theorem, gives us the conclusion that the economically meaningful solution for $\vec{p_\pi}$ is the left eigenvector associated with the highest eigenvalue obtained from solving such problem. The dominant eigenvalue of $\mathbf{H}$ then gives us a number (x), which can be easily used to calculate the homogeneous rate of profit as shown below:

$$\frac{1}{\pi} = x \rightarrow \pi = \frac{1}{x} \quad (24)$$

Once $\vec{p_\pi}$ has been obtained from the eigenvalue problem in equation (17), it is normalised according to a condition analogous to that in equation (12). The normalisation condition for prices of production establishes that the sum of sectoral outputs valued at prices of production be equal to such sum valued at market prices, that is:

$$\overrightarrow{p_\pi}\vec{Q} = \vec{m}\vec{Q} \quad (25)$$

If we let $\vec{p_\pi^*} \equiv (1 \times N)$ row vector of normalized prices of production, then we can derive a normalization constant (λ) such that $\vec{p_\pi^*} \equiv \lambda\vec{p_\pi^*}$. Following the same reasoning employed to determine the constant of proportionality for direct prices, λ is given by:

$$\overrightarrow{p_\pi}\vec{Q} = \vec{m}\vec{Q} \rightarrow \left(\lambda\overrightarrow{p_\pi^*}\right)\vec{Q} = \vec{m}\vec{Q} \rightarrow \lambda = \frac{\vec{m}\vec{Q}}{\overrightarrow{p_\pi}\vec{Q}} \quad (26)$$

The vector of normalized prices of production can thus be obtained from:

$$\overrightarrow{p_\pi^*} = \lambda\overrightarrow{p_\pi} = \frac{\vec{m}\vec{Q}}{\overrightarrow{p_\pi}\vec{Q}}\overrightarrow{p_\pi} \quad (27)$$

Once these price vectors have been calculated with the use of input-output data, the turn can be made to measure their degree of proximity as measured by vector distance formulas. It should be noted that it is preferable for the purpose of analysing individual price-value deviations to determine the tightness of their relation according to measures of vector distance rather than cross-sectional linear regressions, since, from the very nature of the construction of input-output tables, the vector of unit market prices features no variation among its elements, thereby making a regression between unit market prices and unit labour values[4] meaningless.

This paper focuses on two measures of vector distance: a weighted measure[5], the mean absolute weighted deviation (MAWD), and a non-weighted one, the normalised vector distance (NVD), both of which give percentages reflecting the degree of proximity of the price vectors under scrutiny. The MAWD can be seen as having an analytical advantage over the NVD pertaining its controlling for each industry's weight on the entire economy, which has the effect of reducing the potentially distortionary effects that small industries with large deviations (and vice-versa) may have on the total sum of intersectoral deviations. If we take the market price and direct price vectors for illustration purposes, these two measures can be calculated from the following formulas:

$$\%\mathrm{MAWD} = \sum_{i=1}^{n} \left| \frac{m_i - p_{\delta,i}}{p_{\delta,i}} \right| \frac{Q_i}{\sum_{i=1}^{n} Q_i} \quad (28)$$

$$\%\mathrm{NVD} = \frac{\sqrt{\sum_{i=1}^{n} \left[(m_i Q_i) - (p_{\delta_i} Q_i)\right]^2}}{\sqrt{\sum_{i=1}^{n} (p_{\delta,i} Q_i)^2}} \quad (29)$$

Having formalized the pertinent price vectors and established the methodology by which the distance existing between them is measured, we can now turn to the empirical evidence.

4.2. Cross-Sectional Evidence

The empirical results presented in this subsection are obtained from the Spanish economy and cover sixty-three industries using input-output tables for the years 2010, 2015, and 2016. The analysis proceeds by measuring, in turn, the cross-sectional distances between market prices and direct prices, between market prices and prices of production, and between direct prices and prices of production, using the MAWD and NVD distance measures introduced above.

As shown in Table 1, direct prices are found to be remarkably close to observed market prices across all three years. The average deviation over the period amounts to 18.6% according to the weighted measure (MAWD) and 23.1% according to the non-weighted measure (NVD). These magnitudes are noteworthy given the multitude of short-run and sector-specific factors that may affect market prices in actual economies and potentially drive them away from underlying production costs.

Table 1.Market Prices-Direct Prices Deviations

Year	%MAWD	%NVD
2010	16.4	20.1
2015	17.6	22.2
2016	21.7	27.1
Average	**18.6**	**23.1**
Spain 2000 (Benchmark)	**11.0**	**13.2**

Notes: Results for 2010-2016 are computed from Spanish input-output data in this paper. The row "Spain 2000" reports benchmark results from Sánchez and Nieto (2010).

