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Beam moments and angular momentum in non-uniformly polarized beams

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ABSTRACT

The angular momentum of non-uniformly totally polarized beams is investigated using methods from the beam characterization approach. The relationship between the elements of the beam matrix for the two components of the field and the angular momentum is given. The unconventional distribution of the polarization across the beam profile could result in contributions to both the spin and orbital terms of the angular momentum. To illustrate this, a particular example with a vortex beam is considered.

Electromagnetic fields can transport energy over a given space and transfer momentum (linear and angular) onto matter. Angular momentum is receiving a special attention not only as a fundamental aspect of light, but also because it can be used to control light matter interaction [1–11].

Within the paraxial approach, it is found that the total angular momentum of polarized light splits naturally in two components: an intrinsic (or spin) part related to polarization, and an extrinsic component, also called orbital angular momentum, related to vortices, twisted irradiance beams, general astigmatism and associated phenomena, as shown in Refs. [1,3,6,11–16]. Several of those papers present experimental evidences of the decoupling of spin and orbital angular momentum and its importance in some applications.

On the other hand, the use of irradiance moments of the beam has proved to be an essential tool in spatial beam characterization. Furthermore, angular momentum for partially coherent beams has been considered in a previous paper [13], in which it was related to some elements of the beam matrix within the beam matrix formalism [14,15,17,18], assuming a uniformly and totally polarized field.

In this paper, we are interested in non-uniformly totally polarized (NUTP) fields. The relation between some characteristic spatial beam parameters, based on the irradiance beam moments, and the angular momentum in NUTP beams is derived. Nowadays, generation and characterization of NUTP beams is a subject of growing interest in optics due to their various applications [19–27]. Finally, a particular example of non-uniformly polarized vortex beam is considered.

Let us begin considering a quasimonocromatic beam of mean angular frequency ω propagating along z-axis with a non-uniform distribution of polarization across the beam profile. Within the paraxial approximation such beam can be written using the Jones formalism as

$$\mathbf{E}(\mathbf{r}) = \begin{pmatrix} E_s(\mathbf{r}) \\ E_p(\mathbf{r}) \end{pmatrix}, \tag{1}$$

where $\mathbf{r} = (x, y)$ and E_s and E_p are the field components along two directions orthogonal to the propagation direction (we are considering that the longitudinal component of the field can be neglected). From the classical electromagnetic theory we know that the flux of the time averaged angular momentum density through a z-plane is given by Allen et al. [1]

$$\bar{J} = c \iint \overline{\mathscr{M}}\, dxdy, \tag{2}$$

where $\overline{\mathscr{M}}(\mathbf{r}) = \mathbf{r} \times \bar{\mathbf{S}}/c^2$ is the angular momentum density of the light beam expressed in terms of the time averaged Poynting vector, $\bar{\mathbf{S}} = (\epsilon_0 c^2/2)\mathrm{Re}[\mathbf{E}(\mathbf{r}) \times \mathbf{B}^*(\mathbf{r})]$, ϵ_0 being the vacuum permittivity and c the speed of light.

Using Eq. (2), the z-component of the flux of the time averaged angular momentum density through a transverse plane, $\bar{J}_z$, results

$$\bar{J}_z = \bar{J}_z^L + \bar{J}_z^S, \tag{3}$$

with

$$\bar{J}_z^L = \frac{P_s}{c}(\langle xv\rangle_s - \langle yu\rangle_s) + \frac{P_p}{c}\left(\langle xv\rangle_p - \langle yu\rangle_p\right), \tag{4}$$

$$\bar{J}_z^S = \epsilon_0 c\, \mathrm{Im} \iint E_s^* E_p\, dxdy = \frac{1}{\omega}\hat{s}_3, \tag{5}$$

where it is clear the separation between the orbital and the polarization contributions. We also find that the orbital angular momentum term is divided into s and p components, as expected in the paraxial regime. In Eq. (4) we have used P_i, with $i = s, p$, for the power of each component

$$P_i = \frac{\epsilon_0 c}{2} \iint E_i^* E_i\, dxdy, \tag{6}$$

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and $\langle xv\rangle_i$ and $\langle yu\rangle_i$ for the crossed irradiance moments used in the beam matrix formalism [14,15,17,18], also for each component

$$\langle xv\rangle_i = \frac{\epsilon_0 c}{2kP_i}\iint x\,\mathrm{Im}\left(E_i^*\frac{\partial E_i}{\partial y}\right)dxdy, \tag{7}$$

$$\langle yu\rangle_i = \frac{\epsilon_0 c}{2kP_i}\iint y\,\mathrm{Im}\left(E_i^*\frac{\partial E_i}{\partial x}\right)dxdy. \tag{8}$$

