Proton decay in a nucleus: Nonrelativistic treatment of nuclear effects

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In this paper, proton decay in a large nucleus is studied in the framework of SU(5) grand unification theory (GUT). By using a method based upon the Green's-function technique of many-body physics, nuclear effects on spectator and pole terms are computed. The decay width in the nucleus is found to be practically the same as in free space. However, nuclear effects are of considerable importance concerning the positron spectrum. A density-correlation expansion is introduced which is useful for carrying out a systematic study of nuclear effects in proton decay in a large nucleus. The method presented here can be easily extended to other GUT's or supersymmetric GUT's.

I. INTRODUCTION

During the last few years, a considerable amount of effort, both theoretical and experimental, has been devoted to the study of proton decay, a remarkable phenomenon predicted, as is known, in grand unification theories (GUT's).[1] Due to the fact that the corresponding experiments are carried out with materials made (mainly) out of complex nuclei, the question of the relevance of nuclear effects in the decay is, no doubt, interesting. Concerning just the decay width, it is expected, in principle, that said nuclear effects are of little importance. The physical reason is that the process is highly incoherent in the nuclear sense, and therefore one expects $\Gamma_A = A\Gamma_p$, where Γ_p is the proton decay width in free space and A is the number of nucleons ($\Gamma_n = \Gamma_p$ has been assumed). However, it has been pointed out recently that this is not necessarily so because of the existence of the three-quark-fusion process (i.e., direct conversion of three quarks into a positron) which, according to Ref. 2, greatly increases the proton decay width in a complex nucleus. As a matter of fact, a sizable reduction of the proton lifetime in ^{16}O as compared with that of a free proton is predicted.[2] Nevertheless, although the physical arguments exposed in Ref. 2 are not wrong, their actual computation of proton lifetime in a complex nucleus is not correct because some unjustified extrapolations are made throughout the calculation. This is discussed at length in a previous paper by two of the present authors.[3] Also in said paper, a treatment of nuclear effects in proton decay in a large nucleus, based upon the Green's-function technique in many-body physics, has been introduced. This treatment allows us to study nuclear effects in proton decay in a systematic way. Some preliminary (and rough) numerical estimates were presented,[3] according to which nuclear effects could be of relevance for complex nuclei, as opposed to the deuteron case wherein they appear to be almost negligible.[3,4] For ^{4}He, some explicit estimates by Dover *et al.*[4] indicate that nuclear effects in the decay width are small, and, moreover, they make the guess that those effects should not be very important for heavier nuclei.[4] On the other hand, in Ref. 5, a substantial (50% for $A=60$) increase of the proton decay width in complex nuclei has been predicted. This comes mainly from (virtual) pion absorption in the nucleus giving place to a Δ state. As the situation seems to be rather confusing, we think a detailed treatment of proton decay in a nucleus is worth presenting, following our previously introduced method.[3]

In Sec. II, we discuss some general aspects of proton decay in the framework of SU(5) GUT that we shall need in the nuclear case. We show how to eliminate the quark fields in the effective Lagrangian for the decay, replacing them by interpolating proton and pion fields. This turns out to be quite useful when dealing with proton decay in a nucleus.

In Sec. III, the Green's-function method is developed further and our results for the decay width and positron spectrum are given. Nuclear effects are shown to be very small in what concerns the decay width; however, they are certainly important in the positron spectrum.

Finally, in Sec. IV, we introduce a density-correlation type of expansion in the nucleus which allows us to prove the consistency of the method developed in this paper, in the sense that the computed decay width for a proton in nuclear matter reduces to that of a proton in free space when the nuclear density goes to zero.

To end this introduction, we would like to remark that the treatment of nuclear effects in proton decay presented here, although applied to SU(5) GUT for convenience, is quite general and can be easily extended to other GUT's without essential modifications.

II. SOME GENERAL ASPECTS OF PROTON DECAY

In the framework of SU(5) GUT, the effective Lagrangian for proton decay (with a positron in the final state) can be written as[1]

$$\mathscr{L}_{\rm eff}(x)=\frac{g_{\rm GUT}{}^2}{2M_x{}^2}\xi\bar{e}^{+}(x)p(x)\;, \qquad (2.1)$$

where $g_{\rm GUT}$ and M_X are the GUT coupling constant mass, respectively, ξ is the so-called enhancement factor, which is related to renormalization effects of strong and electroweak interactions, and $p(x)$ is a composite operator, which, in terms of elementary quark fields, reads

$$p(x)=\epsilon_{ijk}\left[\bar{u}_k(x)\gamma^{\mu}\left[\frac{1+\gamma_5}{2}\right]u_j(x)\right]\times\gamma_{\mu}\left[\frac{3+\gamma_5}{2}\right]d_i(x)\;. \qquad (2.2)$$

The total width for the decay of a proton in a nucleus in its ground state $|A>$ with emission of a positron with four-momentum K is[2,3]

$$\Gamma_A=\sum_F\Gamma(A\rightarrow e^{+}F)=\left[\frac{g_{\rm GUT}{}^2}{2M_X{}^2}\right]^2 A_s(f)\frac{\langle A\mid A\rangle}{2m_A}\times\sum_{\alpha,\beta=1}^{4}\int\frac{d^3K}{2K^0}(\gamma^0K)_{\beta\alpha}\Lambda_{\alpha\beta}\;, \qquad (2.3)$$

where $A_s(f)=\xi^2$, and

$$\Lambda_{\alpha\beta}=\langle A\mid A\rangle^{-1}(2\pi)^4\times\sum_F\delta^{(4)}(P_A-K-p_F)\langle A\mid p_\beta^{\dagger}(0)\mid F\rangle\times\langle F\mid p_\alpha(0)\mid A\rangle\;. \qquad (2.4)$$

By using completeness, translation invariance, and the known spectral properties of the nuclear states, Eq. (4) can be cast as

$$\Lambda_{\alpha\beta}\simeq\theta((m_A/A)-|\vec{K}|)\langle A\mid A\rangle^{-1}\times\int d^4x\,\exp iKx\times\left\langle A\left|\left[p_\beta^{\dagger}\left[-\frac{x}{2}\right],p_\alpha\left[\frac{x}{2}\right]\right]\right|A\right\rangle\;. \qquad (2.5)$$

At this stage, it is convenient to introduce the following three-quark operator:

$$\eta(x)=[\bar{u}_i^c(x)\gamma_\mu u_j(x)]\gamma_5\gamma^{\mu}d_k(x)\epsilon_{ijk}\;. \qquad (2.6)$$

It is shown in Ref. 6 that this operator can be regarded as a suitable current for the proton (see Ref. 6 for a detailed discussion). Now it is easy to establish the relationship between operators p and n,

$$p(x)=-\tfrac{1}{4}(3\gamma_5-1)\eta(x)\;. \qquad (2.7)$$

The main point lies, however, in whether $\eta(x)$ can be properly considered as an interpolating proton field. As is known, the existence of relativistic fields for bound states, made out of their elementary constituents, is an old and difficult problem of standard quantum field theory.[7] We only point out here that if one introduces $\eta(x)$ as an effective proton field and computes Eq. (4) (with $A=1$) in the nonrelativistic limit [assuming, for instance, an SU(6) wave function for the proton], one is then computing the nonrelativistic pole diagram, as, for example, in Ref. 8 (see also Ref. 9 for a relativistic treatment of pole terms in the framework of the QCD sum rules of Shifman, Vainshtein, and Zakharov[10]).