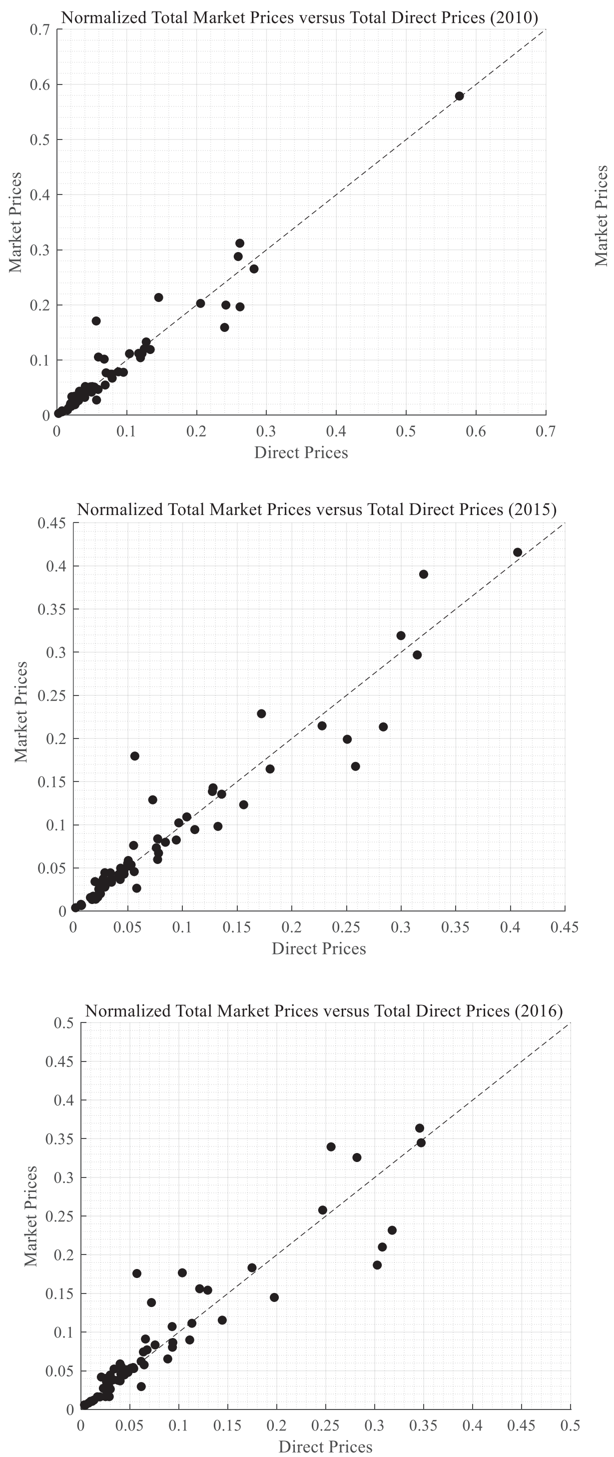

Figure 1. Market Prices vs Direct Prices Graphical Deviations

[4] Or unit direct prices and/or unit prices of production for that matter.

[5] Where the weights are the share of each industry's output on the total sum of inter-industrial outputs.

Figure 1 provides a graphical representation of these deviations by plotting, for each year, the normalised vectors of total market prices against the corresponding normalised vectors of total direct prices. Normalisation sets the length of each price vector equal to one, allowing for a direct comparison of their relative structures. The dotted 45-degree line indicates points at which market prices and direct prices would coincide. The tight clustering of observations around this line visually confirms the relatively small distances reported in Table 1.

The same methodology can be employed to assess the proximity of market prices to prices of production. Table 2 reports the distances between market prices and prices of production evaluated at the observed rate of profit. The average MAWD over the period is 20.2%, while the corresponding NVD is 22.9%. These figures indicate that prices of production also lie at a relatively short distance from market prices, consistent with their interpretation as long-run centres of gravity under capitalist competition.

Table 2. Market Prices-Prices of Production Deviations at the Observed Rate of Profit

Year	%MAWD	%NVD
2010	21.2	22.6
2015	20.8	23.7
2016	18.6	22.6
Average (2010-2016)	**20.2**	**22.9**
Spain 2000 (Benchmark)	**18.9**	**20.6**

Notes: Results for 2010-2016 are computed from Spanish input-output data in this paper. The row “Spain 2000” reports benchmark results from Sánchez and Nieto (2010).

Figure 2 depicts the relationship between market prices and prices of production in normalised form. As in Figure 1, the closeness of observations to the 45-degree line provides visual confirmation of the tight relationship between the two price vectors. Despite the presence of persistent disequilibrating forces in real economies, the competitive mechanisms described in Section 3 appear to exert sufficient discipline to keep market prices within a limited range of their theoretical centres of gravity. These results provide empirical support not only for the classical theory of relative prices, but also for the notion of competition upon which it is built.

The analysis can be extended by measuring the deviations between direct prices and prices of production, which were also theorised in Section 3 to be moderate in magnitude. As reported in Table 3, the average distance between these two price vectors amounts to 23.8% according to the MAWD and 27.8% according to the NVD.

Figure 3 represents these deviations graphically and, in line with the previous figures, shows that direct prices and prices of production remain closely aligned across all three years. This empirical proximity is consistent with the theoretical result that prices of production represent a systematic, though perturbed, transformation of labour-value-based prices under capitalist competition.

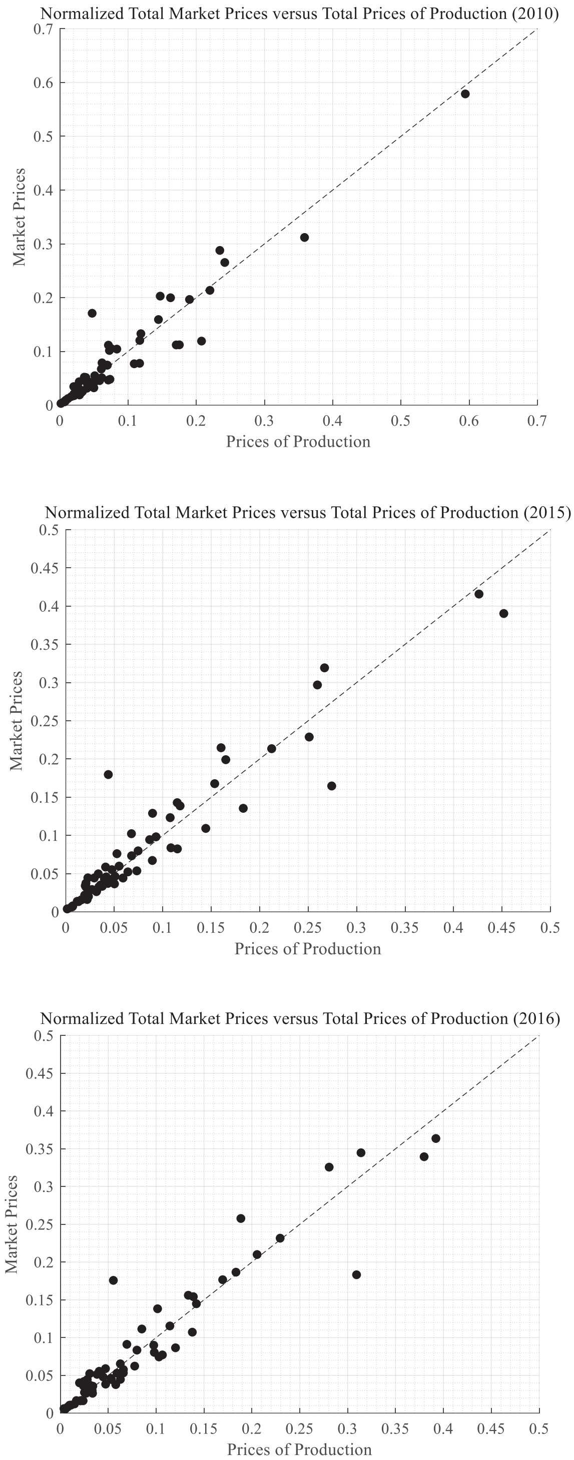

Figure 2. Market Prices vs Prices of Production Graphical Deviations

Table 3. Direct Prices-Prices of Production Deviations at the Observed Rate of Profit