It should be noted that from Eq. (4) we can infer that the orbital angular momentum can be obtained, in an easy way, from the measurements of the beam irradiance moments. In this case, since we are dealing with crossed terms $\langle xv\rangle_i$ and $\langle yu\rangle_i$, measurements in free space are not enough, and an auxiliary cylindrical lens is needed [12,14]. On the other hand, the polarization term $\bar{J}_z^S$ is proportional to the integrated $s_3(x,y)$ Stokes parameter. It can be measured using standard polarization techniques, with a polarizer, a quarter wave plate and a CCD camera (see, for example, Refs. [20,28]). Note that this term is closely connected to a new parameter, $\tilde{\rho}$, recently introduced in the literature to characterize the circular or linear polarization content of light across the beam profile in such regions where the irradiance is significative [28].

For the beam in Eq. (1) the photon flux (photons per unit time) is $P/\hbar\omega$, P being the total power of the beam, $P = P_s + P_p$, and $\hbar$ being the normalized Planck's constant. In terms of these parameters the angular momentum per photon j_z is found to be

$$j_z = \bar{J}_z\frac{\hbar\omega}{P} = k\frac{P_s(\langle xv\rangle_s - \langle yu\rangle_s) + P_p(\langle xv\rangle_p - \langle yu\rangle_p)}{P}\hbar + \frac{\hat{s}_3}{P}\hbar. \tag{9}$$

where $k = \omega/c$ is the modulus of the wave vector. Note that by using this expression we can calculate the angular momentum of non-uniformly polarized beams.

Eqs. (3) and (9) are also important, since they give us the different weights of orbital and spin contributions to the total angular momentum. In that sense it should be remarked that $\hat{s}_3$ normalized by the total power P, as appears in Eq. (9), has a value between -1 and 1, while the orbital part has no bounds. In any case both terms in Eq. (9) should be understood as averaged values, since we are considering beams with a non-uniform distribution of the angular momentum contributions.

In order to illustrate how we can obtain the angular momentum by means of Eqs. (3)–(5), we will study a particular example of beam with a non-uniform polarization distribution. Let us consider a non-uniformly totally polarized field defined as

$$\mathbf{E}(r,\theta) = \frac{f(r)}{\sqrt{2}}\begin{pmatrix}1\\ e^{in\theta}e^{i\delta}\end{pmatrix}, \tag{10}$$

where $f(r)$ is a radial function, $\exp(in\theta)$ is a helical phase, which represents a single vortex of charge n, θ being the azimuthal angle, and δ a constant phase factor. With those phase factors the polarization state changes from point to point across the transverse section of the beam, as shown in Fig. 1. In that figure we have used ellipses and arrows to represent the local polarization state with its handedness in several cases.

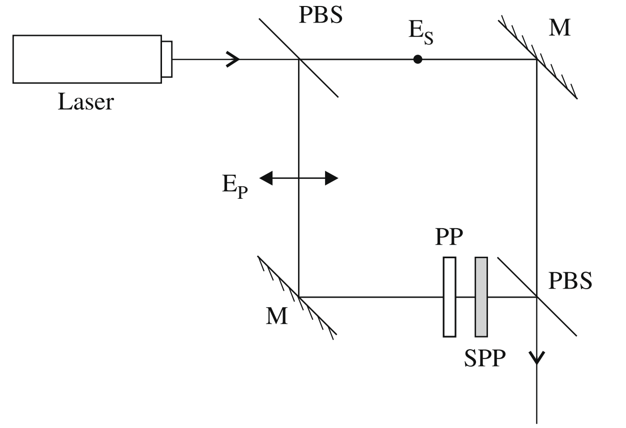

Fig. 2. Schematic Mach–Zehnder setup proposed to obtain the field given in Eq. (10). PBS: polarized beam splitters; M: mirrors; PP: phase plate; SPP: spiral phase plate.

The field proposed in Eq. (10) can be synthesized by means of a Mach–Zehnder interferometer with polarized beam splitters, as suggested in Fig. 2. Starting with a linearly polarized beam at 45°, the polarized beam splitters give us orthogonal polarizations at each arm with an adequate power balance. The additional phase terms can be introduced by means of a phase plate and a spiral phase plate placed in one of the interferometer arms. The phase plate is used to introduce the δ phase. On the other hand, spiral phase plates generate a vortex in a beam [29–33]. Spiral phase plates (and analogous systems) are used in a wide range of applications, and can even be designed with a fractional order n [34–39]. Fixed spiral phase plates can be made using micromachining, lithography or holographic methods (see, for example, Refs. [31–33]). Another approach is to use spatial light modulators (SLM) with the adequate design [36–38].

