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Coulomb Gauge Hybrid Meson Calculation

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We report a relativistic many-body calculation for the hybrid meson spectrum utilizing a QCD inspired Coulomb gauge Hamiltonian. Our approach is comprehensive and unifies the quark and gluon sectors. The vacuum and quasiparticle properties are generated by a BCS transformation, whereas the meson and glueball spectra are described by the TDA and RPA approximations. Using the quark and gluon gap solutions, we formulate hybrid mesons as states composed of three quasiparticles (quark, antiquark and gluon) and compute the mass spectrum variationally with trial wavefunctions. We predict the important exotic 1^{-+} states to have masses above 2 GeV in rough agreement with lattice and flux tube models, implying the recently observed resonances are not hybrids.

One of the signature predictions of QCD are exotic hadron states with quantum numbers not possible in any $q\bar{q}$ model. Two such states have recently been observed [1] at 1.4 and 1.6 GeV with $J^{PC} = 1^{-+}$ whose detailed structure is quite controversial. Theoretically the situation is unclear as both flux tube and lattice results [2] indicate that the lightest exotic hybrid meson has a mass of 2.0±0.2 GeV, while somewhat dated bag model calculations predicted exotic excitations with explicit gluonic degrees of freedom in the mass range around 1.5 GeV. In this communication we investigate if these states can be described as hybrids within a many-body constituent approach that has successfully described both conventional meson [3] and glueball [4] systems.

Our starting point is the QCD Coulomb gauge Hamiltonian (see for example Ref. [5]). For calculational purposes, we simplify it to

$$\begin{aligned} H = & \int d\mathbf{x}\, \Psi^\dagger(\mathbf{x})(-i\alpha\cdot\nabla + \beta m)\Psi(\mathbf{x}) \\ & -\frac{1}{2}\int d\mathbf{x}\, d\mathbf{y}\, \rho^a(\mathbf{x}) V(|\mathbf{x}-\mathbf{y}|)\rho^a(\mathbf{y}) \\ & -g_s \int d\mathbf{x}\, \Psi^\dagger(\mathbf{x})\alpha\cdot\mathbf{A}(\mathbf{x})\Psi(\mathbf{x}) \\ & +\mathrm{Tr}\int d\mathbf{x}(\mathbf{\Pi}^a\cdot\mathbf{\Pi}^a + \mathbf{B}^a\cdot\mathbf{B}^a), \end{aligned}$$

containing both quark, Ψ, and gluon, $\mathbf{A}$, fields with color density $\rho^a = \Psi^\dagger \frac{\lambda^a}{2}\Psi + f^{abc}\mathbf{A}^b\cdot\mathbf{\Pi}^c$. We adopt the standard current quark mass, m, values for the u, d, s and c flavors; $m_u = m_d = 5$ MeV, $m_s = 150$ MeV and $m_c = 1200$ MeV. We also set $g_s = 0$, allowing us to neglect the minimal coupling term whose treatment is detailed in [4] for the glueball case. This term should be important for calculating hybrid meson hadronic decays, but can be neglected in the present spectroscopic study. This approximation also modifies the non-Abelian color magnetic field $\mathbf{B}^a = \nabla\times\mathbf{A}^a + \frac{g_s}{2} f^{abc}\mathbf{A}^b\times\mathbf{A}^c$, reducing it to Abelian form. Confinement and leading canonical interactions are represented by the instantaneous potential V, that we model as a Cornell-type potential $V = \sigma r - \alpha_s/r$, where the string tension, $\sigma = 0.18$ GeV^2, is specified by lattice calculations and sets the hadronic scale. Through extensive numerical calculations with the Coulomb term, we find that most observables can be described using just the linear potential. We also exclude the Coulomb potential in the gap equation solutions since it diverges and requires renormalization.

We now apply three different many-body techniques to approximately diagonalize our model Hamiltonian. The first step is to perform a canonical transformation to a new quasiparticle basis. This is provided by the Bardeen-Cooper and Schrieffer (BCS) approximation that generates a mass gap of about 100 MeV for the u/d quarks [3], 250 MeV for the s quark and 800 MeV for the gluon [4]. The quasiparticle gap energies are plotted in Fig. 1. We use a variational proce-

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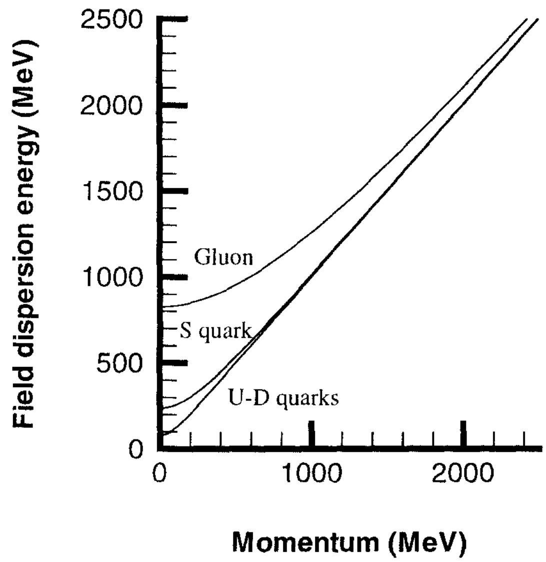