Year	%MAWD	%NVD
2010	22.9	24.7
2015	23.9	27.9
2016	24.7	30.8
Average	**23.8**	**27.8**
Spain 2000 (Benchmark)	**19.0**	**20.5**

Notes: Results for 2010-2016 are computed from Spanish input-output data in this paper. The row “Spain 2000” reports benchmark results from Sánchez and Nieto (2010).

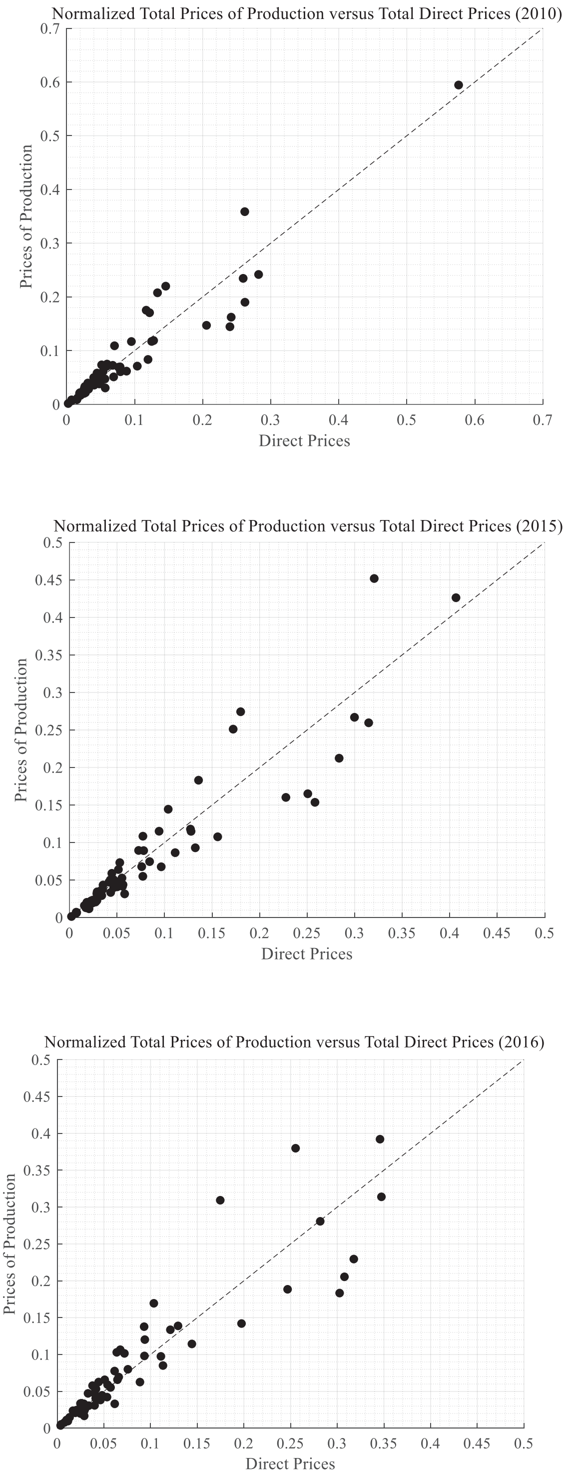

Figure 3. Prices of Production vs Direct Prices Graphical Deviations

The magnitudes of the deviations reported in Tables 1-3 are fully consistent with those obtained in previous empirical studies of the classical theory of relative prices. In particular, the Spanish benchmark results reported by Sánchez and Nieto (2010) for the year 2000, reproduced in the final rows of each table, are of comparable magnitude to those obtained here for the period 2010-2016, despite differences in data vintage, sectoral aggregation, and methodological implementation. This intertemporal stability suggests that the observed proximity between market prices, direct prices, and prices of production is not a transitory feature of a specific dataset, but a robust empirical regularity of the Spanish economy.

More broadly, the distances obtained in this study fall well within the range documented in the international literature. Using Italian and U.S. input-output data, Shaikh (1984) reports mean absolute deviations between market prices and both direct prices and prices of production typically lying between 17% and 25%. Employing weighted and normalised distance measures, Ochoa (1989) finds similarly moderate deviations for the U.S. economy when fixed capital is considered. The Spanish results presented here are therefore quantitatively in line with those reported for other advanced capitalist economies, reinforcing the conclusion that the empirical strength of the classical theory of relative prices is not country-specific but reflects a general feature of competitive market economies.

A potential concern arising from these results is that the deviations between market prices and direct prices appear to be slightly smaller than those between direct prices and prices of production. Given the two-stage determination of market prices outlined in Section 3 (whereby direct prices regulate prices of production, which in turn serve as the centres of gravity of market prices) it might be expected that market prices would lie closer to prices of production than to direct prices. This intuition can be reinforced by noting that, for each industry *i*, the difference between its market price and its direct price can be decomposed as:

$$m_i - p_{\delta,i} \equiv (p_{\pi,i} - p_{\delta,i}) + (m_i - p_{\pi,i}), \quad \forall i \in N \tag{30}$$

However, as shown by Shaikh (2016) through a numerical example, this algebraic relationship does not carry over straightforwardly to vector distance measures. It is formally possible for the distance between market prices and direct prices to be smaller than that between market prices and prices of production, while still satisfying Equation (30) at the level of individual industries. The present results therefore do not contradict the two-stage theory of price determination developed in Section 3 but rather reflect the properties of the distance metrics employed.