It has been discussed very recently whether the pole term should be added to the "traditional" spectator diagram (i.e., $qq\rightarrow\bar{q}e^{+}$) in order to properly calculate the decay width, or, on the contrary, whether both mechanisms represent just different approximate ways of computing the decay width (the problem of double counting). In the case of a nonrelativistic approximation for the pole term, it seems that there is no double counting.[11] In any case, it is not our purpose here to enter into that discussion, for our present work is devoted to *nuclear effects* in the decay. Moreover, since we will finally make a nonrelativistic approximation for proton states within the nucleus (see below), it seems appropriate to consider pole and spectator terms and, consequently, to study nuclear effects in both cases.

By using an SU(6) wave function for the proton, it is easy to relate the matrix element of $\eta(x)$ between the vacuum and a proton state $|\tilde{p}(p,\lambda)\rangle$ with that of an interpolating proton field $\psi(x)$, the latter being defined as

$$\langle 0 \mid \psi(0) \mid \tilde{p}(p,\lambda) \rangle = u_\lambda(\vec{p}),$$

$u_\lambda(\vec{p})$ being the corresponding Dirac spinor. We obtain

$$\langle 0 \mid \eta(x) \mid \tilde{p}(p,\lambda) \rangle = 6[2^{1/2}\psi(0,0)] \times \langle 0 \mid \psi(x) \mid \tilde{p}(p,\lambda) \rangle , \qquad (2.8)$$

where $\psi(0,0)$ is the spatial wave function of the proton when all three quarks are at the same point. In this way, we are able to construct an effective Lagrangian for the pole term, which reads

$$\mathscr{L}^{(1)}_{\text{eff}}(x) = \frac{g_{\text{GUT}}{}^2}{2M_X{}^2}\xi \sum_{\alpha,\beta=1}^{4} \bar{e}^{\dagger}_{\alpha}(x) F^{(1)}_{\alpha\beta}\psi_\beta(x) , \qquad (2.9)$$

where the matrix $F^{(1)}$ is given by

$$F^{(1)} = -\frac{\psi(0,0)}{2^{1/2}}(q\gamma_5 - 3) .$$

Going now to the spectator term, and considering just the dominant process $p \to e^+\pi^0$, its effective Lagrangian can be written as

$$\mathscr{L}^{(2)}_{\text{eff}}(x) = \frac{g_{\text{GUT}}{}^2}{2M_X{}^2}\xi \sum_{\alpha,\beta=1}^{4} \bar{e}^{\dagger}_{\alpha}(x) F^{(2)}_{\alpha\beta}\psi_\beta(x)\phi(x) , \qquad (2.10)$$

where $\phi(x)$ is the pion field and the matrix $F^{(2)} = i\kappa(-3+\gamma_5)$, κ being a (model-dependent) constant that, in fact, measures the relative strength between pole and spectator terms (see Ref. 8 and our further comments below).

Now, upon setting $\mathscr{L}_{\text{eff}} = \mathscr{L}^{(1)}_{\text{eff}} + \mathscr{L}^{(2)}_{\text{eff}}$ and comparing with Eq. (1), we can express $p(x)$ in the following way:

$$p_\alpha(x) = \sum_{\beta=1}^{4} F^{(1)}_{\alpha\beta}\psi_\beta(x) + \sum_{\beta=1}^{4} F^{(2)}_{\alpha\beta}\psi_\beta(x)\phi(x) \qquad (2.11)$$

Notice that no quark fields whatsoever appear in Eq. (11). (Though probably unnecessary, we recall that the spectator term we are considering corresponds to the process $p \to e^+\pi^0$). This fact will prove to be quite useful in the nuclear case as shown in what follows.

III. NUCLEAR EFFECTS ON PROTON DECAY IN A LARGE NUCLEUS

A. General treatment

In the following we shall consider only nonrelativistic nucleons. As discussed previously,[3] we believe that such a restriction will not essentially modify the main physical features of proton decay in a nucleus. Then, $p(x)$ as well as the proton field $\psi(x)$ become two-component operators. Moreover, we shall directly give the final results in the many-body limit [$A \to \infty$, nuclear volume $V \to \infty$, with $A/V \to (2/3\pi^2)K_F{}^3$, K_F being the Fermi momentum]. After some algebra, Eqs. (2.3), (2.5), and (2.11) (together with the preceding expressions for $F^{(1)}$ and $F^{(2)}$) yield the following result for the decay rate per nucleon:

$$\Gamma_A/A = \frac{g_{\text{GUT}}{}^4}{4M_X{}^4}A_s(f)\frac{3\pi^2}{2K_F{}^3}\frac{1}{(2\pi)^3}\int \frac{d^3K}{2K^0}\theta((m_A/A) - |\vec{K}|)90K^0 \times \sum_{\alpha,\beta=1}^{2}\left\{\left[\frac{|\psi(0,0)|^2}{2}I(K)_\alpha + |\kappa|^2 I_s(K)_\alpha\right]\delta_{\alpha\beta} + \frac{|\psi(0,0)|}{2^{1/2}}2\,\text{Re}\left[\kappa^* i\left[\frac{\bar{\sigma}K}{K^0}\right]_{\beta\alpha} I_{\text{int}}(K)_{\beta\alpha}\right]\right\} , \qquad (3.1)$$

where

$$I(K)_\alpha = \frac{1}{\langle A \mid A \rangle}\int d^4x \exp iKx \left\langle A \left| \left[\psi^{\dagger}_{\alpha}\left[-\frac{x}{2}\right], \psi_\alpha\left[\frac{x}{2}\right]\right] \right| A \right\rangle , \qquad (3.2)$$

$$I_s(K)_\alpha = \frac{1}{\langle A \mid A \rangle}\int d^4x \exp iKx \left\langle A \left| \left[(\psi_\alpha\left[-\frac{x}{2}\right]\phi\left[-\frac{x}{2}\right])^{\dagger}, \psi_\alpha\left[\frac{x}{2}\right]\phi\left[\frac{x}{2}\right]\right] \right| A \right\rangle , \qquad (3.3)$$

$$I_{\text{int}}(K)_{\beta\alpha} = \frac{1}{\langle A \mid A \rangle}\int d^4x \exp iKx \left\langle A \left| \left[(\psi_\beta\left[-\frac{x}{2}\right]\phi\left[-\frac{x}{2}\right])^{\dagger}, \psi_\alpha\left[\frac{x}{2}\right]\right] \right| A \right\rangle . \qquad (3.4)$$