Figure 1. Gap energy solutions for the u/d, s quarks and gluon quasiparticles.

dure which minimizes the ground state (vacuum) energy with respect to the BCS angle. Plotting the ground state energy versus angle produces a Mexican-hat shape curve.

As detailed in our previous letters [3,4], the meson and glueball spectra are reasonably described by the Tamm-Dancoff approximation (TDA) in which a glueball is represented as $g^\dagger g^\dagger|\Omega\rangle$, and a meson by $q^\dagger \overline{q}^\dagger|\Omega\rangle$, where $g^\dagger$, $q^\dagger$ and $\overline{q}^\dagger$ are, respectively, the gluon, quark and anti-quark quasiparticle creation operators. The notable exception is the light pseudoscalar sector, which required an improved, random phase approximation (RPA) treatment to adequately describe the Goldstone boson nature of the pion due to spontaneous chiral symmetry breaking in our BCS vacuum. Again, this has been detailed previously [3].

Finally we formulate the hybrid meson wavefunction as

$$|hybrid\rangle = q^\dagger \overline{q}^\dagger g^\dagger |BCS\rangle$$
$$\equiv [[q^\dagger \otimes \overline{q}^\dagger]_8 \otimes g^\dagger]_0 |BCS\rangle,$$

now involving quark color octet states. The resulting TDA equation is

$$\langle hybrid|[H, q^\dagger \overline{q}^\dagger g^\dagger]|\Omega\rangle = M\langle hybrid|q^\dagger \overline{q}^\dagger g^\dagger|\Omega\rangle.$$

Instead of solving this formidable nonlocal problem in 6 dimensional momentum space, we evaluate the hybrid mass variationally using an exponential radial wavefunction for each of the two independent momentum variables.

There are two corresponding orbital angular momentum, L_+, L_-, and we adopt an LS coupling representation. The lowest lying exotic states will have a combination of one s and one p-wave generating the three states 1^{-+}, 0^{--} and 3^{-+}. In the center of momentum frame the Hamiltonian matrix element reduces to a 9-D integral that we evaluate using Monte Carlo techniques with the computer code VEGAS on a Cray J-90.

Fig. 2 summarizes the main results of this work. Here we compare our exotic hybrid mass prediction with three other theoretical models for the light and charmed quark flavors. Notice that with the exception of the outdated bag model, all theoretical approaches generally agree, but disagree with observation. This clearly suggests the ob-

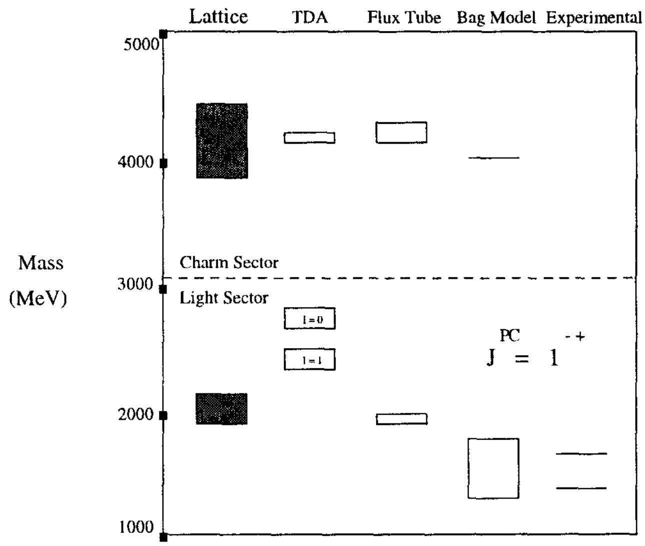

Figure 2. Light and charmed exotic 1^{-+} hybrid mesons. The TDA calculation is comparable to the lattice results but both disagree with observed resonances.

served resonances have an alternative structure which we speculate is of four quark nature, perhaps quark molecules (e.g. two $q\bar{q}$ color singlets). Two other interesting predictions from our model are: (i) four 1^{--} states in the J/ψ channel that may help explain the overpopulation of experimental states; (ii) an isospin splitting of $\simeq$ 300 MeV between the $I = 0$, $I = 1$ light hybrids for states where the $q\bar{q}$ spins are aligned and annihilate into a gluon, in precise analogy with ortho positronium.

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