Finally, before turning to the dynamic analysis in Section 5, it is worth briefly considering the implications of the simplifying assumptions adopted here. In particular, the present results are obtained within a pure circulating-capital framework. As noted by Shaikh (1984), abstracting from fixed capital, depreciation, and turnover times tends to overestimate the general rate of profit. Since deviations between prices of production and labour values decrease as the rate of profit falls (a point examined in more detail in the next section) this abstraction is likely to overstate, rather than understate, the degree of proximity between prices of production and labour values. Incorporating fixed capital would therefore be expected to yield deviations that are at least as small as those reported here, thereby strengthening the empirical conclusions.

5. Evidence on the Linearity of Prices of Production as Functions of the Rate of Profit

The conceptual discussion of the classical theory of relative prices has thus far been focused on the competitive forces that bring about the regulation of market prices by production costs, as well as the factors that complicate their relationship. There is nevertheless a further question to be addressed in relation to the variations within the classical/Marxian system of prices of production as changes are made to the distribution between wages and profits.

As it turns out, by allowing the profit rate to move along a given path, we bring about changes in the deviations between prices of production and labour values, which are of great importance to the components of the classical/Marxian argument regarding the issue of distribution and the surplus-value transfers between different parts of an economy. In relation to this question, Sraffa (1960, 13-19) makes the case for the potential complex behaviour of relative prices of production as the profit rate changes, which, if real, would directly lead to the proposition that individual price paths are non-linear, exhibiting curvature and direction changes instead (these manifest in the form of twisted price curves at a graphical level). Since prices of production are the theorised centres of gravity of market prices, it follows that their potentially anarchic behaviour leaves the door open to indeterminacies regarding the relationship of prices to labour values, and ultimately to the existence of technique reswitching and capital reversal.

Originally, phenomena of this nature were employed by Sraffian economists as part of a broader critique of neoclassical economics. However, they soon also turned to applying such critical insights into the classical approach to the problems of value and distribution. The main concern that twisted price curves gave rise to was the fact that the complex behaviour of individual prices in the face of distribution changes may lead to the reversal of the ranking of production techniques in price terms, thereby making technique ranking in terms of labour values virtually irrelevant for producers' decision-making, which would in turn make any attempt at analysing broader economic patterns on the basis of value (and surplus-value for that matter) an unproductive enterprise at best.

For the classical theory of relative prices to be a solid analytical ground from which to build a rigorous understanding of political economy, it is thus of especial importance to study the empirical patterns pertaining the behaviour of individual price curves and the direction of price-value deviations along a determined path for the profit rate. We can test this by employing the linear approximation of the classical/Marxian price system developed by Shaikh (1998, 231-232)[6]. If the model of prices of production contained in equation (16) is taken as a starting point, then it can be rearranged it into the following form, where $\mathbf{F} = \mathbf{A}(\mathbf{I} - \mathbf{A})^{-1}$:

$$\overrightarrow{p_\pi} = w\vec{v} + \pi w\vec{v} + \pi\overrightarrow{p_\pi}\mathbf{F} = [w(1+\pi)\vec{v} + \pi\vec{v}\mathbf{F}] + \pi(\overrightarrow{p_\pi} - \vec{v})\mathbf{F} \quad (31)$$

This expression consists of two main terms. The first one ($[w(1+\pi)\vec{v} + \pi\vec{v}\mathbf{F}]$), which Shaikh calls the Marx component, represents the vertically integrated version of the transformation procedure put forth by Marx. The second one ($\pi(\overrightarrow{p_\pi} - \vec{v})\mathbf{F}$), referred to by Shaikh as the Wicksell-Sraffa term, illustrates the effects on the stocks of capital of deviations between prices of production and labour values.

This linear approximation, assuming that the Wicksell-Sraffa term is mainly unimportant[7], requires the setting of the condition that $\vec{v} = \overrightarrow{p_\pi}$ in the portion of the equation containing such term, thereby leaving us with the expression:

$$\overrightarrow{p_\pi} = w(1+\pi)\vec{v} + \pi\vec{v}\mathbf{F} \quad (32)$$

Now, by allowing the profit rate to vary from zero to its maximum value (π_{max})[8], we can calculate the price paths for all industries to test whether they feature pronounced curvature. But before making any such calculation, we would need to specify how the wage behaves as well, since the wage rate is itself also a function of the profit rate, going from $w = w_{max} = 1$ when $\pi = 0$, to w = 0 at the point where $\pi = \pi_{max}$. In the case of the classical/Marxian pure circulating capital model, the standard wage-profit curve it entails is non-linear, and takes the form:

$$w = \left[1 - \frac{\pi}{\pi_{max}}\right](1+\pi)^{-1} \quad (33)$$

Furthermore, the approximation of prices of production in Equation (32) also leads us to an approximation of the output-to-capital ratios that characterize this system. If we establish that $\vec{K} \equiv (1 \times N)$ row vector of total capital advanced in the approximated pure circulating capital model, then it is given by $\vec{K} \equiv w\vec{v} + \vec{v}\,\mathbf{F}$. The $(1 \times N)$) row vector of sectoral output-capital ratios as functions of the rate of profit $(\overrightarrow{QK(\pi)})$ would in turn be:

$$\overrightarrow{QK(\pi)} = \overrightarrow{p_\pi} \,./\, \vec{K} = [w(1+\pi)\vec{v} + \pi\vec{v}\mathbf{F}] \,./\, [w\vec{v} + \vec{v}\mathbf{F}] \quad (34)$$

For the i[th] industry at any given wage and profit rates, their output-capital ratio ($QK(\pi)_i$) would correspond to the i[th] entry of such vector. It should be noted that including the paths of output-capital ratios into the analysis is pertinent because their behaviour is indicative of the shape that individual price paths may exhibit.

Lastly, our attention is drawn to the formalization and calculation of actual aggregate wage shares, whose paths also prove relevant to the issues pertaining the Sraffian critique of the classical/Marxian theory of value and distribution. For any given input-output year of a certain economy, we can start formalizing the aggregate wage share from the expression of prices of production given by the first equality of Equation (31). This expression can be then lightly rearranged and multiplied by the vector of net sectoral outputs ($\vec{Y}$) to obtain:

[6] The difference between the approximation presented in this paper and that in Shaikh (1998) is that Shaikh approximates the fixed capital model, whereas we do so for the pure circulating capital model.