(See Ref. 3 for similar calculations when $\kappa=0$.) Similarly, we shall find it useful to introduce

$$\widetilde{G}(K)=-\frac{1}{\langle A\,|A\rangle}\int d^4x\,\exp iKx\left\langle A\left|T\left[\psi_\alpha\left[\frac{x}{2}\right]\psi_\alpha^\dagger\left[-\frac{x}{2}\right]\right]\right|A\right\rangle , \tag{3.5}$$

$$\widetilde{G}_s(K)=-\frac{i}{\langle A\,|A\rangle}\int d^4x\,\exp iKx\left\langle A\left|T\left[\psi_\alpha\left[\frac{x}{2}\right]\phi\left[\frac{x}{2}\right](\psi_\alpha(-\frac{x}{2})\phi\left[-\frac{x}{2}\right])^\dagger\right]\right|A\right\rangle , \tag{3.6}$$

$$\widetilde{G}_{\rm int}(K)=\sum_{\beta=1}^{2} i\left[\frac{\bar{\sigma}K}{K^0}\right]_{\beta\alpha}\widetilde{G}_{\rm int}(K)_{\alpha\beta} , \tag{3.7}$$

where

$$\widetilde{G}_{\rm int}(K)_{\alpha\beta}=-\frac{i}{\langle A\,|A\rangle}\int d^4x\,\exp iKx\left\langle A\left|T\left[\psi_\alpha\left[\frac{x}{2}\right](\psi_\beta\left[-\frac{x}{2}\right]\phi\left[-\frac{x}{2}\right])^\dagger\right]\right|A\right\rangle , \tag{3.8}$$

the symbol T denoting the time-ordered product. Some calculations establish the connections between I's and $\widetilde{G}$'s:

$$I(K)_\alpha=2\,{\rm Im}\widetilde{G}(K),\quad I_s(K)=2\,{\rm Im}\widetilde{G}_s(K) , \tag{3.9}$$

$$\sum_{\beta=1}^{2} i\left[\frac{\bar{\sigma}K}{K^0}\right]_{\beta\alpha} I_{\rm int}(K)_{\beta\alpha}=2\,{\rm Im}\widetilde{G}_{\rm int}(K) . \tag{3.10}$$

General invariance arguments in the many-body limit do show that the right-hand sides of Eqs. (3.5)–(3.7), as well as the left-hand sides of Eqs. (3.9) and (3.10), are indeed independent of α. This fact can also be verified by explicit many-body perturbation theory calculations. Upon combining Eqs. (3.1) and (3.9) and (3.10), we get the final result for the decay rate per nucleon:

$$\Gamma_A/A=\frac{g_{\rm GUT}{}^4}{4M_x{}^4}A_s(f)\frac{3\pi^2}{2K_F{}^3}\frac{90}{(2\pi)^3}\int d^3K\,\theta((m_A/A)-|\vec{\rm K}|)$$
$$\times\left[|\psi(0,0)|^2{\rm Im}\widetilde{G}(K)+2|\kappa|^2{\rm Im}\widetilde{G}_s(K)+\frac{|\psi(0,0)|}{2^{1/2}}4\,{\rm Re}\kappa\,{\rm Im}\widetilde{G}_{\rm int}(K)\right] \tag{3.11}$$

which generalizes Eq. (19) in Ref. 3, when the spectator term and its interference with the pole term are also taken into account.

We shall require a meson-theoretical pion-nucleon interaction Hamiltonian. We shall use, for convenience, the so-called cloudy-bag model (CBM),[12] which contains pions, nucleons, and Δ's, including three-momentum conservation and recoil corrections. Then, the quantities $\widetilde{G}(K)$, $\widetilde{G}_s(K)$, and $\widetilde{G}_{\rm int}(K)$ can be obtained by applying the many-body perturbation theory, in terms of the assumed CBM interaction Hamiltonian.[12] We shall require, for that purpose, the following standard free nucleon hole, π, and Δ propagators $[l=(l^0,\vec{\rm l})]$:

$$\widetilde{G}^{(0)}(l)=\left[l^0-\left[m_p+\frac{\vec{\rm l}^2}{2m_p}\right]+i\epsilon\right]^{-1}+2\pi i\delta\left[l^0-\left[m_p+\frac{\vec{\rm l}^2}{2m_p}\right]\right]\theta(K_F-|\vec{\rm l}|) , \tag{3.12}$$

$$\widetilde{D}^{(0)}(l)=[(l^0)^2-\vec{\rm l}^2-\mu^2+i\epsilon]^{-1} ,$$
$$\widetilde{\Delta}(l)=[l^0-(m_\Delta+\vec{\rm l}^2/2m_\Delta)+i\epsilon]^{-1} . \tag{3.13}$$

Notice that the Δ is regarded as a stable particle with $m_\Delta=1232$ MeV.

As suggested by the work of other authors for

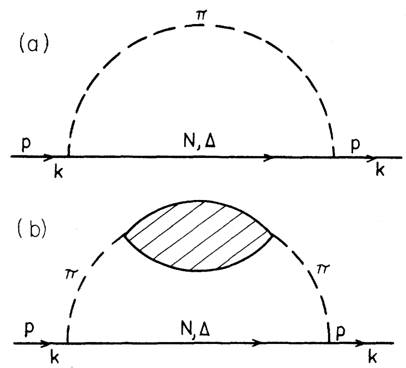

FIG. 1. Nucleon self-energy diagrams. In (b) the blob represents pion polarization.

isolated-proton decay,[8,9,11] we shall accept that the main contribution comes from the pole term, namely, $\tilde{G}(K)$. Accordingly, we shall present in more detail the analysis of the contribution from the CBM many-body Feynman diagrams for $\tilde{G}(K)$ to Γ_A/A, although we shall also consider more briefly those for $\tilde{G}_s(K)$ and $\tilde{G}_{\rm int}(K)$.

B. Pole term in nuclear matter

The proton Green's function $\tilde{G}(K)$, which determines the pole contribution to Γ_A/A, can be written in terms of the proton proper self-energy in the infinite nucleus $\Sigma^*(K)$ as

$$\tilde{G}(K)=[\tilde{G}^{(0)}(K)-\Sigma^*(K)]^{-1} . \tag{3.14}$$

We shall evaluate both individual many-body Feynman diagrams as well as certain infinite sets of them for $\Sigma^*(K)$.