By inserting the beam defined in Eq. (10) into Eq. (9) we find that

$$j_z = j_z^L + j_z^S = \frac{n}{2}\hbar + \frac{\sin(n\pi)\sin(n\pi+\delta)}{n\pi}\hbar, \tag{11}$$

where the first term corresponds to the orbital angular momentum, while the second is the polarization contribution. Note that we have normalized to the input power. If we analyze the previous expression a first conclusion is that even though there are two separate terms, the polarization term includes both contributions from the device that modifies the polarization (the phase plate), and from

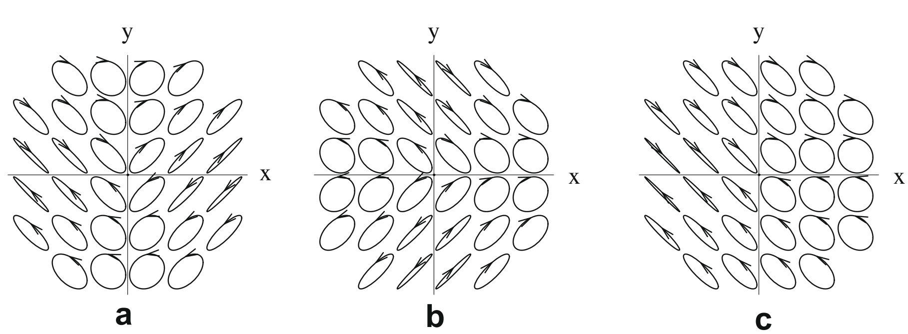

Fig. 1. Local polarization state of the field defined by Eq. (10) in cases (a) $n = 1, \delta = 0$, (b) $n = 1, \delta = \pi/2$ and (c) $n = 1/2, \delta = \pi/2$.

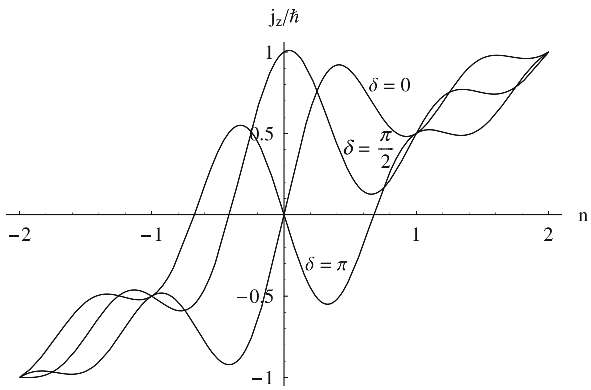

Fig. 3. Angular momentum per photon for the non-uniformly totally polarized beam defined in Eq. (10) versus spiral phase plate order n and phases $\delta = 0, \delta = \pi/2$ and $\delta = \pi$.

the device that modifies the orbital angular momentum (the spiral phase plate). From Eq. (11) it is easy to see that if we restrict ourselves to integer values of the phase step n the polarization term is cancelled independently of the phase δ. We would have a non-uniformly polarized beam with no polarization term in the angular momentum. But if we use a fractional phase step we can obtain different values for the total angular momentum per photon j_z. In such case the phase δ introduced by the phase plate can be used as an additional control over the angular momentum, as shown in Fig. 3. In that figure we have plotted Eq. (11) for several values of δ and for values of n ranging from -2 to 2. We see that j_z presents maxima and minima, and for certain values of n the orbital and spin terms compensate, resulting in an angular momentum null. In summary, we have achieved a control over j_z by modifying n and δ.

Finally, we can compare the previous result with that obtained for a uniformly and totally polarized beam. If we propagate a linearly polarized beam through an spiral phase plate and a waveplate (retarder) we will obtain

$$\mathbf{E} = \frac{f(r)}{\sqrt{2}} e^{in\theta} \begin{pmatrix} 1 \\ e^{i\delta} \end{pmatrix}. \quad (12)$$

Note that this beam has the same irradiance profile as the previous one, but it is a uniformly polarized beam, i.e., the polarization state is the same everywhere in a transversal plane. If we calculate the angular momentum for this beam using Eq. (9) we find that

$$j_z = j_z^L + j_z^S = n\hbar + \sin\delta\,\hbar. \quad (13)$$

In this case the waveplate controls the polarization term of the angular momentum, independently of the spiral phase plate order n. To conclude, we could say that the angular momentum of non-uniformly totally polarized fields reveals physical features that would not be appreciated if only uniformly totally polarized fields are considered.

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