[7] Assumption that Shaikh (1998) justifies empirically.

[8] π_{max} is obtained from setting the wage rate equal to zero (all value added goes to profits) in Equation (22), substituting π for π_{max} and calculating it as the dominant eigenvalue of the subsequent eigenvalue problem.

$$\overrightarrow{p_\pi}\vec{Y} = [w(1+\pi)\vec{v}]\vec{Y} + (\pi\overrightarrow{p_\pi}\mathbf{F})\vec{Y} \quad (35)$$

Since net sectoral outputs are defined as $\vec{Y} = (\mathbf{I}-\mathbf{A})\vec{Q}$, aggregate employment is given by $L = \vec{l}\vec{Q}$, and we know from earlier that $\vec{v} = \vec{l}\,(\mathbf{I}-\mathbf{A})^{-1}$, we can express the actual wage share as a function of the profit rate ($W^*(\pi) \equiv \frac{wL}{\overrightarrow{p_\pi}\vec{Y}}$) for the system of prices of production according to the procedure below:

$$\overrightarrow{P_\pi}\vec{Y} = wL(1+\pi) + (\pi\overrightarrow{p_\pi}\mathbf{F})\vec{Y}$$

$$\rightarrow 1 = \frac{wL}{\overrightarrow{p_\pi}\vec{Y}}(1+\pi) + \frac{(\pi\overrightarrow{p_\pi}\mathbf{F})\vec{Y}}{\overrightarrow{p_\pi}\vec{Y}} \quad (36)$$

$$\rightarrow \frac{wL}{\overrightarrow{p_\pi}\vec{Y}} = \left[1 - \frac{\pi}{\left(\frac{\overrightarrow{p_\pi}\vec{Y}}{(\overrightarrow{p_\pi}\mathbf{F})\vec{Y}}\right)}\right](1+\pi)^{-1}$$

With these expressions in mind, we can proceed to analysing the empirical shape of output-capital ratios, individual price paths and the actual wage share characterizing the Spanish economy. The input-output year of 2015 is taken for illustration purposes, although similar patterns obtain for the other years.

Firstly, we analyse the shape of the sectoral output-capital ratios as functions of the rate of profit in its movement from 0 to π_{max}. As shown in Figure 4, all paths converge to the endpoint (1,1) when $\pi = \pi_{max}$, since at such point the output-capital ratio for any industry becomes $QK(\pi)_i = \pi_{max} \cdot \frac{K_i}{K_i}$, which, if divided by π_{max}, yields a ratio equal to 1. The paths reported are, for the most part, either linear or near-linear, although there are certain industries that exhibit more pronounced curvature. Nevertheless, even the more curvilinear paths are smooth and do not feature any complexities beyond such curvature. The main implication of linear and near-linear output-capital ratios is the enhanced likelihood of finding linear and near-linear price paths, but the existence of certain industries featuring more curvature may give rise to concerns about their behaviour being transferred to such industries' price paths.

As far as this last question is concerned, however, Figure 5 shows that even in the case of the industries featuring more pronounced curvature in the paths of their output-capital ratios, the shape of their price paths is either linear or almost exactly linear. The plots contained in this figure represent the ratios of sectoral prices of production to labour values as functions of the rate of profit ($pv(\pi)_i$). From these ratios we can, on the one hand, observe the paths of individual prices, and, on the other, make sense of the behaviour of individual price-value deviations in the face of larger (or smaller) profit rates.

If complex price behaviour were the norm rather than an anomaly, we would expect to see pronounced curvature along the paths of these ratios, as well as shifts in the direction of individual price-value deviations. These shifts, which Shaikh (1998) calls "Marx-reswitching" instances, would take the form of paths that start either above (or below) 1, and then fall below (or rise above) such level. However, Figure 5 reports no instances of Marx-reswitching, containing instead price paths that are almost exactly linear and show no wiggles even at profit rates approximating the limit $\pi = \pi_{max}$. We can also observe that all paths start at the point (1,1), which is owed to the fact that, when $\pi = 0$ and $w = w_{max}$, prices of production are exactly equal to labour values. Furthermore, although price-value deviations increase steadily at higher profit rates, the graphs show that such increments are moderate.

Finally, with the preceding results in mind, we could naturally conclude that the expected shape for the actual aggregate wage share as a function of the rate of profit will approximate linearity, since the underlying factors that compose it are themselves characterized by their yielding almost exactly linear paths. Figure 6 presents the standard wage-profit curve ($W(\pi)$) given in Equation (33) for the 2015 π to π_{max} range, alongside such input-output years' actual wage-profit curve ($W^*(\pi)$), calculated from the expression developed in Equation (36). As the graph shows, both wage-profit curves start at the point (0,1), where $w = w_{max} = 1$ and $\pi = 0$. Both curves also end at the same point (1,0), where $w = 0$ and $\pi = \pi_{max}$ (thereby making their ratio equal to 1). An analysis of these extremes is not, however, the most relevant part of such wage-profit curves[9], since no economy is ever at either one of them. The result for the standard wage-profit curve corresponds to the fact that the wage-profit relationship implicit in the classical/Marxian system is non-linear, although only mildly curved. Regarding the actual wage-profit curve, we can appreciate from the graph the near-linearity it exhibits, product of the behaviour of individual price paths recorded in Figure 5.

This result, together with those that preceded it, presents further evidence in support of the robustness of the classical theory of relative prices, and provides additional grounds on which to argue against Sraffian critiques centred around the potential for complex price behaviour, technique reswitching and capital reversal.

---

[9] The same was true for the output-capital ratios and standard price-value ratios.