First, we shall consider the diagram in Fig. 1(a), when both nucleon and Δ appear as intermediate states. Their contribution to the proper self-energy in the CBM framework reads

$$\Sigma^*(K)=\frac{3if_{\rm CBM}{}^2}{\mu^2}\int\frac{d^4l}{(2\pi)^4}\tilde{D}^{(0)}(K-l)[\tilde{G}^{(0)}(l)+\tfrac{32}{25}\tilde{\Delta}^{(0)}(l)]\,|\vec{K}-\vec{l}\,|^2[u(|\vec{K}-\vec{l}\,|R)]^2-\delta m_p , \tag{3.15}$$

$f_{\rm CBM}$ and u being the CBM coupling constant and cutoff function, respectively, and μ being the pion mass. On the other hand, δm_p is the finite mass-renormalization counterterm for the proton in CBM, in lowest-order perturbation theory. We have included trivial recoil corrections in it.

By looking at the position of the poles in the l^0 complex plane and carrying through the corresponding residue integration, one sees that (i) the Δ intermediate state only contributes to $\mathrm{Re}\Sigma^*(K)$, and (ii) the only contribution to $\mathrm{Im}\Sigma^*(K)$ comes from the hole term in $\tilde{G}^{(0)}(l)$.

One finds $\mathrm{Re}\Sigma^*(K)\simeq m_p/7$ (practically independent of the nuclear density) as well as

$$|\mathrm{Im}\Sigma^*(K)| \ll |\tilde{G}^{(0)}(K)^{-1}-\mathrm{Re}\Sigma^*(K)| .$$

Consequently, for the diagrams under consideration,

$$\begin{aligned}\mathrm{Im}\tilde{G}(K)&\simeq\frac{\mathrm{Im}\Sigma^*(K)}{[\tilde{G}^{(0)}(K)^{-1}-\mathrm{Re}\Sigma^*(K)]^2}\\&=\frac{1}{1-[\mathrm{Re}\Sigma^*(K)]\tilde{G}^{(0)}(K)^2}\\&\quad\times[\tilde{G}^{(0)}(K)\mathrm{Im}\Sigma^*(K)\tilde{G}^{(0)}(K)] .\end{aligned}$$

Since $\tilde{G}^{(0)}(K)\mathrm{Im}\Sigma^*(K)\tilde{G}^{(0)}(K)$ is the first contribution to $\mathrm{Im}\tilde{G}(K)$ in the Dyson series, we see that $\mathrm{Re}\Sigma^*(K)$ takes into account only higher-order corrections, which are small.

The contribution of the diagram in Fig. 1(a) to the normalized e^+ spectrum, defined as

$$S_{e^+}=(\Gamma_f A)^{-1}d\Gamma_A/d\,|\vec{K}\,|$$

is shown in Fig. 2 (dashed line). Γ_f is given by

$$\begin{aligned}\Gamma_f&=\tfrac{1}{2}[\Gamma(p\to e^+\pi^0)+\Gamma(n\to e^+\pi^-)]\\&=\tfrac{3}{2}\Gamma(p\to e^+\pi^0) .\end{aligned}$$

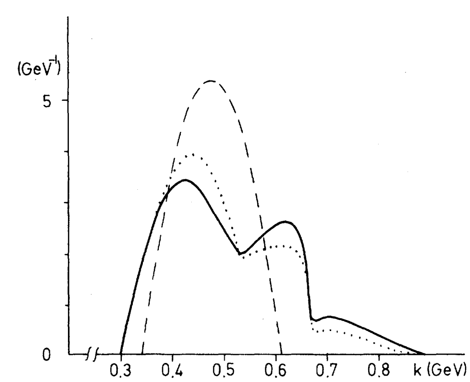

FIG. 2. Normalized positron spectrum S_{e^+} (see text). Dashed and continuous curves show, respectively, the results from diagrams 1(a), and 1(b), for the pole-term contribution. Dotted curve shows the corresponding result [from diagram 1(b)] for the spectator term with $Rp=0.87$ fm.

As discussed in Ref. 3, the free decay width, Γ_f, is obtained upon taking the limit $K_F \to 0$ in Eq. (3.11) (with $\kappa=0$, as we are considering only the pole term for now) (see also Sec. IV for a general discussion). Now the relevant quantity in our work is the nuclear correction to the free width, i.e., $\Gamma_A/A-\Gamma_f$. We have found that the influence on nuclear effects coming from diagrams in Fig. 1(a) is quite negligible, since we get

$$(\Gamma_A/A-\Gamma_f)/\Gamma_f \simeq -0.01 .$$

Notice that those corrections are due only to the nuclear density dependence, but that they do not account for the correlations among nucleons.

In order to somehow incorporate such correlation effects, we will consider diagrams like (b) in Fig. 1, which may contain nucleon-hole loops. Notice that in Fig. 1(b) the blob represents the pion polarization in the nuclear medium, which can be computed by means of Dyson's equation in terms of the proper pion polarization. Then, diagram (b) is obtained from (a) in Fig. 1 by just replacing $\widetilde{D}^{(0)}$ (free-pion propagator) by $\widetilde{D}$ (pion propagator in a medium).

We shall limit ourselves to computing the imaginary part of the contribution from the diagrams in Fig. 1(b) using the CBM. An adequate treatment of the real part of those diagrams should require a proper relativistic treatment for the nucleon, as well as the inclusion of short-range nuclear correlations, as given by σ, ω, and ρ exchanges. This lies beyond the scope of the present work.

In any case, some numerical estimates show that the contribution of the real part coming from Fig. 1(b) is not very important. (Recall that $\mathrm{Re}\Sigma^*$ accounts just for second-order effects.) Thus, we have taken the real part as given only the diagram in Fig. 1(a). We have tested the reliability of such an approximation by noticing that variations of ± 30 MeV in $\mathrm{Re}\Sigma^*$ only change the width in less than 10% (as is known, 30 MeV is about the mean potential energy of a bound nucleon in a nucleus).