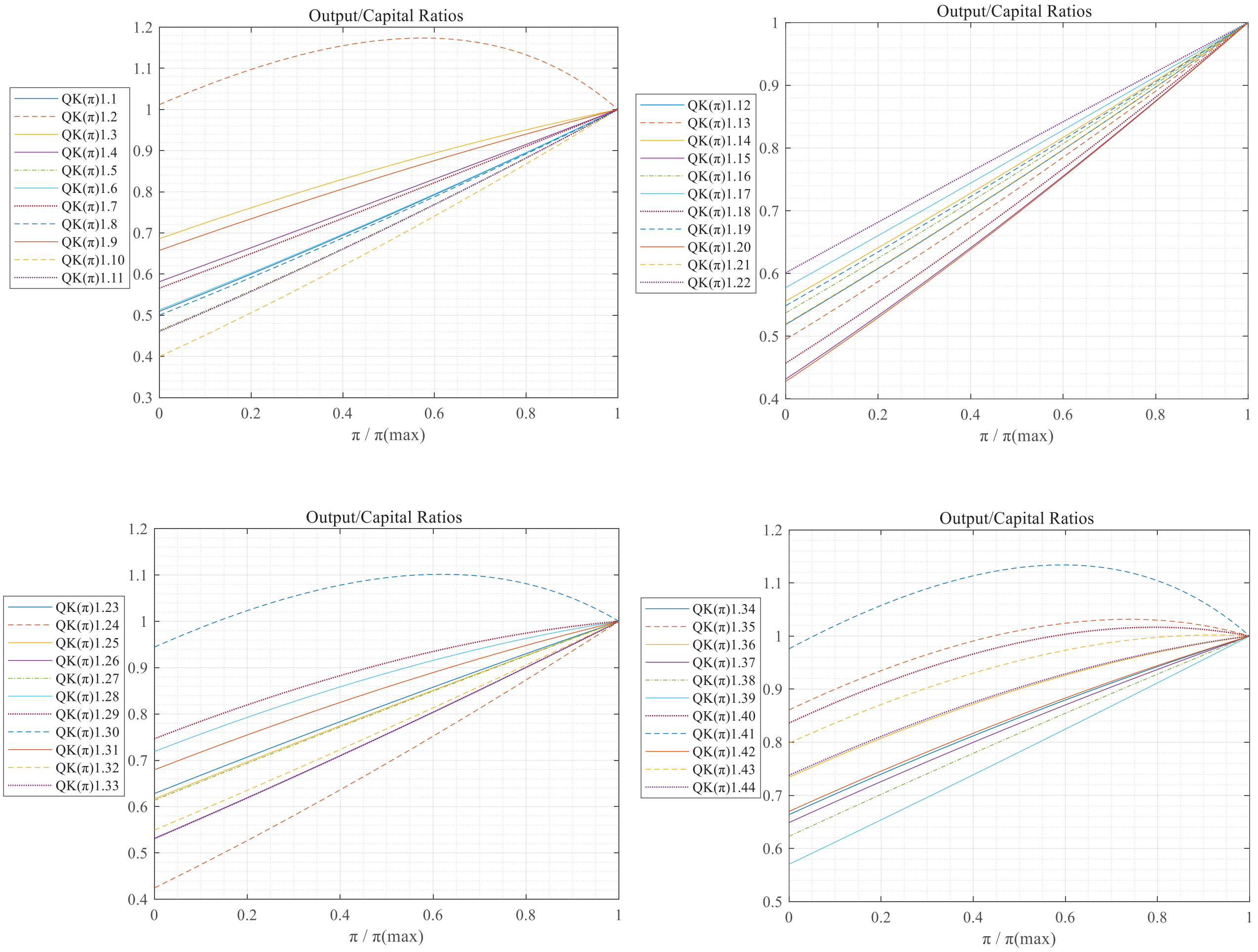

Figure 4. Vertically Integrated Output-Capital Ratios (2015)

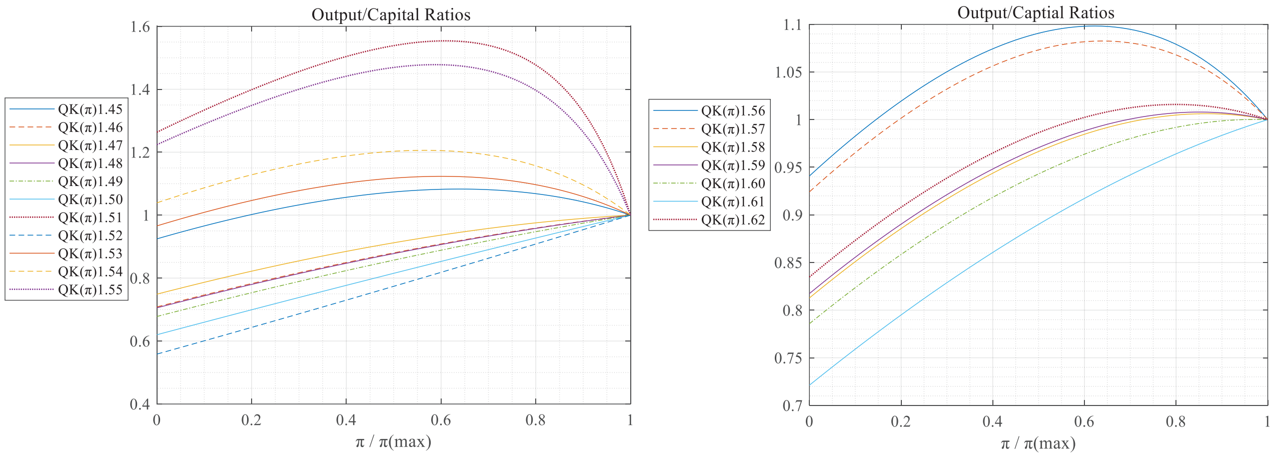

Figure 5. Standard Price-Labour Values Ratios (2015)

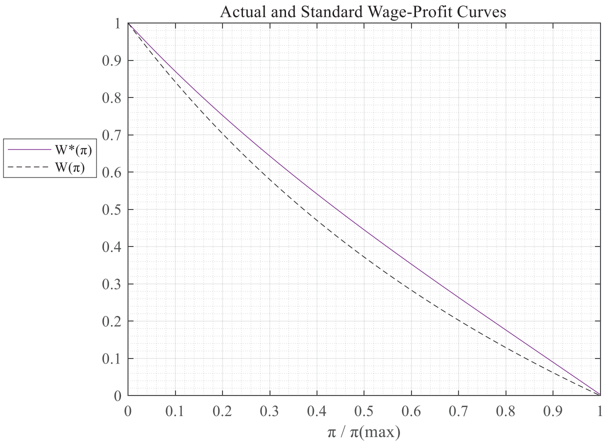

Figure 6. Actual and Standard Wage-Profit Curves (2015)

6. Conclusion

This paper has examined the classical theory of relative prices and its empirical strength in the Spanish economy by combining a rigorous theoretical reconstruction with a systematic analysis of input-output data. The theoretical discussion proceeded from first principles, clarifying the mechanisms through which competitive forces regulate market prices and identifying the objective magnitudes that act as their long-run centres of gravity.