After some calculations for $\mathrm{Im}\Sigma^*$ from the diagrams in Fig. 1(b), performing a Wick rotation (see Ref. 13 for similar calculations) and realizing that the K_F-independent term in $\widetilde{G}^{(0)}$ [first term in Eq. (3.11)] gives a vanishing contribution to $\mathrm{Im}\Sigma^*(K)$, one arrives at

$$\mathrm{Im}\Sigma^*(K) = -\frac{3 f_{\mathrm{CBM}}^2}{(2\pi)^2\mu^2}\int_0^\infty d|\vec{q}|\,|\vec{q}|^2\int_{-1}^{+1} dt\,\theta(k_F-|\vec{K}-\vec{q}|)\,\mathrm{Im}\left[\frac{|\vec{q}|^2[u(|\vec{q}|R)]^2}{(q^0)^2-\vec{q}^{\,2}-\mu^2-\Pi^*(q^0,\vec{q})+i\epsilon}\right], \tag{3.16}$$

where $t=\cos(\vec{k},\vec{q})$ and $q^0=K^0-[m_p+(\vec{k}-\vec{q})^2/2m_p]$. The proper pion polarization $\Pi^*(q^0,\vec{q})$ is given in terms of the Lindhard function $U(q^0,\vec{q})$ by

$$\Pi^*(q^0,\vec{q}) = \frac{f_{\mathrm{CBM}}^2}{\mu^2}|\vec{q}|^2[u(|\vec{q}|R)]^2 U(q^0,\vec{q}) . \tag{3.17}$$

An explicit expression for the Lindhard function can be found, for instance, in Ref. 14.

After evaluating numerically the integral in Eq. (3.16), our result for the positron spectrum S_{e^+} is displayed in Fig. 2 (continuous line). One may notice that, in spite of the fact that the positron spectra are rather different for the two cases considered [associated to the diagrams (a) and (b) in Fig. 1], the integrated spectra, i.e., the widths per nucleon Γ_A/A are quite similar in both cases and, hence, also similar to the free-proton width Γ_p. This amounts to saying that, at least within our present approximations [namely, the restrictions to the diagram (b) in Fig. 1 in the CBM framework], the proton decay width is scarcely affected by its being bounded in a nucleus. To be more concrete, we find (Γ_f being defined as above)

$$(\Gamma_A/A-\Gamma_f)\Gamma_f \simeq -0.02 .$$

Notice that the above ratio does not depend either on $\psi(0,0)$ or on the assumed GUT values, but only on the CBM interaction Hamiltonian.

The positron spectrum corresponding to diagram 1(a) (dashed line of Fig. 2) shows the effect of Fermi motion in the decay $p(n)\to e^+\pi^0(\pi^-)$ inside the nucleus. (Recall that $\rho,\omega,\ldots$ mesons are not taken into account.) Obviously, the positron spectrum should reduce to a δ function centered at $K\simeq m_p/2$ when $K_F\to 0$, i.e., for proton decay in free space. Of course, this occurs in the present calculation (see also Ref. 3 and Sec. IV below). Now when correlations among nucelons are accounted for, through the mechanism depicted in Fig. 1(b), we find the positron spectrum shown by the continuous line of Fig. 2. The latter spectrum has several peaks which are due to the following physical effects. The first peak, at $K\simeq 0.42$ GeV, corresponding to quasifree nucleon decay, that is, the decay of an uncorrelated nucleon inside the nucleus. The second peak, at $K \simeq 0.63$ GeV, comes from absorption of the emitted (mainly virtual) pions by nucleons giving rise to Δ's, i.e., from the reaction $NN \to e^+\Delta$ in the nucleus. We have to point out that this peak would appear somewhat wider, i.e., the dropoff at $K \simeq 0.65$ GeV would be less pronounced if the Δ width were taken into account (in any case, this would not appreciably change the nuclear decay width). Finally, the small peak (or rather sort of a hump) at $K \simeq 0.7$ GeV, is due to (virtual) pion absorption without nucleon excitation, i.e., to the reaction $NN \to e^+N$ in the nucleus. In a similar manner as occurs in ^{4}He,[4] the above mechanisms "conspire" among themselves to produce a decay width for a bound proton in a nucleus nearly equal to the free-decay width, in spite of their being responsible for an appreciable distortion of the e^+ spectrum. In the case of a large nucleus here considered, we get a more conspicuous distortion than that found for ^{4}He in Ref. 4. This is presumably due to our accounting for off-shell effects in π propagation and πN interaction, which are important in nuclear matter and are neglected in Ref. 4 when dealing with ^{4}He decay. (At this point, we would like to point out the "power" of the many-body Green's function techniques to that end.) In any case, we should remark that this kind of calculation turns out to be sensitive to the details of the πN form factor characterizing the CBM. This is so especially in what concerns the positron spectrum. In fact, we have calculated the same diagrams mentioned above with a Veneziano-type form factor[15] and found that the shape of the e^+ spectrum differs appreciably from the one previously found with the CBM for factor (the main differences being a decrease of the first peak and an increase of the second one). The integrated spectra (widths), however, turn out to be rather similar again, as we get, with the new form factor $\Gamma_A/A = 1.2\Gamma_f$ (for brevity, we omit details).

C. Spectator and interference terms

Next, we shall consider the Green's function for the pure spectator term. By using Wick's theorem together with a diagrammatic analysis, one shows that any CBM many-body Feynman diagram contributing to $\widetilde{G}(K)$ gives rise to one contributing to $\widetilde{G}_s(K)$ through the following prescriptions:

(i) Delete the two external free nucleon-hole lines.

(ii) Replace, in the diagram so resulting, the CBM vertices at the two end points by the effective πN overlap function $\phi_1(K)$ times the kinematic factor $(2\omega_{\vec{K}})^{1/2}$ (see Ref. 8), all the remaining vertices being left unchanged.

Conversely, any CBM many-body Feynman diagram for $\widetilde{G}_s(K)$ can be obtained from one for $\widetilde{G}(K)$ according to the above prescriptions.

As for the interference term $\widetilde{G}\,'_{\rm int}(K)_{\alpha\beta}$, the corresponding Feynman rules are similar to the ones just sketched for $\widetilde{G}_s(K)$, the main difference being that the above modifications (i) and (ii) should be made only at *one* end point.

By using the above rules (and by adding suitable factors of 3 in order to take into account the $\pi^+ n$ coupling), one has the following lowest-order expressions for the spectator and interference contributions:

$$\widetilde{G}_s(K) \simeq \frac{3i}{(2\pi)^4}\int d^4l\,\widetilde{G}^{(0)}(K-l)\widetilde{D}^{(0)}(l) \times[\phi_1(l)]^2 2\omega_l\ , \tag{3.18}$$

$$\widetilde{G}\,'_{\rm int}(K)_{\alpha\beta} = \frac{3i}{(2\pi)^4}\int d^4l\,\widetilde{G}^{(0)}(K-l)\widetilde{D}^{(0)}(l) \times\phi_1(l)(2\omega_l)^{1/2} i(\vec{\sigma}\vec{l})_{\alpha\beta} \times u(|\vec{l}|R)\frac{f_{\rm CBM}}{\mu}\widetilde{G}^{(0)}(K)\ . \tag{3.19}$$

By using the preceding equations, we can compute the nuclear effects on spectator and interference terms, in a similar way as done before for the pole term. Concerning the spectator term, the corresponding normalized e^+ spectrum is shown in Fig. 2 (dotted curve); we again remark that it does not depend on the GUT parameters. In fact, the most relevant parameter now is the chosen value for the proton radius (for instance, in a harmonic-oscillator wave function). We have carried out computations for several values of this radius, ranging from 0.35 to 0.87 fm and found that the nuclear corrections to the free decay width vary from -8 to $+6\%$, i.e., they are of little importance for all the significant values of the proton radius. The e^+ spectrum shown by the dotted curve of Fig. 2 corresponds to $R = 0.87$ fm which yields the greatest difference with respect to the pole-term contribution.