At the most abstract level, the analysis showed that in a system of simple commodity production characterised by reproducible techniques and competitive mobility, the tendential equalisation of income per unit of labour time leads market prices to gravitate around magnitudes proportional to vertically integrated labour requirements. These labour-time magnitudes provide a technical anchor for relative prices and establish the core intuition of the classical theory of value.

Once capitalist relations of production are introduced, this initial result requires further specification. The separation between workers and the means of production, together with intersectoral competition and profit-rate equalisation, implies that prices of production no longer stand in a relation of direct proportionality to labour values. Instead, relative prices of production incorporate a systematic disturbance factor reflecting differences in vertically integrated profit-wage ratios across sectors. The paper showed, however, that this modification does not undermine the labour-time regulation of prices. On the contrary, by analysing the structure of vertically integrated profit-wage ratios, it was argued that their dispersion is likely to be limited, implying that deviations between labour values and prices of production are bounded and law-governed rather than arbitrary.

The discussion of Marx's transformation problem further reinforced this conclusion. By interpreting Marx's original transformation as the first step of a logically coherent adjustment process and by appealing to its iterative completion, the paper clarified that the transformation problem does not reflect a logical inconsistency, but rather the comparison of magnitudes defined at different levels of abstraction. In this context, recent work by Ian Wright was argued to provide a compelling resolution by extending labour-time accounting to encompass the reproduction of capitalist consumption. Once labour costs are defined at this broader level of social reproduction, prices of production and labour-time magnitudes are once again shown to be directly proportional.

These theoretical results were confronted with empirical evidence from Spanish input-output tables for the years 2010, 2015, and 2016. Using both a weighted (MAWD) and a non-weighted (NVD) measure of vector distance, the paper showed that market prices lie at relatively short distances from both direct prices and prices of production, and that the latter two price vectors are themselves closely aligned. The magnitudes obtained are stable across years and comparable to those reported in previous studies, including earlier evidence for Spain. Taken together, these findings provide strong empirical support for the classical theory of relative prices and for the conception of competition upon which it is based.

Finally, the paper examined claims concerning the potential complexity of price behaviour within the Sraffian framework, particularly those related to technique reswitching, capital reversal, and highly nonlinear price paths. The empirical evidence presented in Section 5 (covering output-capital ratios, individual price paths, and the observed wage share) suggests that such phenomena, while formally possible, are of limited empirical relevance in actual economies. The near-linearity of price curves and the stability of price-value rankings observed in the data further reinforce the conclusion that the classical-Marxian account of relative price formation provides a coherent and empirically grounded framework for the analysis of capitalist economies.

7. References

[1] Bienenfeld, M. 1988. "Regularities in Price Changes as an Effect of Changes in Distribution." *Cambridge Journal of Economics* 12 2: 247-255.

[2] Bródy, A. 1970. *Proportions, Prices and Planning: A Mathematical Restatement of the Labor Theory of Value*. Akademiai Kiadó.

[3] Cantillon, R. 1755. *Essay on the Nature of Trade in General*. Liberty Classics.

[4] Cockshott, P., A. Cottrell, and G. Michaelson. 1995. "Testing Marx: Some New Results from UK Data." *Capital and Class* 19 1: 103–130.

[5] Cockshott, P., and A. Cottrell. 1997. "Labour Time Versus Alternative Value Bases: A Research Note." *Cambridge Journal of Economics* 21 4: 545–549.

[6] Cockshott, P., and A. Cottrell. 2005. "Robust Correlations Between Prices and Labour Values: A Comment." *Cambridge Journal of Economics* 29 2: 309–316.

[7] Cockshott, Paul, Allin Cottrell, and Alejandro Valle Baeza. 2014. "The Empirical Aspect of Value Theory: A Response to Nitzan and Bichler." *Investigación Económica* 73 (287): 121–142. https://doi.org/10.1016/S0185-1667(14)72609-8

[8] Fröhlich, N. 2012. "Labour Values, Prices of Production and the Missing Equalisation Tendency of Profit Rates: Evidence from the German Economy." *Cambridge Journal of Economics* 37 5: 1107-1126.

[9] Işıkara, G., & Mokre, P. 2022. Price-Value Deviations and the Labour Theory of Value: Evidence from 42 Countries, 2000–2017. *Review of Political Economy*, *34*(1), 165–180. https://doi.org/10.1080/09538259.2021.1904648

[10] Kliman, A. J. 2002. The law of value and laws of statistics: sectoral values and prices in the US economy, 1977-97. *Cambridge Journal of Economics*, 26(3), 299–311. https://doi.org/10.1093/cje/26.3.299

[11] Krelle, W. 1979. "Basic Facts in Capital Theory: Some Lessons from the Controversy in Capital Theory." *Revue d'Economie Politique* 89 5: 282-329.

[12] Marx, K. 1999. *Capital, vol. 3*. New York: Random House

[13] Montibeler, E., and C. Sánchez. 2014. "The Labour Theory of Value and the Prices in China: Methodology and Analysis."

[14] Morishima, M. 1973. *Marx's Economics: A Dual Theory of Value and Growth*. Cambridge University Press.

[15] Nitzan, J., & Bichler, S. 2009. *Capital as Power: A Study of Order and Creorder*. Routledge.

[16] Ochoa, E. 1984. "Labor Values and Prices of Production: An Interindustry Study of the U.S. Economy, 1947-1972." PhD diss., New School for Social Research.

[17] Ochoa, E. 1989. "Values, Prices, and Wage-profit Curves in the US Economy." *Cambridge Journal of Economics* 13 3: 413-29.