Concerning the interference term, we find nuclear effects on it are in between those corresponding to pole and spectator terms. Consequently, we do not expect any appreciable changes for any "realistic" model of proton decay. For instance, we have computed the positron spectrum with the so-called hybrid model[8] and found it to be practically indistinguishable from that corresponding to the pole term shown in Fig. 2.

To close this section, we shall consider some features of the behavior of the emitted pions on their way through the nucleus and their departure across the nuclear surface.[16] Our discussion will now be restricted to the pole term, though the inclusion of spectator and interference terms, through, for instance, the hybrid model referred to above does not change the main features appreciably.

A simple treatment of π propagation in nuclear matter suggested by the study of diagram 1(b) tells us the following. (i) The π's related to the region of the e^+ spectrum called before quasifree (which corresponds to π momentum 0.4 GeV $\lesssim |\vec{q}| \lesssim 0.6$ GeV) will be little absorbed in the nucleus as $\text{Im}\Sigma^* \simeq 0$ in that region. Moreover, one such π will go out from the nucleus nearly back-to-back with respect to the corresponding emitted e^+. A simple optical-model calculation (based upon the behavior of the π wave function at the nuclear surface) shows that the average angle between e^+ and π tracks is about 160°. One may estimate (see Fig. 2, continuous line) that such π's amount to about 45% of the total number of produced pions inside the nucleus. (ii) On the other hand, the π's related to the part of the e^+ spectrum around the second peak ($|\vec{K}|=0.6$ GeV) are strongly absorbed after being produced, as $\text{Im}\Pi^*$ is rather large in this case. They give rise to Δ's, the momentum of which is around 0.5 GeV. This region of the spectrum contains about 40% of the total number of decays. Neglecting Δ rescattering in the nucleus, one can study the angular distribution of the pions coming from the decay of the corresponding Δ's. At first sight, it is not easy to distinguish a genuine event of nuclear decay in this kinematic region from another coming from nuclear Δ production by neutrinos. Nevertheless, since the Δ which is produced after nucleon decay has, as said above, an appreciable momentum in the opposite direction to the emitted e^+, it is in principle possible to distinguish between both kinds of events just by means of a detailed analysis of the correlated $e^+-\pi$ spectrum. Of course, this requires a certain number of events to have a reliable sample. (iii) Finally, the π's associated to the small peak at $K \simeq 0.7$ GeV, for which $\text{Im}\Pi^*$ is still rather large, are considerably absorbed by nucleons but, in this case, without Δ excitation. In fact, this corresponds physically to nucleon decay in the nucleus without emitted π's outside the nucleus, the latter being strongly excited and, most probably, decaying through the ejection of a fast nucleon. This latter part of the spectrum amounts to about 15% of the total number of decay events (we recall again that no final states with $\rho,\omega,\ldots,$ etc., mesons are considered here).

IV. DENSITY-CORRELATION EXPANSION FOR THE NUCLEON SELF-ENERGY IN AN INFINITE NUCLEUS AND ANALYSIS OF $\lim\Gamma_A/A$ AS $K_F \rightarrow 0$

The analysis in the previous sections about the importance of nuclear effects on proton decay indicates the interest of carrying through, as well, a purely theoretical study on them.

For simplicity and without an essential loss of generality, we shall consider only the pole-term contribution to the decay probability of a proton, that is, we shall set $\kappa=0$ throughout the following study. The inclusion of the spectator and/or interference term(s) would only amount to unnecessary complications in our arguments.

We shall continue assuming that the πN interaction is described by the CBM (including three-momentum conservation and nonrelativistic recoil).

For later use, we shall start by giving a general formula for the decay rate $\Gamma(p \rightarrow e^+ + r \text{ pions})$ for an isolated nonrelativistic proton, when $\kappa=0$ and the CBM are used. (Notice that no final-state interaction among the emitted pions is taken into account.)

The physical proton state $|\tilde{p}\rangle$ can always be expanded in terms of the states $[1/r!]^{1/2} a^\dagger_{q_1} \cdots a^\dagger_{q_r} |\alpha\rangle$, which consist of a bag nucleon (N) or Δ state $|\alpha\rangle$ with given momentum and spin-isospin projections, as well as r pions with given three-momentum and isospin projections ($a^\dagger$ being the corresponding π creation operators). Thus,

$$|\tilde{p}\rangle = Z^{1/2}|p\rangle + \sum_{r=1}^{\infty}\sum_{\alpha}\sum_{q_1\cdots q_r} c_r(\alpha;q_1\cdots q_r)\left[\frac{1}{(r!)^{1/2}} a^\dagger_{q_1}\cdots a^\dagger_{q_r}|\alpha\rangle\right], \tag{4.1}$$

where $|p\rangle$ is a bare (bag) nucleon state, Z is the probability that $|\tilde{p}\rangle$ is a bare bag, c_r is the probability amplitude for finding r pions together with a bag state $|\alpha\rangle$ in $|\tilde{p}\rangle$, and $\sum_{q_i}$ is a shorthand notation for both a summation over the isospin and an integration over the three-momentum $\vec{q}_i$ of the ith pion.

Let $|F\rangle$ be an r-pion state with any possible values for isospins and three-momenta. Then, one evaluates the two matrix elements in Eq. (2.4) using the nonrelativistic form of the pole approximation

$[\mathscr{L}_{\rm eff}(x)\simeq\mathscr{L}^{(1)}_{\rm eff}(x)]$, the fact that the field $\Psi_\alpha(x)$, $\alpha=1,2$, destroys one bare nonrelativistic proton and Eq. (4.1). From this, and upon integrating the positron three-momentum in Eq. (2.3) for $|A\rangle=|\tilde{p}\rangle$ and a definite number r of final π's, one gets

$$\Gamma(p\to e^+ + r\ \text{pions})=\frac{g_{\rm GUT}{}^4}{4M_x{}^4}45\pi\,|\psi(0,0)|^2A_s(f)\,.$$
$$\times\sum_{s=1}^{2}\sum_{q_1\cdots q_r}|c_r(\text{protons},\,s;q_1\cdots q_r)|^2\delta\left[m_p-\sum_{i=1}^{r}\omega_i-\left|\sum_{i=1}^{r}\vec{q}_i\right|\right], \tag{4.2}$$

where s is the bare-proton spin projection and ω_i is the energy of the ith pion.