[18] Pasinetti, L. 1977. *Lectures on the Theory of Production*. New York: Columbia University Press.

[19] Petrović, P. 1987. "The Deviation of Production Prices from Labour Values: Some Methodological and Empirical Evidence." *Cambridge Journal of Economics* 11 3: 197–210.

[20] Petty, W. 1662. *A Treatise of Taxes and Contributions.* Kelly.

[21] Quesnay, F. 1760. *Physiocratie*. Flammarion.

[22] Ricardo, D. 2001. *On the Principles of Political Economy and Taxation*. Batoche Books.

[23] Sánchez, C., and M. Nieto. 2010. "[Values, Prices of Production, and Market Prices from Spanish Economic Data]." *Investigación Económica* 69 (274): 87–118. https://doi.org/10.22201/fe.01851667p.2010.274.24259

[24] Shaikh, A. 1977. "Marx's Theory of Value and the 'Transformation Problem'." In J. Schwartz, ed., *The Subtle Anatomy of Capitalism*, 106-139. Santa Monica, CA: Goodyear Publishing.

[25] Shaikh, A. 1984. "The Transformation from Marx to Sraffa: prelude to a critique of the neo-ricardians." In E. Mandel, ed., *Marx, Ricardo, Sraffa*. The Langston memorial volume, London: Verso.

[26] Shaikh, A. 1998. "The Empirical Strength of the Labour Theory of Value." In *Marxian Economics: A Reappraisal, vol. 2*, edited by R. Bellofiore. New York: St. Martin's Press.

[27] Shaikh, A. 2012. "The Empirical Linearity of Sraffa's Critical Output-Capital Ratios." In Christian Gehrke, Neri Salvadori, Ian Steedman, and Richard Sturn, eds., *Classical Political Economy and Modern Theory: Essays in Honour of Henry Kurz* 89-102. Abingdon, UK: Routledge.

[28] Shaikh, A. 2016. *Capitalism: Competition, Conflict, Crises*. Oxford: Oxford University Press.

[29] Smith, A. 2007. *An Inquiry into the Nature and Causes of the Wealth of Nations*. MetaLibri.

[30] Sraffa, P. 1960. *Production of Commodities by Means of Commodities*. Cambridge: Cambridge University Press.

[31] Tsoulfidis, L. 2008. "Price-Value Deviations: Further Evidence from Input-Output Data of Japan." *International Review of Applied Economics* 22 6: 707–724.

[32] Tsoulfidis, L., and D. Paitaridis. 2009. "On the Labor Theory Value: Statistical Artefacts or Regularities?" *Research in Political Economy* 25: 209–232

[33] Tsoulfidis, L., and D. Paitaridis. 2016. "Monetary Expressions of Labour Time and Market Prices: Theory and Evidence from China, Japan and Korea." *Review of Political Economy* 29 1: 111-132.

[34] Tsoulfidis, L. and D.-M. Rieu. 2006. "Labor Values, Prices of Production, and Wage-Profit Rate Frontiers of the Korean Economy." *Seoul Journal of Economics* 19 3: 275–95.

[35] Tsoulfidis, L. and T. Maniatis. 2007. "Labour Values, Prices of Production and the Effects of Income Distribution: Evidence from the Greek Economy." *Economic System Research* 19 4: 425-437.

[36] Tsoulfidis, L. and T. Maniatis. 2002. "Values, Prices of Production and Market Prices: Some More Evidence from the Greek Economy." *Cambridge Journal of Economics* 26 3: 359–369.
[37] Wright, Ian. 2014. "A Category-Mistake in the Classical Labour Theory of Value." *Erasmus Journal of Philosophy and Economics* 7: 27–55.
[38] Wright, Ian. 2017. "The General Theory of Labour Value." Working paper. https://www.semanticscholar.org/paper/The-general-theory-of-labour-value-Wright/268aa26b47ee86ee2a554c081a590fab7cd14daa
[39] Zachariah, D. 2004. "Testing the labor theory of value in Sweden April."
[40] Zachariah, D. 2006. "Labour value and equalisation of profit rates: a multi-country study." *Indian Development Review* 4.

Appendix A. Data Sources and Homogenization of Labour Coefficients

All input-output data is taken or estimated from the tables made available by the Spanish *Instituto Nacional de Estadística* (INE) website (www.ine.es). These tables are at the 64-order level of detail, but no data is contained within the cells of the 64th industry (services provided by extraterritorial organisations and bodies), so it was discarded from the computations. Furthermore, industry N°63 (services of households as employers; undifferentiated goods and services produced by households for own use) features non-market activities and is not used in the production of any other industry's output. It thus produced a linear but unconventional output-capital ratio path and was in turn discarded from the analysis of individual price curvature.

Although no direct data is made available by these tables for the total amount of hours worked in each industry, there is a way to estimate them by using the information given on total (full-time and part-time) jobs, which are defined by the ESA 2010 as the ratio of total hours worked in a certain industry to the average annual number of hours worked in the entire economic territory. With the use of data on the average number of hours worked in Spain provided by the OECDiLibrary (www.oecd-ilibrary.org), the sectoral total hours worked were then calculated by multiplying each industry's total jobs by the average number of hours worked in the entire economy. This procedure was undertaken for the three years analysed in the study.

Differences in skill across industries were also accounted for by homogenizing the direct labour coefficients. The homogenization procedure is done by computing the ratio of each industry's wage to the economy's minimum wage and multiplying it by the vector of sectoral total hours worked, that is, $\frac{w_i}{w_{min}} \cdot \overrightarrow{\text{Hours Worked}}\ \forall i \in N$. The homogenized total hours worked are then dot divided by the transpose of the vector of sectoral outputs to obtain the vector of homogenized direct labour coefficients ($\vec{l}$).

Data on the column vector of the basket of goods that workers normally consume with their money wage ($\vec{b}$) is obtained from the column of final consumption expenditure by households. This vector is given by:

$$\left[\frac{(\text{Final consumption expenditure by households})_i}{\sum_{i=1}^{N}(\text{Final consumption expenditure by households})}\right] \times w_{min}$$