The physical proton state $|\tilde{p}\rangle$ and, hence, the relevant probability amplitudes c_r for the CBM can be constructed perturbatively by following the procedure in Sec. III A of Ref. 12, with direct modifications regarding energies of intermediate states in order to include nonrelativistic recoil.

Next, we shall turn to the infinite nucleus and recall that $\Sigma^*(K)$ is given by the sum of all many-body Feynman diagrams (actually, an infinite set). Each of the latter depends on several proton propagators like $\tilde{G}^{(0)}(q)$ which, in turn, contains $\theta(K_F-|\vec{q}\,|)$ [see the right-hand side of Eq. (3.12)], q being an integrated four-momentum in the Feynman integral associated to the diagram. Consequently, nuclear effects imply that $\Sigma^*(K)$ is a functional of $\theta(K_F-|\vec{q}\,|)$, where $\vec{q}$ now represents generically an infinite set of three-momenta which are necessarily integrated over. Thus, it makes sense to consider the functional Taylor (density-correlation) expansion for $\Sigma^*(K)$,

$$\Sigma^*(K)=\Sigma_0^*(K)+\sum_{n=1}^{\infty}\frac{1}{n!}\int\left[\prod_{i=1}^{n}d^3\vec{q}_i\,\theta(K_F-|\vec{q}_i\,|\right]\times\sigma_n(K;\vec{q}_1\cdots\vec{q}_n)\,, \tag{4.3}$$

$$\Sigma_0^*(K)=\lim_{k_F\to 0}\Sigma^*(K)\,, \tag{4.4}$$

$$\sigma_n(K;\vec{q}_1\cdots\vec{q}_n)=\lim_{K_F\to 0}\frac{\delta^n\Sigma^*(K)}{\delta\theta(K_F\,|\vec{q}_1\,|)\cdots\delta\theta(K_F-|\vec{q}_n\,|)}\,. \tag{4.5}$$

An important property is the following: $\Sigma_0^*(K)$ coincides exactly with the proper self-energy function of an isolated nonrelativistic proton (i.e., infinitely apart from other protons) in the CBM, with recoil corrections included. Recall that the latter self-energy function vanishes on the mass shell for a physical proton with mass m_p. That property of $\Sigma_0^*(K)$, even if it may look correct *a priori*, requires an adequate justification. We shall sketch below a proof that every many-body Feynman diagram contributing to $\Sigma^*(K)$ and containing at least one nucleon-hole loop [like the one displayed generically in Fig. 1(b)] does vanish when $K_F=0$.

Let $\Sigma_1^*(K)$ be the sum of all many-body Feynman diagrams for $\Sigma^*(K)$, each of which has neither nucleon-hole nor Δ-hole loops [like the ones in Fig. 1(a)]. Similarly, $\Sigma_2^*(K)$, each of which has, at least, one nucleon-hole or a Δ-hole loop [like the generic diagram in Fig. 1(b)], so that

$$\Sigma^*(K)=\Sigma_1^*(K)+\Sigma_2^*(K)\,. \tag{4.6}$$

First, let us consider the Feynman integrals associated to diagrams for $\Sigma_2^*(K)$ when $K_F=0$. By looking at the integration over energies corresponding to internal four-momenta in the blob, one shows that all the Feynman integrals vanish, by residue integration (as poles distribute themselves so that there are always half-planes free of singularities). This property is particularly easy to establish for the diagram in Fig. 1(b), when the blob reduces to just a nucleon-hole loop. Then, one has $\Sigma_2^*(K)=0$ for $K_F=0$ to all orders, for any K.

Second, for $K_F=0$, the set of all Feynman diagrams for $\Sigma_1^*(K)$ in the CBM can be shown to coincide with the set of all old-fashioned perturbation-theory graphs for the proper self-energy of any isolated proton in the same model (with recoil). In fact, by performing similar residue integrations over the energies corresponding to internal lines in the Feynman diagrams for $K_F=0$, one arrives at the contribution for the self-energy function of the isolated proton dictated by old-fashioned perturbation theory. Moreover, the integrals appearing in the latter are never singular when $K^2\simeq 0$, $|\vec{K}|\simeq K^0<m_p$. Then, under the latter conditions characterizing proton decay, $\lim_{K_F\to 0}\Sigma_1^*(K)=\Sigma_0^*(K)$ is real.

We now turn to the first-order term in Eq. (4.6).

It will be useful to consider

$$\sigma_{1,i}(K;\vec{q}_1)=\lim_{K_F\to 0}\frac{\delta\Sigma_i^*(K)}{\delta\theta(K_F-|\vec{q}_1|)},\quad i=1,2, \tag{4.7}$$

$$\sum_{i=1}^{2}\sigma_{1,i}(K;\vec{q}_1)=\sigma_1(K;\vec{q}_1)\ . \tag{4.8}$$

Equation (3.12) yields, by direct functional differentiation, the basic relation for the analysis of $\sigma_{1,i}$:

$$\frac{\delta\widetilde{G}^{(0)}(q)}{\delta\theta(K_F-|\vec{q}_1|)}=2\pi i\delta^{(3)}(\vec{q}-\vec{q}_1)\times\delta(q^0-(m_p+\vec{q}^{\,2}/2m_p))\ . \tag{4.9}$$

The set of all Feynman diagrams for $\sigma_{1,1}(K;\vec{q}_1)$ is obtained from that for $\Sigma_i^*(K)$ with arbitrary K_F as follows. A diagram contributing to Σ_i^*, which contains n internal fermion propagators $\widetilde{G}^{(0)}$, gives rise, by applying the functional differentiation (4.9) and setting $K_F=0$, to n different Feynman diagrams for $\sigma_{1,1}$. Then, in the latter, one should carry out all integrations over internal energies, by residues. The application of those recipes to the diagram in Fig. 1(a), when the intermediate state contains just a nucleon, yields the following contribution to $\mathrm{Im}\sigma_{1,1}$:

$$2\pi\sum_{s=1,2}\sum_{l}\delta^{(3)}(\vec{q}_1-\vec{k}-\vec{l})\delta(K^0+\omega_l-m_p)\times|(V_l^{N\alpha})^*|^2\ , \tag{4.10}$$

$V_l^{N\alpha}$ is the CBM vertex function (for a $\pi N\alpha$ vertex, $\alpha=N,\Delta$), the π having three-momentum $\vec{l}$. An inductive argument to higher perturbative orders leads us to infer that, in the CBM (with recoil)

$$\mathrm{Im}\sigma_{1,1}(K;\vec{q}_1)=2\pi\sum_{r=1}^{\infty}\sum_{s=1,2}\sum_{l_1\cdots l_r}\delta^{(3)}\left[\vec{q}_1-\vec{k}-\sum_{i=1}^{r}\vec{l}_i\right]\delta\left[K^0+\sum_{i=1}^{r}\omega_i-m_p\right]\times\left|\left[\sum_{i=1}^{r}\omega_i+\left[\sum_{i=1}^{r}\vec{l}_i\right]^2\Big/2m_p+\Sigma_0^*\left[m_p-\sum_{i=1}^{r}\omega_i,\sum_{i=1}^{r}\vec{l}_i-\vec{q}_1\right]\right]c_r(\text{proton},s,l_1\cdots l_r)\right|^2\ . \tag{4.11}$$

The set of all Feynman diagrams for $\Sigma_2^*(K)$ for any K_F determines that for $\sigma_{1,2}(K;\vec{q}_1)$ in the following way. A given diagram contributing to Σ_2^* contains (i) n $\widetilde{G}^{(0)}$'s along the "external fermion line" [namely, the one having four-momentum K before and after all interactions; see Fig. 1(b)], as well as (ii) n' $\widetilde{G}^{(0)}$'s *outside* that "external fermion line," corresponding necessarily to internal fermion lines in the nucleon-hole loops [say, in the blob of Fig. 1(b)]. Upon performing the functional differentiation (4.9) in any of the $\widetilde{G}^{(0)}$'s along the external fermion line, setting $K_F=0$, and performing residue integrations over the internal energies in all nucleon-hole loops, one necessarily arrives at a vanishing result [the same argument led to $\Sigma_2^*(K)=0$ when $K_F=0$]. It is only when one applies the functional differentiation (4.9) to fermion lines *outside* the external fermion lines that one obtains n' different Feynman diagrams contributing to $\sigma_{1,2}(K;\vec{q}_1)$.

It is possible to relate $\sigma_{1,2}(K;\vec{q}_1)$ to an off-shell forward proton-proton elastic scattering amplitude as we shall see. For that purpose, we shall start by considering the diagram in Fig. 1(b) when the blob is just the *proper* pion self-energy $\Pi^*(q)$. Notice that only two pion lines join the (one-particle irreducible) blob to the external fermion line in Fig. 1(b). Then one can prove to all perturbative orders that

$$\lim_{K_F\to 0}\frac{\delta\Pi^*(q)}{\delta\theta(K_F-|\vec{q}\,'|)}=4\pi f_s(q,q')\ ,\qquad q_0'=m_p+\vec{q}\,'^2/2m_p\ , \tag{4.12}$$

where f_s is the off-shell extension of the forward πN elastic scattering amplitude in the CBM, the on-shell one being normalized so that

$$\mathrm{Im}f_s(q,q')=\frac{|\vec{q}|}{4\pi}\sigma_{\mathrm{tot}},\quad |\vec{q}|=|\vec{q}\,'|\ ,\qquad q^0=(\mu^2+\vec{q}^{\,2})^{1/2},\ q'^0=m_p+\vec{q}\,'^2/2m_p\ . \tag{4.13}$$

When the blob in Fig. 1(b) is just a nucleon-hole loop, the proof of Eq. (4.12) proceeds by direct application of Eq. (4.9) and identification of the off-shell forward πN scattering amplitude in lowest-order perturbation theory. An inductive argument establishes the correctness of Eq. (4.12) to all perturbative orders.

Next, upon considering further Feynman diagrams corresponding also to $\Sigma_2^*(K)$, where the nucleon-hole loops are joined to the external fermion line by $n>2$ pion lines, by applying Eq. (4.9) suitably, setting $K_F=0$, performing residue integrations over all internal energies, and comparing with typical old-fashioned perturbation-theory graphs for the forward proton-proton elastic scattering amplitude, one gets

$$\sigma_{1,2}(K;\vec{q}_1)=4\pi T(K,q_1)_{\text{non-spin-flip}}\ ,\quad q_1^0=m_p+\vec{q}_1^{\,2}/2m_p\ . \tag{4.14}$$

This relation plays a role for the present situation analogous to the one relating the second virial coefficient to the elastic scattering amplitude in quantum statistical mechanics.[17]

Upon collecting the previous results, dropping all terms of order $n\geq 2$ in the expression (4.3), and keeping only linear terms in $\theta(K_F-|\vec{q}\,|)$ ($\vec{q}$ being an integrated momentum), Eqs. (3.11), (3.14) (with $\kappa=0$) yield

$$\Gamma_A/A\simeq\frac{{g_{\rm GUT}}^4}{4{M_X}^4}\,|\psi(0,0)|^2A_s(f)\frac{3\pi^2}{2{K_F}^3}\frac{90}{(2\pi)^3}\times\int d^3K\,\theta(m_p-|\vec{K}\,|)\left[\left[K^0-(m_p+\vec{K}^{\,2}/2m_p)-\Sigma_0^*(K)\right]\right]^{-2}\int d^3q\,\theta(K_F-|\vec{q}\,|)\times\sum_{i=1}^{2}{\rm Im}\sigma_{1,i}(K;\vec{q}\,)\ . \tag{4.15}$$

Next, we notice the following fact. Let us assume for a moment that ${\rm Im}\sigma_{1,2}(K;\vec{q}\,)=0$. Then, Eqs. (4.11) and (4.2) imply directly, as $K_F\to 0$ that

$$\lim_{K_F\to 0}\Gamma_A/A=\sum_{r=1}^{\infty}\Gamma(p\to e^+ + r\ \text{pions})\ , \tag{4.16}$$

where $\Gamma(p\to e^+ + r$ pions) is given in Eq. (4.2). Such a result, namely, the equality between the total decay rate per nucleon for zero nuclear density and the total decay rate for an isolated nucleon is physically natural. Notice that the divergent ${K_F}^{-3}$ term in Eq. (4.15) cancels with a similar factor coming from $\int d^3q\,\theta(K_F-|\vec{q}\,|)$. It is easy to check Eq. (4.16) by using Eqs. (4.11) and (4.15) and (4.1) and (4.2) (with $Z=1$). Actually, this lowest-order calculation was already given in Ref. 3.

The consistency condition (4.16) fails to be true if ${\rm Im}\sigma_{1,2}(K;\vec{q}\,)\neq 0$. We have calculated this quantity using Feynman diagrams of low order and found that, indeed, for those diagrams, ${\rm Im}\sigma_{1,2}(K;\vec{q}\,)=0$. We omit the details of said calculation when the blob in Fig. 1(b) is just a nucleon-hole loop. To carry through a general proof to all perturbative orders seems to be a formidable task. We shall end by conjecturing that such a relation *has to be true* to all orders if the natural consistency condition (4.16) holds.

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